Solitons in a Bose-Einstein Condensate

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Introduction

The atoms in a Bose-Einstein condensate are all described by the same wavefunction, since they are in the same quantum state, which includes each atom's interaction with the other atoms in the condensate. Introducing an pseudopotential term $g|\psi|^2$ which characterises the interactions between the particles into the dimensionless Schrödinger equation yields the Gross-Pitaevskii equation []

$$i\frac{\partial\psi}{\partial t} = -\frac{\partial^2\psi}{\partial x^2} + (V + g|\psi|^2)\psi, \tag{1}$$

where ψ is the atomic wavefunction, t and x are the rescaled time and length variables, V is the external potential (if present) and g is the interaction parameter. It is usual to define ζ as a parameter to characterise width, with units of inverse length. The normalised wavefunction is then

$$\psi(x) = \sqrt{\frac{\zeta}{2}}\,\mathrm{sech}\,(\zeta x)e^{ivx+\phi}, \tag{2}$$

where v is the velocity of the soliton and ϕ is a phase factor. Substituting the $v = 0$ case into the time-independent Schrodinger equation and setting E to be ζ^2, we find $g = 4\zeta$.

Propagating a Soliton

The Gross-Pitaevskii equation was solved numerically using the split-step Fourier method for rescaled time t from 0 to either 20 or 40 [UNITS]. The box length used was initially 20 [UNITS], which was increased to 40 for repeated soliton collisions. [put in repeated collisions section?] A box length of 20 meant that a single soliton could have a velocity of up to 5 [UNITS] in the limit of zero width up to $t = 20$, without boundary effects causing unrealistic distortion. 4000 space points and 4000 time points were used to minimise discretization effects.

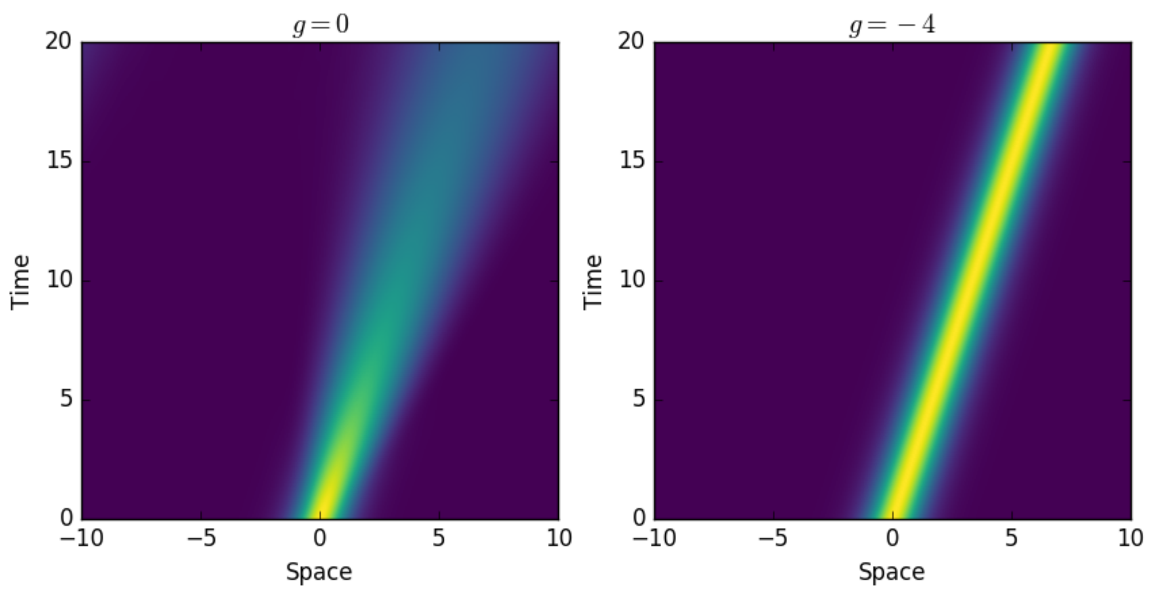

Figure 1: A graph showing the effect of inter-atom interactions $g|\psi^2|$ on a soliton model. The family parameter $\zeta = 1$ hence $g = -4$ perfectly confines the soliton with velocity $v = 5/3$.

