Assessment of model performance

- Likelihood
- AIC
- Absolute measures of model fit
  Chi-squared test
  Qq plots
  Kolmogorov-Smirnov and Cramér-von Mises tests

Likelihood

$f(x)$ = probability density function of x

$f(x)\,dx$ = Pr (animal was between x and $x+dx$ from the line, given it was detected between 0 and w) for small dx

When distances are exact, the **likelihood** is given by

$$L = \prod_{i=1}^{n} f(x_i) = f(x_1) \times f(x_2) \times \ldots \times f(x_n)$$

x_i = distance of i^{th} detected animal from the line.

We fit $f(x)$ by finding the values for the parameters of $f(x)$ (or equivalently $g(x)$) that maximize L (or $\log_e(L)$).

Akaike's Information Criterion

$$AIC = -2\log_e(L) + 2q$$

L is the maximized likelihood (evaluated at the maximum likelihood estimates of the model parameters)

and q is the number of parameters in the model.

- Models need not be special cases of one another
- Select the model with smallest AIC
- Gives a relative measure of fit

Limitations of AIC

Cannot be used to select between models when:

- sample size *n* differs
- truncation distance *w* differs
- data are grouped, and cutpoints differ
- data are grouped in one analysis and ungrouped in the other

Goodness-of-Fit

- Chi-squared test for grouped (interval) data; if data are exact, we must specify interval cutpoints for this test
- Q-Q plots and related tests for exact data

Chi-squared tests

Define u distance intervals, with n_i detections in interval i, i = 1, ..., u.

Then

$$\chi^2 = \sum_{i=1}^{u} \frac{(n_i - n\hat{\pi}_i)^2}{n\hat{\pi}_i}$$

where

$$n = \sum_i n_i$$

and $\hat{\pi}_i$ is the proportion of the area under the estimated pdf, $\hat{f}(x)$, that lies in interval i.

If the model is 'correct': $\chi^2 \sim \chi^2_{u-q-1}$

q = no. of parameters

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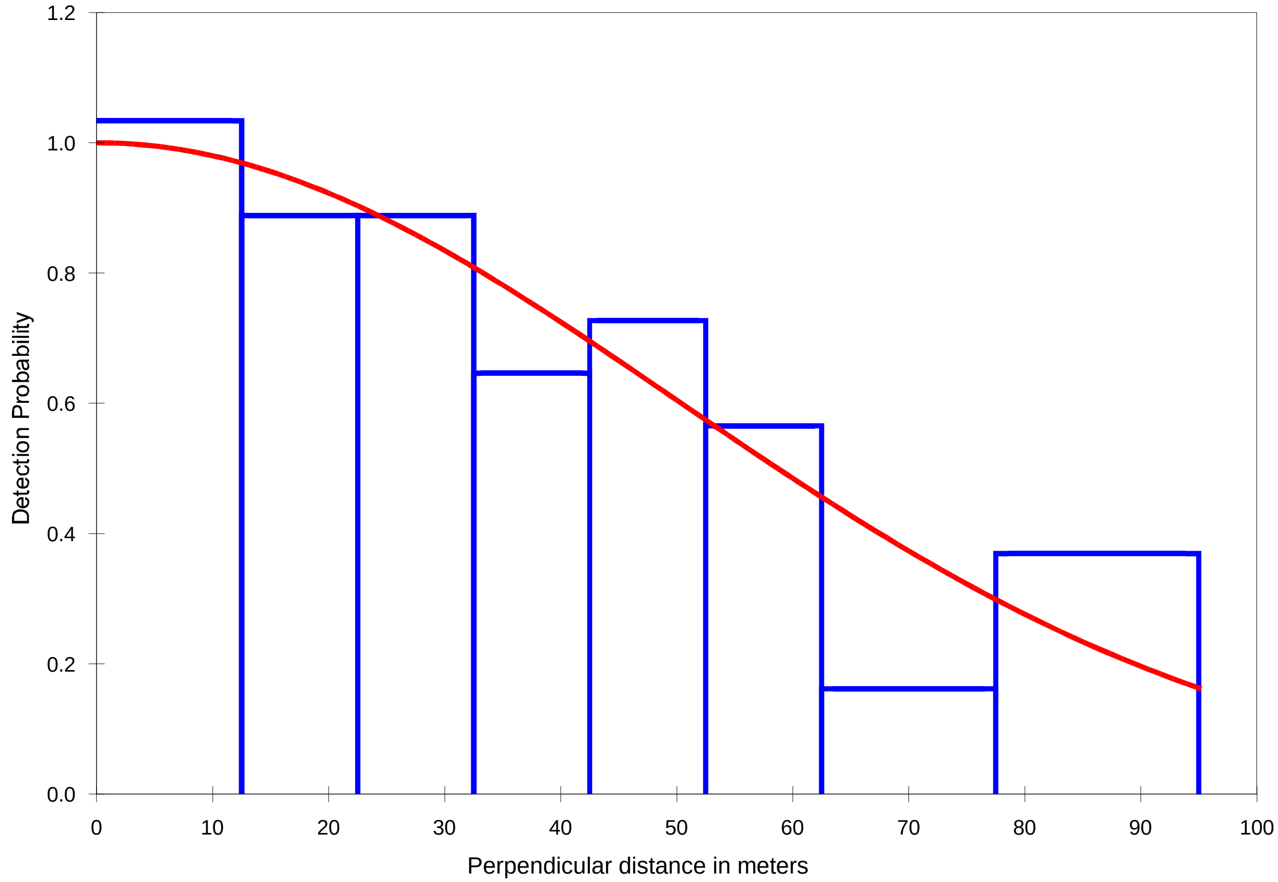
Chaffinch line transect data



