Stretch

Given the following right Cauchy-Green deformation tensor C

$$[C] = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0.36 \end{bmatrix} \tag{1}$$

(a) Find the stretch for the material elements that were in the direction of $\underline{e}_1$, $\underline{e}_2$ and $\underline{e}_3$.

(b) Find the stretch for the material element that was in the direction of $\underline{e}_1 + \underline{e}_2$.

(c) Find $\cos\theta$, where θ is the angle between $d\underline{x}^{(1)}$ and $d\underline{x}^{(2)}$ and where $d\underline{X}^{(1)} = dS_1\underline{e}_1$ and $d\underline{X}^{(2)} = dS_2\underline{e}_1$ deform into $d\underline{x}^{(1)} = ds_1\underline{m}$ and $d\underline{x}^{(2)} = ds_2\underline{u}$.

Solution

(a) Let us recall that

$$\frac{|\underline{v}|}{|\underline{V}|} = \frac{\sqrt{\underline{v} \cdot \underline{v}}}{\sqrt{\underline{V} \cdot \underline{V}}} = \frac{\underline{V} \cdot \underline{\underline{C}} \cdot \underline{V}}{\underline{V} \cdot \underline{V}} = \sqrt{C_{ii}} \tag{2}$$

Thus, stretch for direction $\underline{e}_1$ is $\sqrt{C_{11}} = 3$, stretch for direction $\underline{e}_2$ is $\sqrt{C_{22}} = 2$, stretch for direction $\underline{e}_3$ is $\sqrt{C_{33}} = 0.6$,

(b) To get stretch for direction $\underline{e}_1 + \underline{e}_2$, we define $|\underline{V}| = \frac{1}{\sqrt{2}}(\underline{e}_1 + \underline{e}_2)$, so we have

$$\underline{v} \cdot \underline{v} = \underline{V} \cdot \underline{\underline{C}} \cdot \underline{V} = \frac{1}{2} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0.36 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \frac{13}{2} \tag{3}$$

So the stretch is $\frac{13}{2}$.

(c) To find $\cos\theta$,

$$\sin\theta = \frac{|d\underline{x}^{(1)}| \cdot |d\underline{x}^{(2)}|}{|d\underline{X}^{(1)}| \cdot |d\underline{X}^{(2)}|} = \frac{dS_1\underline{e}_1 \cdot \underline{\underline{C}} \cdot \underline{e}_2 dS_2}{dS_1 dS_2} = C_{12} = 0 \tag{4}$$

Thus no angle variation, $\cos\theta = 1$.