Lecture 2 : Generalized Additive Models

University of
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Overview

- The count model, from scratch

- What is a GAM?

- What is smoothing?

- Fitting GAMs using `dsm`

Building a model, from scratch

- Know count n_j in segment j

- Want :

$$n_j = f([\text{environmental covariates}]_j)$$

- Additive model of smooths s:

$$n_j = \exp\left[\beta_0 + s(y_j) + s(\text{Depth}_j)\right]$$

- model terms

- exp is the *link function*

Building a model, from scratch

- What about area and detectability?

$$n_j = A_j \hat{p}_j \exp \left[\beta_0 + s(\mathrm{y}_j) + s(\mathrm{Depth}_j) \right]$$

- A_j area of segment - "offset"

- $\hat{p}_j$ probability of detection in segment

Building a model, from scratch

- It's a statistical model so:

$$n_j = A_j \hat{p}_j \exp \left[\beta_0 + s(\mathrm{y}_j) + s(\mathrm{Depth}_j) \right] + \epsilon_j$$

- n_j has a distribution (count)

- ϵ_j are errors (differences between model and observations)

That's a Generalized Additive Model!
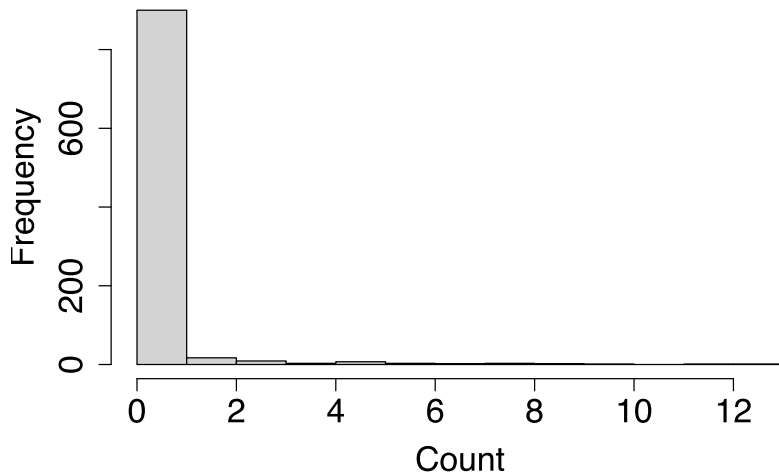
Now let's look at each bit...

Response

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(\mathrm{y}_j) + s(\mathrm{Depth}_j)\right] + \epsilon_j$$

where ϵ_j are some errors, $n_j \sim$ count distribution
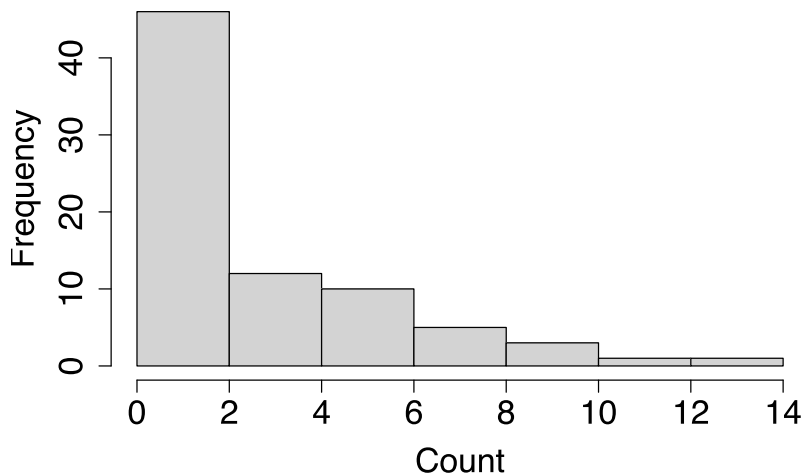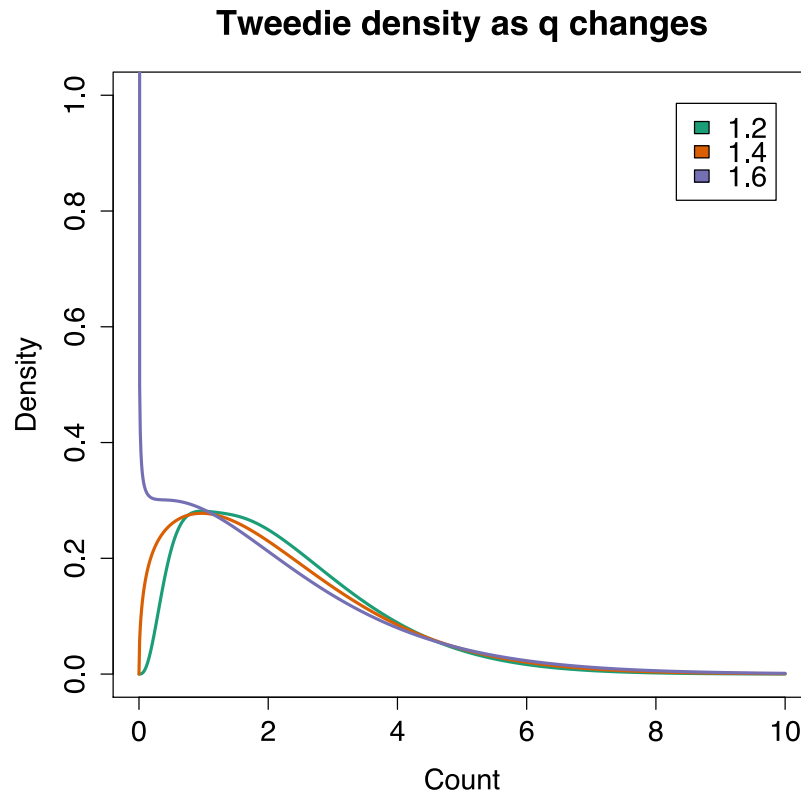
Count distributions