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χ^2 goodness-of-fit test

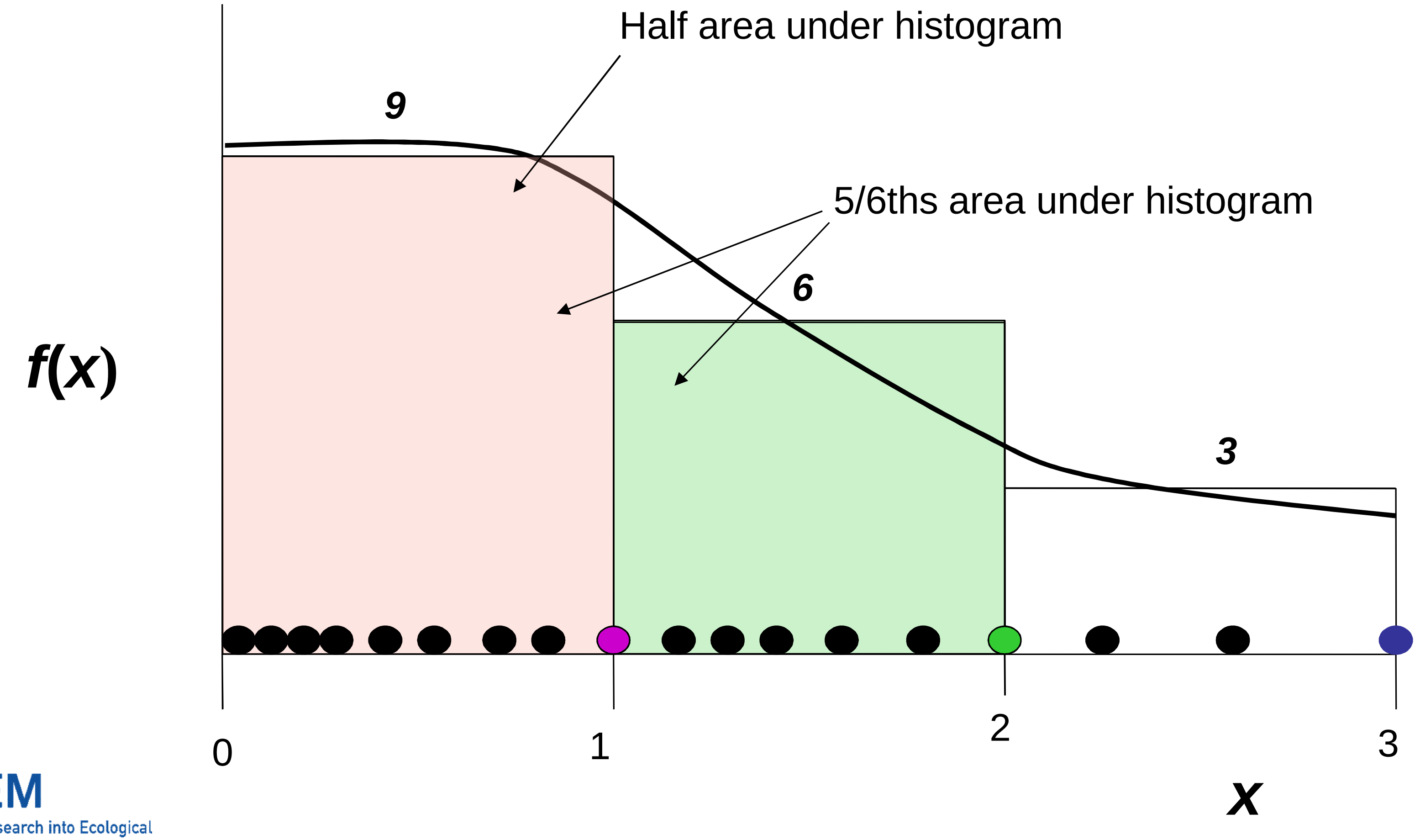
Cell i	Cut Points		Observed Values	Expected Values	Chi-square Values
1	0.000	12.5	16	15.32	0.030
2	12.5	22.5	11	11.63	0.034
3	22.5	32.5	11	10.62	0.013
4	32.5	42.5	8	9.33	0.189
5	42.5	52.5	9	7.87	0.164
6	52.5	62.5	7	6.37	0.062
7	62.5	77.5	3	6.96	2.253
8	77.5	95.0	8	4.91	1.953

