UE22MA241B LINEAR ALGEBRA AND ITS APPLICATIONS

Unit 1: Matrices, Gaussian Elimination and Vector Spaces:

Introduction, The Geometry of Linear Equations, Gaussian Elimination, Singular Cases, Elimination Matrices, Triangular Factors -LU decomposition & Cholesky's Decomposition method and Row Exchanges, Inverses and Transposes, Inverse by Gauss -Jordan method, Vector Spaces and Subspaces (definitions only). Applications.

Self Learning Component: Algebra of Matrices.

Class No.	Portions to be covered
1	Introduction to Linear Algebra
2-3	The Geometry of Linear Equations – Row and Column Pictures
4	Singular cases in two and three dimensions
5-7	Gaussian Elimination
8	**Matlab Class Number 1- Introduction**
9	The breakdown of elimination
10	Elimination(Elementary) Matrices
11-13	Triangular Factors- LU Decomposition, Cholesky's Decomposition, Row Exchanges and Permutation Matrices
14	Inverse by Gauss -Jordan Method, Transposes
15	**Matlab Class Number 2 - Gaussian Elimination**
16-17	Vector Spaces and Subspaces (Definition only)
18	**Applications**

Classwork problems:

1.	Draw the row and column picture of these lines (i) 2x-y=-5 ; -x+2y=4 (ii) x+2y=7; x-y=1 (iii) x+2y=3; 2x+4y=6. Write down the solution if it exists.
2.	Check the following system of equations for consistency and solve if consistent using Gaussian Elimination: (i) x+ 2y+ 3z = 6, 2x -3y+2z =14, 3x +y -z =-2 **Answer:** x=1, y=-2, z=3 (i) x+2y +z =4, y-z =1, x+3y=0 **Answer:** x=2-3k, y=1+k, z=k (ii) x+2y +3z+4w=5 , x+3y+5z+7w =11, x-z-2w=-6 **Answer:** No solution
3.	Find all values of a for which the linear system x + y - z = 3, x - y + 3z = 4, x + y + $(a^2-10)z$ = a has (i)no solution, (ii) a unique solution and (iii)infinitely many solutions. **Answer:** (i) a=-3 (ii) $a \neq \pm 3$ (iii) a=3

4.	Which elementary matrices put A into upper triangular form U? Hence find L and factor A into LU given $A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$ **Answer:** E_{21}, E_{32} & E_{43}.
5.	Compute LU and LDU factorization for the matrix $\begin{pmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{pmatrix}$
6.	Apply elimination to produce the factors L and U for A=$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{pmatrix}$. Is A=LU possible? Explain. Write down the permutation matrices if any. Also find PA=LDU factorization. **Answer:** A=LU is not possible since we need to use permutation matrix P_{23} which gives PA=LU and hence PA=LDU.
7.	For the given matrix $A = \begin{pmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{pmatrix}$. Find A=LDU and discuss how is L and U related and why?
8.	Compute inverse of the following matrices by Gauss Jordan method. (a)$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ (b)$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ **Answer:** (a) $\begin{pmatrix} 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \\ -3/2 & 1 & 1/2 \end{pmatrix}$ (b)$\begin{pmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{pmatrix}$
9.	Find the quadratic polynomial that interpolates the given points (1,2), (3,3), (5,8) **Answer:** p(x)= $(x^2-3x+6)/2$
10.	Producing x trucks and y planes requires x+50y tons of steel, 40x+1000y pounds of rubber, and 2x + 50y months of labour. If the unit costs z_1,z_2,z_3 are \$700 per ton, \$3 per pound, and \$3000 per month, what are the values of one truck and one plane? **Answer:** 6820, 188000

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