Analysing repeated measures with Linear Mixed Models
(Random Effects Models)
(1)
Getting familiar with the Linear Mixed Models (LMM) options in SPSS

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How this document should be used:
This document has been designed to be suitable for both web based and face-to-face teaching. The text has been made to be as interactive as possible with exercises, Multiple Choice Questions (MCQs) and web based exercises.

If you are using this document as part of a web-based course you are urged to use the online discussion board to discuss the issues raised in this document and share your solutions with other students.

This document is part of a series see:
http://www.robin-beaumont.co.uk/virtualclassroom/contents.htm

Who this document is aimed at:
This document is aimed at those people who want to learn more about statistics in a practical way and assumes you have worked through a number documents already.

Any feedback most welcome.   Robin Beaumont

Videos to support material

There are a set of Youtube videos to accompany this chapter which can be found at:
http://www.youtube.com/view_play_list?p=05FC4785D24C6E68

Contents

1. Introduction

Mixed models are theoretically not new, and as with most statistical concepts Fisher first introduced them at the beginning of the 20th century. However only very recently, that is in the last two decades, have they been considered widely as a method of analysing repeated measures data. I will not mention here the many other approaches that have been offered as a method of analysing such data, instead I will try to start with a blank slate.

Repeated measures are those measures that are repeated on more than one occasion, we met an example in the paired t statistic chapter. Taking another example, say we have cholesterol levels measured four times (t_1, t_2, t_3, t_4) over a year for each patient following commencement of a new cholesterol lowering drug. So for each patient we have four measurements which can be depicted thus:

4 observations clustered within each patient   - observations within each cluster may be correlated?
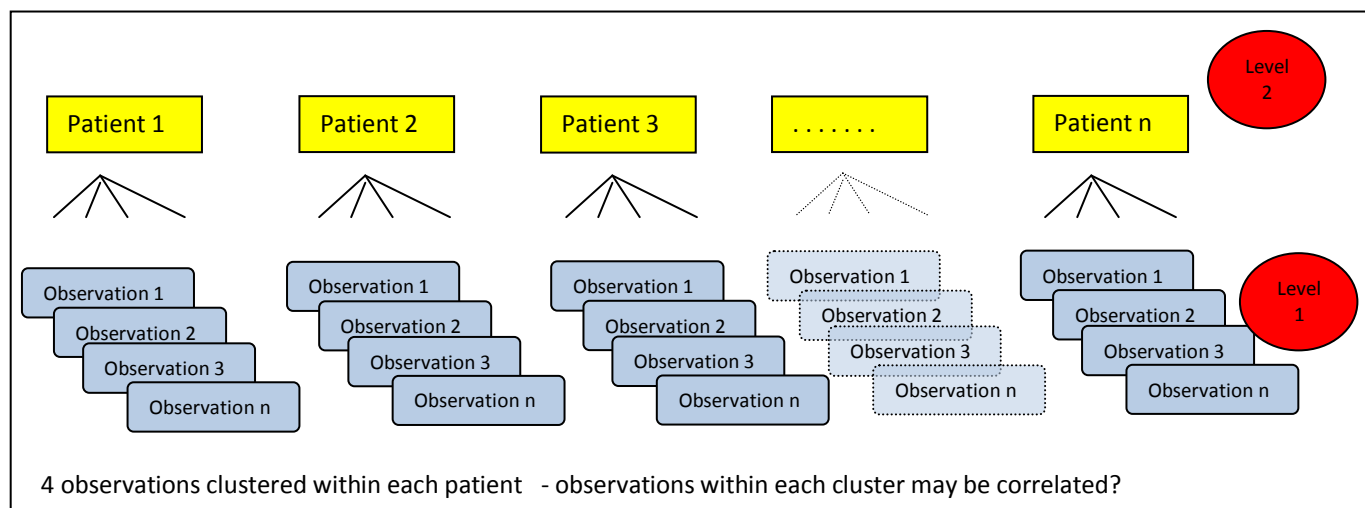
We can say that the observations are the bottom level (level one) while they are clustered within the level two variable patient.

