Estimation & simulation of copulas and analysis of volatility index price evolution

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Outline

1. Estimation and simulation of copulas
2. PCA analysis of volatility index price evolution

Data description

- Date source: http://finance.yahoo.com/
- Selection of stocks: 10 Internet and Software stocks in S&P500
- time period: 2015/05/01-2016/04/25, 248 days
- $R_{t+1} = \ln\left(P_{t+1}/P_t\right)$
- Portfolio construction: A price weighed portfolio, the proportion of each stock is the stock price divided by the sum of stock price at first period.

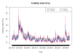
Stock list

Name of the company	Ticker
Google	GOOGL
Ebay	EBAY
Facebook	FB
Yahoo	YHOO
Wester Union	WU
Verisign Inc	VRSN
Netflix Inc.	NFLX
Total System Service	TSS
Fidelity National Information Services	FIS
Automatic Data Processing	ADP

Table 1: Stock list

Estimation of copula parameters

- Gaussian copula
- Student's t-copula with ν degrees of freedom
- Clayton copula
- Gumbel copula

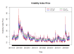


Gaussian copula

- Implicit copula

$$C_\rho(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} exp\{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\} dxdy$$

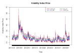
- Symmetric
- Lower and upper tail dependence goes to 0 as variable goes to extreme



Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	1.00	0.52	0.75	0.45	0.44	0.57	0.48	0.56	0.55	0.64
EBAY	0.52	1.00	0.49	0.59	0.48	0.52	0.40	0.57	0.45	0.58
FB	0.75	0.49	1.00	0.43	0.44	0.57	0.46	0.60	0.51	0.60
YHOO	0.45	0.59	0.43	1.00	0.47	0.47	0.40	0.55	0.44	0.52
WU	0.44	0.48	0.44	0.47	1.00	0.58	0.34	0.66	0.60	0.66
VRSN	0.57	0.52	0.57	0.47	0.58	1.00	0.46	0.66	0.64	0.61
NFLX	0.48	0.40	0.46	0.40	0.34	0.46	1.00	0.40	0.36	0.38
TSS	0.56	0.57	0.60	0.55	0.66	0.66	0.40	1.00	0.70	0.72
FIS	0.55	0.45	0.51	0.44	0.60	0.64	0.36	0.70	1.00	0.69
ADP	0.64	0.58	0.60	0.52	0.66	0.61	0.38	0.72	0.69	1.00

Table 2: Rho of Gaussian copula

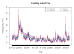


Student's t-copula with ν degrees of freedom

- Implicit copula

$$C_{\rho,\nu}(u,v) = \int_{-\infty}^{t_\nu^{-1}(u)} \int_{-\infty}^{t_\nu^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \{1 + \frac{x^2 - 2\rho xy + y^2}{\nu(1-\rho^2)}\}^{-(\nu+2)/2} ds dt$$

- Symmetric
- Supports joint extreme movements



Estimation results

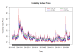
	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	1.00	0.55	0.76	0.46	0.49	0.60	0.49	0.57	0.57	0.67
EBAY	0.55	1.00	0.51	0.60	0.51	0.53	0.42	0.57	0.48	0.59
FB	0.76	0.51	1.00	0.45	0.44	0.57	0.47	0.61	0.52	0.60
YHOO	0.46	0.60	0.45	1.00	0.50	0.48	0.41	0.56	0.46	0.53
WU	0.49	0.51	0.44	0.50	1.00	0.61	0.38	0.67	0.64	0.68
VRSN	0.60	0.53	0.57	0.48	0.61	1.00	0.46	0.68	0.66	0.64
NFLX	0.49	0.42	0.47	0.41	0.38	0.46	1.00	0.40	0.40	0.41
TSS	0.57	0.57	0.61	0.56	0.67	0.68	0.40	1.00	0.72	0.75
FIS	0.57	0.48	0.52	0.46	0.64	0.66	0.40	0.72	1.00	0.74
ADP	0.67	0.59	0.60	0.53	0.68	0.64	0.41	0.75	0.74	1.00

