Solitons in a Bose-Einstein Condensate

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Introduction

The atoms in a Bose-Einstein condensate are all described by the same wavefunction, since they are in the same quantum state, which includes each atom's interaction with the other atoms in the condensate. Introducing an pseudopotential term $g|\psi|^2$ which characterises the interactions between the particles into the dimensionless Schrödinger equation yields the Gross-Pitaevskii equation []

$$i\frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial x^2} + (V + g|\psi|^2)\psi, \tag{1}$$

where ψ is the atomic wavefunction, t and x are the rescaled time and length variables, V is the external potential (if present) and g is the interaction parameter. It is usual to define ζ as a parameter to characterise width, with units of inverse length. The normalised wavefunction is then

$$\psi(x) = \sqrt{\frac{\zeta}{2}}\,\mathrm{sech}\,(\zeta x) e^{ivx+\phi}, \tag{2}$$

where v is the velocity of the soliton and ϕ is a phase factor. Substituting the $v = 0$ case into the time-independent Schrodinger equation and setting E to be ζ^2, we find $g = 4\zeta$.

Propagating a Soliton

The Gross-Pitaevskii equation was solved numerically using the split-step Fourier method [].

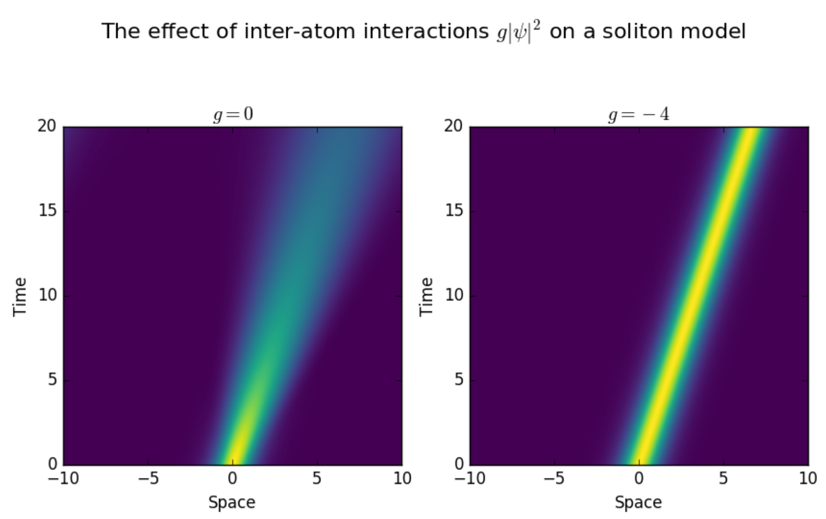

Figure 1: $n = 50$, $p = 0.6$

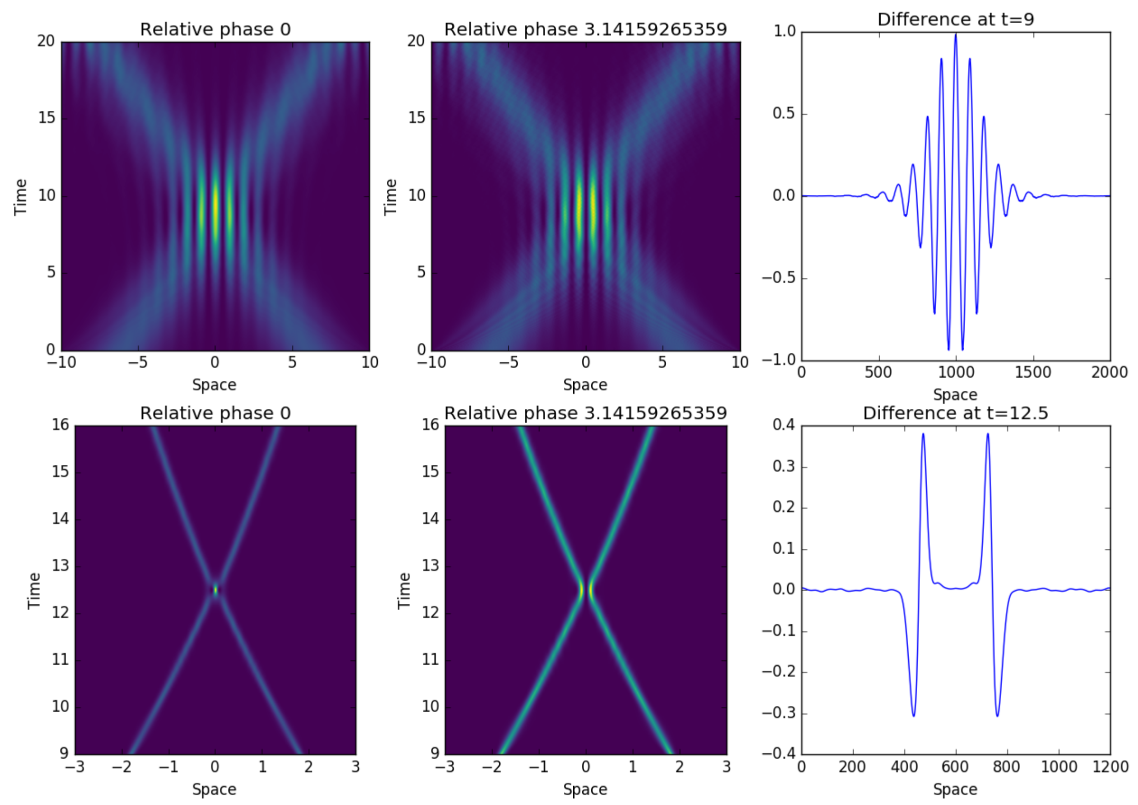

Figure 2: Caption

Repeated Collisions

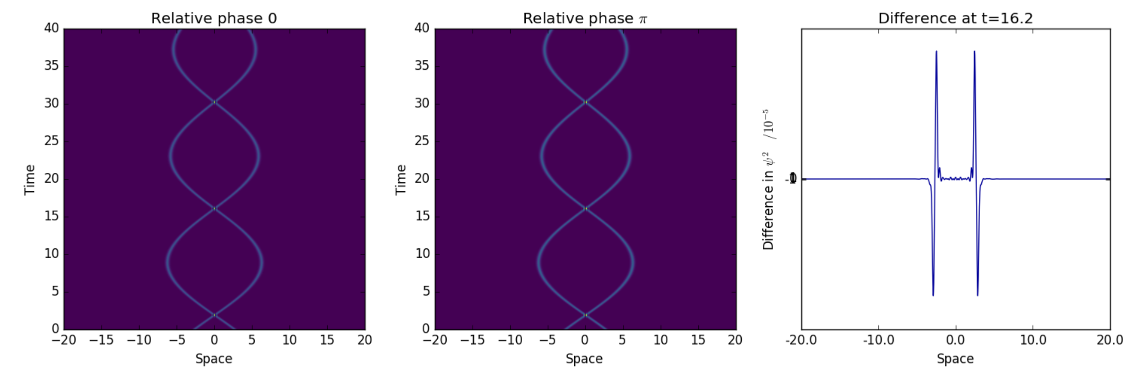

Figure 3: $n = 50$, $p = 0.6$

Conclusion

References

[1] D. J. Korteweg and G. de Vries (1895), "On the Change of Form...", *Philosophical Magazine* **39**(240) pp. 422-443.

[2] H. Segur (1973), "The Korteweg de Vries equation and water waves, part 1", *Journal of Fluid Mechanics* **59** pp. 721-736.

[3] C. S. Gardner et al. (1967), "Method for Solving the Korteweg de Vries Equation", *Physical Review Letters* **19**(19) pp. 1095-1097.