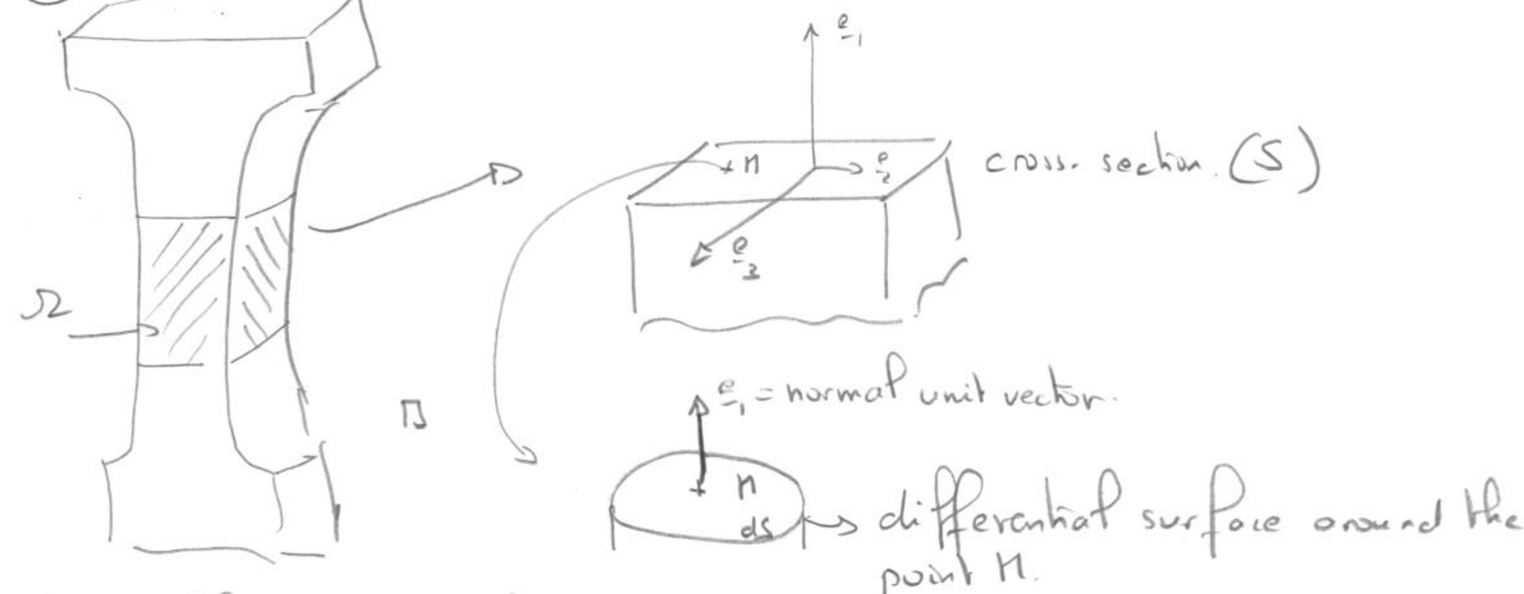
Problem 2 $\qquad \underline{\underline{\sigma}} = \sigma_{11}\, \underline{e}_1 \otimes \underline{e}_1$ with $\sigma_{11} = 100\,\text{Mpa}.$

Q1



By definition the elementary force applied on the surface is:

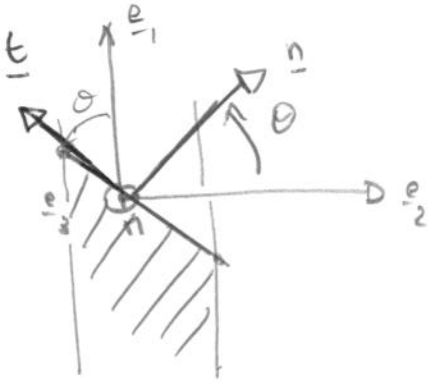
$$d\underline{F} = (\underline{\underline{\sigma}} \cdot \underline{n})\, ds$$

$$\hookrightarrow \quad \underline{F} = \int_S \underline{\underline{\sigma}} \cdot \underline{n}\, ds = \int_S \sigma_{11}\, \underline{e}_1\, ds = (S\, \sigma_{11})\, \underline{e}_1$$

