APPENDIX: REVISION OF DC CIRCUITS AND ELECTROMAGNETISM

A.1 ESSENTIAL CONCEPTS

You are recommended to read these notes in conjunction with your school physics books. [For example, Whelan and Hodgson (Essential Principles of Physics), or Duncan (Physics)].

A.1.1 Electric Charge: unit of charge is the Coulomb [C]

Charge is positive or negative. The electron has a charge of $-1.6\ 10^{-19}\ C$. Like charges repel, and unlike charges attract one another. Charge is not created or destroyed but is conserved so that the total charge remains constant. Common symbols for charge: q or Q.

A.1.2 Current: unit of current is the Ampere [A]

Current is the rate of flow of charge: one ampere flowing for one second transfers one coulomb between the terminals. Current has to have a closed loop or 'mesh' for its flow. Common symbols for current are J, I, or i. (You will find that in engineering, $j = \sqrt{-1}$ so we do not often use j for current).

A.1.3 Energy: unit of energy is the Joule [J]

The concept of energy is fundamental to all science and engineering. From Mechanics, kinetic energy and potential energy will be well known to you. Electrical energy is yet another form of energy which can be stored in batteries, capacitors and inductors. In direct current circuits we are particularly interested in electrical potential energy, provided for example by a battery.

A.1.4 Potential Difference: unit of p.d. is the volt [V]

Three terms are used in engineering to describe essentially the same quantity – voltage, potential difference (*p.d.*) and electromotive force (*e.m.f.*).

The analogy is often made between water current flow and electrical current flow. The analogous quantity to the pressure of the water is the electrical voltage. The current flows from high voltage or high pressure to low voltage or low pressure. The difference in voltage is known as the potential difference.

The potential difference between two points 'a' and 'b' is $1V$ if $1J$ of energy is dissipated between a and b when $1C$ of charge flows from a to b. Equally $1J$ must be supplied to drive $1C$ from b to a (from the lower to higher potential).

A common symbol for voltage is V or V_{ab} indicating the potential difference between the points 'a' and 'b'. One often just writes V_a, indicating the potential of the point with respect to some clear reference point. Sometimes a reference point is defined as 'earth'. [For the 'earth' symbol see Figure E1 in Section 3.1]. All circuits with an earth defined then have a common potential for that earthed point. Points of common potential may be considered to be that earthed point. They may be considered to be effectively connected together by a conductor, but no current flows between such points, because there is no potential difference between them.

An ideal voltage generator is shown in Figure A1(a) or (b). Any battery or generator is assumed to be ideal, unless otherwise stated, supplying an electromotive force of V volts independent of the current flow. Such an ideal voltage generator will have across its output terminals a fixed potential difference, V, which is not changed by any finite current drawn from these two terminals. It is not then possible to measure any difference in the voltages across AB between the generator shown in Figure A1(c), with a finite value of r, and those ideal generators of Figure A1(a) and (b).

A car battery is a good practical realisation of an ideal voltage generator as it usually provides a voltage which is nearly independent of the current drawn from the battery over a useful range of currents. The long bar of a battery symbol indicates the positive terminal of the battery – see Figure A1(d).

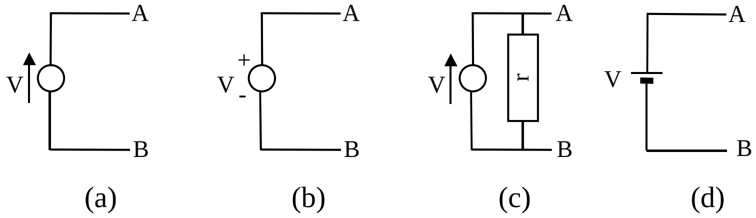


Figure A1

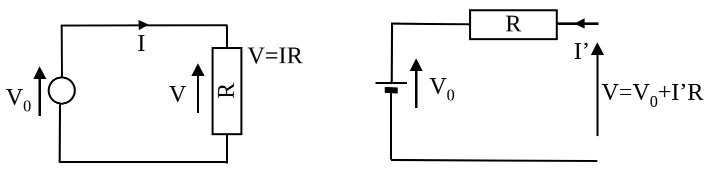
We see then three different expressions – voltage, potential difference and electromotive force; all have a similar meaning and the same units for their measurement. In circuit problems, electromotive force (abbreviated to *e.m.f.*) always refers to the voltage supplied by a battery or generator. The term 'potential' is usually short for potential difference and is used to describe the voltage between two points. The potential of a point, with no apparent reference point, usually refers to the potential difference between that point and 'earth'.

