Practice:
Static and Kinematic Admissibility

1 Problem 1

A homogeneous continuum as in Fig. 1 is clamped on surface $S_3^-(z=0)$ and submitted to a uniform pressure p_0 over the surface $S_1^+(x=l_1)$.

Question 1: Write the static admissibility equations!

Question 2: Write the kinematic admissibility equations!

Question 3: Assuming we use the Cartesian coordinate system, please give the expansion of these equations!

Question 4: Calculate $\int_{S_3^-} \underline{\underline{\sigma}} \cdot \underline{n} dS$

2 Problem 2

Let us consider a sphere with cavity (exterior radius R_e and internal radius R_i). The external surface S_e is free of loading and the interior surface S_i is submitted to a pressure p_0. The material is assumed to be isotropic linear elastic with young modulus E and Poisson ratio ν.

Question 1: Write the static admissibility equations!

Question 2: Please expand this equation using spherical polar coordinates system!

Question 3: Let's assume that the following stress tensor

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{\varphi\varphi} \end{bmatrix} \tag{1}$$

with

$$\sigma_{rr} = -A\left[\frac{B}{r^3} - 1\right] \tag{2}$$

and

$$\sigma_{\theta\theta} = \sigma_{\varphi\varphi} = A\left[\frac{B}{2r^3} + 1\right] \tag{3}$$

Please calculate the coefficient A and B so that $\underline{\underline{\sigma}}$ could be a solution of the problem!

Question 4: Please calculate the strain tensor $\underline{\underline{\varepsilon}}$? Does it satisfy the compatibility equation? Conclusion?

3 Problem 3

Let us consider a domain Ω, simply supported by a rigid basis (Γ). Ω is submitted to gravity volume forces $\underline{f} = \rho g \underline{y}$ and to uniform pressure $p > 0$ over every vertical side. We assume plane stresses in the plane of $x - y$ and the material is isotropic linear elastic with Young modulus E and Poisson ratio ν.

Question 1: Write down the full problem! (kinematic, static and constitutive equations)

Question 2: We assume an Airy function $\phi(x, y) = ax^2 + cy^2 + hy^3$. Find a, c, h so we define a statistically admissible stress field.

Question 3: Find the related strain field using the constitutive equation. Does that satisfy the compatibility equation? Conclusion?

Question 4: Integrate the strain field to get the displacement at every point. Can we find the exact solution of the problem using this Airy function?

4 Problem 4

Let us consider a classical test in material engineering that make possible to identify the oedometric modelus $\hat{E}$. A cylinder material domain Ω is compressed due to the prescribed motion of the top surface $\underline{u} = -U\underline{e}_z$. All surfaces are assumed to be perfect contact without friction. Note that $\underline{P} = -P\underline{e}_z$ the resulting force that is necessary to press the materials (Young modulus E and Poisson ratio ν.

Question 1: Write the complete problem: kinematic + static + constitutive equation!

Question 2: Assume that $\underline{u} = -\frac{U}{h} z \underline{e}_z$. Is this kinematically admissible?

Question 3: Find the corresponding stress $\underline{\underline{\sigma}}$. Is it statistically admissible?

Question 4: Find oedometric modulus $\hat{E}$ as a function of E and ν.