Principal Stress

The stress state for a material point is given in a Cartesian basis $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ by:

$$\underline{\underline{\sigma}} = \begin{bmatrix} 2 & 10 & 0 \\ 10 & 0 & 8 \\ 0 & 8 & 0 \end{bmatrix} \tag{1}$$

Question Calculate the principal stresses and the principal directions for this stress tensor.

Answer: The principal stresses are the Eigen values of the stress tensor.

$$\det(\underline{\underline{\sigma}} - \lambda \underline{\underline{1}}) = 0 \rightarrow \begin{vmatrix} 2-\lambda & 10 & 0 \\ 10 & -\lambda & 8 \\ 0 & 8 & -\lambda \end{vmatrix} = 0 \tag{2}$$

Then,

$$\det(\underline{\underline{\sigma}} - \lambda \underline{\underline{1}}) = (2-\lambda)(\lambda^2 - 64) + 100\lambda = -\lambda^3 + 2\lambda^2 + 164\lambda - 128 = 0 \tag{3}$$

The equation above also know as the characteristic equation. By solving the polynomial order 3 above, we found that the Eigenvalues are: $\lambda_1 = -12.246$ MPa,$\lambda_2 = 0.776$ MPa,$\lambda_3 = 13.470$ MPa.

The principal directions are the Eigenvectors of the stress tensor. Note that $\underline{P}_I$ is associated with λ_I if $\underline{\underline{\sigma}} \cdot \underline{P}_I = \lambda_I \underline{P}_I$. So $\underline{P}_I$ is a solution of $(\underline{\underline{\sigma}} - \lambda \underline{\underline{1}})\underline{P}_I = 0$. By convention, we usually always normalize $\underline{P}_I$ as a unit vector so $||\underline{P}_I|| = 1$