

# Practice:

## Static and Kinematic Admissibility

### 1 Problem 1

A homogeneous continuum as in Fig. 1 is clamped on surface  $S_3^-(z = 0)$  and submitted to a uniform pressure  $p_0$  over the surface  $S_1^+(x = l_1)$ .

**Question 1:** Write the static admissibility equations!

**Question 2:** Write the kinematic admissibility equations!

**Question 3:** Assuming we use the Cartesian coordinate system, please give the expansion of these equations!

**Question 4:** Calculate  $\int_{S_3^-} \underline{\underline{\sigma}} \cdot \underline{n} dS$

### 2 Problem 2

Let us consider a sphere with cavity (exterior radius  $R_e$  and internal radius  $R_i$ ). The external surface  $S_e$  is free of loading and the interior surface  $S_i$  is submitted to a pressure  $p_0$ . The material is assumed to be isotropic linear elastic with young modulus  $E$  and Poisson ratio  $\nu$ .

**Question 1:** Write the static admissibility equations!

**Question 2:** Please expand this equation using spherical polar coordinates system!

**Question 3:** Let's assume that the following stress tensor

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{\varphi\varphi} \end{bmatrix} \quad (1)$$

with

$$\sigma_{rr} = -A \left[ \frac{B}{r^3} - 1 \right] \quad (2)$$

and

$$\sigma_{\theta\theta} = \sigma_{\varphi\varphi} = A \left[ \frac{B}{2r^3} + 1 \right] \quad (3)$$

Please calculate the coefficient  $A$  and  $B$  so that  $\underline{\underline{\sigma}}$  could be a solution of the problem!

**Question 4:** Please calculate the strain tensor  $\underline{\underline{\varepsilon}}$ ? Does it satisfy the compatibility equation? Conclusion?

### 3 Problem 3

Let us consider a domain  $\Omega$ , simply supported by a rigid basis ( $\Gamma$ ).  $\Omega$  is submitted to gravity volume forces  $\underline{f} = \rho g \underline{y}$  and to uniform pressure  $p > 0$  over every vertical side. We assume plane stresses in the plane of  $x - y$  and the material is isotropic linear elastic with Young modulus  $E$  and Poisson ratio  $\nu$ .

**Question 1:** Write down the full problem! (kinematic, static and constitutive equations)

**Question 2:** We assume an Airy function  $\phi(x, y) = ax^2 + cy^2 + hy^3$ . Find  $a, c, h$  so we define a statistically admissible stress field.

**Question 3:** Find the related strain field using the constitutive equation. Does that satisfy the compatibility equation? Conclusion?

**Question 4:** Integrate the strain field to get the displacement at every point. Can we find the exact solution of the problem using this Airy function?

### 4 Problem 4

Let us consider a classical test in material engineering that make possible to identify the oedometric modulus  $\hat{E}$ . A cylinder material domain  $\Omega$  is compressed due to the prescribed motion of the top surface  $\underline{u} = -U \underline{e}_z$ . All surfaces are assumed to be perfect contact without friction. Note that  $\underline{P} = -P \underline{e}_z$  the resulting force that is necessary to press the materials (Young modulus  $E$  and Poisson ratio  $\nu$ ).

**Question 1:** Write the complete problem: kinematic + static + constitutive equation!

**Question 2:** Assume that  $\underline{u} = -\frac{U}{h} z \underline{e}_z$ . Is this kinematically admissible?

**Question 3:** Find the corresponding stress  $\underline{\underline{\sigma}}$ . Is it statistically admissible?

**Question 4:** Find oedometric modulus  $\hat{E}$  as a function of  $E$  and  $\nu$ .