All segments

Counts > 0

- Response is a count

- Often, it's mostly zero

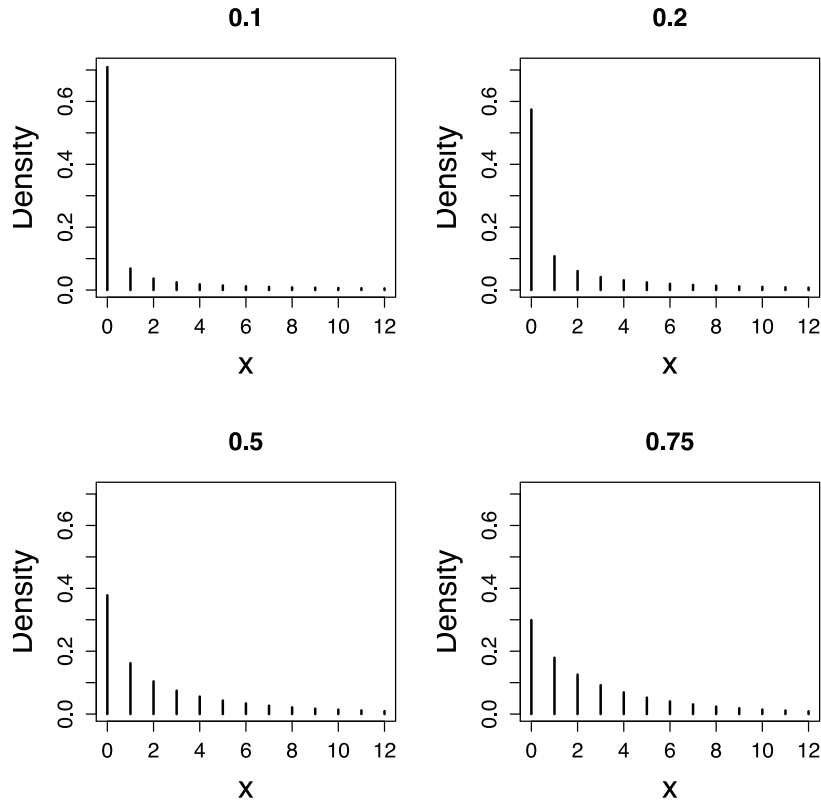- Flexible mean-variance relationship

- (Poisson isn't good at this)

Tweedie distribution

Tweedie density as q changes

Density / Count plot with legend:
- 1.2
- 1.4
- 1.6

(NB there is a point mass at zero not plotted)

- $\mathrm{Var}\,(\mathrm{count}) = \phi\mathbb{E}(\mathrm{count})^q$

- Poisson is $q = 1$

- We estimate q and ϕ

Negative binomial distribution

- $\text{Var}\,(\text{count}) = \mathbb{E}(\text{count}) + \kappa\mathbb{E}(\text{count})^2$

- Estimate κ

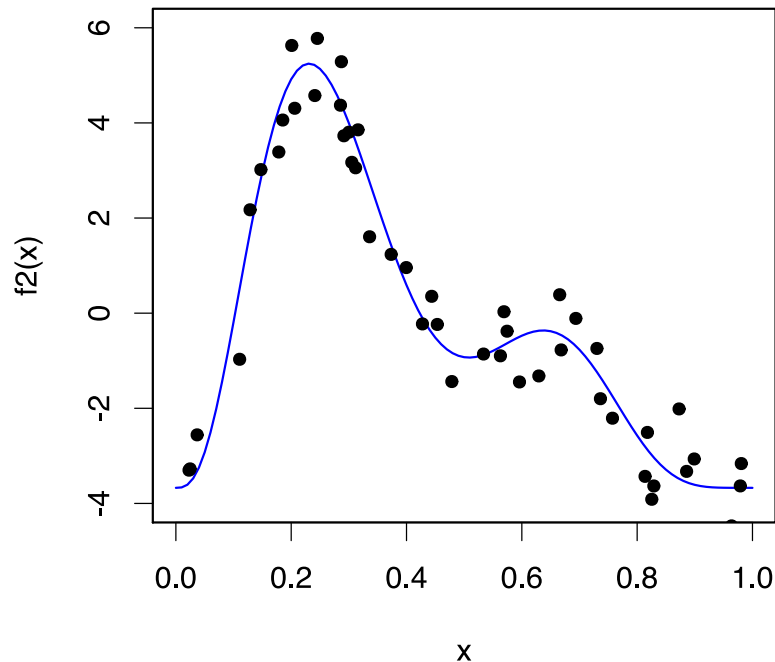- (Poisson: $\text{Var}\,(\text{count}) = \mathbb{E}(\text{count})$)

Smooths

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(\mathrm{y}_j) + s(\mathrm{Depth}_j)\right] + \epsilon_j$$

