Term Paper: Three-factor Constant Elasticity of Substitution Production function: Identifying the Role of Energy in India

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Introduction

Utilizing energy is essential to building wealth within an economy. All transformative processes use energy in one way or another to bring about changes that create value. Using new energy sources has sparked numerous important improvements in economic output, productivity, and technology. Moreover, one of the major problems that civilizations must deal with is increased energy consumption as a result of economic activity and growth, not only in their efforts to conserve energy but also with regard to environmental issues and geopolitical worries about obtaining energy from nations with shaky governments. At the macroeconomic level, the elasticity of substitution is crucial to many dynamics.

Using data for India, we use this to investigate how energy inputs work as a third production element alongside capital and labour in an aggregate production function. We show that estimating the parameters of a layered CES function with the fewest limiting assumptions is possible by retaining the elasticity of the nested process fixed at an empirically motivated value.

Literature Review

We can now more accurately predict how factor use will change as well as, more significantly, the potential effects of energy restrictions on growth thanks to improvements in aggregate energy consumption modeling. This underlines the reality that energy use is not only an environmental issue but also a crucial economic one. Energy restrictions' possible impact on economic output has been studied before: Jevons explains in 1865 how a fast industrializing and expanding British economy would face the problem of running out of coal.

The subject reappeared 40 years ago with the oil crisis, where the potential effects on growth were examined by individuals such as Dasgupta and Heal (1974), Solow, and others (1974) , Stiglitz, too (1974). In an aggregated growth model, Saunders (1992) explains the significance of energy use and how it might be used in place of other elements. Energy as a production input is anticipated to have a very low elasticity of substitution in comparison to other production factors, according to Ayres' (2007) explanation of the physical justifications. Stern and Kander (2012) specifically include energy as a production element in their attempt to go deeper into the subject of endogenous growth.

Since Solow (1957) hinted and Arrow et al. formalized the CES model's functional form for two elements, it has been a potent research tool (1961). It is frequently used as a production function for examining how two input elements relate to one another, not only on a macroeconomic level as an aggregated production function but also on a sectoral level and in panel data. Chirinko (2008) demonstrates that the preponderance of empirical evidence supports substitution elasticities between capital and labor that are less than unity. In contrast to other methods like Cobb-Douglas, which only permits a unitary interpretation of the parameters, the CES function may describe these various elasticities while still enabling a precise economic interpretation of the parameters.

The difficulty of estimating them stands in stark contrast to the attraction of n-input CES production functions for simulating complicated relationships between inputs. This develops when factor-biased technical change is taken into account as more urgent. The significance of taking into account the implicit technological influence of the elasticity of substitution is one of de la Grandville's (1997) most important conclusions. The dependence of growth on the relationship between elasticities and productivity is already demonstrated by Solow (1956): In some cases, even in the absence of productivity advances, a very high elasticity can permit endogenous growth of per capita output. The linear Kmenta approximation is commonly used to estimate CES production functions. Although the linear approximation is simple to estimate, it has some limitations: Because of its (i) elasticity estimations that are skewed towards unity, (ii) strong neutrality assumptions must be made for technical change, and (iii) its inability to work for nested models with extra production elements.

Methodology

We employ the concept of an aggregated three-factor CES production function to illustrate how energy is used and how it might be replaced by other production factors. A CES production function relates the endogenous variable Y (output) to the exogenous variables x_i (input factors), and is defined as

$$Y = A\,(\gamma x_1^{-\rho} + (1-\gamma)x_2^{-\rho})^{-1/\rho} \qquad \text{eq. (1)}$$

Factor A is a (Hicks-neutral) measure of productivity, $\gamma \in (0, 1)$ is the equilibrium factor share of inputs, and $\rho \in (-1, 0) \cup (0, \infty)$ determines the elasticity of substitution $\sigma = 1/(1 + \rho)$. An advantage is that the CES formulation contains as its limits the Leontief production function when $\sigma \rightarrow 0$ ($\rho \rightarrow \infty$) and Cobb-Douglas functions when $\sigma \rightarrow 1$ ($\rho \rightarrow 0$).

Incorporating more than two factors can be done by nesting a CES production function. This means that two inputs are combined in a CES production function, which is then nested in a further CES production function, either of a single input for a three-factor formulation or with another aggregate for four or more inputs. The nested formulation then, for three factors, looks like this:

$$Y = A\,(\gamma V_1^{\rho/\rho_1} + (1-\gamma)x_3^{-\rho})^{-1/\rho} \qquad \text{eq. (2)}$$

Here, the ρ_i determine the elasticity of substitution within the factors of process V_i.