There are several aspects of this situation we might be interested in investigating:

> The difference between the first and last observation for each patient
> The trend over time for the observations for each patient taking into account the initial value. Do they follow a straight line or some type of curved one?
> The relationship between the repeated observations, are they correlated or independent, do they become less correlated as the observations become more distant, that is, is t_1 to t_2 more strongly correlated than that between t_1 and t_4
> How spread out are the scores at each time point, and do they become more spread out as time goes on or is the variance constant.
> How much of the variability in the observations is accounted for within the clusters compared to that between the clusters

All the above questions are common for this type of situation which suggests that an appropriate analysis here is rather an involved process. It is important to realise that some of the above questions are more valid than others an issue we will discuss latter. As is usual a largely graphical initial exploratory data analysis (EDA) is always the first stage of any analysis, both Singer & Willett (2003) and Diggle & Heagerty (2003) devote a whole chapter at the start of their books to graphical techniques. Unfortunately both SPSS and R do not make life easy when it comes to developing various graphs for repeated measures, Heck Thomas & Tabata 2010 attempt various graphs in SPSS, while using workarounds, while Singer & Willett 2003 provide SPSS syntax.

Before we can do anything we need to consider the structure of the data as this affects our ability to analyse it.

2. Wide and long data formats

In most of the analyses we have carried out so far we have arranged the data so that a single case (i.e. patient/subject) represents a row of data, and specifically if we had some repeated measures we would represent each repeated measure as a separate column. In contrast in a long format structure the data is structured in such a way so that each observation time represents a row.

Long format = one observation time for a specific case, per row

Wide format = one case (subject) per row

Wide format = one case per row

	id	health.1	health.2	health.3	health.4	lifestyle.1	lifestyle.2	lifestyle.3	lifestyle.4
1	1	4.2	3.9	3.9	3.6	2.51	2.10	2.16	2.26
2	2	4.4	4.2	4.6	4.1	2.48	2.34	2.45	2.57
3	3	3.7	4.0	3.3	3.6	2.17	2.39	2.44	2.42
	4	4.3	4.1	3.8	3.4	2.58	2.68	2.84	3.00
	5	4.2	4.1	4.1	4.5	2.33	2.02	2.00	2.27
	6	4.1	3.8	3.6	3.7	2.05	2.09	2.25	2.24

One case per row

Long format = one observation (per case) per row

id	health	lifestyle	time
1	4.2	2.51	1
1	3.9	2.10	2
1	3.9	2.16	3
1	3.6	2.26	4
2	4.4	2.48	1
2	4.2	2.34	2
2	4.6	2.45	3
2	4.1	2.57	4

One observation time for a single case

Data from Twisk 2006:

twisk_longitudinal_wide.sav

twisk_longitudinal_long.sav

My Youtube videos at http://www.youtube.com/view_play_list?p=05FC4785D24C6E68 describe how to restructure data using either SPSS or R. Also my introduction to R chapter describes the various restructure commands in R.

3. Graphing change in SPSS

The simplest way is to produce a scatter plot of the variable you are interested in over time, this is also called a profile or spaghetti plot.

id	time	result
1.00	1	25.00
1.00	2	27.00
2.00	1	2.00
2.00	2	12.00
3.00	1	8.00
3.00	2	9.00
4.00	1	10.00
4.00	2	10.00
5.00	1	7.00
5.00	2	9.00
6.00	1	8.00
6.00	2	7.00
7.00	1	24.00
7.00	2	30.00
8.00	1	30.00
8.00	2	31.00

t_test_paired_long_format.sav
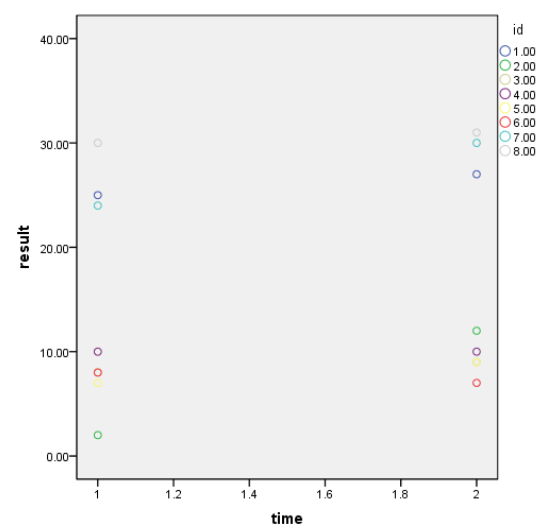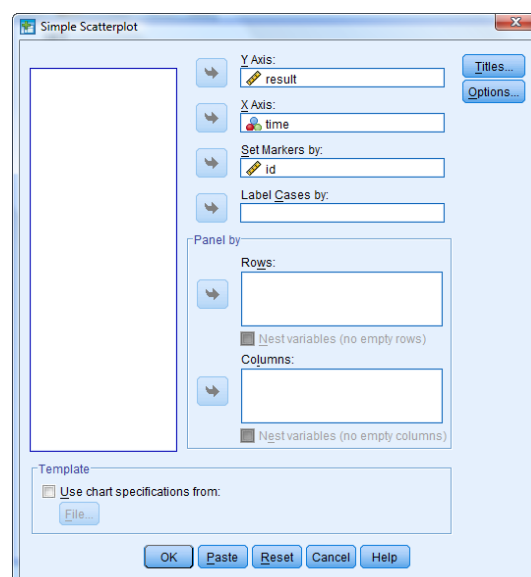
While the above details offer a solution of sorts, see http://www.medicalnerds.com/producing-spaghetti-plots-using-r/#more-36 for a discussion of the problems in SPSS, it is not ideal as you can see from the final result over the page.