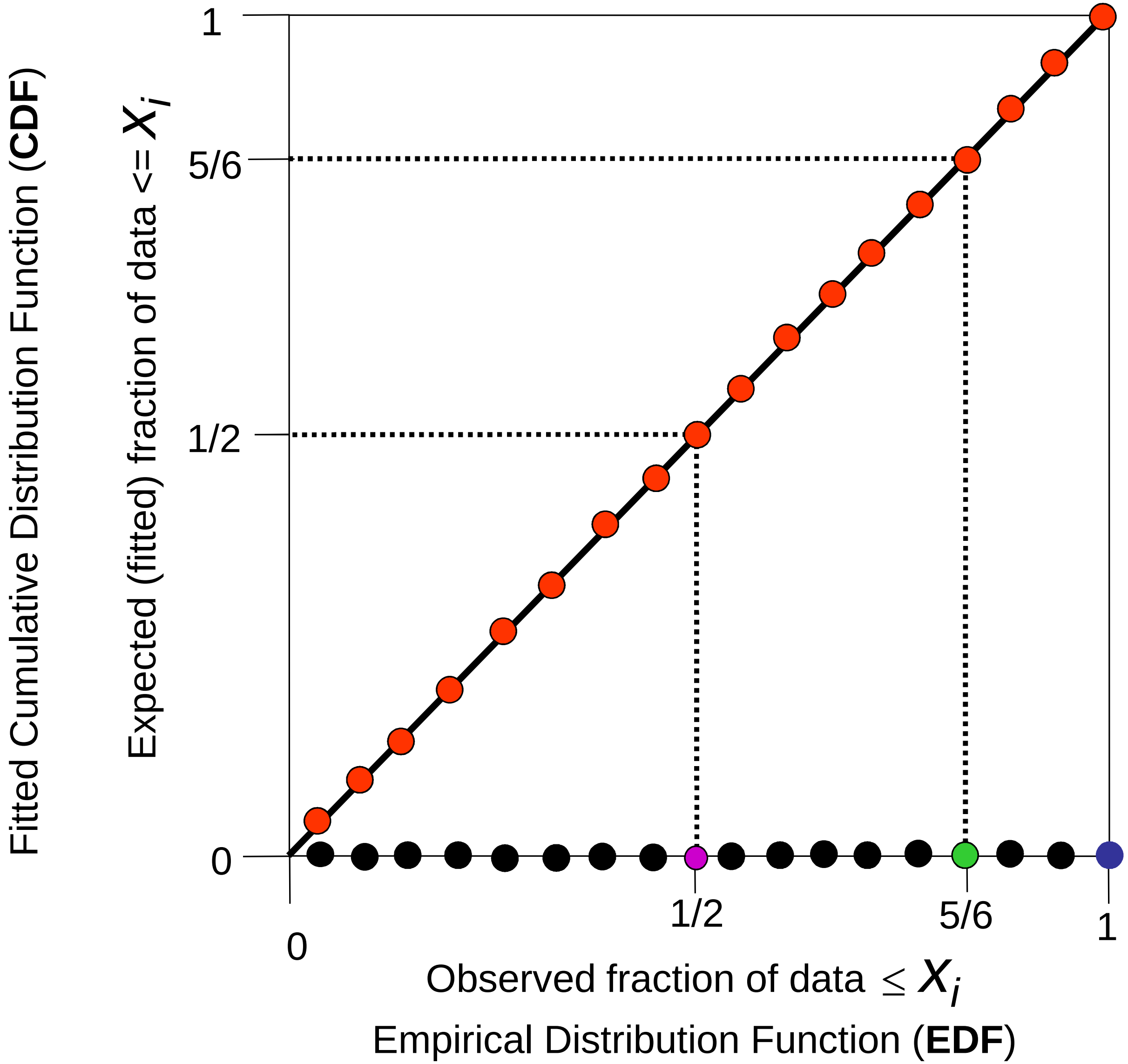
Total Chi-square value = 4.6970 Degrees of Freedom = 6.00

