ME 211A: Stretch

October 2, 2018

3.70 Question

Given the following right Cauchy-Green deformation tensor at a point

$$[C] = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0.36 \end{bmatrix} \tag{1}$$

(a) Find the stretch for the material elements that were in the direction of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$.

(b) Find the stretch for the material element that was in the direction of $\mathbf{e}_1 + \mathbf{e}_2$.

(c) Find $\cos\theta$, where θ is the angle between $d\mathbf{x}^{(1)}$ and $d\mathbf{x}^{(2)}$ and where $d\mathbf{X}^{(1)} = dS_1\mathbf{e}_1$ and $d\mathbf{X}^{(2)} = dS_2\mathbf{e}_1$ deform into $d\mathbf{x}^{(1)} = ds_1\mathbf{m}$ and $d\mathbf{x}^{(2)} = ds_2\mathbf{n}$.

Solution

(a) We remind that

$$\frac{|\underline{v}|}{|\underline{V}|} = \frac{\sqrt{\underline{v}\cdot\underline{v}}}{\sqrt{\underline{V}\cdot\underline{V}}} = \frac{\underline{V}\cdot\underline{\underline{C}}\cdot\underline{V}}{\underline{V}\cdot\underline{V}} = \sqrt{C_{ii}} \tag{2}$$

Thus, stretch for direction $\underline{e}_1$ is $\sqrt{C_{11}} = 3$, stretch for direction $\underline{e}_2$ is $\sqrt{C_{22}} = 2$, stretch for direction $\underline{e}_3$ is $\sqrt{C_{33}} = 0.6$,

(b)

To get stretch for direction $\underline{e}_1 + \underline{e}_2$, we define $|\underline{V}| = \frac{1}{\sqrt{2}}(\underline{e}_1 + \underline{e}_2)$, so we have

$$\underline{v}\cdot\underline{v} = \underline{V}\cdot\underline{\underline{C}}\cdot\underline{V} = \frac{1}{2}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0.36 \end{bmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix} = \frac{13}{2} \tag{3}$$

So the stretch is $\frac{13}{2}$.

(c)

$$\sin\theta = \frac{|\underline{dx}^{(1)}| \cdot |\underline{dx}^{(2)}|}{|\underline{dX}^{(1)}| \cdot |\underline{dX}^{(2)}|} = \frac{dS_1 \underline{e}_1 \cdot \underline{\underline{C}} \cdot \underline{e}_2 dS_2}{dS_1 dS_2} = C_{12} = 0 \tag{4}$$

Thus no angle variation, $\cos\theta = 1$.