

Bank Heterogeneity and Discount Window Access: Theory and Evidence

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Abstract

The federal funds rate is constructed as a volume-weighted average and masks significant heterogeneity in individual federal funds transactions. This paper explores one source of that heterogeneity: discount window access. Data from transaction-level discount window borrowing and quarter call reports from 2010 to 2020 show clear differences between banks with and without access to discount window lending. On average, banks that can access the discount window hold fewer reserves, lend less on the federal funds market, and borrow at a lower federal funds rate. This paper develops a search theoretical framework to explain the observed heterogeneity taking discount window access as exogenous. In the model, the discount window insures banks against matching failures in the interbank market and provides an outside option that limits the rents that lender banks can capture. The outside option role shows up as a lower interest rate in the federal funds market, and the insurance role explains lower reserve holdings and lower lending volume as a result.

JEL Code: E41, E43, E44, E52, E58

1 Introduction

The discount window was opened along with the Federal Reserve banks in 1913 to serve as a lender of last resort during times of crisis. Over the past century of its operation, the pros and cons of its operation have been well debated.¹ Although there has been a long debate, a consensus still has not been reached on whether the facility is net positive or negative. Evidence of this can be seen in Figure 1, which shows the heterogeneity of access to discount window lending across Federal Reserve districts. Districts 1 (Boston) and 12 (San Francisco) allow the majority of its member banks to borrow, while districts 9 (Minneapolis) and 11 (Dallas) are more restrictive. Since access to discount window lending is generally granted to financial institutions that the Federal Reserve considers 'sound', there are differences in the interpretation across different districts. This paper's purpose is to explore the implications of discount window access and how it can affect bank behavior in the federal funds market by documenting the facts, theoretically uncovering the channels, and testing the model-implied correlations.

To answer whether access to the discount window affects bank behavior, we first have to show that there are differences between banks with and without access. Transaction-level data of discount window borrowing has only recently become available due to the Dodd-Frank Act of 2010.² I combine this data-set with quarterly call reports filed by banks to the Federal Financial Institutions Examination Council (FFIEC) that contains balance sheet information, federal funds transactions, reserves, capital risk ratio, and other key characteristics to create a consolidated sample that spans Q1 2010 to Q4 2020.³ In the aggregate, the share of banks that has access to the discount window has gone up from 2010 (28%) to 2020 (33%). Banks with access to the discount window hold 1.5 percentage points more reserves as a proportion of their assets than those that cannot access the window. As a sector, assets held by banks have increased from \$16.1T in Q1 2010 to \$26.9T in Q1 2020, a 67% increase. Over the same period, CPI has only increased 30%. Breaking this statistic down into banks with and without access, we find that all of the growth has been driven by banks with access to the discount window (92%) and not those without access (.9%). In terms of quantity, banks without discount window access have decreased from 5,187 to 3,427 (-34%) and those with access have decreased from 2,037 to 1,700 (16.5%) over the same period. On the federal funds' side, Figure 7 shows federal funds lending volume both as a share of the bank's assets as well as the proportion of banks that choose to lend on the market. As a share of assets, banks without access to the window lend 1 percentage point

¹For a comprehensive review, see Ennis (2017).

²Data can be found at <https://www.federalreserve.gov/regreform/discount-window.htm>. Information from before 2010 is unavailable due to fear of discount window stigma.

³For the summary statistics, as well as figures supporting the following statements, see Appendix A.

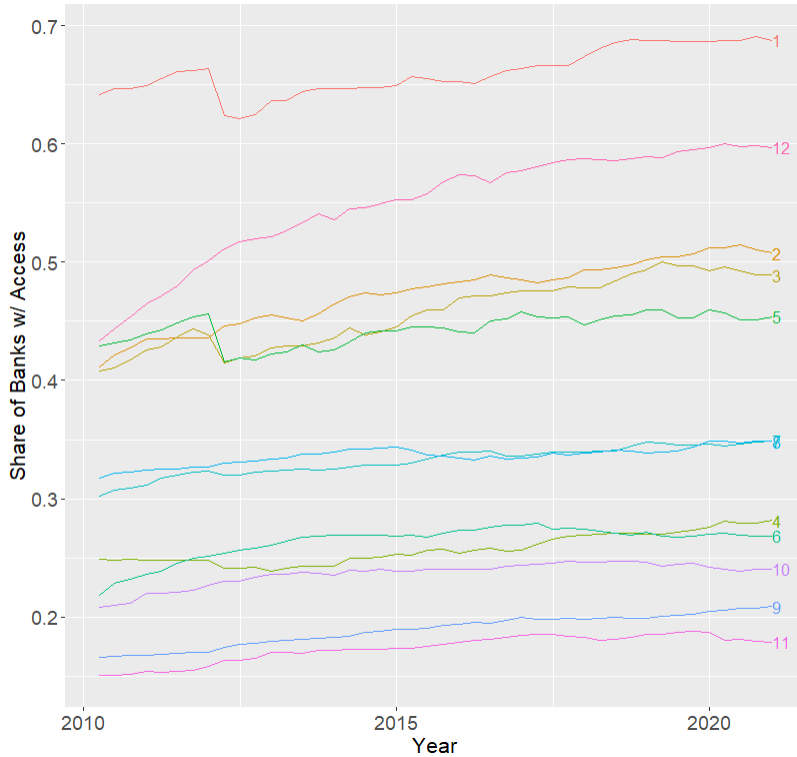


Figure 1: Share of banks that can access the discount window across Federal Reserve districts from 2010-2020. Data is compiled using transaction-level discount window lending information. If a bank borrowed from the discount window, then they are assumed to have access for the duration of the sample. Changes over time are due to the entry/exit decisions of individual banks.

higher than those with access. A higher proportion of banks without access also choose to lend on the market for a particular quarter than those with access. All this goes to show that discount window access draws an important line of separation between banks.

To uncover the mechanisms underpinning differences in reserve balances, federal funds lending behavior, and federal funds interest rate, I develop a search theoretic model that includes an interbank market with matching frictions, a lender of last resort, and agents (which I interpret as banks) that can and can't access the lender of last resort. In this model, agents first choose their money holdings given an expected consumption demand, as well as expected profits from lending in the interbank market. Consumption demand is interpreted as shocks to deposit outflows or reserves required for settlements. After the choice of money holdings and realization of the shock, agents face an interbank market with matching frictions, where banks that have excess liquidity (furthermore referred to as surplus banks) will be lenders, and banks that require liquidity will be borrowers. In this market, the lending facility plays two key roles for agents with access. One, it insures against matching

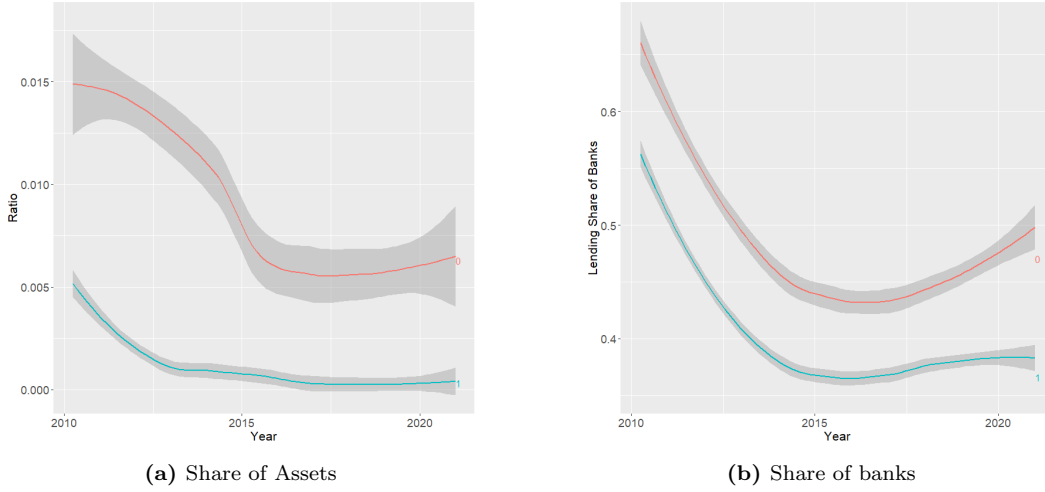


Figure 2: (a) shows the ratio of federal funds volume lent over assets for each quarter. (b) shows the proportion of banks that lend on the federal funds market for a particular quarter.

failures when an agent who requires liquidity is unmatched in the interbank market. Two, it provides an outside option that limits the capturable rents that surplus banks can seek. The first role implies that banks who cannot access this lending facility carry more reserves to self-insure against matching frictions in the interbank market, and the second implies that banks with access are charged a lower equilibrium interest rate in the interbank market.

