The Role of Public Lending as an Outside Option in Private Markets

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Abstract

This paper constructs a New Monetarist model with public and private lending to analyze whether the discount window (DW) operated by the Federal Reserve serves a role aside from their primary purpose of last resort lending and interest rate control, and under which conditions the DW improves aggregate welfare. The theory concludes that the DW reduces the market power of lender banks in the interbank market, and is welfare improving if borrowing relaxes monetary constraints. The main mechanism driving these results is the DW's ability to serve as an outside option for borrowers during the interbank bargaining, and the window's ability to provide external liquidity.

1 Introduction

Banks hold reserves to meet reserve requirements, fund investment opportunities, and act as a means of payment to settle transactions.[1] They can acquire these reserves by holding deposits, borrowing from other banks through the Federal Funds (interbank) market, or borrowing from the central bank. To ensure that banks have sufficient liquidity, the discount window (DW) was opened in 1913 to serve as a lender of last resort and to place a ceiling on interest rates.[2] While its usefulness during crises is unquestionable, issues with the operation of the DW during normal (absent of aggregate shocks) times have been extensively documented.[3] Furthermore, recent policy changes have made the primary roles of the DW obsolete.[4] If the two main purpose of the DW during normal times is obsolete, does the discount window still serve a role, or should the Federal Reserve entertain the possibility of discretionary operation of the public lending facility?

This paper uses a New Monetarist framework surveyed in Lagos et al. (2017) and Rocheteau and Nosal (2017) to explore an externality of the DW that is overlooked in the literature. In this model, agents face idiosyncratic consumption shocks that cannot be paid for by credit due to anonymity and lack of commitment, therefore requiring the use of an immediate medium of exchange. We introduce a monitored lending market for private lending (similar to the interbank market) followed by a public lending facility (DW) operated

[1]For most of recent history, reserve requirements in the US have been 10%. Reserve's role as a means of payment is demonstrated through the daily transaction of around USD 3.3 trillion dollars and involves around 10,000 banks made through the Federal Reserve Wire Network (Fedwire). For a model that has these features, see Bianchi and Bigio (2017).

[2]Any banks that cannot borrow from another bank can borrow from the central bank by posting collateral to meet their liquidity demand. As for its role as an interest rate control mechanism, borrowers would have no incentive to borrow from a private bank if the interest rate offered is above the discount window rate. Therefore, the discount window is often seen as a ceiling on the interbank rate.

[3]Schwartz (1992) shows that banks that do borrow from the window in the 1980s were mainly insolvent and used 'almost daily' to delay bankruptcy, putting the burden of repayment on taxpayers instead of the responsible institution. Ennis and Price (2015) examines a case study where BoNY had to borrow $22.6 billion dollars from the window due to software failure, equivalent to paying a fine of $\tilde{5}$ million dollars, and asks whether intervention was justified given that it might lead to inadequate safeguards taken by banks against failures. Ennis and Klee (2021) finds that banks with access to the discount window hold lower reserves along with riskier asset portfolios than their counterparts after controlling for size and other salient characteristics, implying that the DW promotes moral hazard.

[4]In April of 2021, the Federal Reserve reduced its reserve requirements for banks from 10% to 0%, subsequently eliminating one of the banks' primary reason for holding reserves. Additionally, Figure 8 in the Appendix shows that the interest rate difference between the discount rate and the interbank rate from 2003 to 2020 is positive, therefore the DW rate is non-binding as an interest rate control mechanism.

by the government to be used as a last resort before consumption opportunities. Agents in the model have bargaining power when conducting private trade, and the inclusion of the public lending facility will affect equilibrium allocations even if the public rate is non-binding. We show that there exist three possible monetary regimes aside from the Friedman Rule depending on the parameters.[5] Under expansionary monetary policy (low interest rates), the cost of borrowing from the DW is lower than the cost of holding money, and the DW increases aggregate consumption through the provision of credit. Under contractionary monetary policy (high interest rates), public borrowing is more costly than holding money, agents hold no public debt, and the DW only affects the distribution of surpluses between borrowers and lenders but not aggregate consumption. This result implies that monetary policy might be ineffective for welfare if raised past a critical level, since the change in the DW rate would have no effect on aggregate consumption. Under the third, the cost of holding money and borrowing from the DW is equal, and agents uses both privately held money and publicly borrowed money to pay for the consumption good.

We conclude that the welfare impact of the DW depends on the nominal interest rate, which is determined by the asset portfolio of banks. When banks have access to high return investments, they borrowing money from the DW rather than holding reserves to settle transactions. On the other hand, if banks only invest in low return securities, their cost of holding reserves are low, and they use held reserves to settle transactions instead of borrowing from the DW. This theoretical result confirms empirical findings by Drechsler et al. (2016) and Ennis and Klee (2021) that banks with access to the DW hold lower reserves along with riskier asset portfolios (higher real returns) relative to their counterparts after controlling for size and other salient characteristics. As for the question of interest rate control, access to the discount window affords borrowers a higher outside option, and limits the rent seeking behavior of lenders during private negotiations.[6] Therefore, even if the

[5] Friedman (1969) explains that the optimal rate of deflation is equal to the discount rate to minimize the cost of holding money. Issues with this assumption was shown in Andolfatto (2013), since the assumptions that make money essential can also makes it difficult to enforce taxes. The monetary authority might also not have full control of money growth due to fiscal spending constraints, shown in Sargent and Wallace (1981), Leeper (1991), and Davig and Leeper (2011). One prominent example of this constraint was the 1970s, where fiscal spending caused persistent high inflation.

[6] Choi and Rocheteau (2021) shows this more extensively under a continuous time New Monetarist model

interbank rate is at a level where the constraint is non-binding, a change in the DW rate still affects the equilibrium private rate and controls the distribution of surplus between lender banks and borrower banks in the interbank market.[7] Surprisingly, we find that bargaining power does not affect equilibrium trade quantities, only allocations.

