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Successive Differentiation

$y = f(x)$

$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots \frac{d^ny}{dx^n}$

$Dy, D^2y, \dots D^ny$

$y', y'', \dots y^{n}$

$y_1, y_2, \dots y_n$

Calculation of n^{th} derivative

i) x^m

Let $y = x^m$

$y_1 = mx^{m-1}$

$y_2 = m(m-1)x^{m-2}$

$y_3 = m(m-1)(m-2)x^{m-3}$

$\vdots$

$$y_n = m(m-1)(m-2)\dots\{m-(n-1)\}x^{m-n}$$

$m = n.$

$= D^m x^m = m(m-1)(m-2)\dots\{m-m+1\}x^{m-m}$

$= m(m-1)(m-2)\dots 1$

$= m!$

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2. e^{ax}. Let $y = e^{ax}$

$y_1 = ae^{ax}$

$y_2 = a^2 e^{ax}$

$y_3 = a^3 e^{ax}$

$\vdots$

$y_n = a^n e^{ax}$

$$D^n e^{ax} = a^n e^{ax}$$

3. a^{mx}. Let $y = a^{mx}$

$y_1 = m a^{mx} (\log a) = (m \log a)\, a^{mx}$. ($m \log a$: constant)

$y_2 = (m \log a)^2 a^{mx}$.

$y_3 = (m \log a)^3 a^{mx}$.

$\vdots$

$y_n = (m \log a)^n a^{mx}$

$$D^n a^{mx} = (m \log a)^n a^{mx}$$

4. $\frac{1}{ax+b} = (ax+b)^{-1}$ = let y

$y_1 = (-1)(ax+b)^{-2} \cdot a$

$y_2 = (-1)(-2)(ax+b)^{-3} a^2$

$y_3 = (-1)(-2)(-3)(ax+b)^{-4} a^3$.

$\vdots$

$y_n = (-1)^n n! (ax+b)^{-(n+1)} a^n$

$$D^n\left(\frac{1}{ax+b}\right) = \frac{(-1)^n n! a^n}{(ax+b)^{(n+1)}}$$

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i) $D^9 x^{10}$

ii) $D^4 e^{mx}$.

iii) $D^3 2^{10x}$.

iv) $D^{30}(2x+3)^{-1}$

i) $10! x$

ii) $m^4 e^{mx}$

iii) $(10 \log 2)^3 2^{10x}$.

iv). $\frac{(-1)^{30} \cdot 30! \cdot 2^{30}}{(2x+3)^{31}}$

5. n^{th} derivative of $(ax+b)^m$

Let $y = (ax+b)^m$

$y_1 = m(ax+b)^{m-1} \cdot a$

$y_2 = m(m-1)(ax+b)^{m-2} \cdot a^2$

$y_3 = m(m-1)(m-2)(ax+b)^{m-3} a^3$

$\vdots$

$y_n = m(m-1)(m-2) \cdots [m-(n-1)](ax+b)^{m-n} \cdot a^n$

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6 $\log(ax+b)$ Let $y = \log(ax+b)$

$$y_1 = \frac{1}{ax+b} \cdot a$$

$$y_2 = \frac{(-1)}{(ax+b)^2} \cdot a^2$$

$$y_3 = \frac{(-1)(-2)}{(ax+b)^3} \cdot a^3$$

$$\vdots$$

$$y_n = \frac{(-1)^{n-1} (n-1)!}{(ax+b)^n} \cdot a^n$$

7 $\sin(ax+b)$ Let $y = \sin(ax+b)$

$$y_1 = a \cos(ax+b)$$
$$= a \sin\left(\frac{\pi}{2} + ax + b\right)$$

$$y_2 = a^2 \cos\left(\frac{\pi}{2} + ax + b\right)$$
$$= a^2 \sin\left(ax + b + \frac{2\pi}{2}\right)$$

$$y_3 = a^3 \cos\left(ax + b + \frac{2\pi}{2}\right)$$
$$= a^3 \sin\left(ax + b + \frac{3\pi}{2}\right)$$

$$\vdots$$

$$y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$$

$$D^n \cos(ax+b) = a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$$