Due to the heterogeneity in shocks, not all agents hit with the consumption shock will be borrowers. In particular, since agents who cannot access the discount window hold higher levels of reserves, they are also more likely to lend in the interbank market. The theory predicts that as matching frictions become more severe, the gap in reserves held by the two types of agents widens. The model also gives insight into how inflation changes the dynamics in the interbank market. As inflation increases, the marginal cost of holding reserves also increases, which leads to a reduction in the money holdings of both types of agents. Since the money holdings affect the matching function, higher inflation reduces the ratio of lenders to borrowers.

The comparative statics from the theory provides testable implications in the data. Running a fixed effect panel model using banks as individuals at the quarterly frequency, I find that access to the discount window is positively (weakly) correlated with the number of federal funds borrowed and negatively correlated with both the federal funds volume lent and level of reserves. To test the hypothesis that banks with access borrow from the interbank market at a lower rate, I exploit variation in the share of banks with access across Federal Reserve districts and the average interest rate they pay in a given quarter and find those

two variables to be negatively correlated. This implies that districts where a higher share of banks can access the discount window on average borrow at a lower interest rate. Klee (2011) also finds this result using FEDWIRE transactional data and recovered overnight interest rates paid by individual banks using the Furfine (1999) algorithm. She explains this result by linking it to bank-specific 'stigma' levels that lead to a selection effect in the federal funds market; i.e. banks that have a higher propensity to borrow from the discount window pay lower rates in the federal funds market. I propose that the mechanism behind this phenomenon is due to the outside option role of the discount window.

Related Literature

There have only been two papers since 2010 that use the transaction-level discount window lending data. Ackon and Ennis (2018) analyzed data from 2010-2015 and show that conditional on access, banks use the discount window regularly. Ennis and Klee (2021) uses the data to answer the question of who borrows from the discount window during 'normal' times. Their paper indicates that banks adjust their behavior conditional on having access to the window, and would be able to adjust their portfolio holdings even if access to the window was removed during normal times.⁴ I contribute to this field by extending the sample to include data from 2018 to 2020 and studying the normative implications of the discount window and its interaction with the interbank market.

This paper also adds to a long literature on the role that the Central Bank plays as a lender of last resort. Donaldson (1992) uses a strategic pricing model to study panics and show that some banks can exert market power to charge higher than competitive rates in the interbank market. Acharya et al. (2012) show that the discount window, through its role as a lender of last resort, can strengthen the outside option of needy banks to reduce the amount of market power surplus banks can exert. Afonso and Lagos (2015) uses a continuous-time model to study the intra-day trading behavior of banks to study how efficient the interbank market is given search and bargaining frictions. In their model, the discount window is used passively; in the sense that it provides end-of-day funds for banks to meet reserve requirements. In this paper, the discount window plays both the role of a lender of last resort, as well as an outside option for banks with access. I find that the discount rate can be used to discipline the observed federal funds rate, but the strength of the effect depends on the number of banks that has access to the window. Recent works on public provision of liquidity and the connection between monetary policy and the federal funds market includes Bianchi and Bigio (2017); Afonso et al. (2019); Arseneau et al. (2020).

⁴Outside of the US, Drechsler et al. (2016) studies lender of last resort policy by the European Central Bank. They find that banks that are weakly capitalized tend to borrow more from the ECB during the crisis, and later shown by Ennis and Klee (2021) to also happen during 'normal' times.

There is a subset of the discount window lending literature that focuses on the borrowing stigma during the Financial Crisis by Armantier et al. (2010, 2015); Armantier and Holt (2020). Generally, this literature explores the selection effect of those who choose to borrow at the window. Since borrowing from the window could be considered a signal of financial distress, banks are willing to pay a higher rate in the market to avoid the associated stigma cost. This analysis is reliant on the fact that all banks can access the discount window ad-hoc and there is information about which banks choose to access the window. In reality, since transaction-level data was only available after Dodd-Frank, acquiring information about which banks choose to borrow from the window is difficult. Information about individual banks' access to the window is also private information, and cannot be compiled until the transaction is observed. This implies that stigma might not be the only factor driving the bank's borrowing decision to borrow from the window, and other channels could be at play to explain interest rate differentials.

The theoretical portion builds on the literature on liquidity reallocation in search models by Berentsen et al. (2007). They show that any market that can efficiently reallocate liquidity can improve welfare by providing insurance. The interpretation of the liquidity reallocation market and the interbank market comes from Berentsen and Monnet (2008). The microstructure of trading frictions in OTC markets is explored in Duffie et al. (2005) and Lagos and Rocheteau (2009) using a search-theoretic framework. Specific to the federal funds market, Ashcraft and Duffie (2007) shows that 73% of the transactions made through FEDWIRE are done through bargaining, and the rest is brokered. My contribution to this field is by introducing agents heterogeneous in their liquidity needs and matching frictions that are endogenous. The discount window influences the interbank market by affecting the money holding decisions of agents, which endogenously influence the measure of lenders and borrowers in the interbank market. This interaction leads to externalities that have not been explored in the literature on the federal funds market. Because some agents can access the window and others cannot, the interest rate charged in the interbank market differs for the two types of agents through the disagreement point.

My work also marginally connects to search-theoretic models of asset pricing in OTC markets. Duffie et al. (2005) assumes that gains from trade arise from differing valuations for a particular asset. Geromichalos and Herrenbrueck (2016) microfound the DGP framework by adding heterogeneity in asset demand and matching frictions in the asset market. If we think of money as an asset traded in the interbank market, then the federal funds rate would be the price of money. Increasing matching frictions increases the valuation of a match and the marginal utility of lending, therefore incentivizing agents to hold more money.

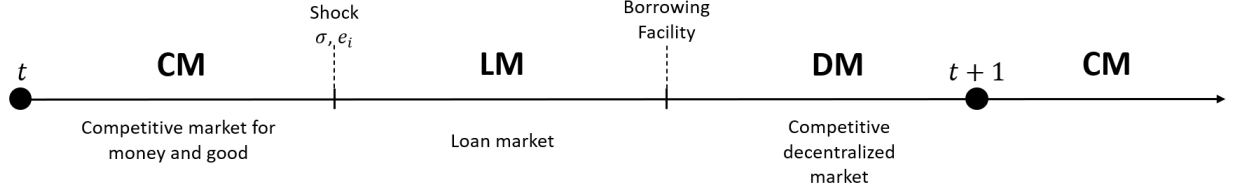


Figure 3: Events in period t

2 The Environment

The model extends the framework of Berentsen et al. (2007). Time is infinite and discrete, with each period consisting of three subperiods. The first (CM) and third stage (DM) is characterized by a frictionless centralized market where agents can trade money for goods. Between these two stages, we introduce a frictional OTC lending market (LM) where agents can lend and borrow from each other.⁵ Figure 3 shows a graphical representation of the events in period t . There are two perishable goods: a good c produced in the CM and taken as the numéraire, and a good q produced in the DM.

