

Put Option Price

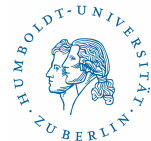
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- Arrow-Dedreu prices λ_n^i (discount risk-neutral probability)
the price of an option that pays 1 in one and only one state i at n th level, and otherwise pays 0.

$$\lambda_{n+1}^1 = e^{-r\Delta t} \{(1 - p_n^1)\lambda_n^1\}$$

$$\lambda_{n+1}^{i+1} = e^{-r\Delta t} \{\lambda_n^i p_n^i + \lambda_n^{i+1} (1 - p_n^{i+1})\}$$

$$\lambda_{n+1}^{n+1} = e^{-r\Delta t} \{\lambda_n^n p_n^n\}$$

- Put option price $P(K, n\Delta t)$
 $P(K, n\Delta t) = \sum_{i=0}^n \lambda_{n+1}^{i+1} \max(K - S_{n+1}^{i+1}, 0)$



with $K = S = S_n^i$ and $S_n^{i+1} < S_n^i < S_{n+1}^{i+1}$ we have:

$$\begin{aligned}
 P(S, n\Delta t) &= e^{-r\Delta t} [\lambda_n^1 (1 - p_n^1) \max(S - S_{n+1}^1, 0) \\
 &\quad + \sum_{j=1}^{n-1} \{ \lambda_n^j p_n^j + \lambda_n^{j+1} (1 - p_n^{j+1}) \} \max(S - S_{n+1}^{j+1}, 0) \\
 &\quad + \lambda_n^n p_n^n \max(S - S_{n+1}^{n+1}, 0)] \\
 &= e^{-r\Delta t} [\lambda_n^1 (1 - p_n^1) (S - S_{n+1}^1) \\
 &\quad + \sum_{j=1}^{i-1} \{ \lambda_n^j p_n^j + \lambda_n^{j+1} (1 - p_n^{j+1}) \} (S - S_{n+1}^{j+1})]
 \end{aligned}$$



$$\begin{aligned}
P(S, n\Delta t) &= e^{-r\Delta t} \{ \lambda_n^1 (1 - p_n^1) (S - S_{n+1}^1) \\
&\quad + [\lambda_n^1 p_n^1 + \lambda_n^2 (1 - p_n^2)] (S - S_{n+1}^2) \\
&\quad + [\lambda_n^2 p_n^2 + \lambda_n^3 (1 - p_n^3)] (S - S_{n+1}^3) \\
&\quad + \dots \\
&\quad + [\lambda_n^{i-1} p_n^{i-1} + \lambda_n^i (1 - p_n^i)] (S - S_{n+1}^i) \} \\
&= e^{-r\Delta t} [\lambda_n^i (1 - p_n^i) (S - S_{n+1}^i) \\
&\quad + \sum_{j=1}^{i-1} \lambda_n^j \{ (1 - p_n^j) (S - S_{n+1}^j) + P_n^j (S - S_{n+1}^{j+1}) \}]
\end{aligned}$$

$$P(S, n\Delta t) = e^{-r\Delta t} \{ \lambda_n^i (1 - p_n^i) (S - S_{n+1}^i) + \sum_{j=1}^{i-1} \lambda_n^j (S_n^i - F_n^i) \}$$