where ϵ_j are some errors, $\quad n_j \sim$ count distribution

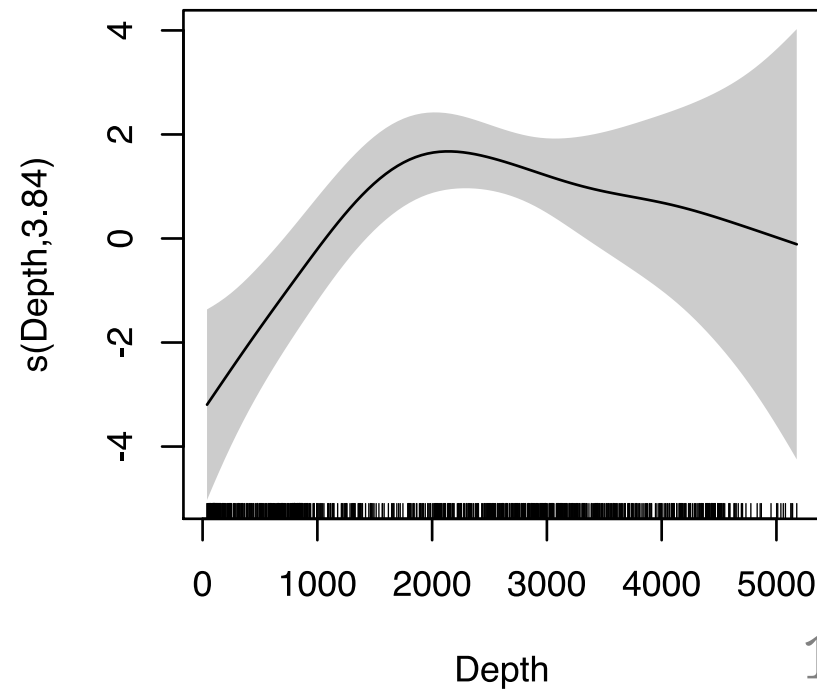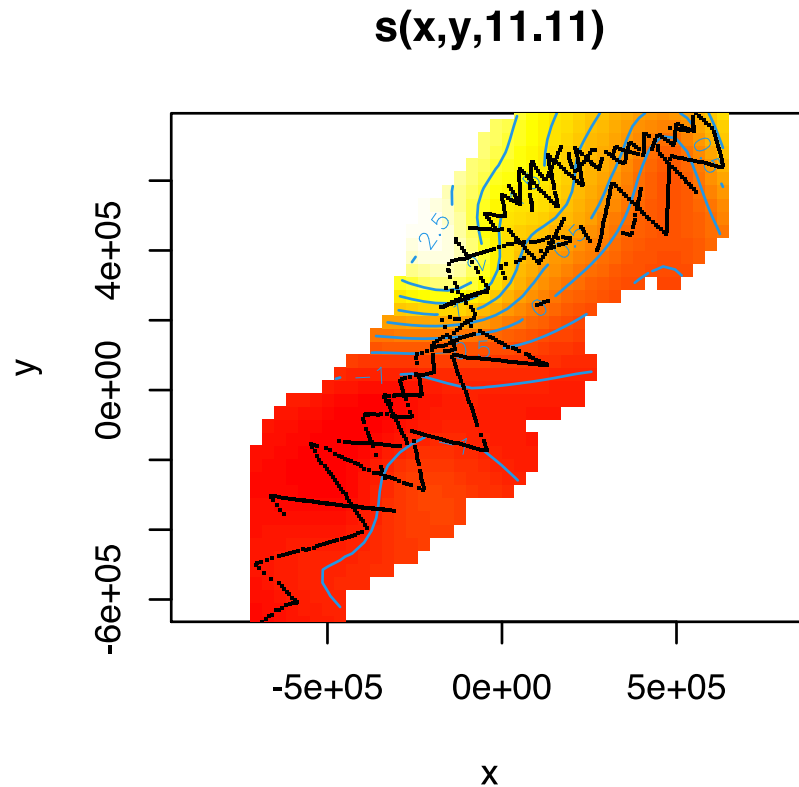# What about these "s" things?

- *Think s=**smooth***

- Want a line that is "close" to all the data

- Balance between interpolation and "fit"
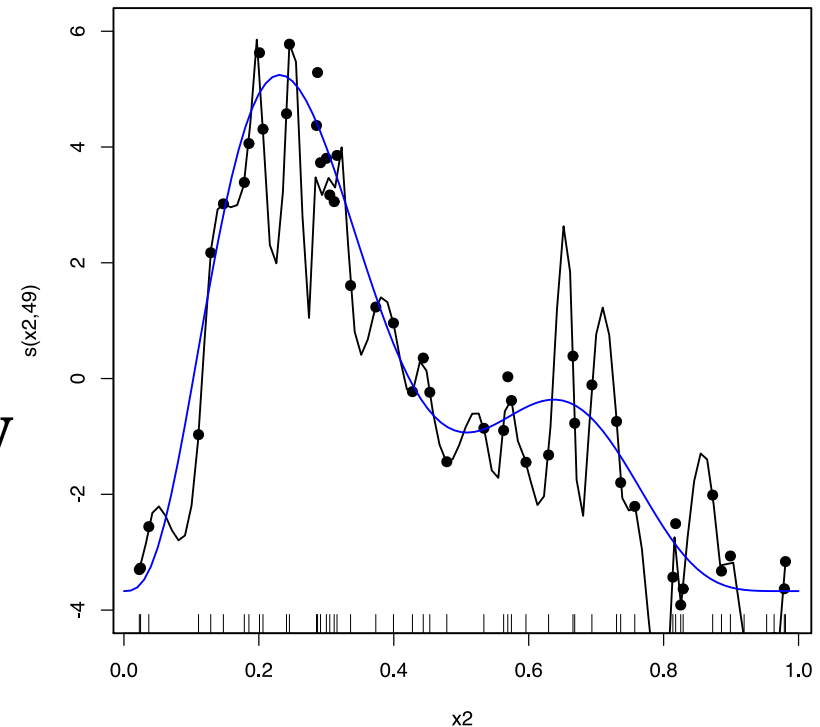
What is smoothing?