Literature Review

The model builds on Berentsen et al. (2007) and Section 8.5 of Rocheteau and Nosal (2017). In the former, liquidity is reallocated through competitive banks after the consumption shock, and the welfare impact arises from payments on interest for lenders. This model removes the banks and matches lenders and borrowers directly. By removing competitive banks, we can include bargaining power between the lender and borrower to see whether changes to bargaining power distorts equilibrium trade quantities. The inclusion of bargaining power in the model environment also allows us to study how the DW channel of monetary policy affects the private lending rate. In the latter, Nosal and Rocheteau shows that the existence of a lending market is welfare improving by allowing agents with preference shocks access to credit, relaxing the liquidity constraint and increasing trade quantities. This model builds off of their work by including the central bank as a lender of last resort during the lending stage.

We view agents as banks and interpret the market where they interact as the interbank market. In Berentsen and Monnet (2008), agents receive a noisy signal of their consumption shock, and lending is conducted based on how reliable the signal is. When the signal is perfectly accurate, they find that agents can fully adjust their portfolio during the lending

where bargaining power is taken endogenously through the arrival rate of matches for agents through the outside option channel. They find that as meeting speed becomes infinite, there exists a sequence of equilibria along which sellers' market power vanishes.

[7] Historical evidence supports the argument that the central bank has a role in limiting the market power of surplus (lender) banks during both normal and unstable times. In 1907, before the formation of the Federal Reserve, collapse of copper stocks in the US caused depositors to run on Mercantile National Bank, while JP Morgan was unaffected due to its good reputation. To help bail out needy banks, JPM leveraged its market power to stage a takeover of MNB assets at a discount. This crisis paved the way for the establishment of a central bank that was welfare maximizing instead of profit maximizing, along with a discount window that would prevent similar crises and promote competition. Donaldson (1992) also show that private lending rates prior to the establishment of the Federal Reserve are substantially higher than after the formation, which can be seen as evidence for market power of surplus banks. See Acharya et al. (2012) Appendix B for more instances.

stage and do not need access to central bank lending. We find that banks may still find it optimal to access the public lending facility even with perfect signalling if the rates that agents can borrow from the CB are lower private rates. The result for central bank lending resembles Acharya et al. (2012) in the sense that government lending can reduce market power of surplus banks in the interbank market, but their finite horizon environment lacks the tools to examine general equilibrium effects and the interplay between government lending and money growth.

Recent works that examines the effects of monetary policy implementation on interbank lending are Rocheteau et al. (2018) and Bianchi and Bigio (2017). Rocheteau et al. (2018) uses the New Monetarist framework that includes reserve and capital requirements to examine the pass-through from monetary policy to entrepreneurs through the interbank market. They find that reserve requirements increase the potency of monetary policy and that an Open Market Purchase by the Central Bank reduces cost of borrowing reserves and incentivize banks to extend loans. Bianchi and Bigio (2017) develops a model with the interbank market from Afonso and Lagos (2015) to examine how monetary policy affects banks by altering the trade-off between profiting from lending and incurring greater liquidity risk. We present a simplified version of the interbank market that still captures the most salient effect of standing facilities; the fact that it can relax liquidity constraints and influence private agreements.

The discount window lending literature is surveyed in Ennis (2017). In this model, the welfare improvement mechanism of the DW comes from its ability to provide liquidity at a lower interest rate than money.

2 The Environment

Time is discrete indexed by $t \in \mathbb{N}$. The economy is populated by a unit measure of agents. Each period is divided into 3 Stages (markets). The first stage is a frictionless centralized

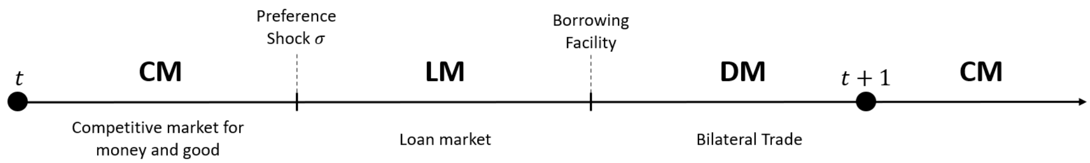

Figure 1: Events in period t

market (CM) and the third stage is a frictional decentralized market (DM). Between these two stages, we introduce an OTC lending market (LM) where agents can readjust their portfolio.[8] Figure 1 shows a graphical representation of the events in period t. There are two perishable goods: a good c produced in the CM and taken as the numéraire, and a good q produced in the DM.

The agent's lifetime utility is:

$$\mathbb{U} = \sum_{t=0}^{\infty} \beta^t [c_t - h_t + u(q_t^b) - q_t^s] \tag{1}$$

where β is the discount factor. All agents have linear utility over c. At the beginning of Stage 2, agents get a preference shock such that they can consume or produce in Stage 3 with probability σ; we refer to these consumers as buyers and producers as sellers. A consumption of q gives the buyer utility $u(q^b)$, and a production of q incurs a cost q^s for the seller. Utility $u(q)$ is strictly concave, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. This paper defines q^* as the level of the specialized good that satisfies $u'(q^*) = 1$.

In Stage 3 trades, agents are anonymous so that trading partners cannot identify the counter-party. Trading histories are private information, therefore credit arrangements are not incentive feasible. There exists a central bank that controls the supply of money. The money stock evolves by $M_{t+1} = \gamma M_t$, where M_t denotes the money stock at time t, and $\gamma > 0$ denotes the gross growth rate of money. The central bank also operates a standing

[8]The OTC structure of the Federal Funds market that we adopt has been empirically highlighted in Ashcraft and Duffie (2007), where they find that in the aggregate, approximately 73% of all loans made through the federal funds market were traded bilaterally.

facility where agents can borrow money before Stage 3 and after Stage 2. An agent who borrows b units of money from the central bank at time t repays $(1 + i^b)b$ units of money in Stage 1 of the following period. In this baseline, we assume that there is a costless enforcement technology operated by the central bank that rules out default. The budget constraint of the government is therefore: $T_t - i^b B_t = (\gamma - 1)M_t$, where T_t is lump-sum transfers given to agents in Stage 1 at period t, and $i_t^b B_t$ is the aggregate interest payment that agents make to the central bank from borrowing.

