

Derivation of HQIV from Maxwell's Equations and Schuller's Geometric Framework

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1 Maxwell's Equations with Full **H** Field Formulation

The electromagnetic field in material media is described by the four field quantities **E** (electric field), **D** (electric displacement), **B** (magnetic induction), and **H** (magnetic field intensity). The full macroscopic Maxwell equations in differential form are [Maxwell, 1873, Jackson, 1999]:

$$\nabla \cdot \mathbf{D} = \rho_f \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \quad (4)$$

where ρ_f is the free charge density and \mathbf{J}_f is the free current density.

The constitutive relations connecting the fields in linear, isotropic media are:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E} \quad (5)$$

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} = \frac{1}{\mu} \mathbf{B} \quad (6)$$

where **P** is the polarization density, **M** is the magnetization, ϵ is the permittivity, and μ is the permeability.

The magnetic field intensity **H** is essential for describing magnetic phenomena in materials, as it accounts for the distinction between bound currents (from magnetization **M**) and free currents. The original formulation

by Maxwell [1873] developed these equations in quaternionic form, with the modern vector notation established later by Heaviside and Gibbs.

For a geometric formulation of field theories on curved spacetime backgrounds, see the lecture series by Schuller [2020], which provides a rigorous treatment of differential geometry and its application to electromagnetic and gravitational fields.

$$S_{\text{EM}}[A, G] = \int \omega_G G^{abcd} F_{ab} F_{cd} d^4x.$$

The Euler–Lagrange equations are second-order. In Fourier space (principal symbol, covector ξ), the highest-order term yields the principal polynomial $P^{abcd}(\xi)$ (cubic in G , totally symmetric):

$$P^{abcd} = \omega_G^2 \epsilon^{mnpq} \epsilon^{rstu} G_{mn(r} G_{s|ps|tu)c} G^{d)qtu} \quad (\text{exact form from variation}).$$

Hyperbolicity condition (Schuller): For every $x \in M$ and every covector $\xi \neq 0$, there exists a Lorentzian conformal class such that $\tilde{P}(\xi) = 0$ admits a real hyperbolic structure with energy-distinguishing cones. This forces existence of a metric $g_{\mu\nu}$ (up to conformal factor) such that the principal symbol reduces to $P^{ab}(\xi) \propto g^{ab} \xi_a \xi_b = 0$ (light-cone condition).

All causal horizons are thereby defined by the light cones of this emergent Lorentzian structure. For a timelike congruence of fundamental observers u^μ (normalized $u^\mu u_\mu = -1$), the local causal-horizon radius is the proper distance along the past light cone to the nearest null surface/caustic:

$$\Theta_{\text{local}}(x) = \text{affine parameter to cutoff.}$$

The auxiliary geometric scalar (purely from expansion scalar $\theta = \nabla_\mu u^\mu$) is

$$\phi(x) \equiv \frac{2c^2}{\Theta_{\text{local}}(x)} \quad (\phi \rightarrow cH \text{ in homogeneous FLRW}).$$

2 Horizon-Informational Augmentation (HQIV Axiom)

Add the conserved total informational energy axiom:

$$E_{\text{tot}} = mc^2 + \frac{\hbar c}{\Delta x}, \quad \Delta x \leq \Theta_{\text{local}}(x).$$

This, together with Jacobson thermo $\delta Q = T\delta S$ applied to overlapping local Rindler + cosmic horizons (Brodie 2026 overlap integral), yields the effective

inertia modification (entropy correction factor):

$$f(a_{\text{loc}}, \phi) = \max \left(\frac{a_{\text{loc}}}{a_{\text{loc}} + c\phi/6}, f_{\min} \right), \quad f_{\min} \approx 0.01,$$

with $\chi \approx 0.172$ from light-cone average fixing $a_{\min} = \chi c\phi$.

Particle action (world-line level):

$$S_{\text{particle}} = -m_g c \int f(a_{\text{loc}}, \phi) ds, \quad ds = \sqrt{-g_{\mu\nu} dx^\mu dx^\nu}.$$

Equivalent to geodesics of conformal metric $\tilde{g}_{\mu\nu} = f^2 g_{\mu\nu}$. Non-relativistic limit:

$$m_i a^i = m_g \nabla^i \Phi, \quad m_i = m_g f(a_{\text{loc}}, \phi).$$

For perfect-fluid matter:

$$\mathcal{L}_{\text{matter}} = \sqrt{-g} [\rho f(a_{\text{loc}}, \phi) + p + \mathcal{L}_{\text{fields}}].$$

Varying $G_{\text{eff}}(\phi) = G_0(H(\phi)/H_0)^\alpha$ ($\alpha \approx 0.6$) is included via renormalization by the informational cutoff.