Probability of a greater chi-square value, P = 0.58322

The program has limited capability for pooling. The user should judge the necessity for pooling and if necessary, do pooling by hand.

Q-Q Plots and Related Tests





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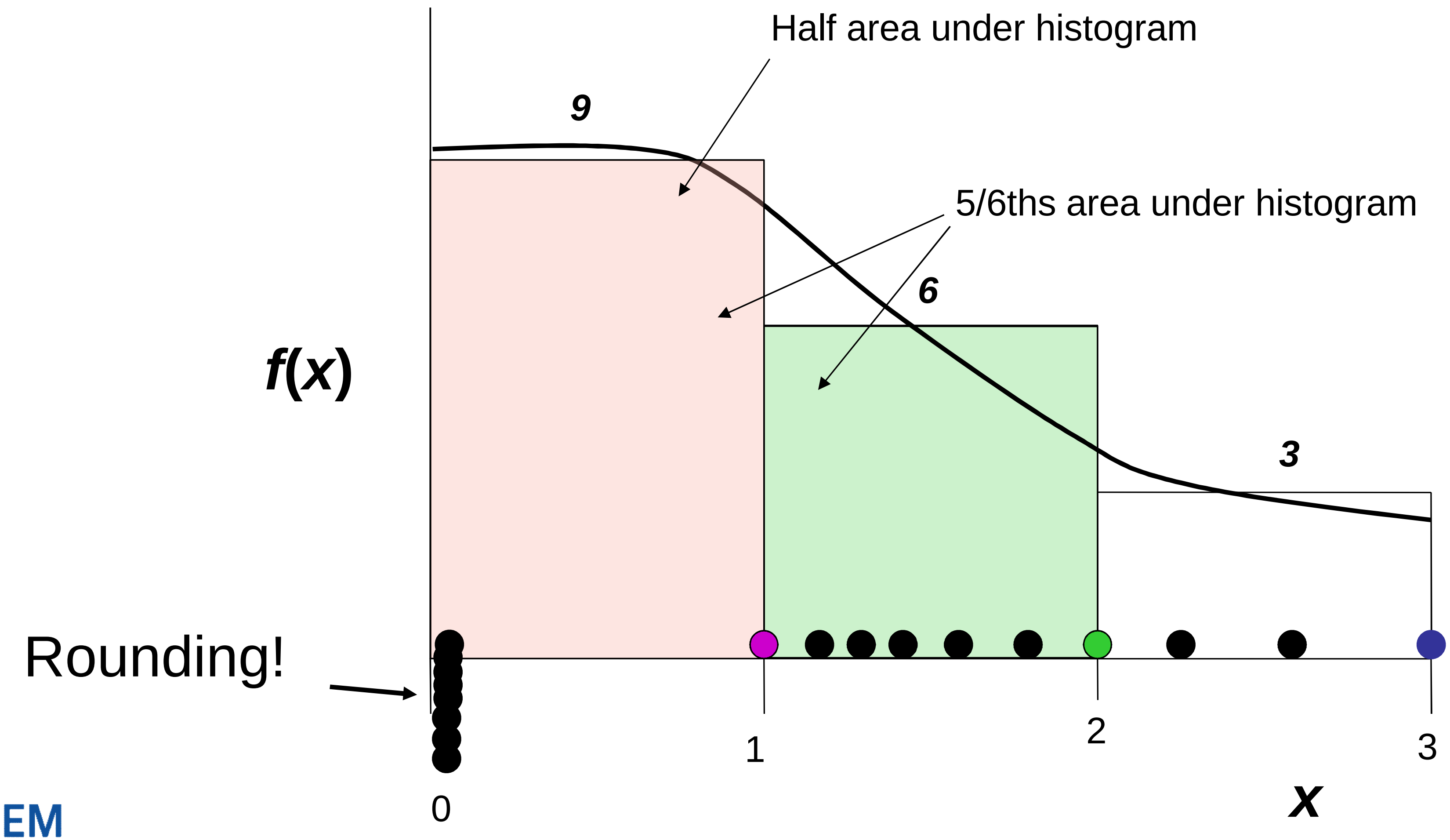
Example: Rounding to zero



