

$$\begin{array}{c}
\frac{}{\cdot p(x) \vdash [p(x)] \cdot \mid p(x)} \\
\frac{}{\cdot p(x) \vdash \cdot \mid \cdot p(x) \mid} \\
\frac{}{\cdot \vdash \cdot p(x)^\perp \mid \cdot p(x) \mid} \\
\frac{}{\cdot \vdash \cdot p(x), p(x)^\perp \mid \cdot \mid} \\
\frac{}{\cdot \vdash \cdot (p(x))\vee^-(p(x)^\perp) \mid \cdot \mid}
\end{array}$$

$$\begin{array}{c}
\frac{}{\cdot p(x) \vdash [p(x)] \cdot \mid (p(x))\vee^+(p(x)^\perp)} \\
\frac{}{\cdot p(x) \vdash [(p(x))\vee^+(p(x)^\perp)] \cdot \mid (p(x))\vee^+(p(x)^\perp)} \\
\frac{}{\cdot p(x) \vdash \cdot \mid \cdot (p(x))\vee^+(p(x)^\perp) \mid} \\
\frac{}{\cdot \vdash \cdot p(x)^\perp \mid \cdot \mid (p(x))\vee^+(p(x)^\perp)} \\
\frac{}{\cdot \vdash [p(x)^\perp] \cdot \mid (p(x))\vee^+(p(x)^\perp)} \\
\frac{}{\cdot \vdash [(p(x))\vee^+(p(x)^\perp)] \cdot \mid (p(x))\vee^+(p(x)^\perp)} \\
\frac{}{\cdot \vdash \cdot \mid \cdot (p(x))\vee^+(p(x)^\perp) \mid} \\
\frac{}{\cdot \vdash \cdot (p(x))\vee^+(p(x)^\perp) \mid \cdot \mid}
\end{array}$$

$$\begin{array}{c}
\frac{}{\cdot p() \vdash [p()] \cdot \mid (p()^\perp)\vee^+(p())} \\
\frac{}{\cdot p() \vdash [(p()^\perp)\vee^+(p())] \cdot \mid (p()^\perp)\vee^+(p())} \\
\frac{}{\cdot p() \vdash \cdot \mid \cdot (p()^\perp)\vee^+(p()) \mid} \\
\frac{}{\cdot \vdash \cdot p()^\perp \mid \cdot \mid (p()^\perp)\vee^+(p())} \\
\frac{}{\cdot \vdash [p()^\perp] \cdot \mid (p()^\perp)\vee^+(p())} \\
\frac{}{\cdot \vdash [(p()^\perp)\vee^+(p())] \cdot \mid (p()^\perp)\vee^+(p())} \\
\frac{}{\cdot \vdash \cdot \mid \cdot (p()^\perp)\vee^+(p()) \mid} \\
\frac{}{\cdot \vdash \cdot (p()^\perp)\vee^+(p()) \mid \cdot \mid}
\end{array}$$

$$\begin{array}{cc}
\frac{}{\cdot a() \vdash [a()] \cdot b() \mid a()} & \frac{}{\cdot b() \vdash [b()] \cdot a() \mid b()} \\
\frac{}{\cdot a() \vdash \cdot \mid b() \cdot a() \mid} & \frac{}{\cdot b() \vdash \cdot \mid \cdot b(), a() \mid} \\
\frac{}{\cdot \vdash \cdot a()^\perp \mid \cdot b(), a() \mid} & \frac{}{\cdot \vdash \cdot b()^\perp \mid \cdot b(), a() \mid} \\
\frac{}{\cdot \vdash \cdot b(), a()^\perp \mid \cdot a() \mid} & \frac{}{\cdot \vdash \cdot b(), b()^\perp \mid \cdot a() \mid} \\
\frac{}{\cdot \vdash \cdot a(), b(), a()^\perp \mid \cdot \mid} & \frac{}{\cdot \vdash \cdot a(), b(), b()^\perp \mid \cdot \mid} \\
\hline
\frac{}{\cdot \vdash a(), b() \cdot (a()^\perp)\wedge^-(b()^\perp) \mid \cdot \mid} & \\
\frac{}{\cdot \vdash \cdot (a())\vee^-(b()), (a()^\perp)\wedge^-(b()^\perp) \mid \cdot \mid} & \\
\frac{}{\cdot \vdash \cdot ((a())\vee^-(b()))\vee^-((a()^\perp)\wedge^-(b()^\perp)) \mid \cdot \mid} &
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\frac{\cdot a() \vdash [a()] \cdot \mid ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))}{\cdot a() \vdash [(a())\vee^+(b())] \cdot \mid ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))}}{\cdot a() \vdash [((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))] \cdot \mid ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))} \\
\frac{\cdot a() \vdash \cdot \mid \cdot ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp)) \mid}{\cdot \vdash \cdot a()^\perp \mid \cdot \mid ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))} \\
\hline
\frac{\cdot \vdash \cdot (a()^\perp)\wedge^-(b()^\perp) \mid \cdot \mid ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))}{\cdot \vdash [(a()^\perp)\wedge^-(b()^\perp)] \cdot \mid ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))} \\
\frac{\cdot \vdash [((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))] \cdot \mid ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp))}{\cdot \vdash \cdot \mid \cdot ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp)) \mid} \\
\frac{\cdot \vdash \cdot \mid \cdot ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp)) \mid}{\cdot \vdash \cdot ((a())\vee^+(b()))\vee^+((a()^\perp)\wedge^-(b()^\perp)) \mid \cdot \mid}
\end{array}$$

FAIL

Definition 1 (Formulae)

$$\begin{aligned} P &= p \mid A \wedge^+ B \mid A \vee^+ B \\ N &= p^\perp \mid A \wedge^- B \mid A \vee^- B \\ A, B &= P \mid N \end{aligned}$$

Definition 2 (Negation)

$(p)^\perp$	$:= p^\perp$	$(p^\perp)^\perp$	$:= p$
$(A \wedge^+ B)^\perp$	$:= A^\perp \vee^- B^\perp$	$(A \wedge^- B)^\perp$	$:= A^\perp \vee^+ B^\perp$
$(A \vee^+ B)^\perp$	$:= A^\perp \wedge^- B^\perp$	$(A \vee^- B)^\perp$	$:= A^\perp \wedge^+ B^\perp$

Definition 3 (System)

$\frac{\Gamma \vdash [A]\Delta \quad \Gamma \vdash [B]\Delta}{\Gamma \vdash [A \wedge^+ B]\Delta} \quad \frac{\Gamma \vdash [A_i]\Delta}{\Gamma \vdash [A_1 \vee^+ A_2]\Delta}$ $\overline{\Gamma, p \vdash [p]\Delta}$ $\frac{\Gamma \vdash N \mid \Delta}{\Gamma \vdash [N]\Delta} N \text{ negative}$
$\frac{\Gamma \vdash A, \Pi \mid \Delta \quad \Gamma \vdash B, \Pi \mid \Delta}{\Gamma \vdash A \wedge^- B, \Pi \mid \Delta} \quad \frac{\Gamma \vdash A_1, A_2, \Pi \mid \Delta}{\Gamma \vdash A_1 \vee^- A_2, \Pi \mid \Delta}$ $\frac{\Gamma \vdash \Pi \mid \Delta, P}{\Gamma \vdash P, \Pi \mid} P \text{ positive} \quad \frac{\Gamma, p \vdash \Pi \mid \Delta}{\Gamma \vdash p^\perp, \Pi \mid} p^\perp \text{ negative atom}$
$\frac{\Gamma \vdash [P]\Delta, P}{\Gamma \vdash \mid \Delta, P}$