We model credit as personal liabilities issued by borrowers to lenders that can be redeemed in the subsequent CM. While the implications to allocations are similar for selling an asset (such as a bond), For this process to function, there exists a costless technology only available in Stage 2 that allows record keeping of financial histories in Stage 2.

In the interbank market, random matches are formed bilaterally following a Leontief matching function; more specifically, if a mass b of borrowers and s lenders are searching, then $m(b, s) = \min\{b, s\}$. In these pairwise meetings, loan size and repayment amount (l, x) are determined according to the proportional solution of Kalai (1977), where the share of surplus received by lenders is $\theta \in [0, 1]$.[9] We define $i = x/l$ as the nominal interest on the loan.

3 Equilibrium

Let ϕ_t be the price of money in the CM in period t. This section characterizes a stationary equilibra where aggregate real balances and allocations are constant; i.e., $\phi_t M_t = \phi_{t+1} M_{t+1}$. Under the assumption that γ is constant, $\phi_t/\phi_{t+1} = M_{t+1}/M_t = \gamma$.

Let $W(\omega)$ denote the expected value from entering Stage 1 with total wealth ω expressed

[9] The Nash (1950) solution has been shown to be problematic when liquidity constraints are binding, see Hu and Rocheteau (2020). In summary, the Nash solution replicates the case where the output is negotiated all at once in an Rubinstein alternating-offer game, while the proportional solution replicates the case where there is an infinite number of negotiations over infinitesimally small bundles. They furthermore show that the liquidity constraint for buyers bind for any $N < +\infty$.

in the numeraire. Let $X^b(m)$ denote the value of entering Stage 2 as a buyer holding m units of money, $X^l(m)$ when entering Stage 2 as a lender, and $V(m, l, b)$ the expected value of entering Stage 3 with m units of money, l units of privately borrowed money, and b units of publicly borrowed money. We examine the individual decision problems at each sub-period in t, then solve the equilibria.

3.1 Stage 1 - Central Market

Consider an agent who holds ω units of wealth. In Stage 1, their value function is:

$$W(\omega) = \max_{c,h,m'} \quad \left\{-h + c + \sigma[X^b(m') + X^l(m')] + (1-2\sigma)\beta W_{t+1}(m')\right\} \tag{2}$$

$$\text{s.t.} \quad h + \omega + T = \phi m' + c \tag{3}$$

where m' is the choice of money holdings brought forth into Stage 2. According to (3), agents must finance their consumption, c, and money holdings, m', with their current wealth, ω, production income, h, and lump-sum transfers from the government (expressed in CM good) T. From the Leontief matching function, the measure of borrowers and lenders are equal, and the remaining unmatched agents moved onto the next period's CM. Agents hit with the consumption shock always want to borrow, therefore the probability that agents become a borrower or lender in the LM is σ.

Rewriting the budget constraint and substituting (3) into (2) yields:

$$W(\omega) = \omega + T$$
$$\max_{m'} \quad \left\{-\phi m' + \sigma X^b(m') + \sigma X^l(m') + (1-2\sigma)\beta W_{t+1}(m')\right\} \tag{4}$$

Due to the linearity of W_{t+1} with respect to m, $W_{t+1}(m') = \phi_{t+1}m' + W_{t+1}(0)$. The first-order condition is given by:

$$\phi_t = \sigma[X^{b\prime}(m') + X^{l\prime}(m')] + (1-2\sigma)\beta\phi_{t+1} \tag{5}$$

The left side of (5) is the marginal cost of holding an extra unit of money, and the right side is the expected marginal benefit from acquiring one extra unit of money. Note that the optimal choice of m' is independent of past history and independent of ω.

3.2 Stage 3 - Decentralized Market

In Stage 3, the terms of trade is determined in a bilateral match between a buyer with m units of money, l units of private loans, and b units of public loans. In these meetings, buyers make a take-it-or-leave-it (TIOLI) offer to the seller, which determines the quantity and payment (q, d). Because of linearity in wealth, the wealth of the seller is inconsequential to the trade because their marginal utility from consumption of the CM good is constant and independent of their wealth. The buyer and seller's individual value functions entering Stage 3 are given by:

$$V^b(m, l^b, b^b) = u[q(m, l^b, b^b)] + \beta W_{t+1}(\omega^b) \tag{6}$$

$$V^s(m, l^s, b^s) = -q + \beta W_{t+1}(\omega^s) \tag{7}$$

The buyer's wealth consists of their money holdings from the CM, as well as any private loan, l, and public debt, b, incurred in Stage 2. The buyer gains $u(q)$ from the consumption of the DM good and the seller incurs a cost $-q$ from production. Under TIOLI, the buyer makes an offer (q, d) that maximizes their consumption utility subject to the seller's participation constraint. Taking into account that one unit of money can be redeemed for ϕ_{t+1} units of CM good in the subsequent period, $\partial\omega/\partial m = \phi_{t+1}$, therefore $W_{t+1}(\omega) = \omega + W_{t+1}(0)$. Using the linearity of W_{t+1}, the offer solves:

$$\max_{q,d} \quad \left[u(q) - \beta\phi_{t+1}d\right] \text{ s.t. } q \leq \beta\phi_{t+1}d \tag{8}$$

$$d \leq m + l + b \tag{9}$$

Where (8) is the offer subject to the seller's participation constraint and (9) is a feasibility constraint that says buyers cannot offer to transact more than their total money holdings; either held from the CM or borrowed from the LM. From (9), we can note that sellers are indifferent between the payment instruments, since their prices are equal in the following

CM. Taking into account that the feasibility constraint of the seller holds at equality from the bargaining formulation, the solution to (8)-(9) is:

$$q = \begin{cases} q^* & \text{if} \quad \beta\phi_{t+1}d \geq q^* \\ \beta\phi_{t+1}d & \text{if} \quad \beta\phi_{t+1}d < q^* \end{cases} \tag{10}$$

$$d = \frac{q}{\beta\phi_{t+1}} \tag{11}$$

The buyer obtains the socially efficient level of trade if they bring enough money to compensate the seller for a production of q^*, otherwise the buyer is liquidity constrained. Given the bargaining solution, the seller receives no surplus in the DM, and the buyer receives a surplus of $\psi(\omega) \equiv u[q(\omega)] - q(\omega)$. Since this is the case, the value of being a seller in the DM is the same as non-participation; i.e, $V_t^s = \beta W_{t+1}$. Note that m, b, and l are interchangeable, since b and l are personal liabilities taken to obtain the medium of exchange, and the only factor that differentiate them is the repayment cost.