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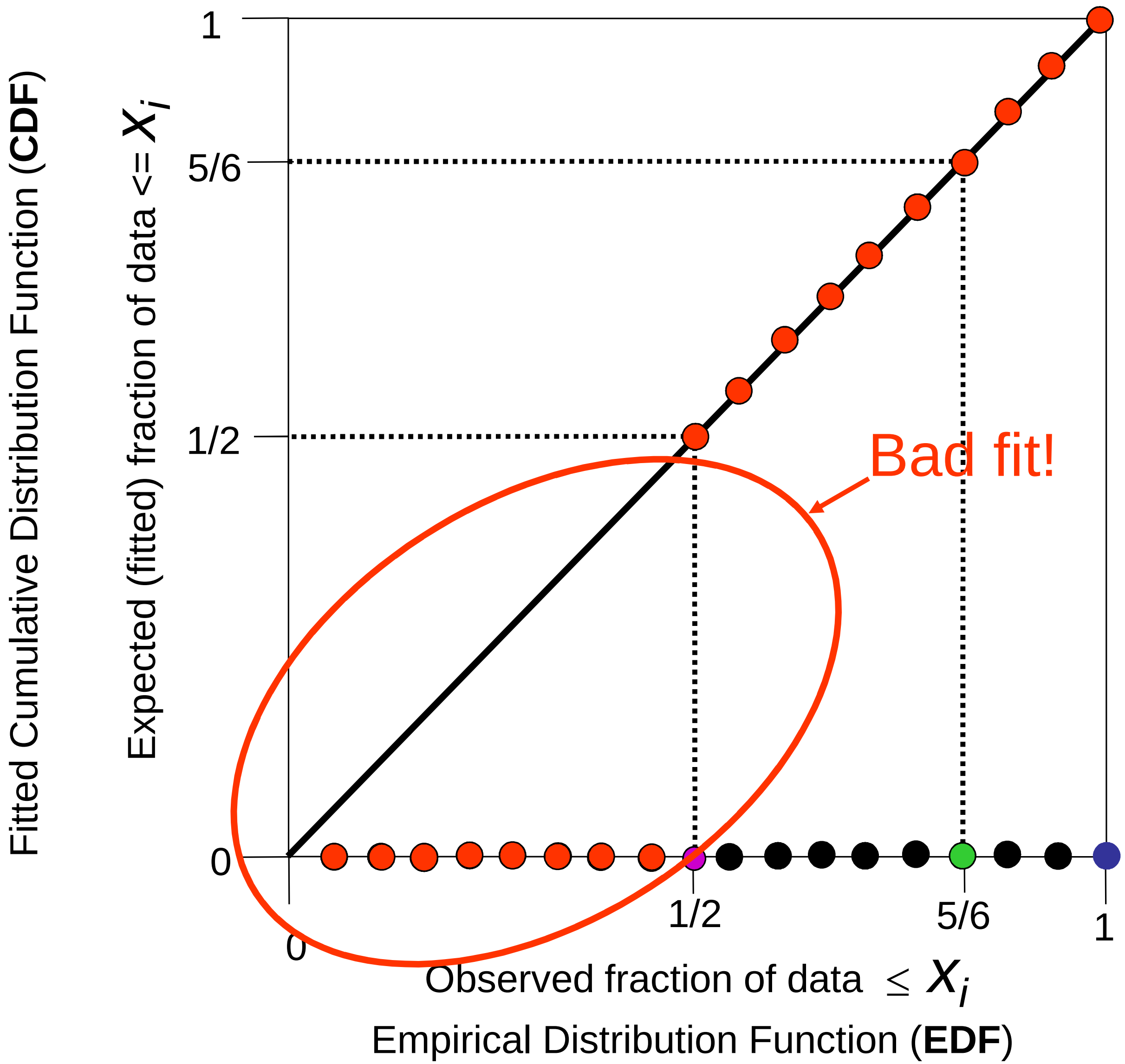

