

$$\begin{array}{c}
\overline{p(x) \vdash [p(x)] \cdot p(x)} \\
\overline{p(x) \vdash \mid p(x) \cdot \cdot} \\
\vdash p(x)^\perp \mid p(x) \cdot \cdot \\
\overline{\vdash p(x), p(x)^\perp \mid \cdot \cdot} \\
\vdash (p(x))\vee^-(p(x)^\perp) \mid \cdot \cdot
\end{array}$$

$$\begin{array}{c}
\overline{p(x) \vdash [p(x)] \cdot \cdot (p(x))\vee^+(p(x)^\perp)} \\
\overline{p(x) \vdash [(p(x))\vee^+(p(x)^\perp)] \cdot \cdot (p(x))\vee^+(p(x)^\perp)} \\
\overline{p(x) \vdash \mid (p(x))\vee^+(p(x)^\perp) \cdot \cdot} \\
\vdash p(x)^\perp \mid \cdot \cdot (p(x))\vee^+(p(x)^\perp) \\
\overline{\vdash [p(x)^\perp] \cdot \cdot (p(x))\vee^+(p(x)^\perp)} \\
\vdash [(p(x))\vee^+(p(x)^\perp)] \cdot \cdot (p(x))\vee^+(p(x)^\perp) \\
\overline{\vdash \mid (p(x))\vee^+(p(x)^\perp) \cdot \cdot} \\
\vdash (p(x))\vee^+(p(x)^\perp) \mid \cdot \cdot
\end{array}$$

$$\begin{array}{c}
\overline{p() \vdash [p()] \cdot \cdot (p())^\perp \vee^+(p())} \\
\overline{p() \vdash [(p())^\perp \vee^+(p())] \cdot \cdot (p())^\perp \vee^+(p())} \\
\overline{p() \vdash \mid (p())^\perp \vee^+(p()) \cdot \cdot} \\
\vdash p()^\perp \mid \cdot \cdot (p())^\perp \vee^+(p()) \\
\overline{\vdash [p()^\perp] \cdot \cdot (p())^\perp \vee^+(p())} \\
\vdash [(p())^\perp \vee^+(p())] \cdot \cdot (p())^\perp \vee^+(p()) \\
\overline{\vdash \mid (p())^\perp \vee^+(p()) \cdot \cdot} \\
\vdash (p())^\perp \vee^+(p()) \mid \cdot \cdot
\end{array}$$

$$\begin{array}{c}
\overline{a() \vdash [a()]b() \cdot \cdot a()} \quad \overline{b() \vdash [b()]a() \cdot \cdot b()} \\
\overline{a() \vdash \mid a() \cdot b() \cdot} \quad \overline{b() \vdash \mid b(), a() \cdot \cdot} \\
\vdash a()^\perp \mid b(), a() \cdot \cdot \quad \vdash b()^\perp \mid b(), a() \cdot \cdot \\
\vdash (a()^\perp) \wedge^-(b()^\perp) \mid b(), a() \cdot \cdot \\
\vdash b(), (a()^\perp) \wedge^-(b()^\perp) \mid a() \cdot \cdot \\
\vdash a(), b(), (a()^\perp) \wedge^-(b()^\perp) \mid \cdot \cdot \\
\vdash (a())\vee^-(b()), (a()^\perp) \wedge^-(b()^\perp) \mid \cdot \cdot \\
\vdash ((a())\vee^-(b()))\vee^-(a()^\perp \wedge^-(b()^\perp)) \mid \cdot \cdot
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\frac{a() \vdash [a()] \cdot \cdot ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))}{a() \vdash [(a()) \vee^+(b())] \cdot \cdot ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))}}{a() \vdash [((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))] \cdot \cdot ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))} \\
\frac{a() \vdash \mid ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp)) \cdot \cdot}{\vdash a()^\perp \mid \cdot \cdot ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))} \\
\hline
\frac{\frac{\vdash (a()^\perp) \wedge^-(b()^\perp) \mid \cdot \cdot ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))}{\vdash [(a()^\perp) \wedge^-(b()^\perp)] \cdot \cdot ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))}}{\vdash [((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))] \cdot \cdot ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp))} \\
\frac{\vdash \mid ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp)) \cdot \cdot}{\vdash ((a()) \vee^+(b())) \vee^+((a()^\perp) \wedge^-(b()^\perp)) \mid \cdot \cdot}
\end{array}$$

FAIL

**Definition 1 (Formulae)**

$$\begin{aligned} P &= p \mid A \wedge^+ B \mid A \vee^+ B \\ N &= p^\perp \mid A \wedge^- B \mid A \vee^- B \\ A, B &= P \mid N \end{aligned}$$

**Definition 2 (System)**

$\frac{\Gamma \vdash [A]\Delta \quad \Gamma \vdash [B]\Delta}{\Gamma \vdash [A \wedge^+ B]\Delta} \quad \frac{\Gamma \vdash [A_i]\Delta}{\Gamma \vdash [A_1 \vee^+ A_2]\Delta}$ $\frac{}{\Gamma, p \vdash [p]\Delta}$ $\frac{\Gamma \vdash N \mid \Delta}{\Gamma \vdash [N]\Delta} \text{ } N \text{ negative}$	
$\frac{\Gamma \vdash A, \Pi \mid \Delta \quad \Gamma \vdash B, \Pi \mid \Delta}{\Gamma \vdash A \wedge^- B, \Pi \mid \Delta} \quad \frac{\Gamma \vdash A_1, A_2, \Pi \mid \Delta}{\Gamma \vdash A_1 \vee^- A_2, \Pi \mid \Delta}$	$\frac{\Gamma \vdash \Pi \mid \Delta, P}{\Gamma \vdash P, \Pi \mid} P \text{ positive} \quad \frac{\Gamma, p \vdash \Pi \mid \Delta}{\Gamma \vdash p^\perp, \Pi \mid} p^\perp \text{ negative atom}$
$\frac{\Gamma \vdash [P]\Delta, P}{\Gamma \vdash \mid \Delta, P}$	