Expanding eq. (1) to allow for factor-biased technical change gives us the following production function:

$$Y_t = C\,[\gamma_L (A_{Lt} L_t)^{\nu-1/\nu} + (1-\gamma_L)(A_{Kt} L_t)^{\nu-1/\nu}]^{\nu/\nu-1} \quad \text{eq. (3)}$$

Likewise, we define the nested three-factor model incorporating energy, biased technical change, and an energy quality index based on eq. (2).

$$Y_t = C\,[\gamma_V V^{(\sigma-1)\nu/(\nu-1)\sigma} + \gamma_E (A_{Et} Q_E E_t)^{\sigma-1/\sigma}]^{\sigma/\sigma-1} \quad \text{eq. (4)}$$

With $$V = [\gamma_L (A_{Lt} L_t)^{\nu-1/\nu} + (1-\gamma_L)(A_{Kt} L_t)^{\nu-1/\nu}]$$

Various variables used:

Eq. (4) consists of a CES process (V) of capital (K) and labor (L) nested within a CES function with energy (E). Y denotes the produced output, C is a productivity parameter, σ is the elasticity of substitution between energy and the capital/labor aggregate, and ν is the elasticity of substitution between capital and labor. The parameters γ_V , γ_L, and γ_E are constants and denote the normalized within-process factor shares with $\gamma_V+\gamma_E = 1$. γ_L is the labor cost share. The productivity parameters, A_i, allow for factor-biased technical change. They are assumed to follow a constant time trend such that $A_{it} = A_{i0} e^{\alpha it}$. The choice of nesting a three-factor production function as (KL)E instead of (LE)K or (KE)L has been examined by Kemfert (1998), and is widely accepted as the standard way of separating the production factors.

Data

We use annual output, capital, and labor data for India and energy data from the yearly energy accounts of India. World Bank Open data is a good source to get the data. Links for the data:

Energy Data: Primarily based on Oil usage in Kg per capita, along with electricity and water-based energy.
https://data.worldbank.org/indicator/EG.USE.PCAP.KG.OE

Labour Force Data:
Year-wise labor force data of India from 1990-2021.
https://data.worldbank.org/indicator/SL.TLF.TOTL.IN?locations=IN

Capital Data (in USD):
GDP of India in US dollars from 1990-2021.
https://data.worldbank.org/indicator/NY.GDP.MKTP.CD?locations=IN

Output Data:
Value added from agriculture and other sources.
https://data.worldbank.org/indicator/NV.AGR.TOTL.ZS?view=chart

Results

Estimation results for the coefficients:

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Coefficients:
        Estimate Std. Error t value Pr(>|t|)
gamma    0.08235    0.78127   0.105    0.916
delta_1  2.28413    3.29024   0.694    0.488
delta    0.34870    1.91669   0.182    0.856
rho_1   -0.04473    0.17437  -0.257    0.798
rho     -0.40235    1.42491  -0.282    0.778

Residual standard error: 1.996989
Multiple R-squared: 0.7588273

Elasticities of Substitution:
               Estimate Std. Error t value Pr(>|t|)
E_1_2 (HM)       1.0468     0.1911   5.479 4.29e-08 ***
E_(1,2)_3 (AU)   1.6732     3.9892   0.419    0.675
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
HM = Hicks-McFadden (direct) elasticity of substitution
AU = Allen-Uzawa (partial) elasticity of substitution
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Estimation results for the “cesCalc” function:

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[1] 21.14180 26.69667 26.15995 28.34016 24.81796 23.99944 23.37701 23.41466
24.46984 23.73234
[11] 24.30095 24.21310 23.92122 21.20274 19.39057 17.84978 17.28183 15.52202
15.74408 15.33820
[21] 14.40075 14.56285 14.89940 15.35144 15.58608 15.98370 16.99964 17.09861
17.51439 18.16596
[31] 18.47297 18.98960
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Interpretation of results:

A nested CES function can produce biassed and unreliable results if a system approach is naively applied since the addition of a third input factor creates a lot of additional difficulties. In this paper, we show that retaining the elasticity of the nested process constant significantly improves the performance of the system approach to estimating a CES function with nested parameters.

The results' ramifications are relevant from an economic and methodological perspective. From an economic standpoint, they show that production factors are excellent complements, both in the traditional two-factor scenario and when applied to additional production factors, such as energy. At least over the medium term, energy, in particular, has limited substitutability to the other production variables.

Conclusion

In order to get better empirical results for multi-factor production analysis, it is hypothesized that a system estimation approach can be successfully integrated with an empirically driven fixed value for the elasticity in nested CES processes. This is particularly true when there is a likelihood that the assumptions of a balanced growth path and some sort of productivity neutrality will be broken. We demonstrate the effectiveness of the concept by using energy as a third production element. Our findings demonstrate how crucial energy is to the production process and how challenging it is to replace.

In conclusion, using nested CES models with many variables is an effective way to examine how different economic sectors and variables would be affected by clean energy legislation. Examining how various inputs are substituted during the production process, these Models can assist decision-makers in understanding the trade-offs and potential advantages of various policy initiatives. The multivariable CES model and the deeply nested CES model offer analytical frameworks for examining intricate manufacturing processes and the interactions between various input components.

References

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