The economy is populated by two types of agents that lives forever. A fraction λ of agents have access to a lending facility at the end of the LM (D-Type), and the corresponding $(1 - \lambda)$ (N-Type) does not. The measure of agents is normalized to the unit and each individual agent indexed by i . Both agents' lifetime utility is:

$$\mathbb{U} = \sum_{t=0}^{\infty} \beta^t [c_t + e_i u(q_t^b) - q_t^s] \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor between periods. All agents have linear utility over c , where $c > 0$ is interpreted as consumption of the numéraire good, and $c < 0$ is interpreted as production of the numéraire good. A consumption of q gives the buyer utility $e_i u(q)$, and a production of q incurs a linear cost for the seller. Utility $u(q)$ is strictly concave, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$.

In the first subperiod, all agents consume and produce a general good c , and have access to a linear technology to trade one unit of labor for one unit of the general good. All liabilities incurred in the previous also are repaid at corresponding interest rates. In this subperiod, a money market also opens up where agents can trade the general good for ϕ_t units of money. The evolution of the money supply is controlled by the government, and evolves by $M_{t+1} = \gamma M_t$, where M_t denotes the money stock at time t , and $\gamma > 0$ denotes the

⁵The OTC structure of the Federal Funds market that we adopt has been empirically highlighted in Ashcraft and Duffie (2007), where they find that in the aggregate, approximately 73% of all loans made through the federal funds market were traded bilaterally.

gross growth rate of money. After agents choose their money holdings taking into account future possibilities of consumption, they move forward to Stage 2.

Before the beginning of the second subperiod, all agents are hit with a preference shock for consumption of the DM good with probability σ . Agents who are not consumers in the DM will produce the goods. I refer to consumers as buyers and producers as sellers. Conditional on receiving the preference shock, a consumer will also draw an idiosyncratic demand scale parameter e_i following a distribution $f(e_i) \in [\underline{e}, \bar{e}]$ with a cumulative distribution function $F(e)$. Once shocks are realized, agents choose to enter the lending market either as a borrower or a lender. In equilibrium, the measure of borrowers is defined as a set \mathcal{B} , and the measure of lenders is \mathcal{L} . Matches are formed bilaterally following a matching function $\mathcal{M}(\mathcal{B}, \mathcal{L})$ with an efficiency parameter A . A low parameter value for A is akin to financially distressed states, where banks have difficulty finding a lending partner. Defining the tightness as $\theta \equiv \mathcal{L}/\mathcal{B}$, the probability that a borrower will match with a lender is $m(\theta)$, and the probability that a lender will match is $m(\theta^{-1})$. In the pairwise meetings between a borrower i and a lender j , loan size and repayment amount $\{l, x\}_{i,j}$ are determined according to the proportional solution of Kalai (1977), where the share of surplus received by lenders is $\eta \in [0, 1]$.⁶ We define $i_i = x_i/l_i$ as the nominal interest on the loan taken by borrower i , and:

$$\bar{i} \equiv \frac{\int_{\mathbb{B}} x_i di}{\int_{\mathbb{B}} l_i di}$$

as the 'federal funds' rate, the volume-weighted average interest rate in the interbank market.

Before trading takes place in the DM, a lending facility opens up for any agents who require additional liquidity. Borrowers can borrow b units of money from the facility and repays $(1+r)b$ units of money in Stage 1 of the following period. The budget constraint of the government is therefore: $T_t - rB_t = (\gamma - 1)M_t$, where T_t is lump-sum transfers given to agents in Stage 1 at period t , and rB_t is the aggregate interest payment that agents make to the central bank from borrowing. The third period is a competitive decentralized market as in Rocheteau and Wright (2005), where the only means of payment is money due to anonymity. Since sellers have a constant marginal cost of production, the price is such that sellers receive no surplus from the trade. The price of the good p_t is set competitively, Buyers and sellers both take prices as given, and supply and demand are determined at the competitive price.⁷

⁶The Nash (1950) solution has been shown to be problematic when liquidity constraints are binding, see Hu and Rocheteau (2020). In summary, the Nash solution replicates the case where the output is negotiated all at once in a Rubinstein alternating-offer game, while the proportional solution replicates the case where there is an infinite number of negotiations over infinitesimally small bundles. They furthermore show that the liquidity constraint for buyers binds for any $N < +\infty$.

⁷Under take-it-or-leave-it offers, the results remain the same. Using the competitive equilibrium framework of Rocheteau and Wright (2005) allows us to abstract away from frictions found in the DM, and sets

This framework allows us to study the federal funds market as a market to reallocate liquidity across banks, and the role that the discount window plays as a lender of last resort.

3 Optimal Solutions

Let ϕ_t be the price of money in the CM in period t . This section characterizes a steady state stationary equilibria where aggregate real balances and allocations are constant; i.e., $\phi_t M_t = \phi_{t+1} M_{t+1}$. Under the assumption that γ is constant, $\phi_t / \phi_{t+1} = M_{t+1} / M_t = \gamma$. D-type and N-type agents are denoted as superscript $\chi \equiv \{D, N\}$.

Let $W_t^X(\omega)$ denote the expected value from entering Stage 1 with total wealth $\omega \equiv \omega(m, l, b)$ expressed as the numeraire. Let $X^\chi(m)$ denote the value of entering Stage 2 as a buyer holding m units of money, and $V^\chi(m, l, b)$ the expected value of entering Stage 3 with m units of money, l units of privately borrowed money, and b units of publicly borrowed money. I examine the individual decision problems at each sub-period in t , then solve the equilibria.

Stage 3 - Decentralized Market

In Stage 3, the seller's problem is represented by:

$$\max_q -q + \beta W_{t+1}(m + p_t q) \quad (2)$$

Such that the seller chooses a level of production q that maximizes their future wealth in Stage 1 of the next period. The production of q units of good nets them a monetary gain of $p_t q$, where p_t is defined as the price of the DM good in terms of money. Since consumption in Stage 1 is linear, the value function can be reduced to $W_{t+1}(m + p_t q) = \phi_{t+1} p_t q + W_{t+1}(m)$. The resulting price that comes from the first-order condition is:

$$p_t = \frac{1}{\beta \phi_{t+1}} \quad (3)$$

Due to constant marginal cost for sellers, supply is perfectly elastic such that the gains from trade are zero for sellers. Therefore, the Bellman equation of the seller in the DM is just the continuation value of entering the CM in $t + 1$. For buyers, they choose a consumption quantity to maximize:

$$V_t(m_2, l, b | e_i) = \max_q e_i u(q) + \beta W_{t+1}(m - p_t q) \quad (4)$$

$$\text{s.t. } p_t q \leq m_2 \quad (5)$$

the main focus of the paper on the liquidity reallocation market.

The arguments of the value function represents all the state variables of agent i conditional on the individual realization of the demand shock e_i . They hold m_2 units of money before entering Stage 3, where the subscript represents their money holdings at the end of stage 2, as well as l amount of private loans and b amount of public loans. The constraint restricts the buyer from transacting more money than they hold, ensuring that the buyer's real wealth is not negative. Define q_i^* as the solution to $e_i u'(q) = 1$. Using (3)-(4), the quantity that an individual buyer chooses to consume in time period t is:

$$q_i = \begin{cases} q_i^* & \text{if } m_{2,i} \geq p_t q_i^* \\ u'^{(-1)}(1/e_i) & \text{if } m_{2,i} < p_t q_i^* \end{cases} \quad (6)$$

Where the first case happens when the buyer is not liquidity constrained, and the second happens in the case that the buyer does not hold enough money. Define $\psi_i(m_2) \equiv [e_i u[q(m_2)] - q(m_2)]$ as the trade surplus of a buyer who enters the DM with m_2 units of money, we can express (4) as:

$$V_t(m_2, l, b | e_i) = \psi_i(m_2) + \beta W_{t+1}(m) \quad (7)$$

Stage 2 - Lending Market

Given a realization of a set of shock $\{\sigma, e_i\}$, an agent's desired money holdings to enter the DM can be represented by $m_{2,i}^* = p_t q_{i,t}^*$. If the choice of money holdings at the end of Stage 1 is below their desired money holdings, then the money demand for agent i is given by $n_i^*(e_i) = m_2^*[q(e_i)] - m_{1,i}$.