A.1.5 Resistance: the unit of resistance is the Ohm [Ω]

One volt will drive one ampere through one ohm. In electronics the kilohm ($k\Omega$) or Megohm ($M\Omega$) will often be more useful than the ohm.

$$\text{Ohm's Law:} \quad V = I\,R$$

R may in reality vary with temperature and with current but often R is assumed to be constant. Common symbols for resistance are r, R, R_{ab} denoting resistance between points a and b. Figure A2 shows an ideal voltage generator feeding a resistance. The arrow on I shows the direction of the current flow, flowing from higher to lower voltage. When a battery or generator is giving out power then the voltage or *e.m.f.* associated with the battery is in the same direction as the current flow – Figure A2(a). If we were charging the battery, then naturally the current would be flowing against the electromotive force provided by the battery – Figure A2(b).



(a) (b)

Figure A2

A.1.6 Conductance: the unit of conductance is the Siemen [S]

Conductance G is reciprocal resistance. One Siemen is the conductance given by one ohm. Millisiemens (mS) will often be the more useful unit. Ohm's law may be equally well written as:

$$I = V\,G$$

Just as the resistor is a component having resistance, so a conductor is a component having conductance. A short-circuit has infinite conductance, or zero resistance. An open-circuit has infinite resistance, or zero conductance.

A.1.7 Power: the unit of power is the Watt [W]

One watt is delivered to a load with $1V$ across it when $1A$ flows:-

$$\text{POWER} = V\,I$$

For a single resistance R, or conductance G ($R = 1/G$) then

$$P = I^2 R = V^2 G$$

One watt flowing for one second gives on Joule or energy, so:

Energy = Power Flow × Time, from which Power Flow = $I\,V\,t$

A.2 KIRCHHOFF'S CIRCUIT LAWS

Junctions of wires are referred to as *nodes*. Any closed path or loop that may be defined in a circuit is referred to as a mesh.

A.2.1 Kirchhoff's First Law

The algebraic sum of the currents flowing into any point of a circuit is zero. Examples (Figure A3):

For the single line at any point P: $I_1 - I_2 = 0$

For the node Q: $I_1 + I_2 + I_3 = 0$. For the node R: $I'_1 + I'_2 - I_3 = 0$.

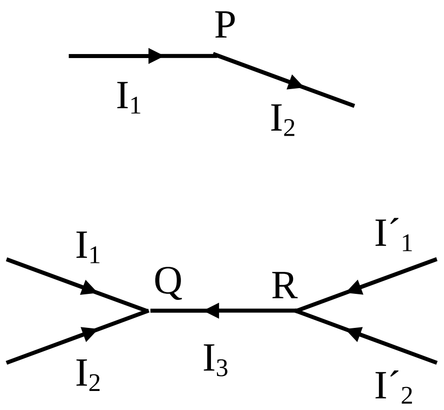


Figure A3

Note that a direction, denoted by an arrow, must always be assigned to the current in each branch. This task of assigning directions to currents when you start to solve a circuit is quite arbitrary. If you happen to have given a current the wrong direction, it will simply turn out to have a negative sign. However having once assigned a direction to a current, you must keep to that sign convention and not change half way through the problem.

A.2.2 Kirchhoff's Second Law

The algebraic sum of all the voltages and *e.m.f.*s, around any mesh, is zero. For example, in Figure A4, around the mesh ACDB we may write:

$V_1 - V_2 - V_3 - V_4 = 0$, where $V_2 = I_2R_2$ and $V_4 = I_4R_4$.

Some prefer to write this circuit law as:

"The sum of the voltage sources around a circuit = the sum of the voltage drops in the components."

This would lead to: $V_1 - V_3 = I_2R_2 + I_4R_4$

Alternatively if we had taken the mesh *AEFB* (which includes the nodes *C* and *D* but not the branch CD containing the generator V_3) then: $V_1 - V_2 - V_5 - V_6 - V_4 = 0$.

Be careful to observe your sign conventions precisely. Kirchhoff's laws hold not just for d.c. or average quantities but hold at all times and consequently will hold when you come to do examples using alternating and transient current flow.