Smoothing

- We think underlying phenomenon is *smooth*
    - "Abundance is a smooth function of depth"
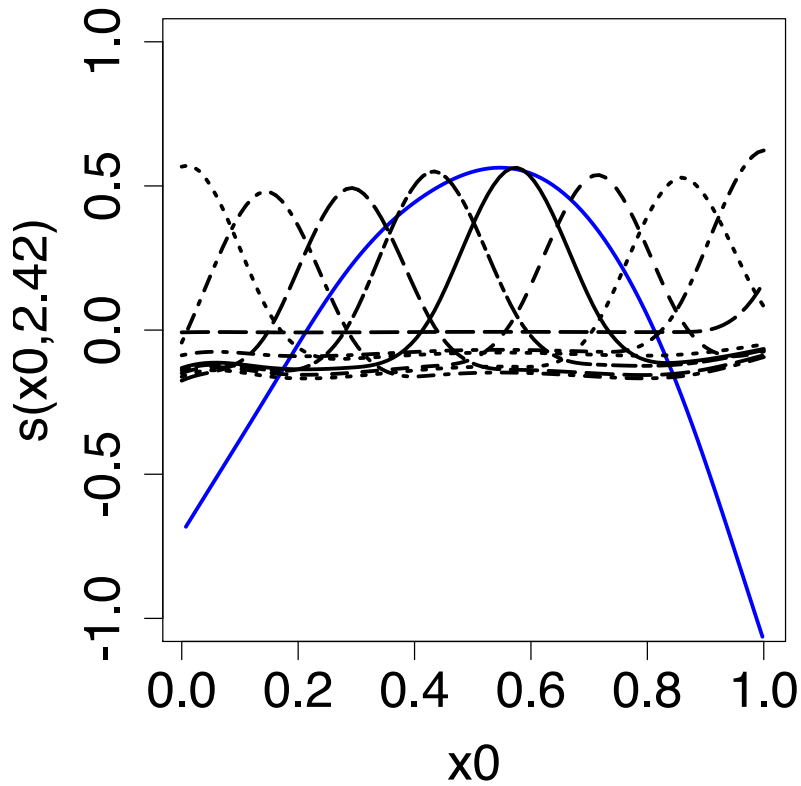
- 1, 2 or more dimensions

Smoothing

- Estimate smooths

- We set:
  - "type": *bases* (made up of *basis functions*)

  - basis size/dimension/complexity -- *maximum wigglyness*
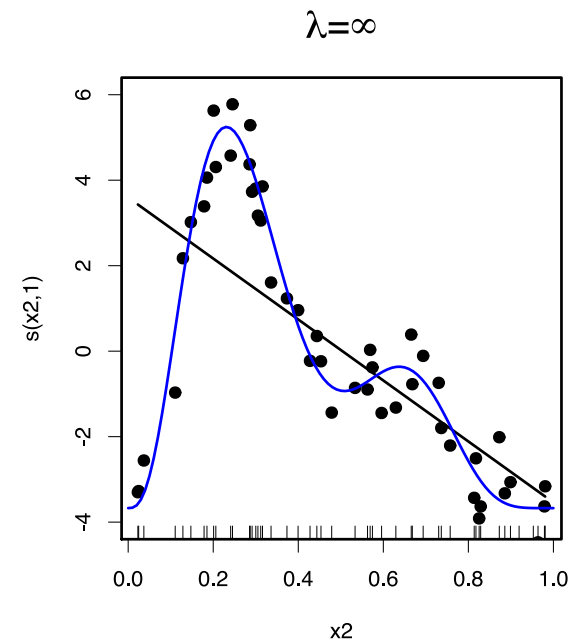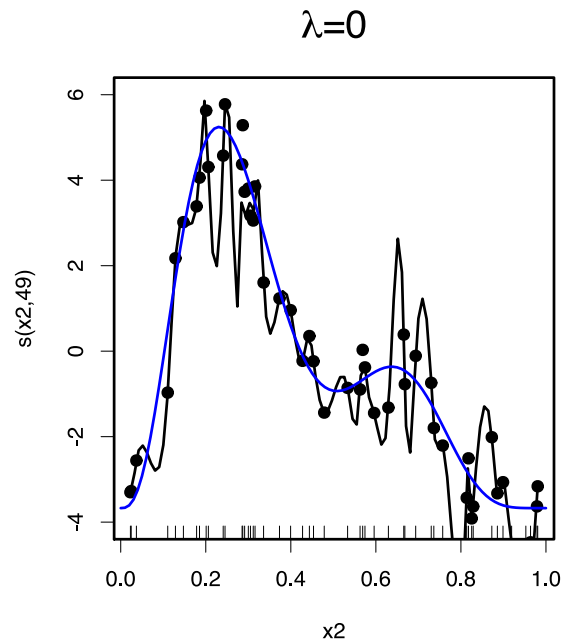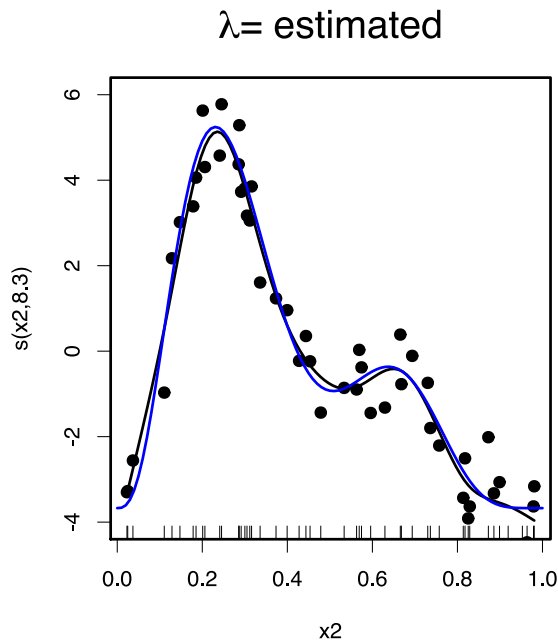
- Automatically
  - *estimate how wiggly it needs to be*