Corollary 3.0.1. *For a type χ agent, there exist two critical values $\{e_\chi^*, e_\Omega\}$ such that if the realization $e_i < e_\chi^*$, then the consumer will also be willing to lend since $m_1^x > m_{2,i}^*$. If the realization $e_i > e_\Omega$, then the D-type consumer will access the lending facility after private borrowing.*

Figure 4 shows the graphical representation of the corollary. Under random search, since some buyers are willing to lend, there exist scenarios such that a D-type borrower i with money demand n_i^D will meet with a lender j that has money holdings less than n_i^D . In that situation, the borrower will borrow the maximum amount from the lender, and then approach the window since they are still lacking liquidity. From this result, the measure of lenders and borrowers in the interbank market is respectively:

$$\begin{aligned} \mathcal{L} &= (1 - \sigma) + \sigma[\lambda F(e_D^*) + (1 - \lambda)F(e_N^*)] \\ \mathcal{B} &= \sigma[(1 - \lambda)F(e_D^*) + (1 - \lambda)F(e_N^*)] \end{aligned}$$

From the matching process, there are two types of borrowers and four types of lenders. The borrower consists of D-type agents and N-type agents, and lenders consist of both types of

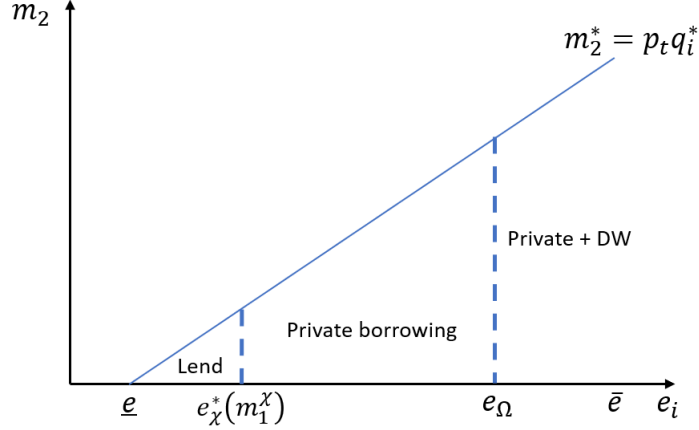


Figure 4: The line represents the demand for m_2 of agent i . There exists two critical values which affects the behavior of agents.

agents that have no use for money, and those that are holding excess money from Stage 1. Unmatched agents either move forward directly with their money holdings if they are N-type or access the discount window if they are D-type. For an N-type agent, the probability that they are not matched in the interbank market is $(1 - m(\theta))$, and the quantity that they consume in the DM is a function of their Stage 1 money holdings $q(m_1^N)$.

For a D-type agent that is unmatched, they move forward to the discount window and borrow b units of money at a rate r . They borrow such that the marginal utility from holding one extra unit of money is equal to the marginal cost of repayment. Their optimal decision is given by:

$$e_i u'[q(m_1^D + b)] - 1 = r \quad (8)$$

Where the left side of the equation is the marginal utility gained from consuming an extra unit of the DM good net of payment, and the right side is the marginal cost of repayment in the future CM. Equation (8) is also used to calculate the disagreement point of matched D-type agents during the bargaining process.

Interbank Behavior

Table 1 and 2 shows all possible types of lenders and borrowers that can appear on the interbank market. After the realization of the consumption and demand shock at the beginning of Stage 2, a borrower i meets a lender j and bilaterally negotiates over a loan contract (l, x) depending on liquidity need. Since there are two choice variables in the

Lender Type	Money Supply	Lender Mass
N-type w/o shock	m_1^N	$(1 - \lambda)(1 - \sigma)$
D-type w/o shock	m_1^D	$\lambda(1 - \sigma)$
N-type w/ shock	$(0, m_1^N - m_2(\underline{e})]$	$\sigma(1 - \lambda)F(e_N^*)$
D-type w/ shock	$(0, m_1^D - m_2(\underline{e})]$	$\sigma\lambda F(e_D^*)$

Table 1: All possible lenders that a borrower entering the LM could face. There is a positive mass of N and D-types without a consumption shock with the money holdings carried over from the CM, and a continuum of N and D-types that have a desire to consume in the period but hold excess money.

Borrower Type	Money Demand	Borrower Mass
N-type	$(0, m_2(\bar{e}) - m_1^N]$	$\lambda\sigma[1 - F(e_N^*)]$
D-type	$(0, m_2(\bar{e}) - m_1^D]$	$(1 - \lambda)\sigma[1 - F(e_D^*)]$

Table 2: All borrower types, the range of their associated loan demand, and mass. Borrowers desire a continuum of funding depending on the realization of e_i .

bargaining game, the system is over-identified. To resolve this issue, I fix l such that:

$$l_{i,j} = \min\{m_j, n_i^*(e_i)\} \quad (9)$$

If borrower i needs more money than the amount held by lender j , then the loan size is restricted to the amount of money that the lender brought into the LM. Otherwise, the match is not liquidity constrained and the borrower achieves the amount of money to purchase q_i^* . Matched agents take the loan size as given, and bargain over the repayment amount x . The bargaining game for a D-type borrower with access to the discount window is:

$$\max_x S_i^D \equiv [\psi_i(m_1^D + l_{i,j}) - \beta\phi_{t+1}x_{i,j}] - [\psi_i(m_1^D + b_i^*) - \beta\phi_{t+1}rb_i] \quad (10)$$

$$\text{s.t.} \quad \eta S_i^D = (1 - \eta)\beta\phi_{t+1}x_{i,j} \quad (11)$$

Where S_i^D is the total surplus from the loan net of their outside option and η is the share of surplus guaranteed to the lender. The first term of the surplus is the total gains in utility in the DM trade using their money holdings from Stage 1 along with the loan amount $l_{i,j}$. The second term in the surplus is the surplus that the buyer would have had given that they borrowed b_i^* from the window if they were an unmatched D-type agent. Using the definition that $x_{i,j} = i_{i,j}l_{i,j}$, the solution to the bargaining game solves:

$$i_{i,j}^D = \frac{\eta[\psi_i(m_1^D + l_{i,j}) - \psi_i(m_1^D + b_i^*)]}{\beta\phi_{t+1}l_{i,j}} + \frac{\eta rb_i^*}{l_{i,j}} \quad (12)$$

Similarly, solving the bargaining solution for an N-type borrower gives us:

$$i_{i,j}^N = \frac{\eta[\psi_i(m_1^N + l_{i,j}) - \psi_i(m_1^N)]}{\beta\phi_{t+1}l_{i,j}} \quad (13)$$

Where the difference between the interest rate that a D-type agent and an N-type agent faces is due to the inclusion of the disagreement point. Naturally, the interest rate faced by a borrower is higher if the bargaining power of the lender increases. A higher realization of e_i also increases the interest repaid due to greater gains from trade. For a given realization of $\{e_i, m_j\}$, the D-type borrower will always borrow at a lower interest rate than an N-type borrower. If we take this to the data, then we should expect that a bank with access to the discount window will borrow at a lower rate on the federal funds market than a bank without access. Since a subset of D-type borrowers will be matched with liquidity constrained lenders, we also expect those agents to borrow from the discount window even if they are matched.