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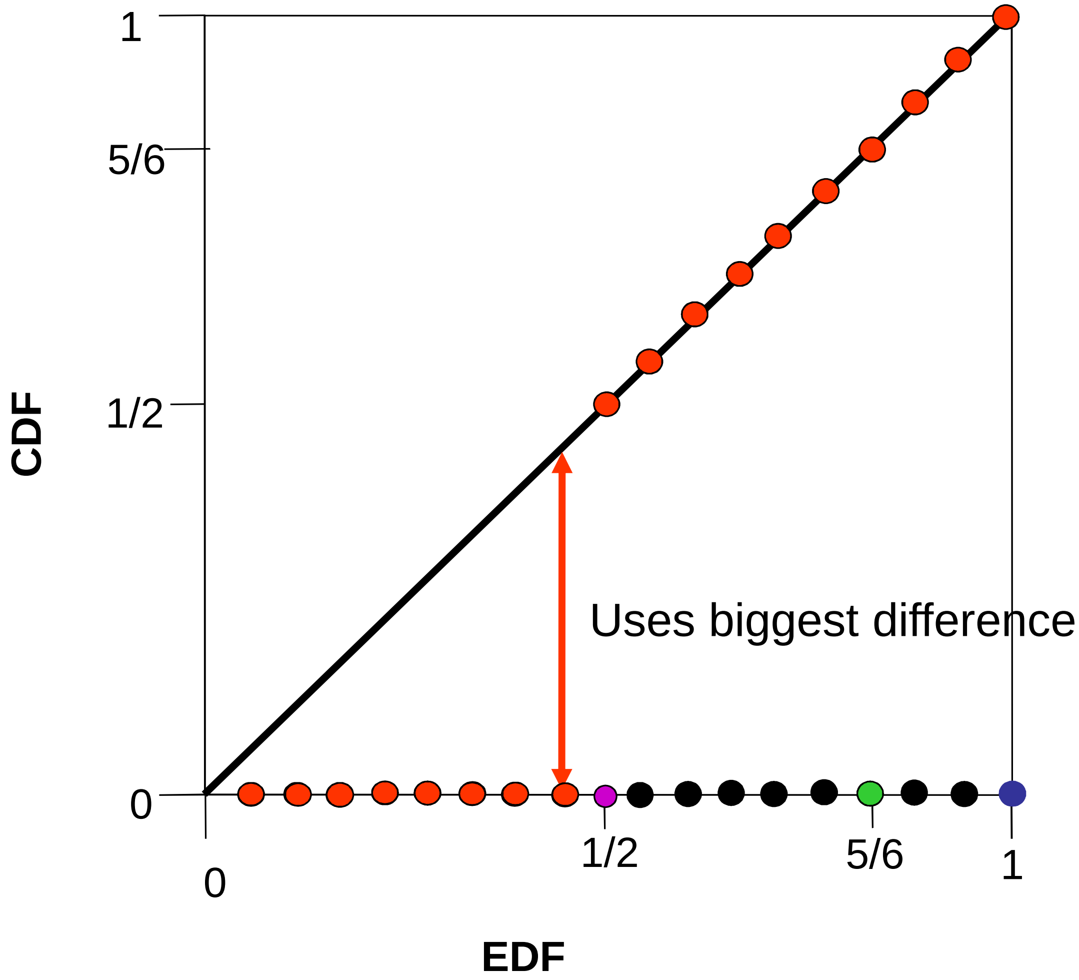
Kolmogorov-Smirnov test