Splines

- Functions made of other, simpler functions

- **Basis functions** b_k, estimate β_k

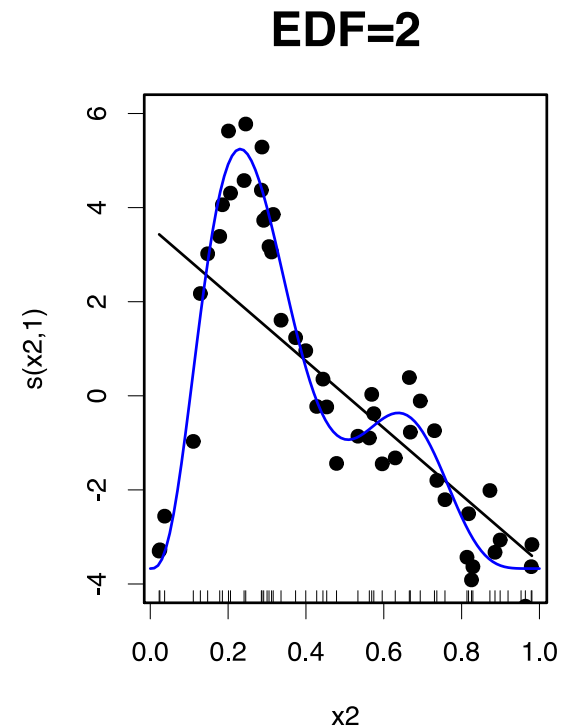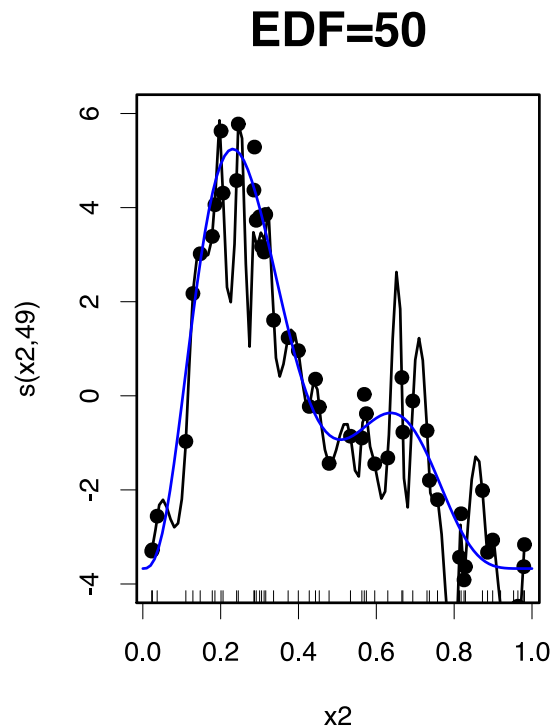- $s(x) = \sum_{k=1}^{K} \beta_k b_k(x)$

Measuring wigglyness

- Visually:
  - Lots of wiggles $\Rightarrow$ *not smooth*
  - Straight line $\Rightarrow$ *very smooth*

How wiggly are things?

- Set **basis complexity** or "size" k

- Fitted smooths have **effective degrees of freedom** (EDF)

- Set k "large enough"
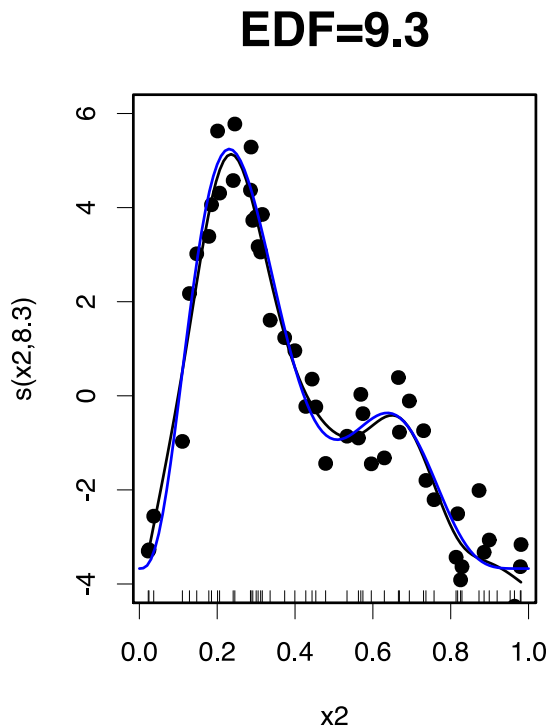
Getting more out of GAMs

- I can't teach you all of GAMs in 1 week

- Good intro book

- (also a good textbook on GLMs and GLMMs)