3.3 Stage 2 - Lending Market

Public Borrowing Decision: Consider an agent that has already conducted private trades. Subsequently, agents have an option to borrow b directly from the public lending facility at a posted interest rate i^b. Since non-buyers have no incentive to borrow, their public borrowing will be zero, and only buyers will need to borrow from the lending facility. We suppress arguments for ω when it is equal to zero. Taking Stage 2 loan size as given, the value function for a borrower in this scenario is given by:

$$\hat{X}^b(m,l) = \max_{b \geq 0} \quad \psi(m,l,b) + \beta W_{t+1}(\omega) \tag{12}$$

The restriction on b comes from the fact that agents can only borrow form the lending facility and cannot make deposits. Using the linearity W_{t+1} and removing terms orthogonal to b, the choice b maximizes:

$$b^* = \underset{b \geq 0}{\operatorname{argmax}} \quad \psi(m,l,b) - \beta\phi_{t+1}[b - (1+i^b)b] \tag{13}$$

Therefore, taking the derivative of (13) with respect to b and (11) with respect to q, the optimal value of b for the agent solves:

$$b = \begin{cases} u'[q(\omega)] - 1 = i^b & \text{if} \quad u'[q(\omega)] - 1 \geq i^b \\ 0 & \text{if} \quad u'[q(\omega)] - 1 < i^b \end{cases} \tag{14}$$

The borrower borrows from the public facility until the point where their marginal benefit equals their marginal cost. The marginal benefit is given by the liquidity premium of Stage 3 transactions, defined as $u'(q) - 1$, and the marginal cost is the repayment interest. While public borrowing relaxes the liquidity constraint in Stage 3 trades by increasing d, it also reduces total wealth ω due to the repayment of interest; therefore, $\partial\omega/\partial b = -\phi_{t+1} i^b$. If private lending can fully satisfy the optimal trade quantity, then it is not necessary for agents to borrow from the discount window and $b = 0$; otherwise buyers still have excess liquidity demand that is not satisfied by private lending and $b > 0$.

Private Borrowing Decision: After the realization of the consumption shock in the beginning of Stage 2, agents can bilaterally negotiate over a loan contract (l, x) depending on liquidity need. Since only buyers have a need for liquidity in Stage 3, they are the only ones who will borrow a positive amount in equilibrium. The value functions of buyers and lenders is given by:

$$X^b(m) = \max_{l,x} \quad \psi(m, l) + \beta \hat{X}^b(m, l) \tag{15}$$

$$X^l(m) = \max_{l,x} \quad \beta W_{t+1}(m, -l) \tag{16}$$

Under the Kalai (1977) bargaining solution, the disagreement point for the borrower is their best outside option, in this case, taking out a public loan of size $b^r = \min\{l, b^*\}$, where b^* fulfills (14). For the lender, the disagreement point is the cost of holding money into the next period's CM. Therefore, the loan contract (l, x) solves:

$$\max_{l,x} \quad S^b \equiv \Big[\psi(m,l) + \beta W_{t+1}(m,l)\Big] - \Big[\psi(m,b^r) + \beta W_{t+1}(m,b^r)\Big] \tag{17}$$

$$\text{s.t.} \quad \theta S^b = (1-\theta)\beta\Big[W_{t+1}(m,-l) - W_{t+1}(m)\Big] \tag{18}$$

$$l \leq m \tag{19}$$

The bargaining solution maximizes the excess surplus of the borrower from taking out a private loan net of the disagreement point, taking into account that the share of the excess surplus θ must go to the lender through transfer x. It is straightforward to show that for $x > 0$, the seller finds it optimal to lend all their money. From the the lending contract, borrowing l units of money privately relaxes the liquidity constraint in Stage 3 trade, but decreases total wealth of the borrower due to repayment x. Constraint (18) represent the share of the excess surplus that goes to each agent and constraint (19) is the maximal lending constraint for the lenders. Define $\Delta\psi \equiv \psi(m,l) - \psi(m,b^r)$ as the difference in surplus between borrowing privately and publicly. Using the linearity of W_{t+1}, (17) and (18) can be reduced to:

$$\max_{l,x} \quad \Delta\psi + \beta\phi_{t+1}[(1+i^b)b^r - (l+x)] \tag{20}$$

$$\text{s.t.} \quad \theta\Big[\Delta\psi + \beta\phi_{t+1}[(1+i^b)b^r - (l+x)]\Big] = (1-\theta)\beta\phi_{t+1}x \tag{21}$$

If constraint (19) is slack, then the loan size is less than the total money holdings that agents bring into the Stage 2 market, and agents can face a smaller loss in utility due to the inflation tax by bringing a marginal unit less. Therefore, in the equilibrium, (19) holds at equality since $l < m$ is not individually rational. The solution to the bargaining game is:

$$x = \frac{\theta\Big[\Delta\psi + \beta\phi_{t+1}[(1+i^b)b^r - l]\Big]}{\beta\phi_{t+1}} \tag{22}$$

$$l = m \tag{23}$$

Figure 2 shows the corridor created by outside option and liquidity constraint. The blue area represents the share of *capturable* surplus captured by lenders, and the green area is the