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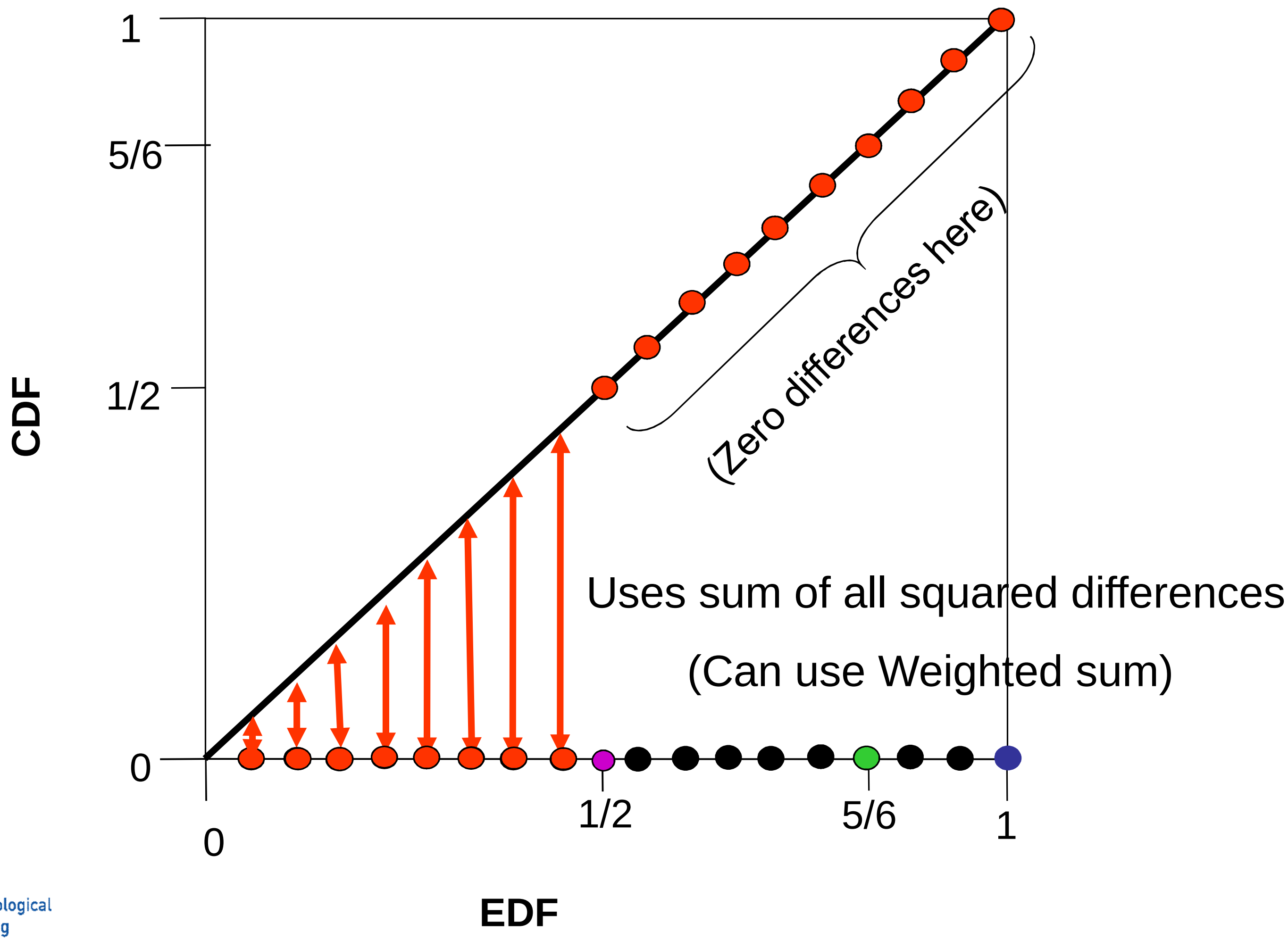
Cramér-von Mises test



