# Static admissibility #3

We consider a closed tube associated with a cylindrical coordinate system $(\underline{O}, \underline{e}_r, \underline{e}_\theta, \underline{e}_z)$.

The tube is subjected to an internal pressure $p_i$. All other surfaces are free of stresses.

**Question:** Write all the equations defining static admissibility for $\underline{\underline{\sigma}}$ (no expansion needed at this time).

**Question:** Expand these equations.

**Answer:** Find $\underline{\underline{\sigma}}$ such that $\underline{\underline{\sigma}} =^T \underline{\underline{\sigma}}, \forall n \in \Omega$

$$\underline{\underline{\sigma}}(r) = \begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} & \sigma_{rz} \\ \sigma_{r\theta} & \sigma_{\theta\theta} & \sigma_{\theta z} \\ \sigma_{rz} & \sigma_{\theta z} & \sigma_{zz} \end{bmatrix} \tag{1}$$

Interior equilibrium equation:

$$\underline{\mathrm{div}\underline{\underline{\sigma}}} = \underline{0}, \forall m \in \Omega \tag{2}$$

Boundary equilibrium equation:

$$\underline{\underline{\sigma}} \cdot \underline{n} = \underline{0}, \forall n \in U^+ \cup L^+ \cup C^+ \tag{3}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = -P_i \cdot -\underline{e}_z, \forall n \in U^- \tag{4}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = -P_i \cdot \underline{e}_z, \forall n \in L^- \tag{5}$$

$$\underline{\underline{\sigma}} \cdot \underline{n} = -P_i \cdot -\underline{e}_r, \forall n \in C^- \tag{6}$$

Expand:
Interior equilibrium equation:

$$\begin{aligned} &\sigma_{rr,r} + (\sigma_{11} - \sigma_{\theta\theta})/r + \sigma_{r\theta,\theta}/r + \sigma_{rz,z} = 0, (2a) \\ &\sigma_{r\theta,r} + 2\sigma_{r\theta}/r + \sigma_{\theta\theta,\theta}/r + \sigma_{\theta z,z} = 0, (2b) \\ &\sigma_{zr,r} + \sigma_{zr}/r + \sigma_{z\theta,\theta}/r + \sigma_{zz,z} = 0, (2c) \end{aligned} \tag{7}$$

Boundary equilibrium equation:

$$U^+ : \begin{array}{l} \sigma_{zz}(r,\theta,l) = 0 \\ \sigma_{zr}(r,\theta,l) = 0 \\ \sigma_{z\theta}(r,\theta,l) = 0 \end{array} \tag{8}$$

$$
L^+ : \begin{array}{l} \sigma_{zz}(r,\theta,0) = 0 \\ \sigma_{zr}(r,\theta,0) = 0 \\ \sigma_{z\theta}(r,\theta,0) = 0 \end{array} \tag{9}
$$

$$
C^+ : \begin{array}{l} \sigma_{rr}(R+e,\theta,z) = 0 \\ \sigma_{r\theta}(R+e,\theta,z) = 0 \\ \sigma_{rz}(R+e,\theta,z) = 0 \end{array} \tag{10}
$$

$$
U^- : \begin{array}{l} \sigma_{zz}(r,\theta,l-h) = -P_i \\ \sigma_{zr}(r,\theta,l-h) = 0 \\ \sigma_{z\theta}(r,\theta,l-h) = 0 \end{array} \tag{11}
$$

$$
L^- : \begin{array}{l} \sigma_{zz}(r,\theta,h) = P_i \\ \sigma_{zr}(r,\theta,h) = 0 \\ \sigma_{z\theta}(r,\theta,h) = 0 \end{array} \tag{12}
$$

$$
C^- : \begin{array}{l} \sigma_{rr}(R,\theta,z) = -P_i \\ \sigma_{rz}(R,\theta,z) = 0 \\ \sigma_{r\theta}(R,\theta,z) = 0 \end{array} \tag{13}
$$

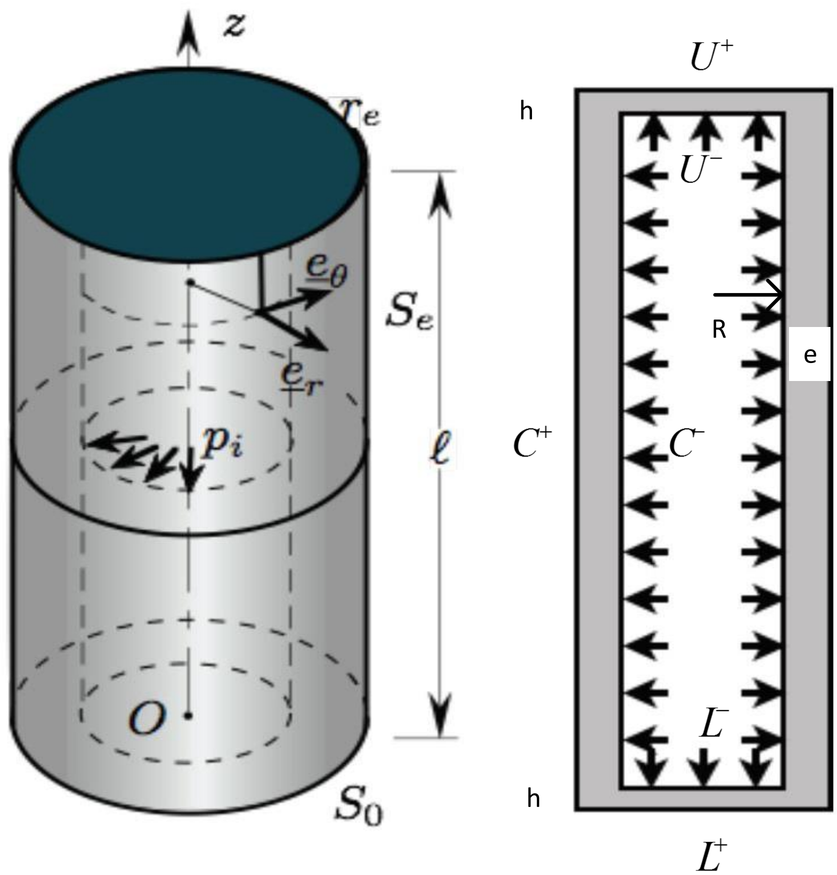


Figure 1: Sketch of the problem