Derivation of Vieta's theorem for solving quadratic equations

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1 Introduction

Most solutions to quadratic equations rely on the application of the discriminant. Although this formula proves to be an effective tool for determining roots, the question arises: is there a simpler approach? This research proposes to use the derivation of Vieta's theorem as an alternative method and aims to provide a more accessible approach to solving quadratic equations.

2 Derivation

We have a system like this

$$\begin{cases} x_1 + x_2 = r_1 \\ x_1 x_2 = r_2 \end{cases}$$

Let $\frac{x_1}{x_2^{-1}} = r_2$, then

$$\begin{cases} x_1 + \frac{1}{x_2^{-1}} = r_1 \\ \frac{x_1}{x_2^{-1}} = r_2 \end{cases}$$

Now we need to express x_1 through x_2, then x_1 will look like this

$$x_1 = \frac{r_2}{x_2^{-1}}$$

We substitute this new expression in place of x_1 in the first expression of the system and solve the equation

$$\begin{aligned} r_1 &= \frac{r_2 + 1}{x_2^{-1}} \\ r_1 x_2^{-1} &= r_2 + 1 \\ x_2 &= \frac{r_1}{r_2 + 1} \end{aligned} \tag{1}$$

Finally, to get the formula for x_1, we divide r_2 by x_2, yielding the following system of formulas (here x_1 and x_2 are swapped, but it makes no difference)

$$\begin{aligned} x_1 &= \frac{r_1}{r_2+1} \\ x_2 &= \frac{r_2}{x_1} \end{aligned} \tag{2}$$

Now we can reduce these formulas, and we get the following

$$\begin{aligned} x_1 &= \frac{-\frac{b}{a}}{\frac{c+a}{a}} = (-\frac{b}{a})(\frac{a}{c+a}) = -\frac{b}{c+a} \\ x_2 &= \frac{\frac{c}{a}}{-\frac{b}{c+a}} = (\frac{c}{a})(-\frac{c+a}{b}) = -\frac{c(c+a)}{ab} \end{aligned} \tag{3}$$

3 Conclusion

Thus, we were able to obtain two formulas that allow us to calculate the roots of a quadratic equation easily and simply. We hope that this work will inspire further research in this area and contribute to a better understanding of quadratic equations and their properties. We thank for attention to our work and encourage discussion of the results presented.