Static admissibility #3

We consider a closed tube associated with a cylindrical coordinate system $(\underline{O}, \underline{e}_r, \underline{e}_\theta, \underline{e}_z)$.

The tube is subjected to an internal pressure p_i. All other surfaces are free of stresses.

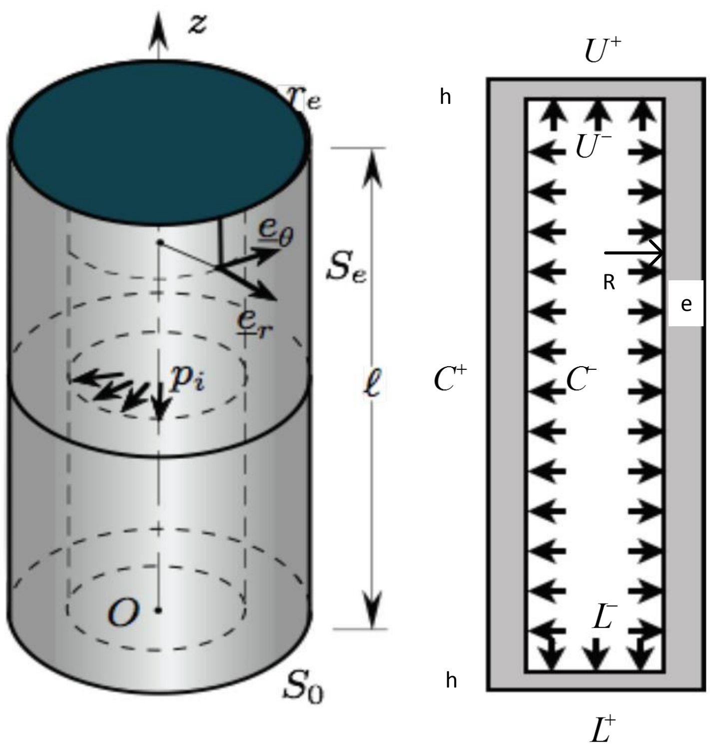

Figure 1: Sketch of the problem

Question: Write all the equations defining static admissibility for $\underline{\underline{\sigma}}$ (no expansion needed at this time). Find $\underline{\underline{\sigma}}$ such that $\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T$, $\forall n \in \Omega$

$$\underline{\underline{\sigma}}(r) = \begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} & \sigma_{rz} \\ \sigma_{r\theta} & \sigma_{\theta\theta} & \sigma_{\theta z} \\ \sigma_{rz} & \sigma_{\theta z} & \sigma_{zz} \end{bmatrix} \tag{1}$$

Interior equilibrium equation:

$$\underline{\mathrm{div}\underline{\underline{\sigma}}} = \underline{0}, \forall m \in \Omega \tag{2}$$

Boundary conditions:

$$\underline{\underline{\sigma}} \cdot \underline{n} = \underline{0}, \forall n \in U^+ \cup L^+ \cup C^+ \tag{3}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = -P_i \cdot -\underline{e}_z, \forall n \in U^- \tag{4}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = -P_i \cdot \underline{e}_z, \forall n \in L^- \tag{5}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = -P_i \cdot -\underline{e}_r, \forall n \in C^- \tag{6}$$

Question: Expand these equations.

Answer: Let us expand the equations above:
Interior equilibrium equation:

$$\begin{gathered}\sigma_{rr,r} + (\sigma_{11} - \sigma_{\theta\theta})/r + \sigma_{r\theta,\theta}/r + \sigma_{rz,z} = 0, (2a) \\ \sigma_{r\theta,r} + 2\sigma_{r\theta}/r + \sigma_{\theta\theta,\theta}/r + \sigma_{\theta z,z} = 0, (2b) \\ \sigma_{zr,r} + \sigma_{zr}/r + \sigma_{z\theta,\theta}/r + \sigma_{zz,z} = 0, (2c)\end{gathered} \tag{7}$$

Boundary conditions:

$$U^+: \begin{array}{l} \sigma_{zz}(r,\theta,l) = 0 \\ \sigma_{zr}(r,\theta,l) = 0 \\ \sigma_{z\theta}(r,\theta,l) = 0 \end{array} \tag{8}$$

$$L^+: \begin{array}{l} \sigma_{zz}(r,\theta,0) = 0 \\ \sigma_{zr}(r,\theta,0) = 0 \\ \sigma_{z\theta}(r,\theta,0) = 0 \end{array} \tag{9}$$

$$C^+: \begin{array}{l} \sigma_{rr}(R+e,\theta,z) = 0 \\ \sigma_{r\theta}(R+e,\theta,z) = 0 \\ \sigma_{rz}(R+e,\theta,z) = 0 \end{array} \tag{10}$$

$$U^- : \begin{aligned} \sigma_{zz}(r,\theta,l-h) &= -P_i \\ \sigma_{zr}(r,\theta,l-h) &= 0 \\ \sigma_{z\theta}(r,\theta,l-h) &= 0 \end{aligned} \tag{11}$$

$$L^- : \begin{aligned} \sigma_{zz}(r,\theta,h) &= P_i \\ \sigma_{zr}(r,\theta,h) &= 0 \\ \sigma_{z\theta}(r,\theta,h) &= 0 \end{aligned} \tag{12}$$

$$C^- : \begin{aligned} \sigma_{rr}(R,\theta,z) &= -P_i \\ \sigma_{rz}(R,\theta,z) &= 0 \\ \sigma_{r\theta}(R,\theta,z) &= 0 \end{aligned} \tag{13}$$