Static admissibility #2

Let us consider a prismatic domain with a Carthesian coordinate system $(\underline{O}, \underline{e}_1, \underline{e}_2, \underline{e}_3)$. The domain is clamped over the plane (z=0). The top surface is subjected to a uniform density of tractions (intensity T about direction $\underline{e}_1$). The other surfaces are free of traction and body forces are neglected.

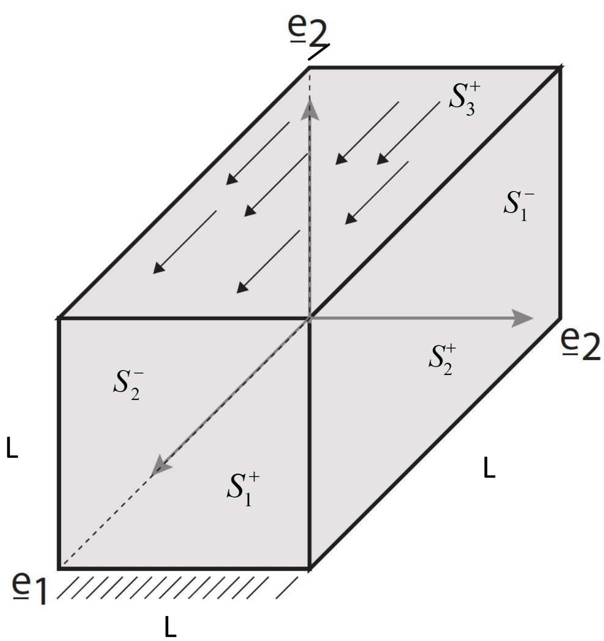

Figure 1: Sketch of the problem

Question: Write all the equations defining static admissibility for $\underline{\underline{\sigma}}$ and expand them.

Answer: Find $\underline{\underline{\sigma}}$ such that $\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T$, $\forall n \in \Omega$

Interior equilibrium equation:

$$\underline{\text{div}}\underline{\underline{\sigma}} = \underline{0}, \forall m \in \Omega \tag{1}$$

Boundary conditions:

$$\underline{\underline{\sigma}} \cdot \underline{n} = \underline{0}, \forall n \in S_1^- \cup S_1^+ \cup S_2^- \cup S_2^+ \tag{2}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = T \cdot \underline{e}_1, \forall n \in S_3^+ \tag{3}$$

Expand:
Interior equilibrium equation:

$$\begin{aligned} \sigma_{11,1} + \sigma_{12,2} + \sigma_{13,3} = 0, (1a) \\ \sigma_{21,1} + \sigma_{22,2} + \sigma_{23,3} = 0, (1b) \\ \sigma_{31,1} + \sigma_{32,2} + \sigma_{33,3} = 0, (1c) \end{aligned} \tag{4}$$

Boundary conditions:

$$S_1^- : \begin{array}{l} \sigma_{11}(0, y, z) = 0 \\ \sigma_{21}(0, y, z) = 0 \\ \sigma_{31}(0, y, z) = 0 \end{array} \tag{5}$$

$$S_1^+ : \begin{array}{l} \sigma_{11}(L, y, z) = 0 \\ \sigma_{21}(L, y, z) = 0 \\ \sigma_{31}(L, y, z) = 0 \end{array} \tag{6}$$

$$S_2^- : \begin{array}{l} \sigma_{12}(x, 0, z) = 0 \\ \sigma_{22}(x, 0, z) = 0 \\ \sigma_{32}(x, 0, z) = 0 \end{array} \tag{7}$$

$$S_2^+ : \begin{array}{l} \sigma_{12}(x, L, z) = 0 \\ \sigma_{22}(x, L, z) = 0 \\ \sigma_{32}(x, L, z) = 0 \end{array} \tag{8}$$

$$S_3^+ : \begin{array}{l} \sigma_{13}(x, y, L) = T \\ \sigma_{23}(x, y, L) = 0 \\ \sigma_{33}(x, y, L) = 0 \end{array} \tag{9}$$

Question: Clearly define the set of of statically admissible stress fields, S^{ad}.

Answer: All stress field that satisfy the condition of the first question are S^{ad}.

Question: Can the following stress field be a viable solution for the problem:

$$\sigma_{13} = \sigma_{31} = T \quad ; \quad \text{otherwise} \quad \sigma_{ij} = 0 \tag{10}$$

Answer: It doesn't satisfy the boundary condition in the face S_1^- and S_1^+, so it can't be a viable solution for this problem.