Put Option Price

Yinan Wu

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://case.hu-berlin.de

- Arrow-Dedreu prices λ_n^i (discount risk-neutral probability) the price of an option that pays 1 in one and only one state i at nth level, and otherwise pays 0.

$$\lambda_{n+1}^{1} = e^{-r\Delta t}\{(1-p_n^1)\lambda_n^1\}$$
$$\lambda_{n+1}^{i+1} = e^{-r\Delta t}\{\lambda_n^i p_n^i + \lambda_n^{i+1}(1-p_n^{i+1})\}$$
$$\lambda_{n+1}^{n+1} = e^{-r\Delta t}\{\lambda_n^n p_n^n\}$$

- Put option price $P(K, n\Delta t)$
$P(K, n\Delta t) = \sum_{i=0}^{n} \lambda_{n+1}^{i+1} \max(K - S_{n+1}^{i+1}, 0)$

with $K = S = S_n^i$ and $S_n^{i+1} < S_n^i < S_{n+1}^{i+1}$ we have:

$$
\begin{aligned}
P(S, n\Delta t) &= e^{-r\Delta t}[\lambda_n^1(1-p_n^1)\max(S - S_{n+1}^1, 0) \\
&\quad + \sum_{j=1}^{n-1}\{\lambda_n^j p_n^j + \lambda_n^{j+1}(1-p_n^{j+1})\}\max(S - S_{n+1}^{j+1}, 0) \\
&\quad + \lambda_n^n p_n^n \max(S - S_{n+1}^{n+1}, 0)] \\
&= e^{-r\Delta t}[\lambda_n^1(1-p_n^1)(S - S_{n+1}^1) \\
&\quad + \sum_{j=1}^{i-1}\{\lambda_n^j p_n^j + \lambda_n^{j+1}(1-p_n^{j+1})\}(S - S_{n+1}^{j+1})]
\end{aligned}
$$

$$
\begin{aligned}
P(S, n\Delta t) &= e^{-r\Delta t}\{\lambda_n^1(1-p_n^1)(S-S_{n+1}^1) \\
&+ [\lambda_n^1 p_n^1 + \lambda_n^2(1-p_n^2)](S-S_{n+1}^2) \\
&+ [\lambda_n^2 p_n^2 + \lambda_n^3(1-p_n^3)](S-S_{n+1}^3) \\
&+ \dots \\
&+ [\lambda_n^{i-1} p_n^{i-1} + \lambda_n^i(1-p_n^i)](S-S_{n+1}^i)\} \\
&= e^{-r\Delta t}[\lambda_n^i(1-p_n^i)(S-S_{n+1}^i) \\
&+ \sum_{j=1}^{i-1} \lambda_n^j\{(1-p_n^j)(S-S_{n+1}^j) + P_n^j(S-S_{n+1}^{j+1})\}
\end{aligned}
$$

$$
P(S, n\Delta t) = e^{-r\Delta t}\{\lambda_n^i(1-p_n^i)(S-S_{n+1}^i) + \sum_{j=1}^{i-1} \lambda_n^j(S_n^i - F_n^i)\}
$$