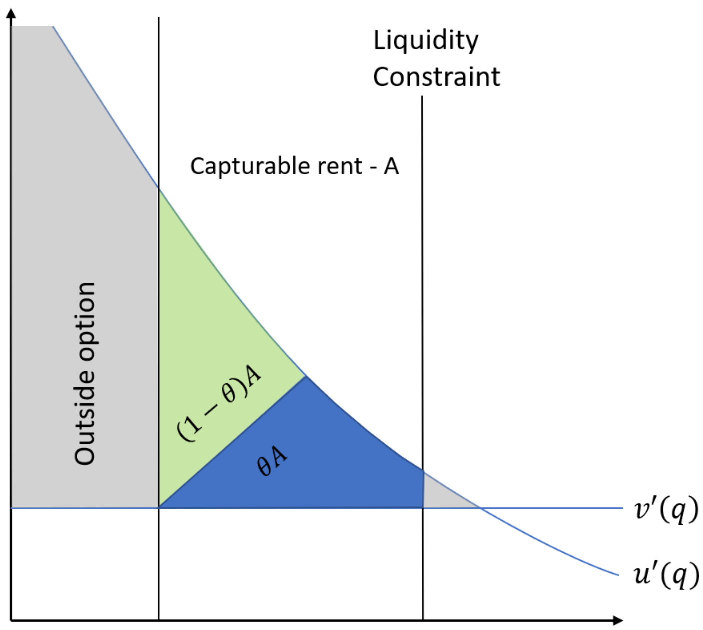

Figure 2: θ controls the size of the rent that can be captured by the lender.

share of *capturable* surplus to borrowers. The public interest rate i^b determines the outside option of borrowers, and increasing i^b shifts the vertical line separating the outside option to the left since it lowers the outside option of the borrower. The vertical line representing the liquidity constraint is captured by the money growth rate γ, and an increase in γ shifts the liquidity constraint leftwards. Increasing the bargaining power of the lender θ increases the amount of *capturable* surplus they are entitled to.

The size of transfers that borrowers give to lenders depend on the lender's bargaining power, and the **total** surplus that can be gained in the Stage 3 trade net of what is guaranteed to the borrower by the outside option. Define $i^l = x/l$ as the interest of the loan paid to lenders (ie: the interbank rate); the following comparative statics table shows how the optimal interbank rate responds to a change in the exogenous variables:

	$\partial\theta$	∂i^b	$\partial\gamma$	$\partial\beta$
∂i^l	+	+	+	-

The rate that lenders can charge borrowers increase with their bargaining power. As the central bank makes it more difficult to acquire liquidity through public means by increasing i^b, lenders have more leeway to capture any trade surplus encountered by borrowers in Stage 3 since it lowers the outside option of the borrower. When money growth is high, the cost of holding money increases, the liquidity premium increases due to the lower money holdings, and lending is more valuable. As agents become more patient, the cost of holding money decreases, the liquidity premium decreases since q is closer to q^*, and lending becomes less valued. Given the solution to the bargaining game, we can rewrite 15 and 16 as:

$$X^b(m) = \psi(m, l, b) - \beta\phi_{t+1}(x + i^b b) + \beta W_{t+1}(m) \tag{24}$$

$$X^l(m) = \beta\phi_{t+1}x + \beta W_{t+1}(m) \tag{25}$$

The borrower gains the trade surplus in the Stage 3 using their whole portfolio net of the future period repayment. The lender, holding m units of money can lend it out for a repayment size of x. The transfer amount, x shows up in both the value function of the borrower and lender. Since an agent ex-ante has equal probabilities to be borrower or lender, in expectation, the transfer size cancels out and money holdings is independent of transfers, implying that money holdings is also independent of bargaining power. This result should be robust to specifications where matches are formed bilaterally.

3.4 Equilibrium Types

Substituting the FOC of 24 and 25 into (5) gives the following money demand:

$$\frac{\gamma - \beta}{2\sigma\beta} = u'[q(m, l, b)] - 1 \tag{26}$$

Definition 1. *An equilibrium is a tuple* $\{m, x, l, b, q\}$ *that satisfies money demand* (26)*, transfers* (22)*, private loan demand* (23)*, public loan demand* (14)*, and DM trade quantity* (10)*.*

The left side of (26) represents the expected cost of holding one extra unit of money, and the right side represents the liquidity premium of the extra unit. The intuition follows that of Berentsen et al. (2007), in which lending reduces the holding cost of money by allowing more agents to extract surplus from the DM trade through monetary transfers x. From (14) and (26), either the cost of borrowing from the public facility is lower than the cost of holding money, in which case $b > 0$ and $m = l = 0$; or the cost of holding money is lower, in which case $b = 0$ and $l = m > 0$. Given the parameter space, there are three types of symmetric steady state equilibria depending on the values of i^b and γ:

Equilibrium Type	Condition	Holdings
Friedman Rule	$\gamma = \beta$	$m = m(q^*)$, $l = b = 0$
Credit	$i^b < i^{b*} \equiv \frac{\gamma-\beta}{2\sigma\beta}$	$l = m = 0$, $b > 0$
Money	$i^b \geq i^{b*}$	$l = m > 0$, $b = 0$

For the intuition of the critical value, take the real rate of return to be equal to the rate of time preference, and use the definition of the Fisher equation $1 + i^n = \gamma(1 + r)$. Substituting this into the left side of (26) gives $1 + \frac{i^n}{2\sigma}$. A graphical representation of the set of equilibria is shown in Figure 3.

The space where $\gamma < \beta$ is infeasible since agents would have positive returns on money and choose to hold an infinite amount. Intuitively, all agents weigh the cost of holding money against what they can gain from trade. Since the price of borrowing and holding money are both fixed cost, agents will choose to only hold the asset that as the highest rate of return.[10] When money growth γ is high, holding fiat money is less valuable, and agents will find it optimal to borrow publicly from the window. In this type of equilibria, agents only work in the CM to repay borrowed funds from the lending facility and hold no cash in their portfolio when exiting the CM. This implies that private lending is shut down and the Stage 2 market becomes obsolete.