Numerical application.

$$\left.\begin{aligned} S &= 10\ \text{mm}^2 \\ \sigma_{11} &= 100\ \text{MPa} \end{aligned}\right\} \quad \boxed{\underline{F} = 10\,(N)\ \underline{e}_1}$$

Q2



$$\underline{n} = \sin\theta\ \underline{e}_1 + \cos\theta\ \underline{e}_2.$$

The stress vector for the point M is equal to:

$$\underline{\underline{\sigma}} \cdot \underline{n} = \sigma_{11}\ \underline{e}_1 \otimes \underline{e}_1 \cdot (\sin\theta\ \underline{e}_1 + \cos\theta\ \underline{e}_2)$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = \sigma_{11}\ \sin\theta\ \underline{e}_1$$

If we rewrite the stress vector in the local basis:

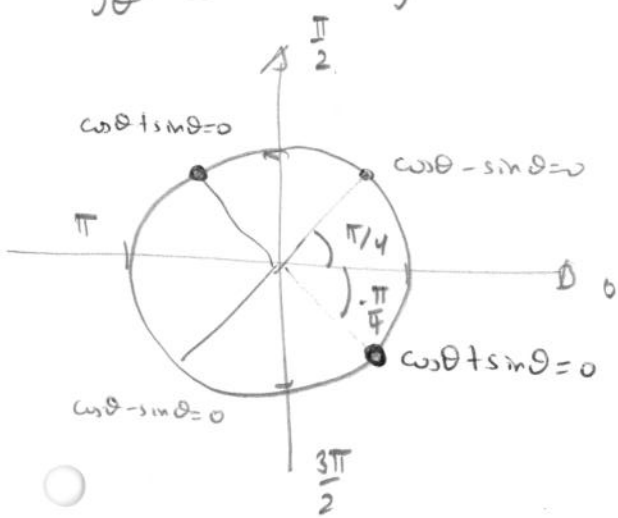
$$\underline{e}_1 = \sin\theta\ \underline{n} + \cos\theta\ \underline{t}$$

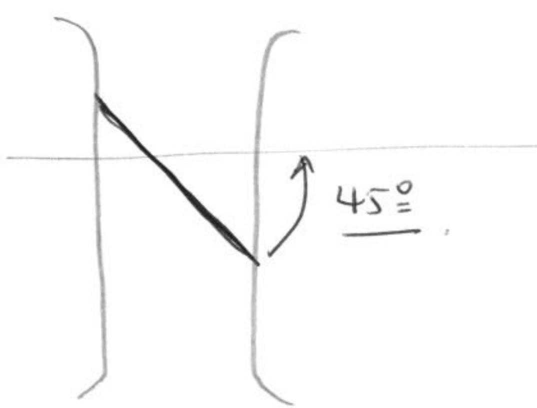
Thus: $$\underline{\underline{\sigma}} \cdot \underline{n} = \underbrace{\sigma_{11} \sin^2\theta\ \underline{n}}_{\substack{\text{Normal component of the} \\ \text{stress vector} \\ (\underline{n} \cdot \underline{\underline{\sigma}} \cdot \underline{n})}} + \underbrace{\sigma_{11} \sin\theta \cos\theta\ \underline{t}}_{\substack{\text{shear component} \\ \text{of the stress} \\ \text{vector.} \\ (\underline{t} \cdot \underline{\underline{\sigma}} \cdot \underline{n})}}$$

Q3

⊳ Let us find the "fictive" plane for which the shear stress is maximum?:

$$\frac{\partial}{\partial\theta}(\sin\theta\cos\theta) = 0 \Rightarrow \cos^2\theta - \sin^2\theta = 0 = (\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$$





(See fig 3.15 - p 80 Text book mage)