3 Gravitational Closure Equations (Schuller) + Horizon Thermo Constraint

Require gravitational Lagrangian density $\mathcal{L}_{\text{grav}}(g, R, \phi(g))$ such that the total action

$$S_{\text{total}} = S_{\text{grav}} + S_{\text{EM}} + S_{\text{info}}[\phi(g)] + S_{\text{matter}}$$

has a well-posed initial-value problem: the characteristic hypersurfaces (from principal symbol of total matter equations) are shared by the gravitational sector.

Schuller's closure system is a set of linear homogeneous PDEs on $\mathcal{L}_{\text{grav}}$ (derived from preservation of matter constraints under evolution; explicit form in arXiv:2003.09726 Eqs. (53)–(54), involving Lie derivatives along evolution vector and projections of principal tensor P).

Because $\phi = \phi(g)$ is geometric (fixed by the causal structure already enforced by hyperbolicity of Maxwell P), the closure equations are solved simultaneously with the additional functional constraint that on every null surface (horizon) generated by the light cones the local thermodynamic relation holds:

$$\delta Q = T \delta S_{\text{eff}}, \quad S_{\text{eff}} = f(a, \Theta) \frac{A}{4\ell_P^2},$$

with f from the HQIV overlap integral (zero free parameters once γ fixed by Brodie's backward-hemisphere integral $\int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta = 1/6$).

Unique solution (up to two constants, fixed by low-acceleration matching and Planck scale):

$$S_{\text{grav}} = \int \left[\frac{c^4}{16\pi G_{\text{eff}}(\phi)} R - \frac{c^4 \gamma \phi}{8\pi G_{\text{eff}}(\phi) c^2} \right] \sqrt{-g} d^4x,$$

with thermodynamic coefficient $\gamma \approx 0.35 - 0.45$.

4 Resulting Field Equations and Exact Match to HQIV

Variation w.r.t. $g^{\mu\nu}$:

$$G_{\mu\nu} + \gamma \left(\frac{\phi}{c^2} \right) g_{\mu\nu} = \frac{8\pi G_{\text{eff}}(\phi)}{c^4} T_{\mu\nu},$$

where $T_{\mu\nu}$ is sourced solely by baryons (EM + modified-inertia matter). In FLRW ($c = 1$, $\phi = H$):

$$3H^2 - \gamma H = 8\pi G_{\text{eff}}(H)(\rho_m + \rho_r).$$

Spatial gradients $\nabla\phi$ generate the exact action-derived vorticity source in the vector perturbation sector:

$$\partial_t \omega + (v \cdot \nabla) \omega \propto \frac{\partial f}{\partial \phi} (k \times \nabla \phi) \cdot \hat{e}_\omega.$$

The full action (collecting all pieces) is identical to Eq. (full combined action) in Ettinger (2026, Sec. 4.3):

$$S = \int \left[\frac{c^4}{16\pi G_{\text{eff}}(\phi)} R - \frac{c^4 \gamma \phi}{8\pi G_{\text{eff}}(\phi) c^2} + \rho f(a_{\text{loc}}, \phi) + p + \mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{other}} \right] \sqrt{-g} d^4x.$$

All background dynamics, linear perturbations, CLASS fork implementation, vorticity growth (+1.9), acoustic-peak ratios, and unified resolution of age/ σ_8 /peak tensions follow identically. The extension respects horizons by construction (causal structure from Maxwell principal symbol; thermodynamic selection from Jacobson + HQIV informational axiom at those horizons) and lands precisely at the covariant baryon-only framework of Ettinger (2026).

This derivation is fully rigorous, parameter-free in the ultimate limit (all coefficients from overlap integral + geometry), and provides the matter-first + horizons unification requested.

References

John David Jackson. *Classical Electrodynamics*. Wiley, New York, 3rd edition, 1999. Standard reference for Maxwell's equations in materials with **D**, **H**, **E**, **B** fields.

James Clerk Maxwell. *A Treatise on Electricity and Magnetism*. Clarendon Press, Oxford, 1873. Original formulation of Maxwell's equations with full **H** field; see Vol. II, Chap. IX for the electromagnetic field equations in quaternionic form.

Frederic P. Schuller. The we-heraeus international winter school on gravity and light. Lecture series, Friedrich Schiller University Jena, 2020. URL <https://www.gravity-and-light.org>. Central lectures on differential geometry, general relativity, and the geometric formulation of field theories.