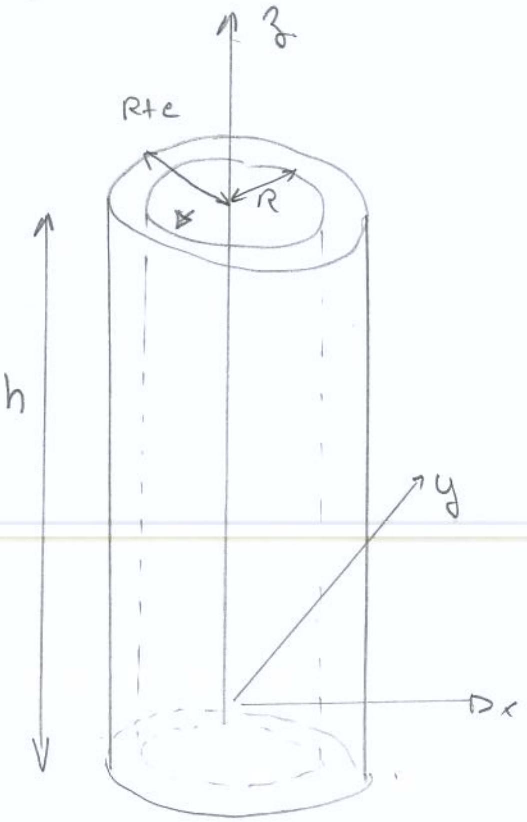
| Practice on stresses |

We consider a circular tube (h, R) thickness $= e$. It is submitted to an internal pressure p, gravity is neglected (as well as atmospheric pressure)



Q1) Verify that a static solution does exist?

We assume that $\underline{\underline{\sigma}}$ does only depend on r (the polar radius)

Q2) Find all the local equations satisfied by $\underline{\underline{\sigma}}(r)$ that define the static admissibility? Prove that $\sigma_{r\theta} = 0$ with the assumption $\underline{\underline{\sigma}}(r)$?

R 1) Existence: $\Rightarrow$ the structure should be under global equilibrium. (We will come back on this when we discuss about variational formulations).

$$\underline{R}_{ext \to \Omega} = \underline{0} \quad \text{(sum of external forces)}$$

$$\underline{\mathcal{M}}_{ext \to \Omega} = \underline{0} \quad \text{(sum of external moments)}.$$

$$\underline{R}_{ext \to \Omega} = \int_{S_{int}} \underline{\underline{\sigma}} \cdot (-\underline{e}_r) \, ds = \int_{S_{int}} p \, \underline{e}_r \, ds = p \int_{z=0}^{z=h} \int_{\theta=0}^{\theta=2\pi} \underline{e}_r \, R \, d\theta \, dz$$

$$= pRh \underbrace{\int_0^{2\pi} \underline{e}_r \, d\theta}_{\underline{0}} = \underline{0}$$

$$\underline{\mathcal{M}}_{ext \to \Omega} = \int_{S_{int}} \underline{OM} \times \underline{\underline{\sigma}} (-\underline{e}_r) \, ds = \int_{z=0}^{z=h} \int_{\theta=0}^{\theta=2\pi} (z \underline{z} + R \underline{e}_r) \times (p \, \underline{e}_r) \, R \, d\theta \, dz$$

$$= pR \int_{z=0}^{z=h} \int_{\theta=0}^{\theta=2\pi} z \, \underline{e}_\theta \, d\theta \, dz = pR \int_{z=0}^{z=h} z \left(\underbrace{\int_{\theta=0}^{\theta=2\pi} \underline{e}_\theta \, d\theta}_{\underline{0}} \right) dz = \underline{0}$$

R2) Interior equilibrium: $\operatorname{div} \underline{\underline{\sigma}} = \underline{0} \quad \forall \underline{M} \in \Omega.$ (1)

Boundary equilibrium: $\underline{\underline{\sigma}}(r) = \begin{bmatrix} \sigma_{rr}(r) & \sigma_{r\theta}(r) & \sigma_{rz}(r) \\ \sigma_{r\theta}(r) & \sigma_{\theta\theta}(r) & \sigma_{\theta z}(r) \\ \sigma_{rz}(r) & \sigma_{\theta z}(r) & \sigma_{zz}(r) \end{bmatrix}$

S_{sup} $\quad \underline{\underline{\sigma}} \cdot \underline{n}\Big|_{S_{sup}} = \underline{\underline{\sigma}} \cdot (\underline{e}_z)\Big|_{z=h} = \underline{0}$

$\hookrightarrow \sigma_{rz}(r) = \sigma_{\theta z}(r) = \sigma_{zz}(r) = 0 \quad \forall r \in [R, R+e]$

(2) a-b-c

S_{inf} $\quad \underline{\underline{\sigma}} \cdot \underline{n}\Big|_{S_{inf}} = \underline{\underline{\sigma}} \cdot (-\underline{e}_z)\Big|_{z=0} = \underline{0}$

$\hookrightarrow \sigma_{rz}(r) = \sigma_{\theta z}(r) = \sigma_{zz}(r) = 0 \quad \forall r \in [R, R+e]$

(3) a-b-c

S_{ext} $\quad \underline{\underline{\sigma}} \cdot \underline{n}\Big|_{r=R+e} = \underline{\underline{\sigma}} \cdot \underline{e}_r\Big|_{r=R+e} = \underline{0}$

$\hookrightarrow \sigma_{rr}(R+e) = \sigma_{r\theta}(R+e) = \sigma_{rz}(R+e) = 0$

4- a.b.c

S_{int} $\quad \underline{\underline{\sigma}} \cdot \underline{n}\Big|_{r=R} = \underline{\underline{\sigma}} \cdot (-\underline{e}_r)\Big|_{r=R} = p\,\underline{e}_r$

$\hookrightarrow \left| \begin{array}{l} \sigma_{rr}(R) = -p \\ \sigma_{r\theta}(R) = \sigma_{rz}(R) = 0 \end{array} \right.$ 5-a-b-c.

From (2) and (3) $\Rightarrow$ $\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} & 0 \\ \sigma_{r\theta} & \sigma_{\theta\theta} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$(1) \Rightarrow \begin{cases} \sigma_{rr,r} + \dfrac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 & (7) \\ \sigma_{r\theta,r} + 2\dfrac{\sigma_{r\theta}}{r} = 0 & (8) \\ 0 = 0 & (9) \end{cases}$$

$$(8) \Rightarrow \frac{1}{r}\left(r^2 \sigma_{r\theta}\right)_{,r} = 0 \Rightarrow \boxed{\sigma_{r\theta} = \frac{K_1}{r}}$$

But $4(b) \Rightarrow \sigma_{r\theta}(R+e) = 0$

$5(b) \Rightarrow \sigma_{r\theta}(R) = 0$

$$\Rightarrow \boxed{\sigma_{r\theta} = 0}$$

Then