We can then construct the *federal funds rate*, \bar{i} , as the volume-weighted average of all interest rates borrowed from in the interbank market:

$$\bar{i} = \frac{\int_{\{i,j\}} x_{i,j} d\{i,j\}}{\int_{\{i,j\}} l_{i,j} d\{i,j\}} \quad (14)$$

And define the corresponding value functions for D and N-type agents respectively:

$$X_t^{b,D}(m_1^D, e_i) = \psi(m_1^D + l_{i,j} + b_i) - \beta\phi_{t+1}(x_{i,j} + rb_i) + \beta W_{t+1}^D(m) \quad (15)$$

$$X_t^{b,N}(m_1^N, e_i) = \psi(m_1^N + l_{i,j}) - \beta\phi_{t+1}x_{i,j} + \beta W_{t+1}^N(m) \quad (16)$$

$$X_t^{l,D}(m_1^D) = \beta\phi_{t+1}x_{i,j} + \beta W_{t+1}^D(m) \quad (17)$$

$$X_t^{l,N}(m_1^N) = \beta\phi_{t+1}x_{i,j} + \beta W_{t+1}^N(m) \quad (18)$$

Where the superscripts consist of b for buyer, l for lender, and the corresponding agent types. The value function takes the behavior in the DM as optimal and incorporates the traded quantities into the ψ function. In order, the equation denotes the value of entering the LM as a borrower with access to the discount window, a borrower without access, a lender with access, and a lender without access. The time subscript t is suppressed for all variables.

Probabilities

This section explicitly outlines the probabilities that an agent face leaving the Stage 1 CM. Conditional on a realization of $e_i > e_D^*$, a D-type agent has the expected value after exiting the CM:

$$\begin{aligned}
\mathbb{E}_t(X_t^D | e_i > e_D^*) &= \sigma \left[m(\theta) \int_{\mathcal{L}} X_t^{b,D}(m_1^D, e_i | j) \, dF(\mathcal{L}) \right. \\
&\quad + [1 - m(\theta)] (\psi_i(m_1^D + b_i^*) - \beta \phi_{t+1} r b_i^* + \beta W_{t+1}^D(m)) \\
&\quad + (1 - \sigma) \left[m(\theta^{-1}) (\beta \phi_{t+1} \int_{\mathcal{B}} x_{i,j} \, dF(\mathcal{B}) + \beta W_{t+1}^D(m)) \right. \\
&\quad \left. \left. + [1 - m(\theta^{-1})] \beta W_{t+1}^D(m) \right] \right] \quad (19)
\end{aligned}$$

The first line of (19) shows the expected value of an agent that has a consumption shock and is matched in the interbank market. The integral takes into account the distribution of all lenders that the agent is expected to face in the LM. The second line is the continuation value of a D-type agent that is unmatched in the interbank market. The third line is the expected gains from lending in the interbank market from holding money given that the agent did not have the desire to consume the DM good in this period. The fourth is the continuation value of non-consumption given that the agent did not find a match in Stage 2. If the agent instead draws a demand shock $e_i \leq e_D^*$, their expected value is:

$$\begin{aligned}
\mathbb{E}_t(X_t^D | e_i \leq e_D^*) &= \sigma \left[m(\theta^{-1}) \left[\psi_i(q_i^*) + \beta \phi_{t+1} \int_{\mathcal{B}} x_{i,j} \, dF(\mathcal{B}) + \beta W_{t+1}^D(m) \right] \right. \\
&\quad \left. + [1 - m(\theta^{-1})] (\psi_i(q_i^*) + \beta W_{t+1}^D(m)) \right] \\
&\quad + (1 - \sigma) \left[m(\theta^{-1}) (\beta \phi_{t+1} \int_{\mathcal{B}} x_{i,j} \, dF(\mathcal{B}) + \beta W_{t+1}^D(m)) \right. \\
&\quad \left. + (1 - m(\theta^{-1})) \beta W_{t+1}^D(m) \right] \quad (20)
\end{aligned}$$

Where the first two lines from (19) are replaced with the corresponding expectation from lending in the interbank market. The first line is the expected value from matching with borrower i , and the second line is the continuation value from a failure to match. Since the draw e_i is low, these agents hold enough money to purchase their optimal quantity of q_i^* .

For an N-type agent with a realization of $e_i > e_N^*$, their expectation takes on the form:

$$\begin{aligned}
\mathbb{E}_t(X_t^N | e_i > e_N^*) &= \sigma \left[m(\theta) \int_{\mathcal{L}} X_t^{b,N}(m_1^N, e_i | j) \, dF(\mathcal{L}) \right. \\
&\quad + [1 - m(\theta)] (\psi_i(m_1^N) + \beta W_{t+1}^N(m)) \\
&\quad + (1 - \sigma) \left[m(\theta^{-1}) (\beta \phi_{t+1} \int_{\mathcal{B}} x_{i,j} \, dF(\mathcal{B}) + \beta W_{t+1}^N(m)) \right. \\
&\quad \left. + [1 - m(\theta^{-1})] \beta W_{t+1}^N(m) \right] \quad (21)
\end{aligned}$$

This mirrors the expectations of the D-type agent, except for the initial money holdings and

the surplus. Since N-types do not have access to the discount window, they only transact with the money holdings that they carry from Stage 1. If the N-type agent has a high realization of e_i , then they have expectations following:

$$\begin{aligned} \mathbb{E}_t(X_t^N | e_i \leq e_N^*) = & \sigma \left[m(\theta^{-1}) \left[\psi_i(q_i^*) + \beta \phi_{t+1} \int_{\mathcal{B}} x_{i,j} dF(\mathcal{B}) + \beta W_{t+1}^N(m) \right] \right. \\ & \left. + [1 - m(\theta^{-1})] (\psi_i(q_i^*) + \beta W_{t+1}^N(m)) \right] \\ & + (1 - \sigma) \left[m(\theta^{-1}) (\beta \phi_{t+1} \int_{\mathcal{B}} x_{i,j} dF(\mathcal{B}) + \beta W_{t+1}^N(m)) \right. \\ & \left. + [1 - m(\theta^{-1})] \beta W_{t+1}^N(m) \right] \quad (22) \end{aligned}$$

Stage 1 - Central Market

Consider a χ type agent who holds m units of money, b units of public loans (if D-type), and l units of private loans. In Stage 1, their value function is:

$$W_t^\chi(m, l, b) = \max_{c, m_1^\chi} c + E_t[X_t^\chi(m_1^\chi)] \quad (23)$$

$$\text{s.t.} \quad \phi(m - (1 + i_{t-1})l_{t-1} - (1 + r)b_{t-1}) + T = \phi m_1^\chi + c \quad (24)$$

where m_1^χ is the choice of money holdings brought forth into Stage 2. According to (24), agents must finance their consumption and money holdings with their wealth carried over from the previous period and lump-sum transfers from the government (expressed in CM good) T . The more liabilities that an agent carries over from the previous period, the more work that they have to do in order to repay the loans. Since the probabilities in the previous section was solved for a given realization of e_i , the expectation term in (23) is the unconditional expectation over all values of e . The FOC for this representative agent is:

$$\phi_t = \beta \phi_{t+1} [E_t[X_t^\chi(m_1^\chi)] + 1]$$

We can rearrange the terms in this equation and take the integral form of the expectation to achieve at our money demand function for a D-type agent:

$$\frac{\gamma - \beta}{\beta} = \int_{\underline{e}}^{\bar{e}} X_t^\chi(m_1^\chi, e_i) dF(e_i) \quad (25)$$

	∂r	$\partial \eta$	$\partial \gamma$	∂A	$\partial \lambda$	∂e	∂m_1^x
∂i^D	+	+	+	+	0	+	-
∂i^N	0	+	+	+	0	+	-
$\partial \bar{i}$	+	+	+	+	-	+	-

Table 3: Interest rate comparative statics.