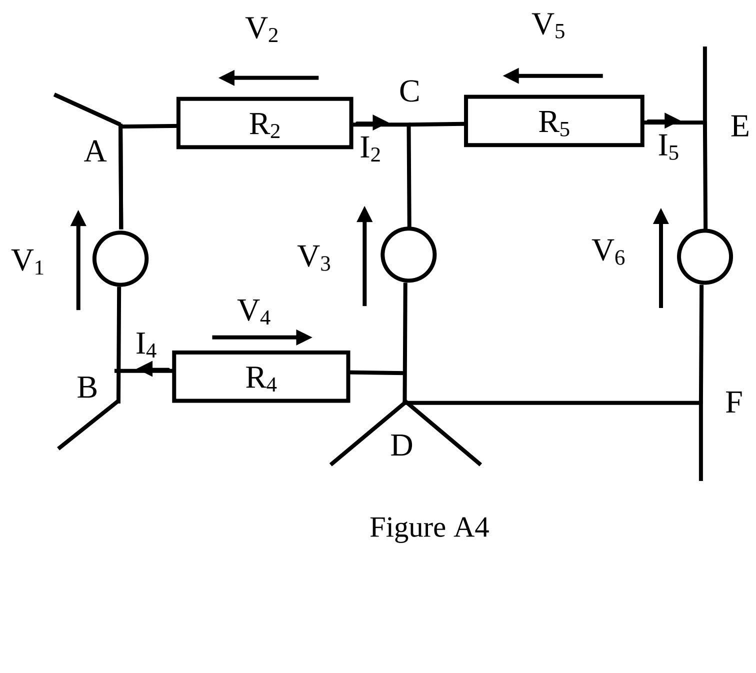


Figure A4

A.3 SIMPLE TECHNIQUES FOR SOLVING CIRCUITS

A.3.1 Combining Resistance and Conductance

Resistors in Series (Figure A5)

From Kirchhoff 1: the same current I flows in both resistors.

From Kirchhoff 2: $V = V_1 + V_2 = I\,R_1 + I\,R_2$

So that by comparing with $V = I\,R_{total}$ it follows that: $R_{total} = R_1 + R_2$.

More generally, any set of N resistances in series has a total resistance given by the sum of all resistances:

$$R_{total} = \Sigma\,[R_m] \text{ where } m = 1 \text{ to } N.$$

Resistors in Parallel (Figure A6)

From Kirchhoff 2: $V = V_1$ ... from mesh *ACDB*
$V_1 = V_2$ ... from mesh *CEFD*

So $V = V_1 = V_2$.

From Kirchhoff 1: $I = I_1 + I_2 = V_1 / R_1 + V_2 / R_2 = V / R_{total}$.

Hence: $1/R_{total} = 1/R_1 + 1/R_2$.

Another way of writing all this is by using conductances [$G = 1/R$] so that conductors in parallel have their conductances added:

$$G_{total} = G_1 + G_2$$

In general with any number of resistors in parallel we may write the result conveniently as the sum of all the conductances:

$$G_{total} = \Sigma\,[G_m]$$

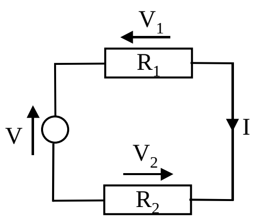


Figure A5

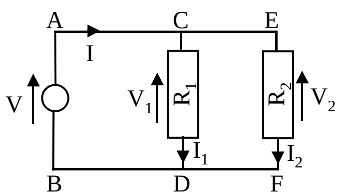


Figure A6

A.3.2 Voltage Division

When there is a set of resistors in series as in Figure A7, then it is possible to write down the voltage across any resistor immediately using "voltage division". We saw that the current through the resistor chain was the same current I. Hence the voltage V_n across the nth resistance R_n is $V_n = [I\,r_n]$. The total voltage is $V = \Sigma\ [I\,R_m]$ so that the voltage across R_n is:

$$V_n = V\,R_n/[\Sigma\,R_m]$$

with the sum taken over all resistances ($m = 1$ to N)

A.3.3 Current Division

When there is a set of resistors in parallel as in Figure A8, then the voltage across each resistor, as seen previously, is the same so that the current I_n through the resistor R_n is $I_n = V/R_n = V\,G_n$. If the total current supplying the set of conductors is I, then: $I = \Sigma\,[V\,G_m]$. Consequently it is possible to write the current I_n as:

$$I_n = I\,G_n/[\Sigma\,G_m]$$

where the summation is carried out over all the conductors (m = 1 to N). If one of the conductors was a short circuit with $R_n = 0$, $G_n = \infty$, then in this example $\infty/\infty = 1$, and the short naturally carries all the current with $I_n = I$.

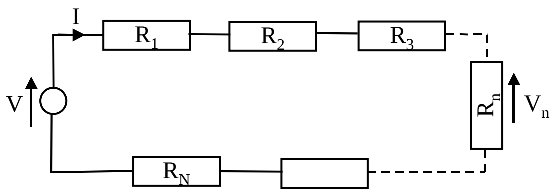


Figure A7

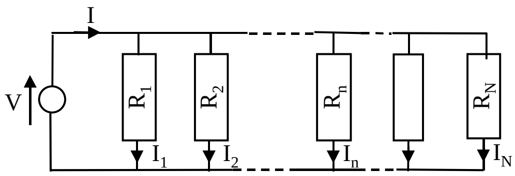


Figure A8

A.4 ELECTROMAGNETISM

A.4.1 Potential and Capacitance

Consider a unit charge in the presence of an electric field **E**. The ***electrostatic potential*** (or voltage) between two points is the work done when a unit positive charge is moved from one point to another. At any point along the path between these points, we must exert a force **-E** to hold the charge in equilibrium.

