PROBLEM

3.70 Given the following right Cauchy-Green deformation tensor at a point

$$[\mathbf{C}] = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0.36 \end{bmatrix}.$$

(a) Find the stretch for the material elements that were in the direction of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$.
(b) Find the stretch for the material element that was in the direction of $\mathbf{e}_1 + \mathbf{e}_2$.
(c) Find $\cos\theta$, where θ is the angle between $d\mathbf{x}^{(1)}$ and $d\mathbf{x}^{(2)}$ and where $d\mathbf{X}^{(1)} = dS_1\mathbf{e}_1$ and $d\mathbf{X}^{(2)} = dS_2\mathbf{e}_1$ deform into $d\mathbf{x}^{(1)} = ds_1\mathbf{m}$ and $d\mathbf{x}^{(2)} = ds_2\mathbf{n}$.

3.71 Given the following large shear deformation:

$$x_1 = X_1 + X_2, \quad x_2 = X_2, \quad x_3 = X_3.$$

(a) Find the stretch tensor $\mathbf{U}$ (*hint:* use the formula given in Prob. 3.68) and verify that $\mathbf{U}^2 = \mathbf{C}$, the right Cauchy-Green deformation tensor.
(b) What is the stretch for the element that was in the direction $\mathbf{e}_2$?
(c) Find the stretch for an element that was in the direction of $\mathbf{e}_1 + \mathbf{e}_2$.
(d) What is the angle between the deformed elements of $dS_1\mathbf{e}_1$ and $dS_2\mathbf{e}_2$?

Some practice on tensorial calculus and manipulation.

PROBLEM

- We assume a continuum with homogeneous infinitesimal strains $\underline{\underline{\varepsilon}}$
- We define $H_\delta(\underline{x})$ the neighborhood of radius δ around a point $\underline{x}$. $\underline{p}$ is a point of $H_\delta(\underline{x})$ and $\underline{\xi} = \underline{p} - \underline{x}$ is the position vector between $\underline{x}$ and $\underline{p}$.
- For any point, we define $\underline{u}(\underline{x})$ the displacement at point $\underline{x}$.

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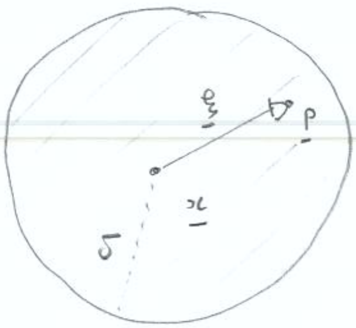
$$u_{\underline{\xi}}(\underline{x}) = \underline{u}(\underline{x}) \cdot \frac{\underline{\xi}}{\|\underline{\xi}\|}$$



$H_\delta(\underline{x})$

We define the quantity:

$$W = \frac{1}{4} \int_{H_\delta(\underline{x})} c \left\{ u_{\underline{\xi}}(\underline{p}) - u_{\underline{\xi}}(\underline{x}) \right\}^2 d\underline{V}_p$$

Prove that you can write W as:

$$\boxed{W = \frac{1}{2} \underline{\underline{\varepsilon}} : \int_{H_\delta(\underline{x})} c \, \frac{\underline{\xi} \otimes \underline{\xi} \otimes \underline{\xi} \otimes \underline{\xi}}{2 \|\underline{\xi}\|^2} \, d\underline{V}_p : \underline{\underline{\varepsilon}}}$$

Idea: use the spectral decomposition of $\underline{\underline{\varepsilon}}$.