4 Equilibrium

Definition 1. A steady-state equilibrium is defined as a set of idiosyncratic component $\{q, b, m_1^x\}_i$ that fulfills (6), (8), and (25), a set of match-specific quantities and interest rates $\{i, l\}_{i,j}$ that satisfies (9), (12), and (13), and the aggregate variable \bar{i} that satisfies (14) for all t .

One finding from the equilibrium equations is that since the cutoff criteria for e_i is a function of the stage 1 money holdings, the money holdings held by agents affect the tightness in the interbank market, but are not internalized when making the decision, leading to a congestion externality.

On the optimal choice of money holdings, the role of the lending facility as a lender of last resort insures D-type agents that are unmatched in the interbank market. Because of this insurance, D-type agents' choice of money holdings is lower than N-types, i.e., $m_1^D < m_1^N$ since those without access to the lending facility need to self-insure. Because the cutoff criteria are dependent on the choice of money holdings, this also implies that $e_N^* > e_D^*$. An economic interpretation of this is that a larger proportion of banks without access to the discount window will lend on the federal funds market, and the average bank without access will also hold higher reserves. The hypothesis of the model matches the stylized facts from the data.

Table 3 gives the comparative statics for the federal funds rate, with row 1 showing the response of the expected interest rate faced by a D-type agent, row two showing the expected rate of an N-type agent, and row three the response of the federal funds rate. As expected, an increase in the discount rate lowers the outside option of the D-type agent and raises the amount of capturable surplus that a lender can capture from the meeting. This also has the effect of increasing the overall federal funds rate, where the pass-through is determined by the share of D-type agents on the market. An increase in the lender's bargaining power raises interest rates across the board, affecting all agents equally. Higher levels of money growth decrease the optimal money holdings from Stage 1, meaning that the marginal utility from acquiring an extra unit of money in the interbank market is increased, allowing lenders to capture a larger surplus. A higher realization of e_i increases money demand, therefore

allowing lenders to charge a higher interest rate.

The efficiency parameter A of the matching function controls the ease of matching on the interbank market. A low level of A implies that agents have a lower matching probability and must hold more money in Stage 1 to insure themselves. If $A = 0$, then the interbank market is shut down. Therefore, an increase in A implies that money is *more* valuable, which increases the surplus from the trade and the interest rate that a lender can charge. Analogous to reality, a low level of A represents financial uncertainty and prevents banks from participating in the interbank market, while a high level shows financial stability. Since low levels of A also imply that there are more unmatched D-type agents, the aggregate borrowing from the discount window in each period also increases.

For λ , the share of agents with access to discount window lending, an increase in that parameter does not affect the expected interest rate conditional on agent type but lowers the federal funds rate in the aggregate. This speaks to the window's ability as a mechanism for interest rate control, even if the federal funds rate is not at the binding level ($\bar{i} = r$). Allowing more agents to access the discount window allows it to play a more relevant role in interbank transactions. If $\lambda = 0$, then the discount window does not affect the equilibrium of the model, and there would be no distinction between D and N-type agents. If $\lambda = 1$, all interbank transactions would be affected by the discount window, and the Federal Reserve would be able to affect more banks. This also leads to a testable conclusion that the average bank in a district with a higher level of λ (a higher share of banks that can access the discount window) pays a lower federal funds rate.

5 Empirical Analysis

In this section, we test the model-implied correlations between discount window access and variables of interest. From the comparative statics of the model, I expect access to discount window lending to be negatively correlated with reserves held, federal funds volume lent, and the interest rate for borrowed funds. From the transaction-level lending data, I create a panel dataset where banks are the individuals, and time is measured in quarters. From this, we can construct two different measures of discount window access. The first measure assumes that if a bank borrowed from the discount window at any time during the sample period, then they have access to the window for the whole period. The second measure assumes that the bank has access to the discount window from the time where they acquire the first loan. The preferred measure of access is the first method since the second method is more accurate for later periods in the sample and does not weigh all periods equally.

Dependent Variables: Model:	FF Lent (1)	Reserves (2)
<i>Variables</i>		
Assets	2.339*** (0.2956)	1.093*** (0.3290)
Liabilities	-1.708*** (0.2936)	-0.2213 (0.3160)
Reserves	-0.2593*** (0.0383)	
Access	-0.4883*** (0.0782)	-0.0541* (0.0280)
<i>Fixed-effects</i>		
Quarter	Yes	Yes
District	Yes	Yes
<i>Fit statistics</i>		
Observations	87,252	184,094
R ²	0.10426	0.73605
Within R ²	0.07857	0.64774
<i>Clustered (District) standard-errors in parentheses</i>		
<i>Signif. Codes: ***: 0.01, **: 0.05, *: 0.1</i>		

Table 4: Regressions following equation (26). Column (1) regresses total federal funds volume lent (in logs) on discount window access and other control variables. Column (2) regresses log reserves on access.

To first test the correlation between access to discount window lending and federal funds volume and reserves, we can run the two way fixed effects regression:

$$Y_{i,t} = \beta_0 + \beta_1 \mathbf{Access}_i + \gamma' \mathbf{X} + \delta_{D(i)} + \delta_t + \epsilon_{i,t} \quad (26)$$

Where Y is the log value of the variable of interest (Reserves, federal funds volume lent), and X is the vector of controls (Assets, Liabilities, Reserves).⁸ Since we use the first method of constructing discount window access, it is not possible to add individual fixed effects since the variable of interest would be perfectly collinear with the fixed effect. To compensate, I add in fixed effects at the federal district level, and the results are reported in Table 4. The first column is a regression of logged federal funds volume lent on key control variables in addition to discount window access. As expected from the theory, access to the discount window reduces the total volume of federal funds that the bank lends in the interbank market controlling for size. The second column is a regression of reserve holdings of the bank on access to the discount window, again with the predicted correlation. I use the interpretation from Halvorsen and Palmquist (1980) since access to the discount window is a binary variable in the form of $100 * (\exp \beta - 1)$. From this sample, banks who access DW hold 5.2% fewer reserves and lend 38.6% less in the federal funds market after controlling for size (using assets as proxy). The same series of regression with individual fixed effects using the alternative measurement method can be found in Appendix B and does not show significantly different results.

Second, I test the theory that banks that have access to the discount window borrow at a lower rate on the federal funds market. One issue with using the call report is that all federal funds lending and interest gained are self-reported. In addition, banks only have to report the aggregate interest paid/received both in the federal funds rate and the reverse repo market, so the interest rate solely due to the federal funds market is not clear. To resolve this issue, I aggregate the federal funds borrowed by each individual bank up to the federal level, and the aggregate interest paid for each quarter. I then constructed an aggregated average interest paid for each federal reserve district, and exploit the variation in the share of banks that can access the discount window in each district. The second specification I run is:

$$\mathbf{i}_{i,t} = \beta_0 + \beta_1 \mathbf{Access_Share}_{i,t} + \delta_i + \delta_t + \epsilon_{i,t} \quad (27)$$

In which I regress the average interest rate in each quarter on the share of banks that can access the discount window in each district controlling for individual (Federal District) and time fixed effects. Table 5 reports the result of the regression. The correlation is negative, implying that on average, a bank in a district that grants higher access to the discount window borrows on the federal funds market at a lower interest rate.

⁸Ennis and Klee (2021) uses a version of this specification, where they regress discount window borrowing in each quarter on bank characteristics.

Dependent Variable:	FF Borrowed Interest
Model:	(1)
<i>Variables</i>	
DW Share	-0.3033* (0.1636)
<i>Fixed-effects</i>	
District	Yes
Quarter	Yes
<i>Fit statistics</i>	
Observations	540
R ²	0.88235
Within R ²	0.00585
<i>Heteroskedasticity-robust standard-errors in parentheses</i>	
<i>Signif. Codes: ***: 0.01, **: 0.05, *: 0.1</i>	

Table 5: A regression of average federal funds interest rate on the discount window share of each district.