Table 3: Rho of Student's t-copula

Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	0.00	5.50	6.36	6.66	4.53	3.67	4.75	11.86	10.12	4.21
EBAY	0.00	0.00	6.25	14.36	5.43	7.02	3.82	14.68	5.71	6.57
FB	0.00	0.00	0.00	5.36	20.03	12.72	4.78	8.37	11.21	4.70
YHOO	0.00	0.00	0.00	0.00	6.79	3.49	19.53	7.57	5.68	7.39
WU	0.00	0.00	0.00	0.00	0.00	5.97	4.30	12.27	4.52	5.04
VRSN	0.00	0.00	0.00	0.00	0.00	0.00	22.65	5.77	5.56	4.64
NFLX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.89	4.17	3.83
TSS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.35	4.13
FIS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.70
ADP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 4: Degree of freedom of Student's t-copula

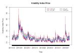


Clayton copula

- The Clayton copula is an asymmetric copula, exhibiting greater dependence in the negative tail than in the positive.

$$C_\delta(u, v) = (u^{-\delta} + v^{-\delta} - 1)^{-1/\delta}$$

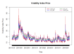
- where $0 < \delta < \infty$ is a parameter controlling the dependence. Perfect dependence is obtained if $\delta \to \infty$, while $\delta \to 0$ implies independence.



Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	999.00	0.90	1.63	0.71	0.69	0.94	0.84	0.99	0.90	1.28
EBAY	0.90	999.00	0.82	1.07	0.85	0.93	0.63	1.05	0.77	1.07
FB	1.63	0.82	999.00	0.81	0.61	0.88	0.84	1.18	0.82	1.05
YHOO	0.71	1.07	0.81	999.00	0.69	0.80	0.56	1.01	0.74	0.89
WU	0.69	0.85	0.61	0.69	999.00	0.99	0.60	1.25	1.13	1.18
VRSN	0.94	0.93	0.88	0.80	0.99	999.00	0.73	1.39	1.35	1.18
NFLX	0.84	0.63	0.84	0.56	0.60	0.73	999.00	0.69	0.60	0.62
TSS	0.99	1.05	1.18	1.01	1.25	1.39	0.69	999.00	1.66	1.73
FIS	0.90	0.77	0.82	0.74	1.13	1.35	0.60	1.66	999.00	1.38
ADP	1.28	1.07	1.05	0.89	1.18	1.18	0.62	1.73	1.38	999.00

Table 5: Delta of Clayton copula



Clayton copula

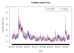
- Kendall's tau

$$\rho_\tau(X, Y) = \frac{\delta}{\delta + 2}$$

- Tail dependence

$$\lambda_u(X, Y) = 0$$

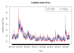
$$\lambda_l(X, Y) = 2^{-1/\delta}$$



Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	1.00	0.31	0.45	0.26	0.26	0.32	0.30	0.33	0.31	0.39
EBAY	0.31	1.00	0.29	0.35	0.30	0.32	0.24	0.34	0.28	0.35
FB	0.45	0.29	1.00	0.29	0.23	0.31	0.30	0.37	0.29	0.34
YHOO	0.26	0.35	0.29	1.00	0.26	0.28	0.22	0.34	0.27	0.31
WU	0.26	0.30	0.23	0.26	1.00	0.33	0.23	0.38	0.36	0.37
VRSN	0.32	0.32	0.31	0.28	0.33	1.00	0.27	0.41	0.40	0.37
NFLX	0.30	0.24	0.30	0.22	0.23	0.27	1.00	0.26	0.23	0.24
TSS	0.33	0.34	0.37	0.34	0.38	0.41	0.26	1.00	0.45	0.46
FIS	0.31	0.28	0.29	0.27	0.36	0.40	0.23	0.45	1.00	0.41
ADP	0.39	0.35	0.34	0.31	0.37	0.37	0.24	0.46	0.41	1.00

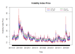
Table 6: Kendall's tau of Clayton copula



Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	1.00	0.46	0.65	0.38	0.37	0.48	0.44	0.50	0.46	0.58
EBAY	0.46	1.00	0.43	0.52	0.44	0.47	0.33	0.52	0.40	0.52
FB	0.65	0.43	1.00	0.43	0.32	0.46	0.44	0.56	0.43	0.52
YHOO	0.38	0.52	0.43	1.00	0.37	0.42	0.29	0.50	0.39	0.46
WU	0.37	0.44	0.32	0.37	1.00	0.50	0.31	0.57	0.54	0.56
VRSN	0.48	0.47	0.46	0.42	0.50	1.00	0.39	0.61	0.60	0.56
NFLX	0.44	0.33	0.44	0.29	0.31	0.39	1.00	0.37	0.31	0.33
TSS	0.50	0.52	0.56	0.50	0.57	0.61	0.37	1.00	0.66	0.67
FIS	0.46	0.40	0.43	0.39	0.54	0.60	0.31	0.66	1.00	0.60
ADP	0.58	0.52	0.52	0.46	0.56	0.56	0.33	0.67	0.60	1.00

Table 7: Lower tail dependence of Clayton copula



Gumbel copula

- The Gumbel copula is also an asymmetric copula, but it is exhibiting greater dependence in the positive tail than in the negative.

$$C_\delta(u, v) = exp(-[(-\log u)^\delta + (-\log v)^\delta]^{1/\delta})$$

- where $\delta \geq 1$ is a parameter controlling the dependence. Perfect dependence is obtained if $\delta \to \infty$, while $\delta = 1$ implies independence.

Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	999.00	1.50	2.09	1.37	1.42	1.65	1.42	1.52	1.54	1.76
EBAY	1.50	999.00	1.44	1.57	1.43	1.44	1.34	1.52	1.39	1.57
FB	2.09	1.44	999.00	1.34	1.37	1.58	1.39	1.60	1.46	1.63
YHOO	1.37	1.57	1.34	999.00	1.44	1.42	1.31	1.50	1.37	1.47
WU	1.42	1.43	1.37	1.44	999.00	1.62	1.26	1.74	1.69	1.83
VRSN	1.65	1.44	1.58	1.42	1.62	999.00	1.36	1.76	1.70	1.69
NFLX	1.42	1.34	1.39	1.31	1.26	1.36	999.00	1.30	1.29	1.32
TSS	1.52	1.52	1.60	1.50	1.74	1.76	1.30	999.00	1.86	2.01
FIS	1.54	1.39	1.46	1.37	1.69	1.70	1.29	1.86	999.00	2.03
ADP	1.76	1.57	1.63	1.47	1.83	1.69	1.32	2.01	2.03	999.00

Table 8: Delta of Gumbel copula

Gumbel copula

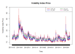
- Kendall's tau

$$\rho_\tau(X,Y) = 1 - \frac{1}{\delta}$$

- Tail dependence

$$\lambda_l(X,Y) = 0$$

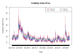
$$\lambda_u(X,Y) = 2 - 2^{1/\delta}$$



Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	1.00	0.33	0.52	0.27	0.30	0.39	0.29	0.34	0.35	0.43
EBAY	0.33	1.00	0.31	0.36	0.30	0.31	0.25	0.34	0.28	0.36
FB	0.52	0.31	1.00	0.25	0.27	0.37	0.28	0.38	0.31	0.39
YHOO	0.27	0.36	0.25	1.00	0.30	0.29	0.23	0.33	0.27	0.32
WU	0.30	0.30	0.27	0.30	1.00	0.38	0.21	0.43	0.41	0.45
VRSN	0.39	0.31	0.37	0.29	0.38	1.00	0.27	0.43	0.41	0.41
NFLX	0.29	0.25	0.28	0.23	0.21	0.27	1.00	0.23	0.23	0.24
TSS	0.34	0.34	0.38	0.33	0.43	0.43	0.23	1.00	0.46	0.50
FIS	0.35	0.28	0.31	0.27	0.41	0.41	0.23	0.46	1.00	0.51
ADP	0.43	0.36	0.39	0.32	0.45	0.41	0.24	0.50	0.51	1.00

