$\boxed{\text{Problem 4}}$ measurement of strain components.

Say $\underline{a}, \underline{b}, \underline{c}$ the reference points

Say $\underline{a}', \underline{b}', \underline{c}'$ their images in the deformed configuration.

We remind that, $\forall \underline{V}$, if $\underline{v}$ is the image of $\underline{V}$ by the transformation $\underline{\phi}$ (such that $\underline{x} = \underline{\phi}(\underline{X}, t)$). Then:

$$\frac{|v|}{|V|} = \frac{\sqrt{v \cdot v}}{\sqrt{\underline{V} \cdot \underline{V}}} = \frac{\sqrt{\underline{V} \cdot \underline{\underline{C}} \cdot \underline{V}}}{\sqrt{\underline{V} \cdot \underline{V}}} \quad (1)$$

Here, we know the change in length for 3 vectors:

- $\underline{ab} = 1\, \underline{e}_1 \xrightarrow{\underline{\phi}} |\underline{a}'\underline{b}'| = 1.5$

  Then from (1): $\dfrac{|\underline{a}'\underline{b}'|}{|\underline{ab}|} = 1.5 = \sqrt{C_{11}} \rightsquigarrow \boxed{C_{11} = 2.25}$

- $\underline{ac} = 1\, \underline{e}_2 \xrightarrow{\underline{\phi}} |\underline{a}'\underline{c}'| = 2$

  Then from (1): $\dfrac{|\underline{a}'\underline{c}'|}{|\underline{ac}|} = 2 = \sqrt{C_{22}} \rightsquigarrow \boxed{C_{22} = 4}$

- $\underline{bc} = \sqrt{2}\, \dfrac{\underline{e}_2 - \underline{e}_1}{\sqrt{2}} = \underline{e}_2 - \underline{e}_1 \xrightarrow{\underline{\phi}} |\underline{b}'\underline{c}'| = 2.8$

  Then from (1): $\dfrac{|\underline{b}'\underline{c}'|}{|\underline{bc}|} = 2.8 = \dfrac{\sqrt{C_{11} + C_{22} - 2C_{12}}}{\sqrt{2}} \rightsquigarrow \boxed{C_{12} = -4}$

Then, $\underline{\underline{e}}$ (the Green-Lagrange strain tensor) is easily defined as