## **Principal Stress**

The stress state for a material point is given in a Cartesian basis  $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$  by:

$$\underline{\underline{\sigma}} = \begin{bmatrix} 2 & 10 & 0 \\ 10 & 0 & 8 \\ 0 & 8 & 0 \end{bmatrix} \tag{1}$$

Question Calculate the principal stresses and the principal directions for this stress tensor.

**Answer:** The principal stresses are the Eigen values of the stress tensor.

$$\det(\underline{\underline{\sigma}} - \lambda \underline{\underline{1}}) = 0 \to \begin{vmatrix} 2 - \lambda & 10 & 0 \\ 10 & -\lambda & 8 \\ 0 & 8 & -\lambda \end{vmatrix} = 0 \tag{2}$$

Then,

$$\det(\underline{\sigma} - \lambda \underline{1}) = (2 - \lambda)(\lambda^2 - 64) + 100\lambda = -\lambda^3 + 2\lambda^2 + 164\lambda - 128 = 0$$
 (3)

The equation above also know as the characteristic equation. By solving the polynomial order 3 above, we found that the Eigenvalues are:  $\lambda_1=-12.246$  MPa,  $\lambda_2=0.776$  MPa,  $\lambda_3=13.470$  MPa.

The principal directions are the Eigenvectors of the stress tensor. Note that  $\underline{P}_I$  is associated with  $\lambda_I$  if  $\underline{\underline{\sigma}} \cdot \underline{P}_I = \lambda_I \underline{P}_I$ . So  $\underline{P}_I$  is a solution of  $(\underline{\underline{\sigma}} - \lambda \underline{\underline{1}})\underline{P}_I = 0$ . By convention, we usually always normalize  $\underline{P}_I$  as a unit vector so  $||\underline{P}_I|| = 1$