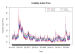
Table 9: Kendell's tau of Gumbel copula



Estimation results

	GOOGL	EBAY	FB	YHOO	WU	VRSN	NFLX	TSS	FIS	ADP
GOOGL	1.00	0.41	0.61	0.34	0.37	0.48	0.37	0.42	0.43	0.52
EBAY	0.41	1.00	0.38	0.44	0.38	0.38	0.32	0.42	0.35	0.44
FB	0.61	0.38	1.00	0.32	0.34	0.45	0.36	0.46	0.39	0.47
YHOO	0.34	0.44	0.32	1.00	0.38	0.37	0.30	0.41	0.34	0.40
WU	0.37	0.38	0.34	0.38	1.00	0.47	0.27	0.51	0.49	0.54
VRSN	0.48	0.38	0.45	0.37	0.47	1.00	0.34	0.52	0.50	0.49
NFLX	0.37	0.32	0.36	0.30	0.27	0.34	1.00	0.29	0.29	0.31
TSS	0.42	0.42	0.46	0.41	0.51	0.52	0.29	1.00	0.55	0.59
FIS	0.43	0.35	0.39	0.34	0.49	0.50	0.29	0.55	1.00	0.59
ADP	0.52	0.44	0.47	0.40	0.54	0.49	0.31	0.59	0.59	1.00

Table 10: Upper tail dependence of Gumbel copula



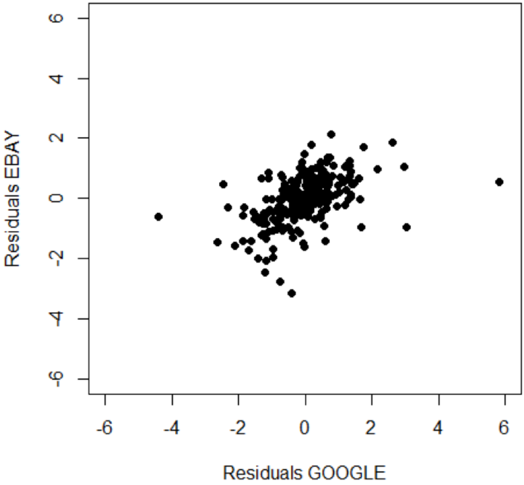


Figure 1: Scatter plot from residuals of GOOGLE and EBAY

Simulations from Gaussian copula

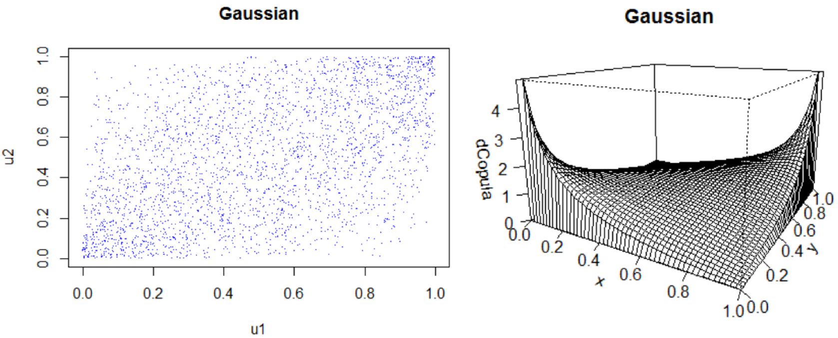


Figure 2: Simulations from Gaussian copula, $\rho = 0.52$

Simulations from Student's t-copula

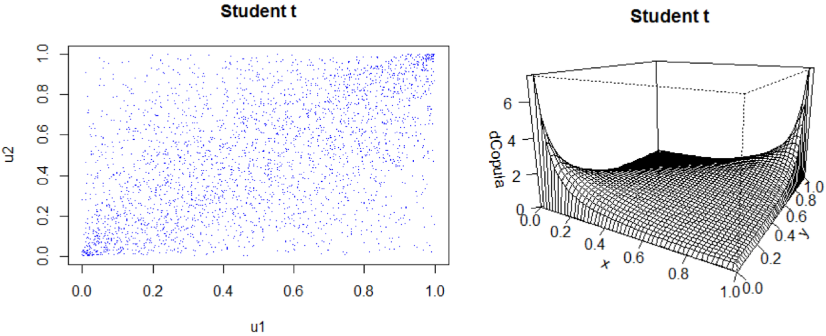
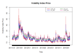