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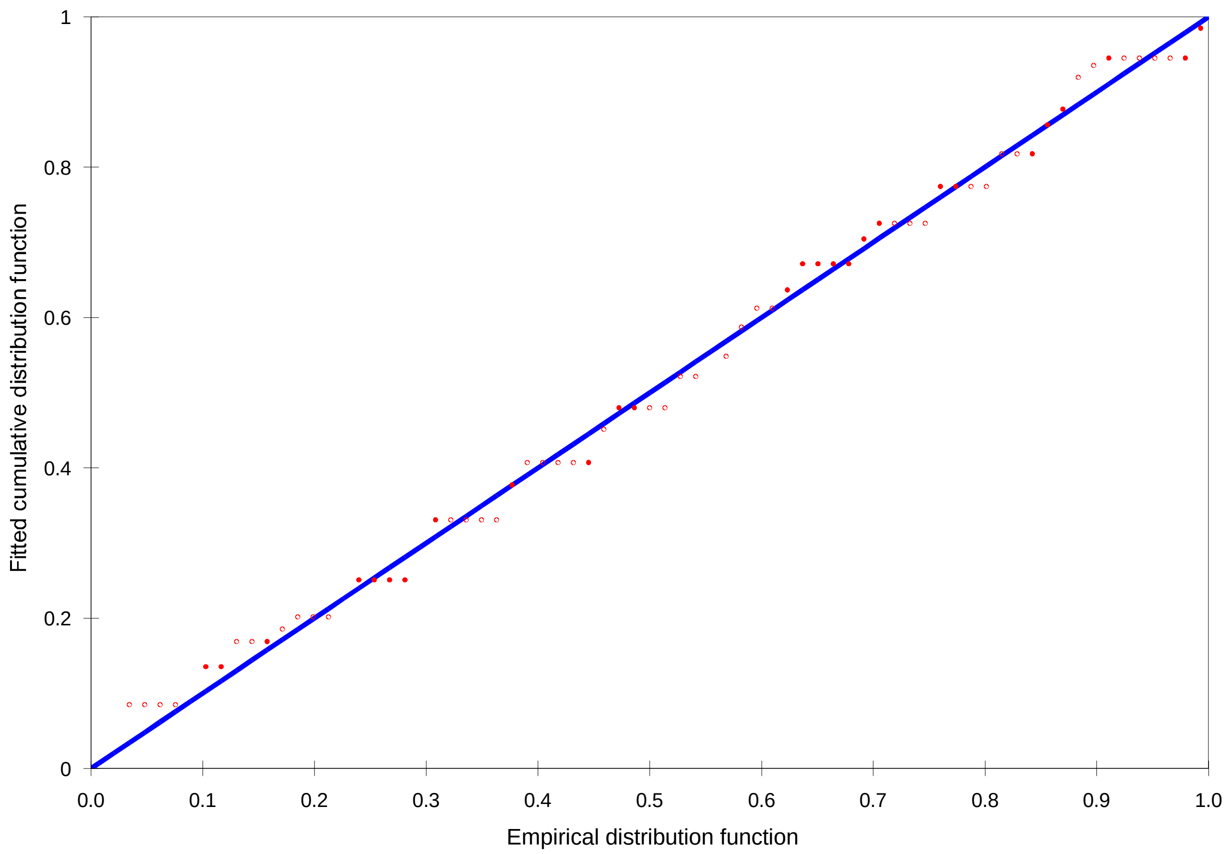
Chaffinch line transect Q-Q plot



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K-S test and Cramer-von Mises test

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Kolmogorov-Smirnov test
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 D_n                        = 0.0573                   p  = 0.9703


 Cramer-von Mises family tests
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 W-sq (uniform weighting) = 0.0368          0.900 < p <= 1.000
   Relevant critical values:
     W-sq crit(alpha=0.900) = 0.0000

 C-sq (cosine weighting)  = 0.0257          0.900 < p <= 1.000
   Relevant critical values:
     C-sq crit(alpha=0.900) = 0.0000
```

Q-Q Plot Summary

- Q-Q plots show goodness-of-fit at “high resolution” – without requiring grouping into intervals
- Kolmogorov-Smirnov test and Cramér-von Mises test are goodness-of-fit tests that do not require grouping
- Cramér-von Mises test can be weighted, to give higher weight to *x* near zero