

Principal stresses

$$[\underline{\sigma}] = \begin{pmatrix} 2 & 10 & 0 \\ 10 & 0 & 8 \\ 0 & 8 & 0 \end{pmatrix} \rightarrow \text{Matrix-like representation of tensor } \underline{\sigma} \text{ in the basis } (\underline{e}_1, \underline{e}_2, \underline{e}_3)$$

Then:

▷ Searching for eigenvalues:

$$\det(\underline{\sigma} - \lambda \underline{1}) = 0 \leadsto \begin{vmatrix} 2-\lambda & 10 & 0 \\ 10 & -\lambda & 8 \\ 0 & 8 & -\lambda \end{vmatrix} = 0$$

⌋ solve for the characteristic equation:
Make sure you know how to calculate a characteristic equation by hand.

$$(2-\lambda)(\lambda^2-64) + 100\lambda = 0 \leadsto \boxed{-\lambda^3 + 2\lambda^2 + 164\lambda - 128 = 0}$$

$$\boxed{3 \text{ eigenvalues: } \lambda_I = -12.246 / \lambda_{II} = 0.776 / \lambda_{III} = 13.470}$$

▷ Searching for eigenvectors:

Remember, \underline{p}_I is associated with λ_I if $\underline{\sigma} \cdot \underline{p}_I = \lambda_I \underline{p}_I$

so \underline{p}_I is a solution of $(\underline{\sigma} - \lambda_I \underline{1}) \cdot \underline{p}_I = \underline{0}$.

and by convention we always take $\|\underline{p}_I\| = 1$.