Figure 3: Simulations from Student's t-copula, $\rho = 0.55$, $\nu = 5.50$



Simulations from Clayton copula

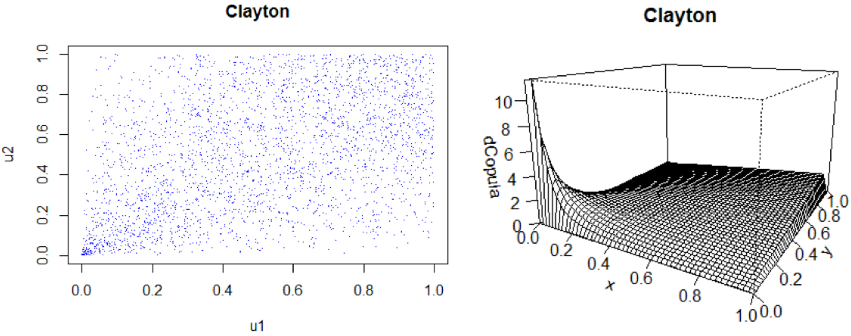
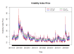


Figure 4: Simulations from Clayton copula, $\delta = 0.9$



Simulations from Gumbel copula

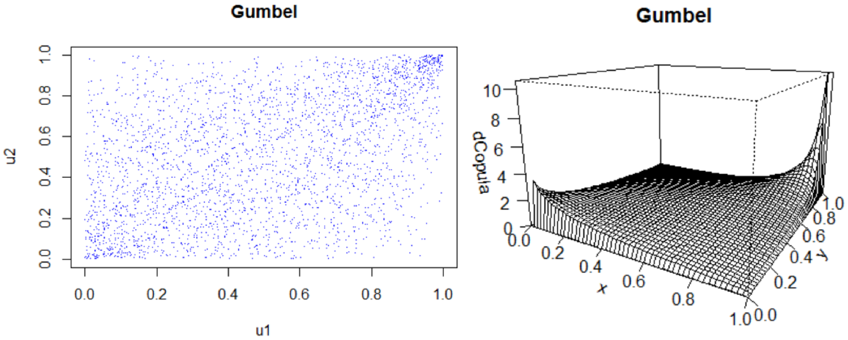
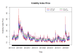


Figure 5: Simulations from Clayton copula, $\delta = 1.50$

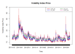


Simulations of portfolio VaR

	1% VaR	5% VaR
Portfolio VaR	-2.106	-1.415

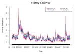
Table 11: Simulations of portfolio VaR

Simulations of portfolio VaR using Gaussian copula with estimated correlation matrix.



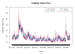
Outline

1. Estimation and simulation of copulas ✓
2. PCA analysis of volatility index price evolution



Data discription

- Date source: Datastream
- 4 types of volatility index prices: VXST, VIX, VXV, VXMT
- Market's expectation of stock market implied volatility over different time periods
- Time period: 2011.01.01-2015.12.31, 1304 obs.



Symbol	Expectation period
VXST	9-day
VIX	30-day
VXV	3-month
VXMT	6-month

Table 12: Different volatility indexes

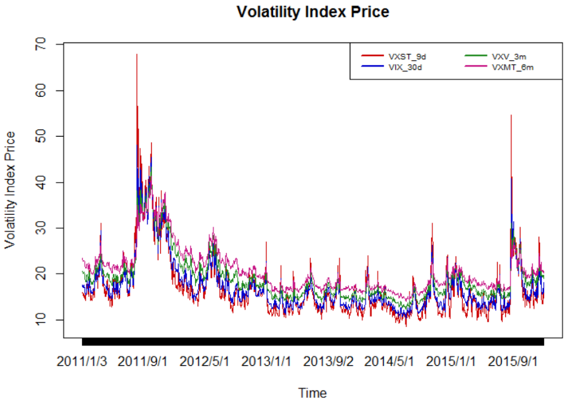
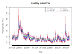


Figure 6: 4 types of volatility index prices from 2011 to 2015



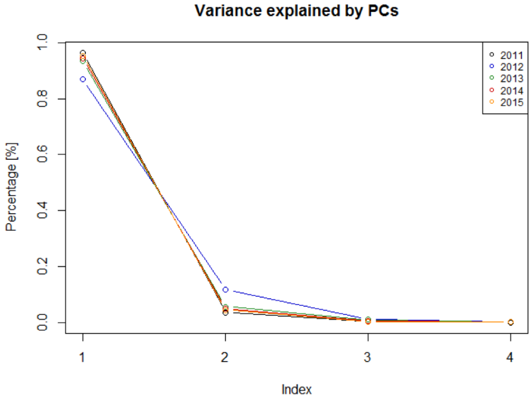
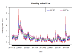