- Quite technical in places

- More resources on course website

Fitting GAMs using dsm

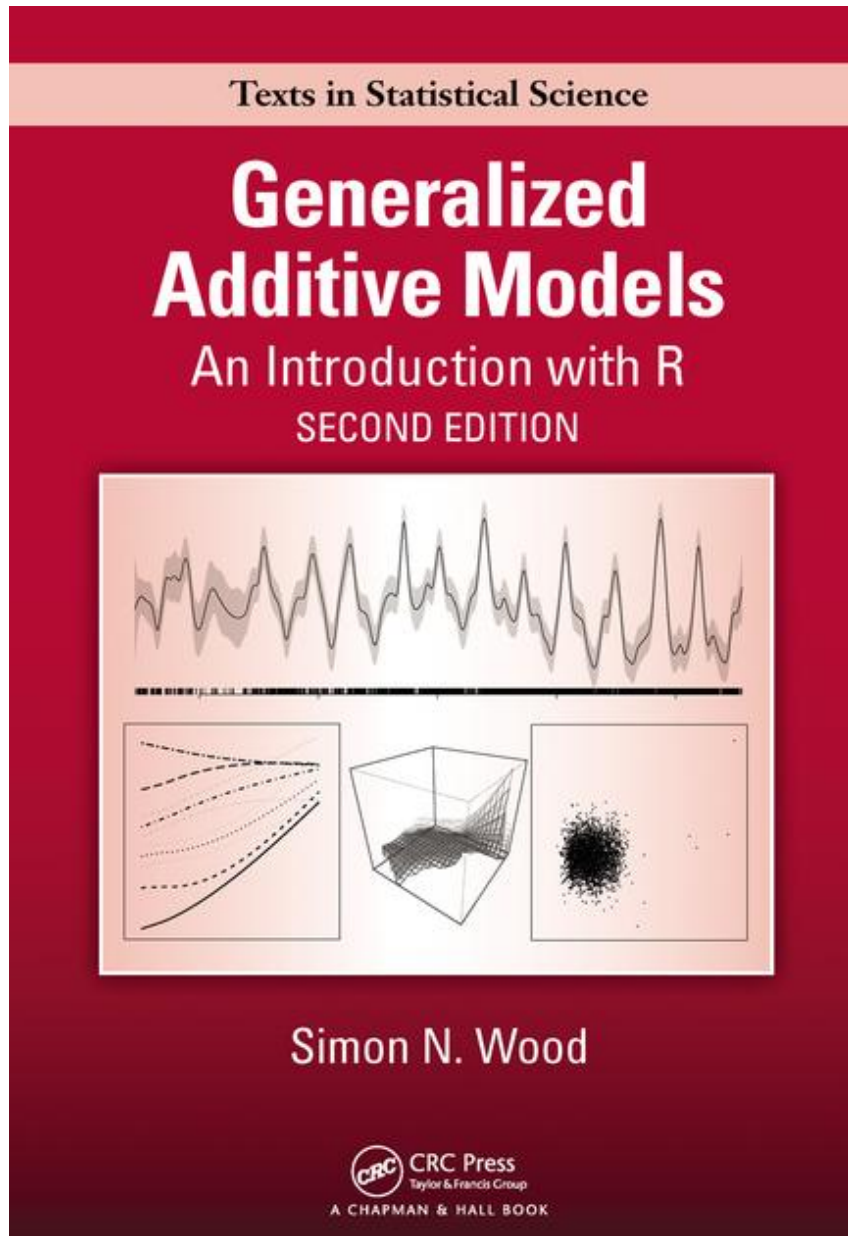# Translating maths into R

$$n_j = A_j \hat{p}_j \exp\left[\beta_0 + s(y_j)\right] + \epsilon_j$$

where ϵ_j are some errors, $n_j \sim$ count distribution

- inside the link: `formula=count ~ s(y)`

- response distribution: `family=nb()` or `family=tw()`

- detectability: `ddf.obj=df_hr`

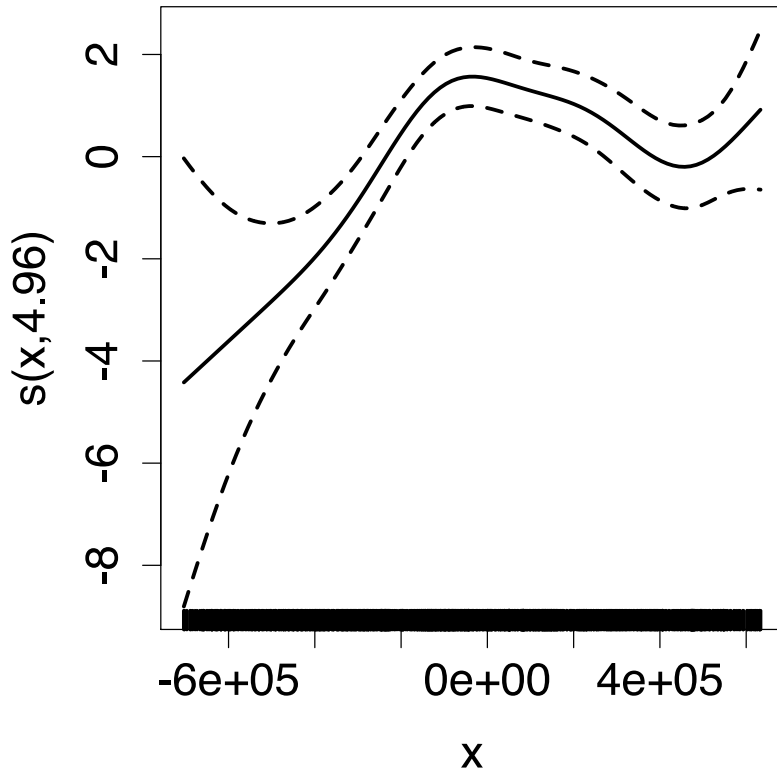- offset, data: `segment.data=segs,`
  `observation.data=obs`

Your first DSM

```
library(dsm)
dsm_x_tw <- dsm(count~s(x), ddf.obj=df,
                segment.data=segs, observation.data=obs,
                family=tw())
```

dsm is based on `mgcv` by Simon Wood

summary(dsm_x_tw)

```
##
## Family: Tweedie(p=1.326)
## Link function: log
##
## Formula:
## count ~ s(x) + offset(off.set)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -19.8115     0.2277  -87.01   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##        edf Ref.df     F  p-value
## s(x) 4.962  6.047 6.403 1.07e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.0283   Deviance explained = 17.9%
## -REML = 409.94  Scale est. = 6.0413    n = 949
```

Plotting

- `plot(dsm_x_tw)`
- Dashed lines indicate +/- 2 standard errors
- Rug plot
- On the link scale
- EDF on y axis