When inflation is low, holding money is less costly than borrowing, and the lending

[10]See Williamson and Wright (2010) for a survey on the New Monetarist literature with two competing assets.

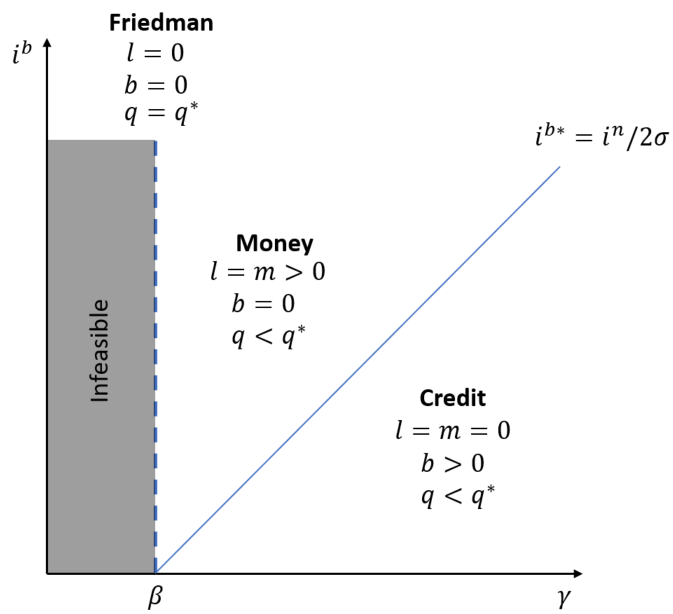

Figure 3: The possible equilibria on (γ, i^b) plane.

facility becomes unused. If meeting probabilities σ are high, the marginal value of holding money increases, and agents do not need to borrow publicly. In terms of nominal interest rates, the nominal interest on money represents how costly it is to hold money instead of an alternative asset. Since the consumption opportunity of agents are determined by their meeting probability, agents are only willing to hold money if the equilibrium interest rate on money is lower than credit, with the risk wedge (defined as $\frac{i^n}{i^b}$) equal to 2σ. When borrowing from the discount window agents are willing to accept a higher premium since they are facing the consumption shock with certainty.

Under the money equilibrium, the Stage 2 lending contract is nonzero. There are two cases for the determination of the transfer depending on the reservation value b^r. From (14), if $y = m$ satisfy the second case, then $b^r = 0$ and the outside option is just money holdings from Stage 1. Because the outside option is independent of interest rates, a change in i^b has no effect on transfers x. Figure 4 shows this phenomenon; in reality, the required borrowing rate is too high to ever make this feasible.

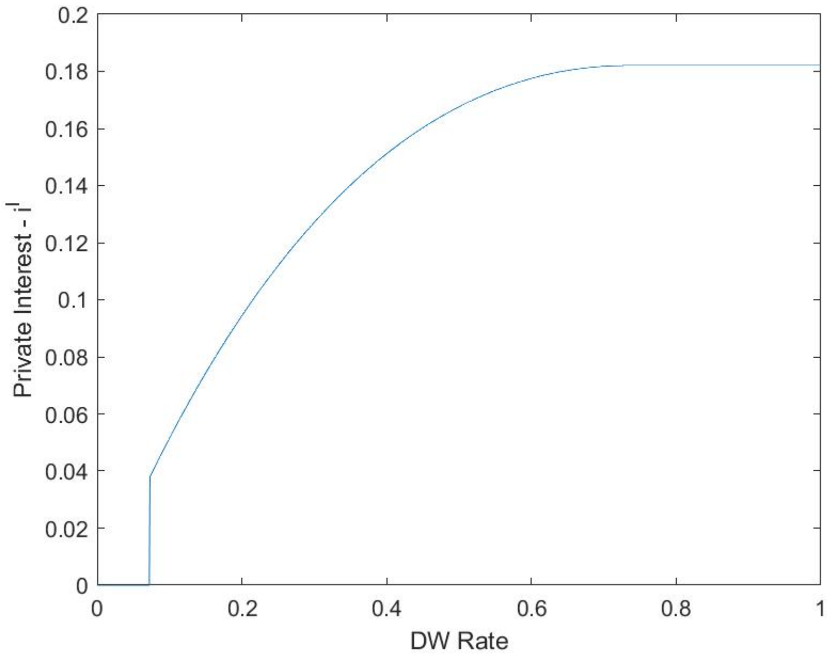

Figure 4: Private lending rate as a function of public rate.

Proposition 3.1. *There exists a critical level $\tilde{i^b} > i^{b*}$ such that if $i^b \in (i^{b*}, \tilde{i^b}]$, then the transfer amount $x = x(i^b)$. If $i^b > \tilde{i^b}$, then private lending is independent of the public lending rate.*

4 Exercises

In this section, we compare the effect of monetary policy through the discount window channel under low and high inflation regimes. To calibrate, we look at the period from July 2010-December 2017, since that is the most recent instance of a 'normal' time and to also match the findings of Ennis and Klee (2021). We assume that $c(q) = q$, and $u(q) = q^{\alpha}$. All calibrated values can be seen from Table 1. We let the discount factor $\beta = 1/(1 + r)$, where r is the real interest rate on the 1-year T-bill. σ is calibrated to the share of large domestic banks that have borrowed from the discount window at least five times within the period.

	Parameter	Value	Target
Coefficient on q	α	.6	Fixed
Discount factor	β	.99	1Y T-Bill
Matching probability	σ	.21	Ennis and Klee (2021)
Lender market power	θ	.52	EFFR
Inflation	γ	1.02	Fed target inflation
Discount window rate	i^b	.02	Variable

Table 1: Parameter Values

4.1 ***Effect of Monetary Policy***

Figures 5 and 6 show the response of key variables when the DW rate varies from 0% to 20% under low and high inflation following the calibrated parameters. In the first figure, the welfare, measured by $\mathbb{W} = \sigma(u(q) - q)$ is plotted in blue, along with the share of the surplus that goes to the lender x in orange. We can see that there is a jump at the critical value i^{b*}, which corresponds to the dual asset equilibrium, and the correspondence of money holdings at the critical value can be seen on the third sub-figure. Values to the left of i^{b*} represents the credit equilibrium and values to the right represents the money equilibrium.