6 Conclusion

This paper provides evidence of the heterogeneity of banks conditional on access to the discount window. In a similar fashion as Ennis and Klee (2021), I construct the dataset using a combination of call reports and transaction-level discount window lending data to find that: (1) The share of banks with access to the discount window is increasing over time, (2) banks without access to the discount window hold a higher portion of their deposit as reserves, (3) a larger proportion of banks without access to the discount window lend on the federal funds market, (4) there has been a sharper decline of banks without access to the discount window than banks that have access, (5) the aggregate assets of banks with access to the window has been increasing since 2010, while the assets of those without access to the window stayed constant.

To explain some of these stylized facts, I develop a search theoretical model *a la* Berentsen et al. (2007) that can shed insights on the mechanism. The main features of the model include an interbank market for liquidity reallocation with search frictions and bilateral bargaining, a share of agents that can access the discount window and those that cannot, and heterogeneous consumption preferences conditional on a consumption shock. In this framework, the discount window insures banks against matching failures in the interbank market and provides an outside option that limits the rents that lender banks can capture. The lender of last resort role implies that agents with access do not need to hold a high

level of reserves to insure themselves, and the outside option role predicts that borrowers with access pay a lower interest rate on the interbank market due to a higher disagreement point.

To test the hypothesis of the model, I run a two-way fixed effects model testing how much access to the discount window affects select variables. I find that banks with access to the window hold 5.2% fewer reserves, lend 38.6% less in the fed funds market, and borrow at a lower rate. This paper contributes to the literature on the discount window by providing stylized facts and uncovering mechanisms behind bank heterogeneity. The natural next step is to explain why some banks choose to request access to the window and some don't by making access an endogenous variable, as well as the interpretation of '*sound*' by each regional Fed.

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Appendix A - Supporting Information

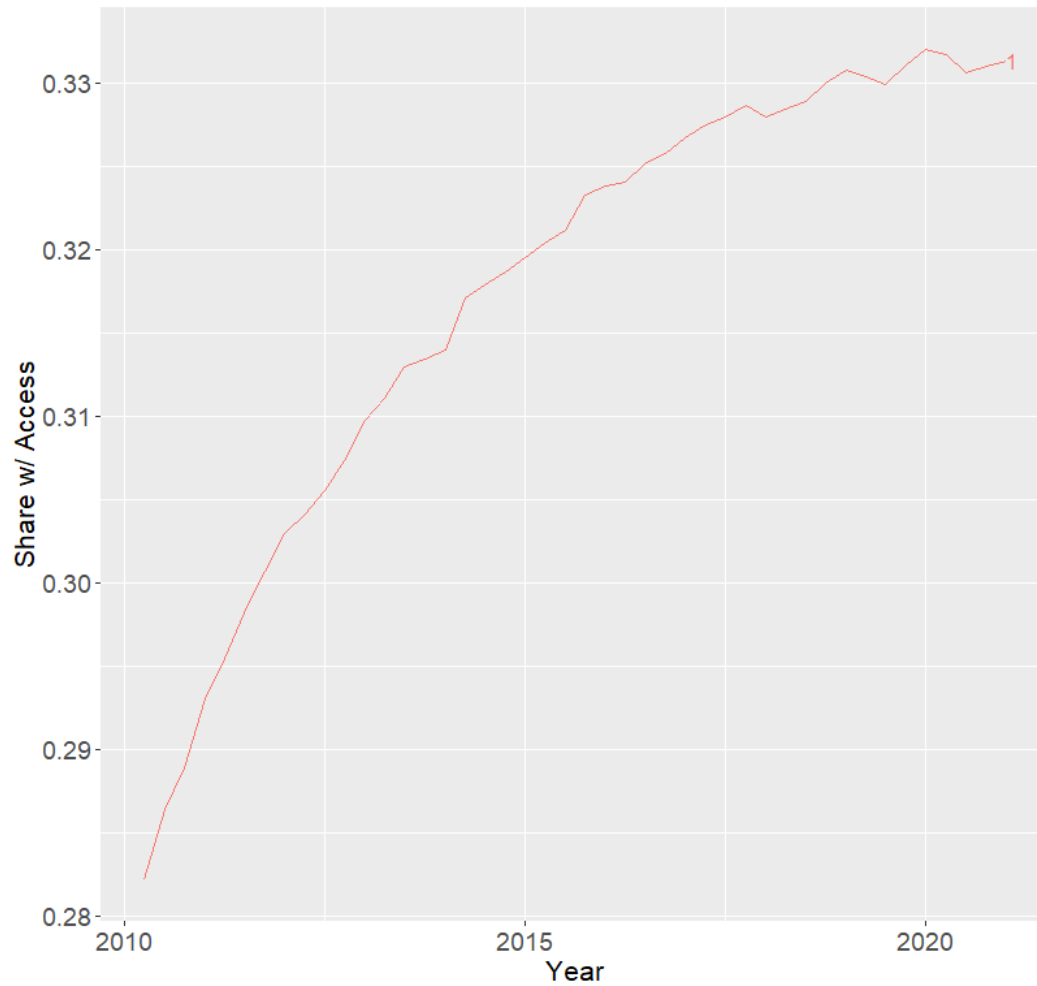


Figure 5: Share of banks that has borrowed from the discount window over time. Data is compiled the same way as in Figure 1.

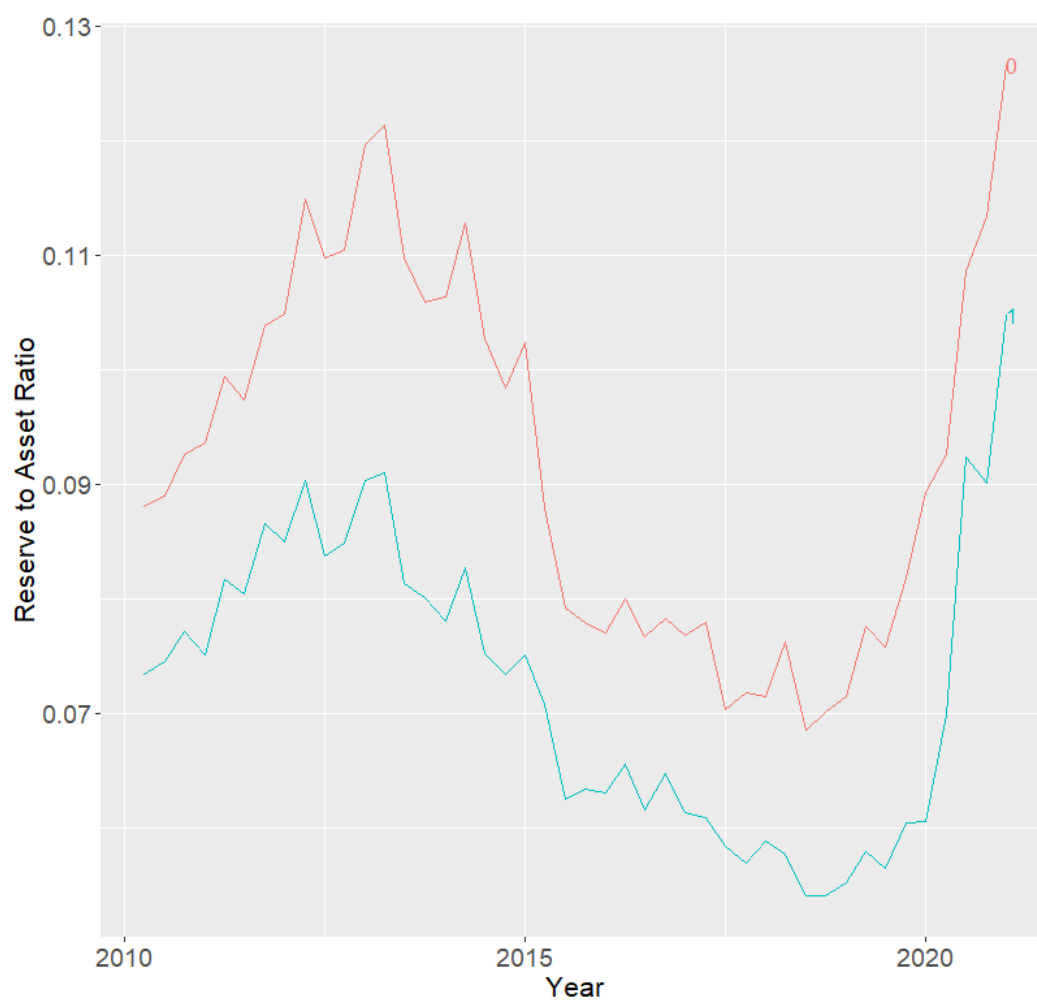
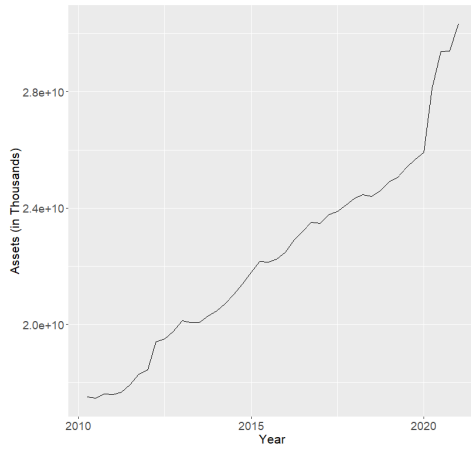
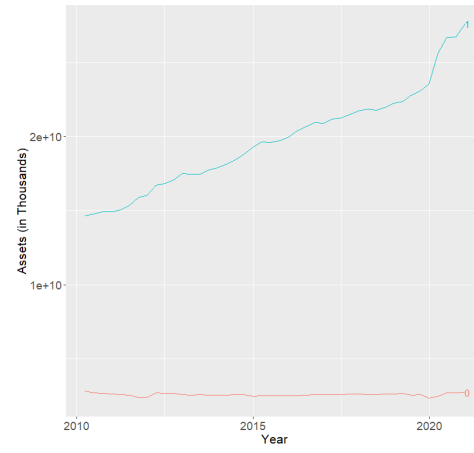