Adding a term

- Just use +

```
dsm_xy_tw <- dsm(count ~ s(x) + s(y),
                 ddf.obj=df,
                 segment.data=segs,
                 observation.data=obs,
                 family=tw())
```
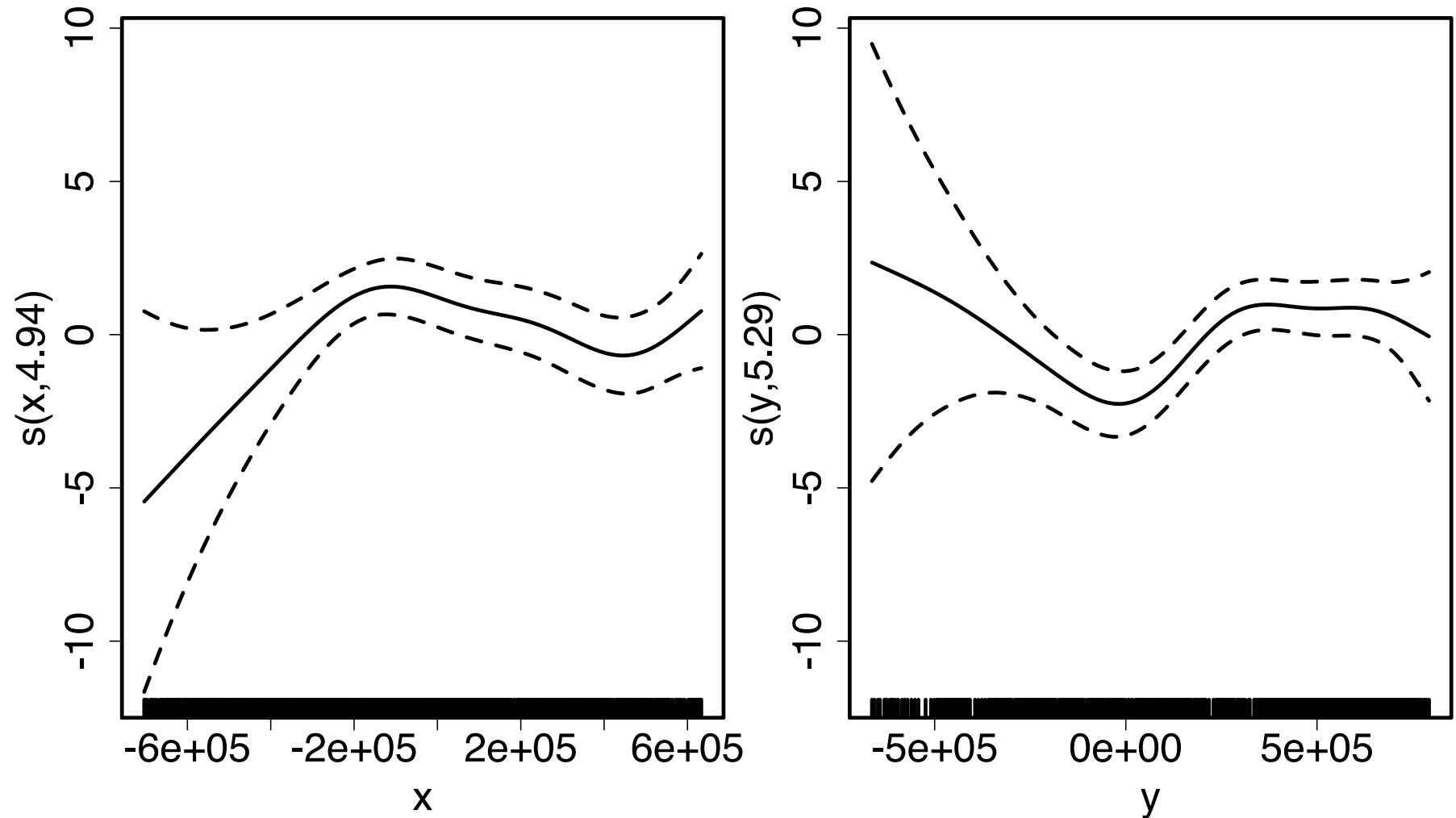
summary(dsm_xy_tw)

```
##
## Family: Tweedie(p=1.306)
## Link function: log
##
## Formula:
## count ~ s(x) + s(y) + offset(off.set)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -20.0908     0.2381  -84.39   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##        edf Ref.df     F  p-value
## s(x) 4.943  6.057 3.224 0.004239 **
## s(y) 5.293  6.419 4.034 0.000322 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.0678   Deviance explained = 27.4%
## -REML = 399.84  Scale est. = 5.3157    n = 949
```

Plotting

```
plot(dsm_xy_tw, pages=1)
```

Bivariate terms

- Assumed an additive structure

- No interaction

- We can specify `s(x,y)` (and `s(x,y,z,...)`)

Bivariate spatial term

```
dsm_xyb_tw <- dsm(count ~ s(x, y),
                  ddf.obj=df,
                  segment.data=segs,
                  observation.data=obs,
                  family=tw())
```