In the credit economy, as the central bank rate increases, the optimal borrowing decreases to match the liquidity premium, with a critical value of $i^{b*} = 7.2\%$ for the low inflation regime, and $i^{b*} = 16.8\%$ for the high inflation regime. Under the low inflation regime, welfare is downward sloping up until the critical value, implying that the discount window is welfare improving (since removal of the DW would put us in the money equilibrium, lowering $\mathbb{W}$), then lies flat when agents do not resort to public borrowing. As we move into the monetary equilibrium, the discount rate only plays a role of outside options for the private bargaining, increasing the surplus of lenders as the value of borrowing decreases. Removal of the outside option would be similar to taking the limit as $i^b \to \infty$, and the transfer x would approach $\theta(u(q) - q)$. Since $\partial x / \partial \theta$ is positive while lending size stays the same, we can conclude that the public rate is a factor in private agreements even if private rates are not at the upper bound.

The last two plots show the interest rate computed by $i = x/l$, as well as the private

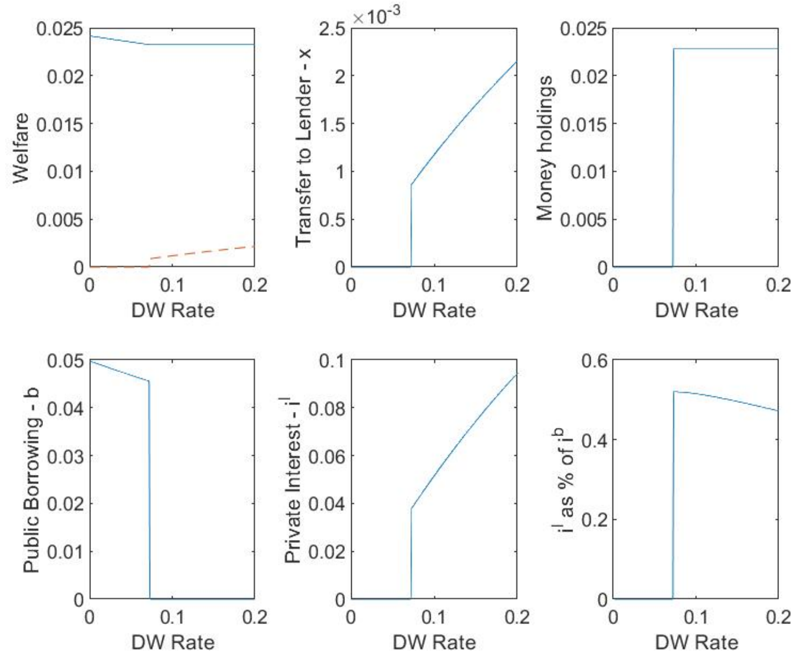

Figure 5: Low inflation - $\gamma = 1.02$

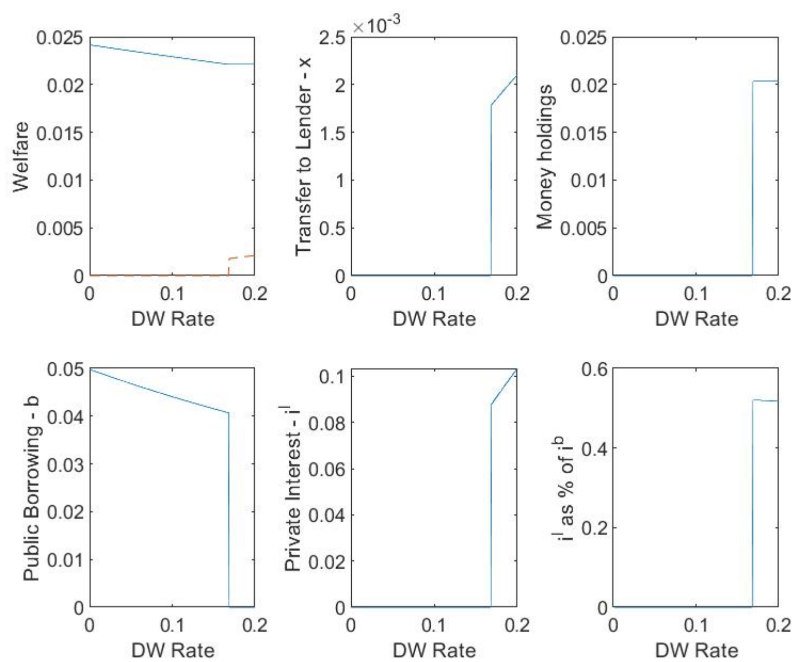

Figure 6: High inflation - $\gamma = 1.06$

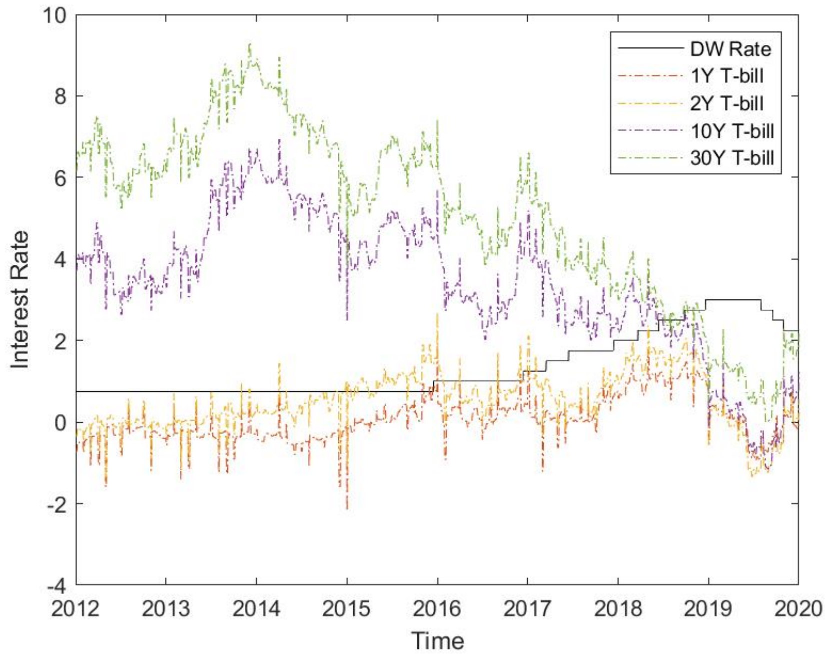

Figure 7: Critical and actual DW value.

interest rate as a percentage of the discount window rate. We can observe that $i/i^b \leq \theta$, where the equality is at the critical value of i^{b*}. From this, we can see that the private interest rate does not need to reach the corridor ceiling i^b, but rather depends on the market power of lenders and borrowers, and can also explain the wedge between the FFR and the DW rate.

Proposition 4.1. *If $i^b < \frac{\gamma-\beta}{2\sigma\beta}$, public lending improves aggregate welfare.*

When public rates are low, borrowing from the lending facility is cheaper than holding onto cash due to the inflation tax. This expands the agent's total money holdings brought into Stage 3, and quantity traded is higher than in the money equilibrium. Can this explanation be seen empirically? Since the discount window for the US is always utilized, we expect that $i^b < i^n/2\sigma$ generally holds true. Figure 7 plots both the discount window rate and the nominal interest rate using different treasury securities for a period of 2012-2020. If the critical value is above the DW rate, then the DW is welfare improving and vice versa.

