DIT UNIVERSITY DEHRADUN

B.TECH (All) MID TERM EXAMINATION, ODD SEM 2022-23 (SEM I)

Roll No. | | | | | | | | | | |

Subject Name: Engg. Mathematics I

Time: 2 Hours **Total Marks: 50**

Note: All questions are compulsory. No student is allowed to leave the examination hall before the completion of the exam.

Q.1) Attempt all Parts :

(a) Evaluate the limit: $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$.

(b) Evaluate n^{th} derivative of the function $sin^2(2x)$.

(c) Find all the asymptotes of the curve $(2x+3)y = (x-1)^2$.

(d) Determine first order partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ of the function $u = y^2 e^{x/y}$ at the point (2, 4).

[4 x 2.5= 10]

Q.2) Attempt all Parts :

(a) Find the value of k for which $f(x) = \begin{cases} 2kx - 1 & when\ x \le 3 \\ x^2 + k & when\ x > 3 \end{cases}$ is continuous at $x = 3$.

(b) Find points of inflection and intervals in which curve is concave upward or concave downward for the function $y = x^4 - 2x^3$.

(c) Find n^{th} derivative i.e. y_n for the function $y = x^2 \log(x)$.

(d) For the function $u(x,y) = \frac{x^3+y^3}{x+y}$ determine the value of $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

[4 x 2.5= 10]

Q.3) Attempt any Two Parts :

(a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, show that $x^2 y_{n+2}(x) + (2n+1)xy_{n+1}(x) + 2n^2 y_n(x) = 0$.

(b) Expand $f(x) = log_e x$ in powers of $(x-1)$ upto degree 5 and hence evaluate $log_e(1.1)$.

(c) If $z = y\, f\,(x^2 - y^2)$ show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{xz}{y}$.

[2 x 5= 10]

Q.4) Attempt any Two Parts :

(a) Examine the continuity of the function $f(x) = \begin{cases} 1 + x^2 & when\ x \le 2 \\ 3x - 1 & when\ x > 2 \end{cases}$ at $x = 2$.

(b) Obtain the Taylor's series expansion of the maximum order for the function $f(x,y) = 2x^2 + y^2 - 3x - 6y + 12$ about the point $(2, 1)$.

(c) If $u(x,y,z) = z\,log(y) + y\,log(z) + xyz$
where $x = sin(t), y = t^2 + 1, z = cos^{-1}(t)$ then evaluate $\frac{du}{dt}$ at $t = 0$.

[2 x 5= 10]

Q.5) Attempt any Two Parts :

(a) Evaluate the limit: $\lim_{x\to 0}\left[\frac{\pi}{4x} - \frac{\pi}{2x(e^{\pi x}+1)}\right]$.

(b) If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

(c) For the function $y = \frac{x}{x^2+1}$, determine following:

(i) Critical points and interval of increasing and decreasing.

(ii) Point of maxima and minima.

[2 x 5= 10]

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