summary(dsm_xyb_tw)

```
##
## Family: Tweedie(p=1.29)
## Link function: log
##
## Formula:
## count ~ s(x, y) + offset(off.set)
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -20.2745     0.2477  -81.85   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##           edf Ref.df     F  p-value
## s(x,y) 16.89  21.12 4.333 3.73e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.102   Deviance explained = 34.7%
## -REML = 394.86  Scale est. = 4.8248    n = 949
```
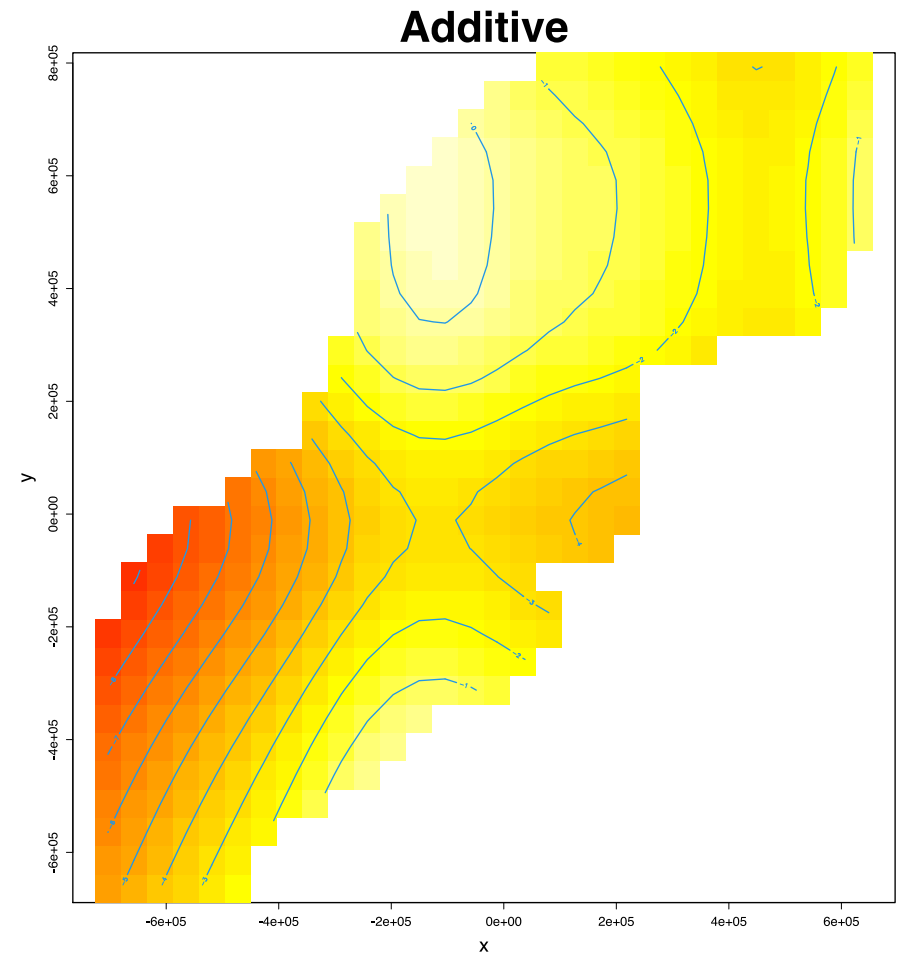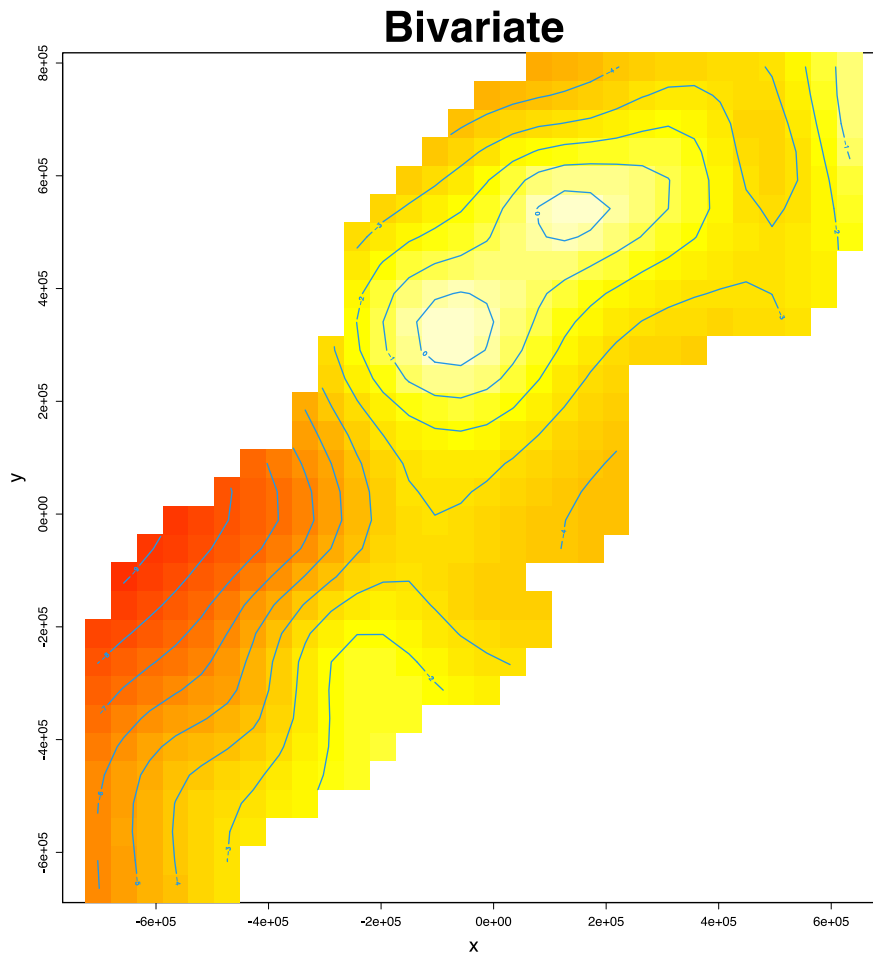
Plotting

```
plot(dsm_xyb_tw, select=1,
      scheme=2, asp=1)
```

- On link scale

- `scheme=2` makes heatmap

- (set `too.far` to exclude points far from data)

s(x,y,16.89)
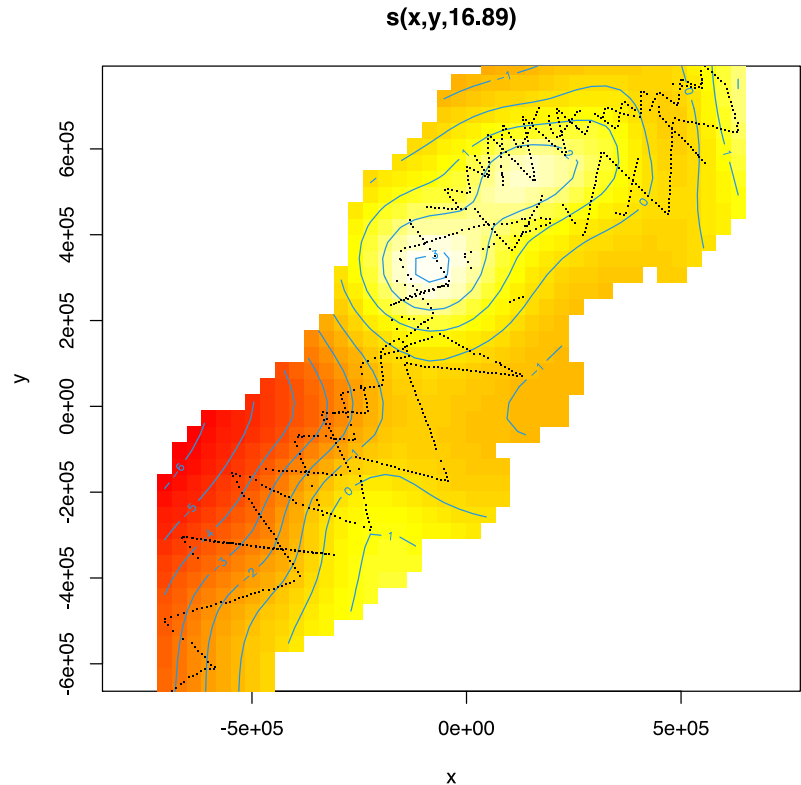
Comparing bivariate and additive models

Let's have a go...