Engineering Mathematics - I.

Limits.

$x \to a$.

$x \to a^+$ | $x \to a^-$
$a + 0.1$ | $a - 0.01$
$a + 0.01$ | $a - 0.001$

$x \to a^-$ $\quad a \quad$ $\underset{a^+}{\leftarrow} x$

Q. Evaluate.

$$\lim_{x \to 1} 2^{\frac{1}{x-1}}$$

RHL.

$$\lim_{h \to 0} 2^{\frac{1}{(h+1)-1}}$$

$$\lim_{h \to 0} 2^{\frac{1}{\cancel{1}+h-\cancel{1}}}$$

$$2^{\frac{1}{0}}$$

$$= \infty$$

LHL.

$$\lim_{x \to 1^-} 2^{\frac{1}{x-1}}$$

$$\lim_{h \to 0} 2^{\frac{1}{\cancel{1}-h-\cancel{1}}}$$

$$2^{-\infty} = \frac{1}{2^\infty} = \frac{1}{\infty}$$

$$= \underline{0}$$

LHL $\neq$ RHL $\quad \therefore$ Limit does not exist.

ii). $$\lim_{x \to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1}$$

RHL

$$\lim_{x \to 0^+} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1}$$

$$\lim_{h \to 0} \frac{e^{\frac{1}{0+h}}}{e^{\frac{1}{0+h}} + 1}$$

$$\Rightarrow \lim_{h \to 0} \frac{\cancel{e^{\frac{1}{h}}}}{\cancel{e^{\frac{1}{h}}}\left(1 + e^{-\frac{1}{h}}\right)}$$

$$\Rightarrow \lim_{h \to 0} \frac{1}{1 + e^{-\frac{1}{h}}}$$

$$\Rightarrow \frac{1}{1 + e^{-\infty}} = \underline{1}$$

$$\frac{1}{e^\infty} = 0$$

Date.
Page No.

iii) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 3x-2, & x<1 \\ 4x^2-3x, & x \geq 1 \end{cases}$

RHL

$\lim_{x \to 1^+} 4x^2 - 3x$.

$\Rightarrow \lim_{h \to 0} 4(1+h)^2 - 3(1+h)$

$= 4(1)^2 - 3(1)$

$= 1$

LHL

$\lim_{x \to 1^-} 3x-2$.

$\Rightarrow \lim_{h \to 0} 3(1-h) - 2$.

$= 3(1) - 2$

$= 1$.

LHL = RHL.

iv) $\lim_{x \to 0} f(x)$ $\quad f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

RHL.

$\lim_{x \to 0^+} \frac{x - |x|}{x}$.

$\Rightarrow \lim_{h \to 0} \frac{0+h - |0+h|}{0+h}$

$\frac{h-h}{h} = 0$.

LHL

$\lim_{x \to 0} \frac{x - |x|}{x}$

$\Rightarrow \lim_{h \to 0} \frac{(0-h) - |(0-h)|}{0-h}$.

$\lim_{h \to 0} \frac{-h-h}{-h} = \frac{+2h}{+h}$

$= 2$.

Q Continuity:- A function $f(x)$ is said to be continuous at $x=a$ if it satisfies three conditions.

i). $f(x)$ should be well defined at $x=a$

ii) $\lim_{x \to a} f(x)$ exists

iii) $\lim_{x \to a} f(x) = f(a)$.

Date. ______
Page No. ______

Q. A function $f(x)$ is defined as,

$$f(x) = \begin{cases} x^2, & 0 < x < 1 \\ x, & 1 \leq x < 2 \\ \frac{1}{4}x^2, & 2 \leq x < 3 \end{cases}$$

Check its continuity at $x=1$ and 2.

Solⁿ. At $x=1$

For continuity

i) $f(1) = 1$

ii) $\lim\limits_{x \to 1} f(x)$

RHL.

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1+h) \Rightarrow \lim\limits_{h \to 0} (1+h)$

$= 1.$

LHL

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} (1-h^2)^2.$

$= 1.$

$\therefore$ limit exists.

$\therefore f(x)$ is continuous at $x=1$.

At $x=2$.

For continuity,

i) $f(2) = \frac{1}{4}x^2 = \frac{1}{4} \times 4 = 1.$

ii) RHL.

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{h \to 0} (2+h) \Rightarrow \lim\limits_{h \to 0} \frac{1}{4}(2+h)^2$

$= \frac{1}{4} \times 4 = 1.$