Capacitance exists between any pair of conductors which are electrically insulated from one another. The value of this capacitance arises from the fact that the voltage between two conductors is proportional to the charge between them:

$$Q = CV$$

This is a fundamental property used in almost every electrostatic problem.

A.4.2 Moving charges – the Lorentz force

Suppose that a current I is flowing in a section l of a conductor in a uniform magnetic flux density B. The force generated by this current derives from the Biot-Savart law, assuming the direction of the current is perpendicular to the magnetic flux density B (Figure A9):

$$F = BI\,l$$

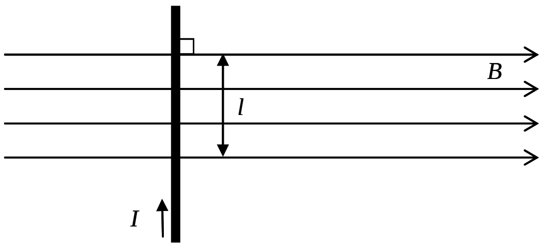


Figure A9

Another way to look at the force on a current carrying conductor in a magnetic field is in terms of the moving charges which constitute the current within the conductor. The current I can be described in terms of a charge q moving at velocity v along the conductor. If the time taken for the charge to move the length l of the conductor is Δt, then I is simply $q/\Delta t$. This allows us to write the force on the current in terms of the charge q and its velocity v through the magnetic field B:

$$F = Bqv$$

This is often called the *Lorentz force*, after the 19th century Dutch physicist Hendrik Anton Lorentz (1853-1928). In fact, the first derivation was by the British electrical engineer Oliver Heaviside (1850-1925) and was published in 1889 in the Philosophical Magazine.

A.4.3 Faraday's law of electromagnetic induction

On August 29, 1831, Michael Faraday, working at the Royal Institution, carried out one of the most famous experiments in the history of electromagnetism. He demonstrated that an electromotive force (emf) e could be produced in a conducting ring by moving it through the lines of magnetic flux of a permanent magnet. He also showed that only the relative motion between the ring and magnet was important. Following this experiment, he showed that *any* changing magnetic field could induce an emf in a circuit, whether the magnetic field was produced by permanent magnets, or electromagnets. Examples of coil geometries that lead to electromagnetic induction are shown in Figure A10.

Faraday established that the size of the emf was proportional to the rate of change of the magnetic flux ϕ, linking the circuit, and this is now known as Faraday's law of electromagnetic induction:

$$e = \frac{d\phi}{dt} = \frac{\Delta\left(NBA\right)}{\Delta t}$$

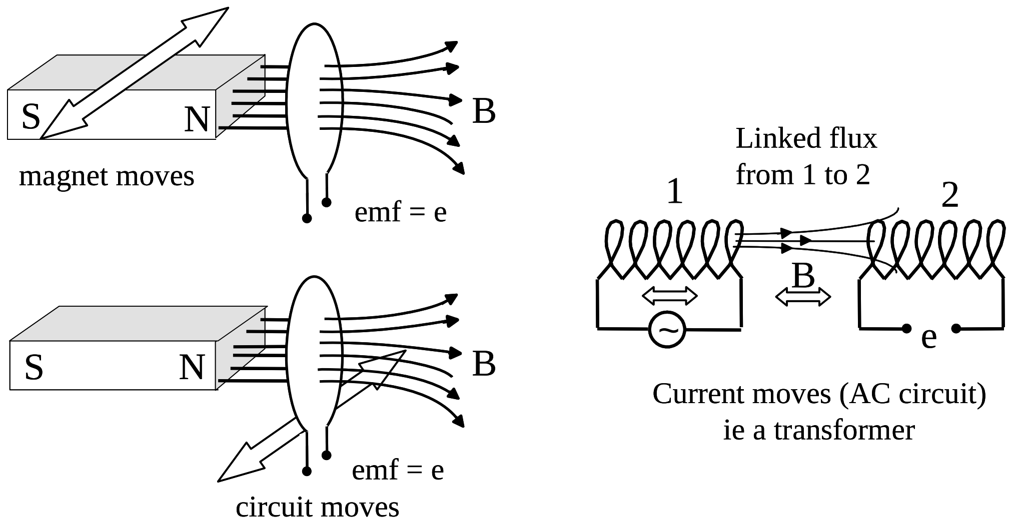


Figure A10