Maximum shear stress

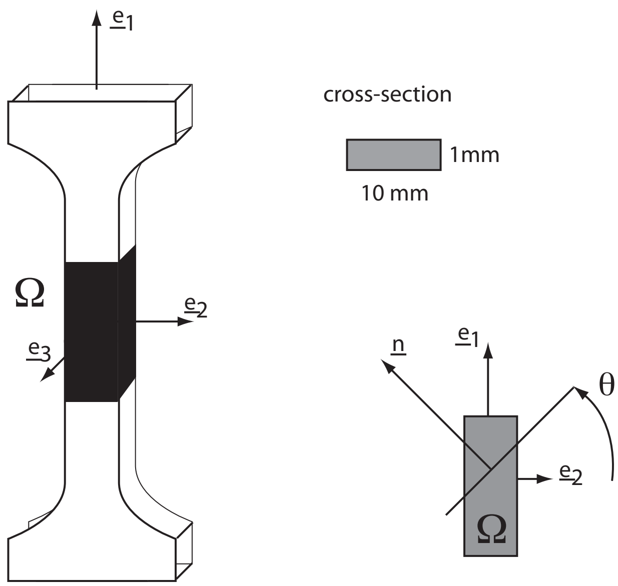

Figure 1: Dogbone sample under uniaxial tensile load

We assume that during a tensile test (Fig. 1), the stress state is homogeneous within the part Ω of a material sample. For any material point of Ω, the local stress tensor is:

$$\underline{\underline{\sigma}} = 100 \cdot \underline{e}_1 \otimes \underline{e}_1 \tag{1}$$

Question 1 Calculate the global force $\underline{F}$ that is applied on the material sample by the experimental tensile device.

Question 2 We cut the sample by a "fictive" plane. The orientation of this plane is denoted by the angle θ between the $\underline{e}_2$ axis and the unit normal vector $\underline{n}$. Calculate as a function of θ the stress vector applied on this surface.

Question 3 Find the value of θ for which the shear stress is maximum.

Solution to "Maximum shear stress"

Question 1 Calculate the global force $\underline{F}$ that is applied on the material sample by the experimental tensile device.

Answer: By definition the elementary force applied on the surface is

$$d\underline{F} = (\underline{\underline{\sigma}} \cdot \underline{n})dS \tag{2}$$

where

$$\underline{F} = \int_S \underline{\underline{\sigma}} \cdot \underline{n}dS = \int_S \sigma_{11}\underline{e}_1 dS = S\sigma_{11}\underline{e}_1 \tag{3}$$

Since $S = 10$ mm^2 and $\sigma_{11} = 100$ MPa, then $\underline{F} = 1000$ (Newton) $\underline{e}_1$

Question 2 We cut the sample by a "fictive" plane. The orientation of this plane is denoted by the angle θ between the $\underline{e}_2$ axis and the unit normal vector $\underline{n}$. Calculate as a function of θ the stress vector applied on this surface.

The normal direction of the fictive plane can be related as follows:

$$\underline{n} = \sin\theta \underline{e}_1 + \cos\theta \underline{e}_2 \tag{4}$$

The stress vector at point M along the fictive plane is given by

$$\underline{\underline{\sigma}} \cdot \underline{n} = \sigma_{11}\underline{e}_1 \otimes \underline{e}_1 \cdot (\sin\theta \underline{e}_1 + \cos\theta \underline{e}_2) \tag{5}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = \sigma_{11} \sin\theta \underline{e}_1 \tag{6}$$

If we transform the global orientation to the local orientation according to the fictive plane, then

$$\underline{e}_1 = \sin\theta \underline{n} + \cos\theta \underline{t} \tag{7}$$

and therefore,

$$\underline{\underline{\sigma}} \cdot \underline{n} = \sigma_{11} \sin^2\theta \underline{n} + \sigma_{11} \sin\theta \cos\theta \underline{t} \tag{8}$$

where the first term is the normal component of the stress vector and the second term is the shear component of the stress vector.

Question 3 Find the value of θ for which the shear stress is maximum.

$$\frac{\partial(\sin\theta \cos\theta)}{\partial\theta} = \cos^2\theta - \sin^2\theta = 0 \rightarrow \theta = 45^\circ \tag{9}$$