Figure 7: Relative proportion of variance explained by principal components



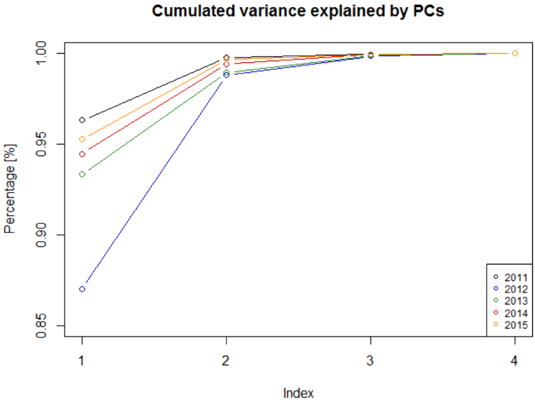
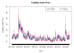


Figure 8: Cumulated variance explained by principal components



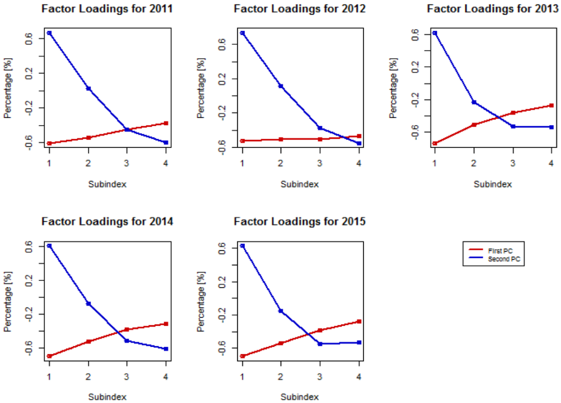
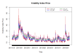


Figure 9: Comparison of Factor loadings across years



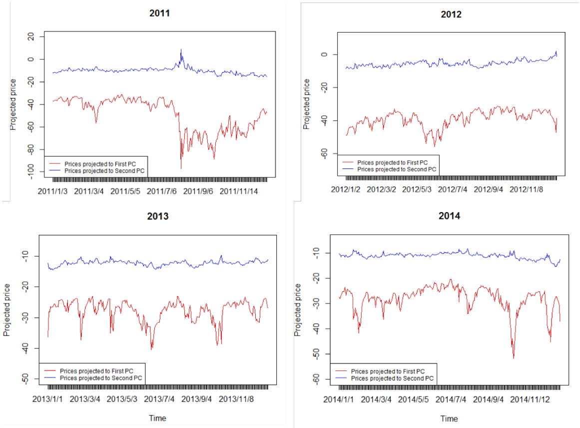
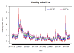


Figure 10: Volatility index prices projected to new coordinates



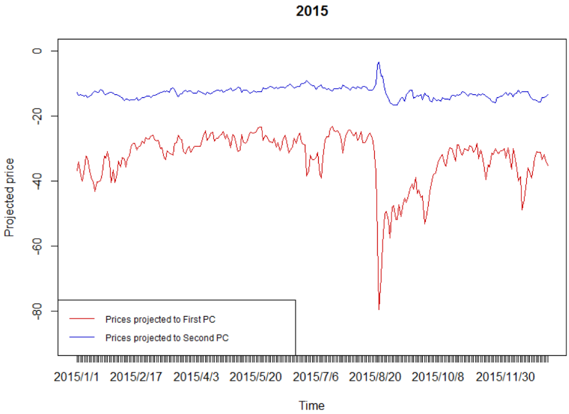
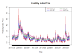


Figure 11: Volatility index prices projected to new coordinates



Conclusions

- The volatility index prices can be different from year to year. 2011 and 2015 have similar pattern due to turmoil in the financial market.
- In the example, the variance of volatility index prices data can be mostly captured by the first PC.
- Short-term and medium-term volatility are more sensitive given a shock to first PC. But they tend to move in opposite directions.
- It seems that the factor loadings of PCs in 2012 are different from the other years.