Soliton Collisions

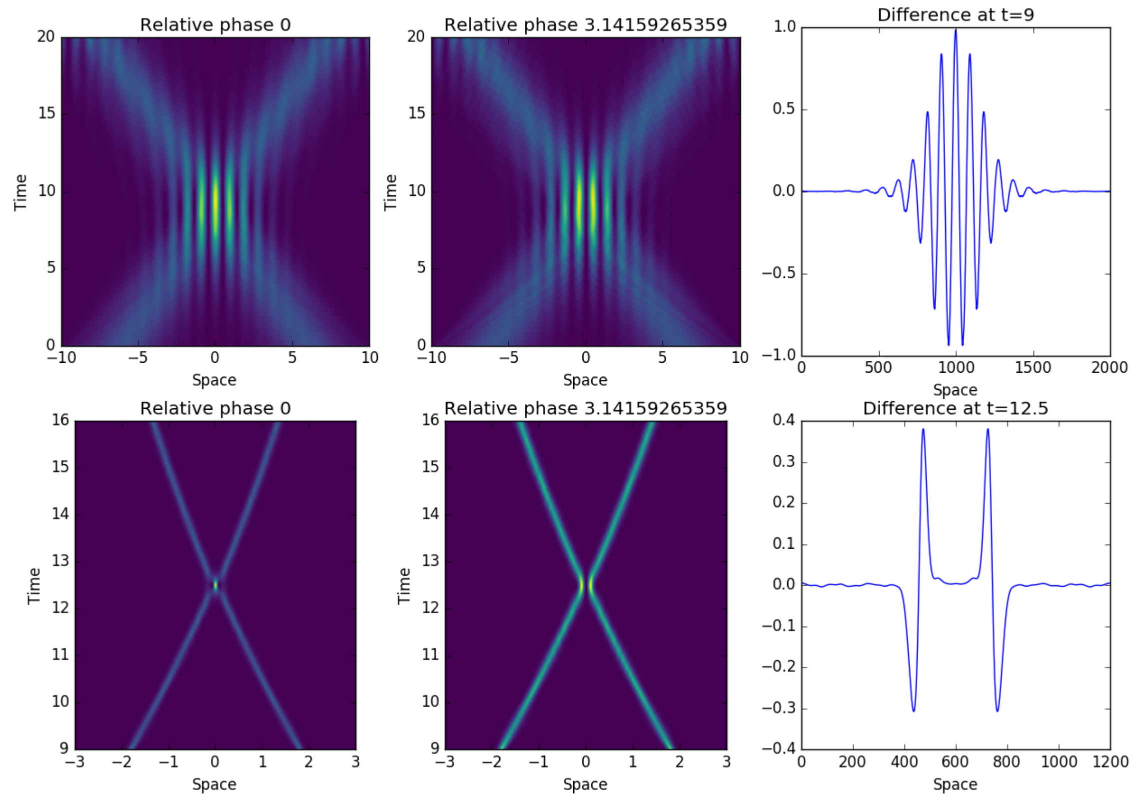

Figure 2: Caption

Repeated Collisions

Introducing a weak harmonic potential to the 1D Bose-Einstein condensate model confines the solitons axially within the box. After a collision the solitons climb a distance up the potential barrier dictated by their momentum (and hence velocity) and return to collide again. The axial potential preserves the relative phase and velocity of the solitons, allowing perfect periodic motion. However, the introduction of this potential places limits on the range of interaction parameters g which lead to soliton-like behaviour.

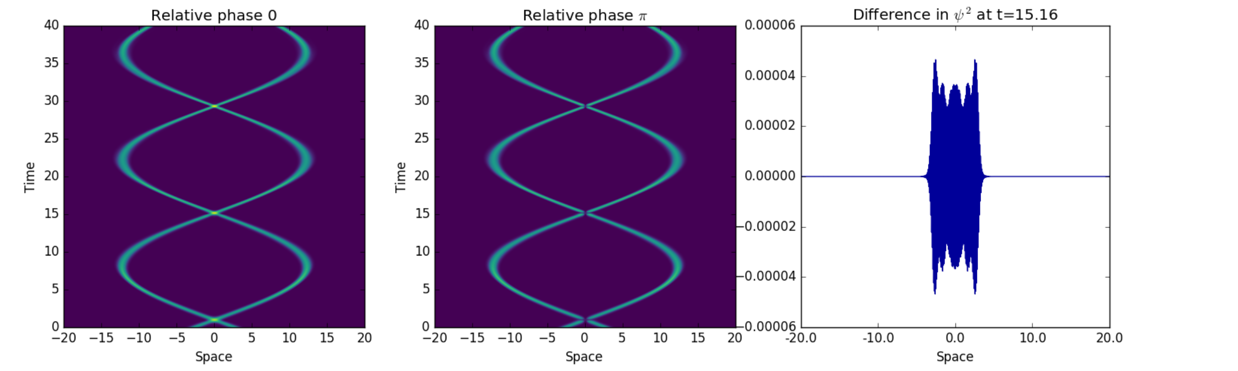

Figure 3: Two solitons with starting positions -4 and 4 and velocities 20 and -20, with relative phase either 0 or π radians. The interaction parameter $g = -5$ thus $\zeta = 1.25$. The images here are plotted on a symmetric log scale with a linear cutoff of $\psi^2 = 0.3$ to improve visibility of the solitons.

It was observed that for $g < 4$, the solitons were affected adversely by the external potential and appeared diffuse. For $g \geq 12$ a numerical limit of the model was reached and the solitons lost energy in subsequent collisions. The model is valid for interaction parameters between these values, corresponding to family parameters $1 < g \leq 3$.

Conclusion

References

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[3] C. S. Gardner et al. (1967), "Method for Solving the Korteweg de Vries Equation", *Physical Review Letters* **19**(19) pp. 1095-1097.