From the theory, we find that for this period, access to the discount window is welfare improving if banks hold long term securities (higher nominal interest rate), and does not affect consumption if banks hold short term securities (lower nominal interest rate). When the central bank enacts contractionary monetary policy and raise the DW rate, then access to public credit does not increase aggregate consumption since the cost of borrowing is higher. When the central bank enacts expansionary monetary policy by lowering the DW rate, then the gains to the trade surplus depends on the asset portfolio of banks.

This theoretical result supports empirical findings by Drechsler et al. (2016), who looks at a panel of countries from 2007 to 2011, and Ennis and Klee (2021), who looks at US banks from 2012-2017. They find that controlling for salient characteristics between banks and across-country variations, banks who access the discount window hold a riskier (higher return) asset portfolio and lower reserves (lower m), which is a relationship that we also find in the model. From a finance perspective, access to a standing facility can relax liquidity constraints and affect the bank's maximization problem, which allows them to put more weight on their risky asset because the public funds can be treated as a risk-less asset.

5 Conclusion

This paper characterizes the equilibrium in a Lagos-Wright economy where there exists one type of agent who can participate in public and private lending. The model finds that: (1) the DW improves aggregate consumption by relaxing the liquidity constraint if the lending rate is sufficiently low, (2) the choice to borrow from the DW is dependent on the asset portfolio of the bank, (3) bargaining power between borrower and lender banks only affect surplus allocations and does not affect trade quantities, (4) access to the DW ensures that buyers capture a larger share of the trade surplus by providing an outside option.

This paper contributes to the literature on monetary policy mechanisms, and shows that public lending can be welfare improving even in the absence of aggregate shocks by providing credit, which acts to relax the liquidity constraints faced by banks. This externality offered by the discount window should be considered for future policy debate.

Further directions that could be explored includes: introducing two types of agents so that bargaining power also affects trade quantities, as well as heterogeneous access to public lending. Switching the matching technology in Stage 2 with a continuous time matching problem akin to Afonso and Lagos (2015) or adding a stochastic demand coefficient to the Stage 3 good would also generate a distribution of trade sizes even with degenerate distribution of money holdings. By including these additions, the federal funds market could be fully stylized.

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Appendix

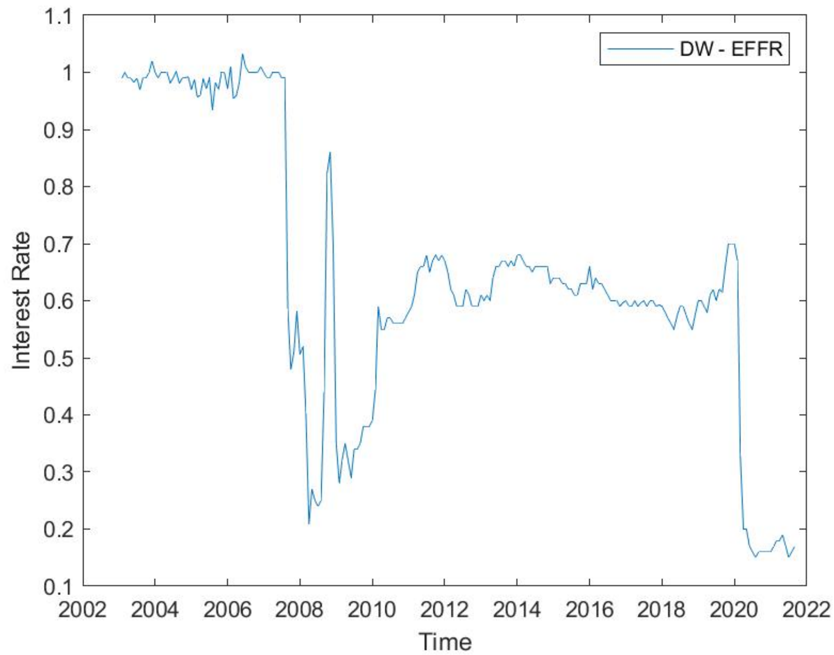

Figure 8: Difference in percentage points between DW rate and fed funds rate (FRED).