Figure 6: Ratio of bank reserves to asset split between banks with and without access. Banks with access (series 1) are denoted in blue and banks without access (series 0) are denoted in red.



(a) Share of Assets



(b) Share of banks

Figure 7: (a) shows the aggregate asset holding of banks in each quarter (in thousands). (b) splits the aggregated information into banks with and without access to the window. Banks with access (series 1) is in blue, banks without access (series 0) is in red.

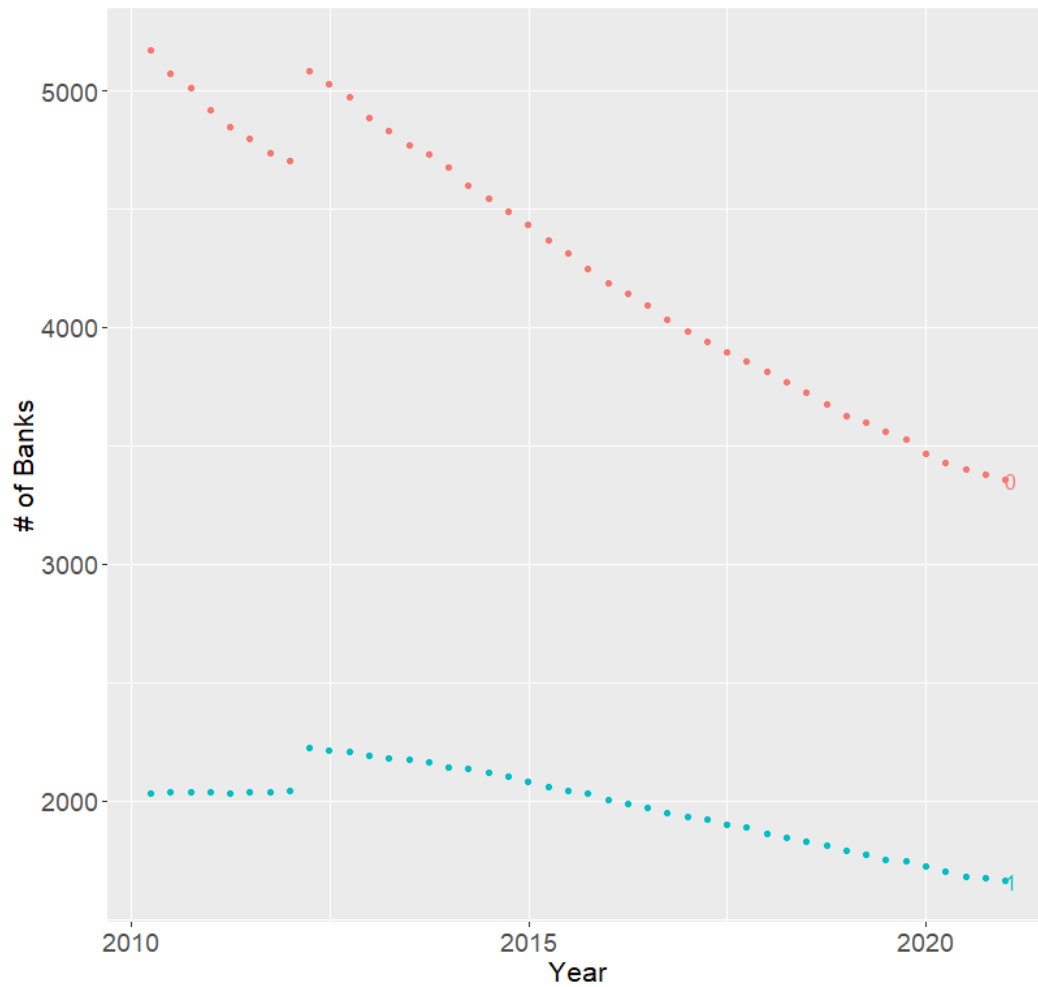


Figure 8: Quantity of banks over the sample. Banks with access (series 1) are denoted in blue and banks without access (series 0) are denoted in red. There is a larger percentage decrease in banks without access to the window. The discontinuity around 2012 comes from changes to reporting requirements.

Appendix B - Alternative Regression

Dependent Variables: Model:	FF Lent		Reserves	
	(1)	(2)	(3)	(4)
<i>Variables</i>				
Assets	2.382*** (0.3508)	-0.0600 (0.3843)	1.174*** (0.2618)	0.8717*** (0.1334)
Liabilities	-1.811*** (0.3371)	0.8182** (0.3609)	-0.2907 (0.2555)	-0.0251 (0.1335)
Reserves	-0.2258*** (0.0356)	-0.2032*** (0.0249)		
Access	-0.6289*** (0.0875)	-0.0927* (0.0493)	-0.1257*** (0.0392)	-0.1083*** (0.0100)
<i>Fixed-effects</i>				
Quarter	Yes	Yes	Yes	Yes
Individual		Yes		Yes
<i>Fit statistics</i>				
Observations	87,252	87,252	184,094	184,094
R ²	0.08302	0.64863	0.72443	0.91934
Within R ²	0.07707	0.01197	0.66768	0.10248
<i>Clustered (District) standard-errors in parentheses</i>				
<i>Signif. Codes: ***: 0.01, **: 0.05, *: 0.1</i>				

Table 6: Columns (1-2) is similar to column (1) from Table 4. The first column only includes time fixed effects, and the second column includes time and individual fixed effects. Columns (3-4) mirrors column (2) from Table 4 with similar specifications. Effects are not significantly different using the alternative measurement method.