We use the *add fit line at subgroups* option to obtain the lines.
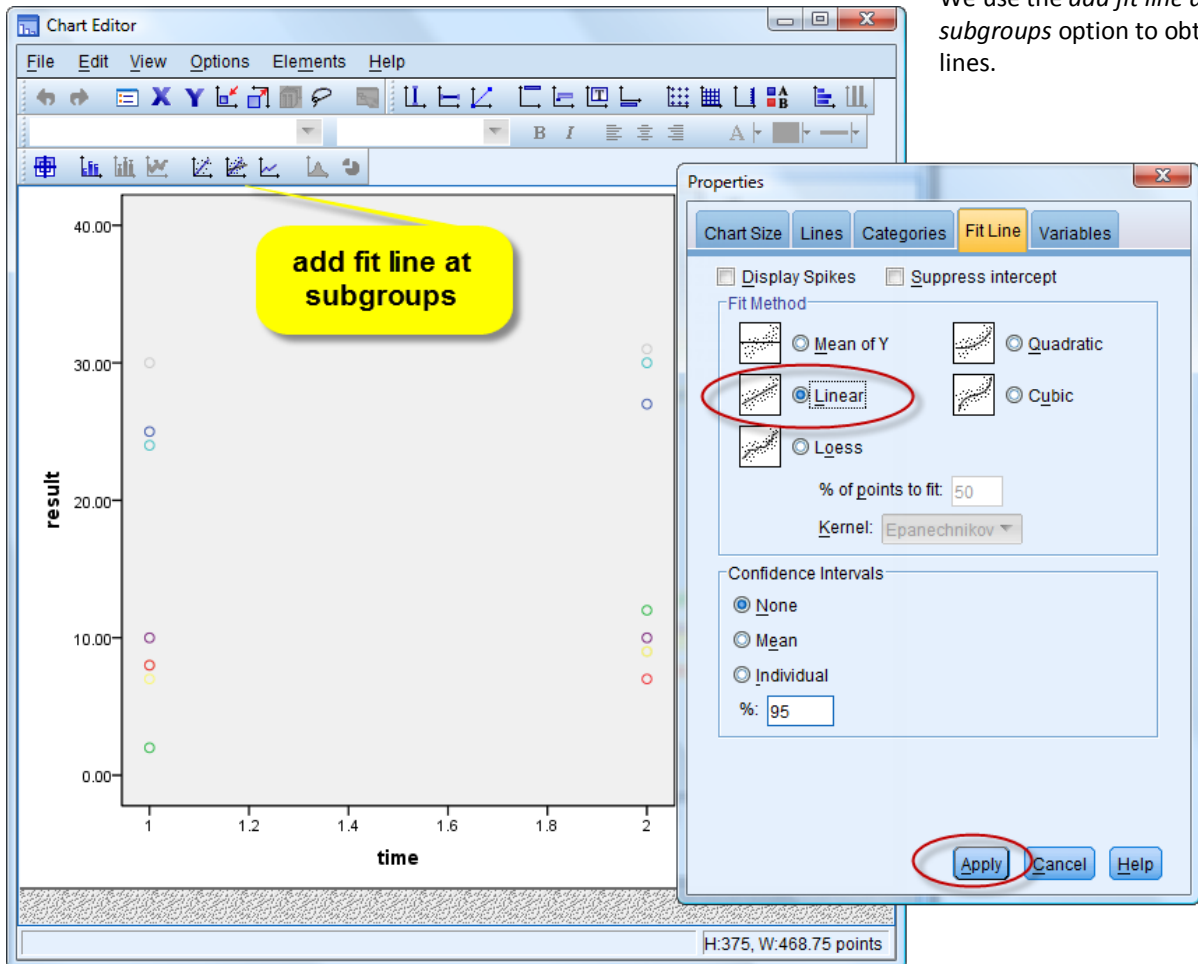
Finally the x axis scale needs to be tidied up to make sense.
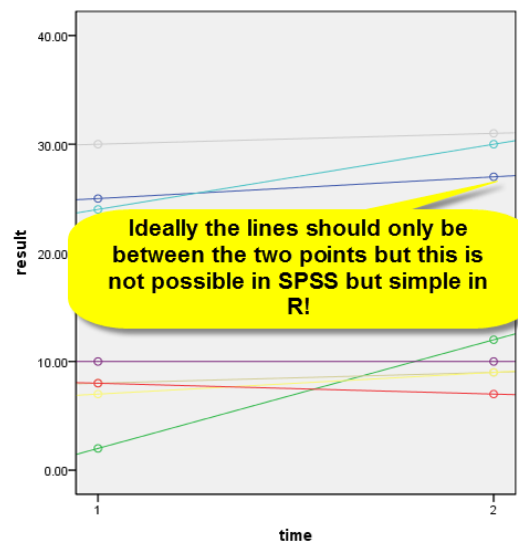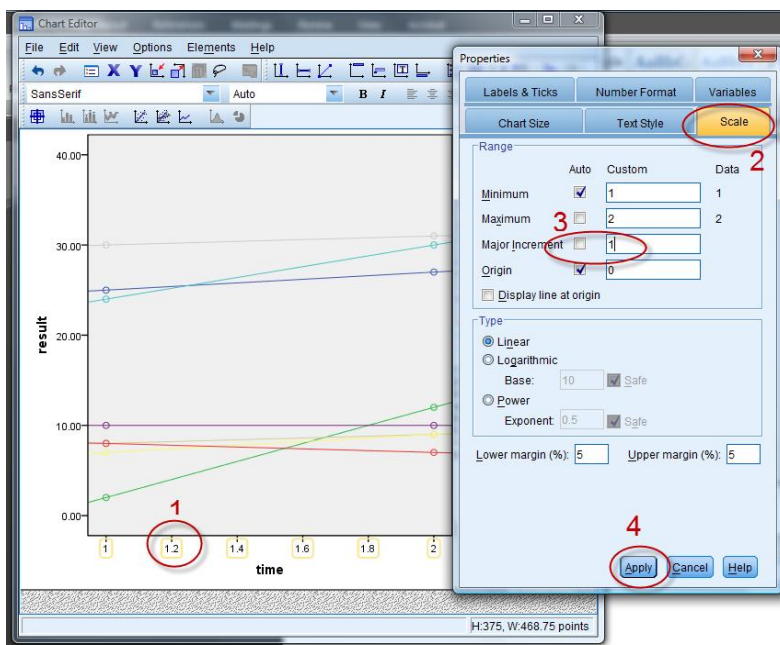
The final result shows how each individual behaved; ideally the lines should only go between the two time periods for which the data was collected. You can achieve this in SPSS by manipulating the data (see the third chapter concerned with random effects models). You can also use SPSS syntax. Such Syntax details can be found at http://www.ats.ucla.edu/stat/spss/examples/alda/ which describes the examples for chapter 2 of Singer & Willett 2003, the same webpage also provides the equivalent (much shorter) R code.

4. Graphing change in R

The data needs to be in long format. To load the data, I'm using the SPSS file into R I use the r commander interface as shown below.
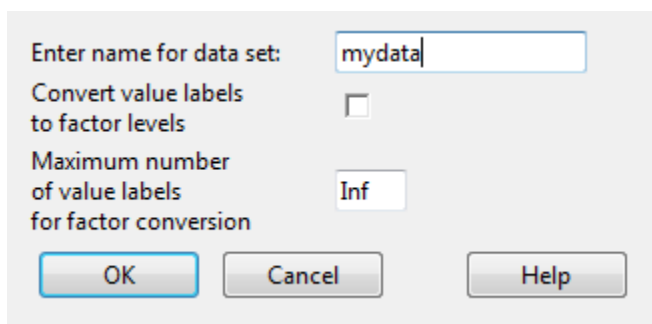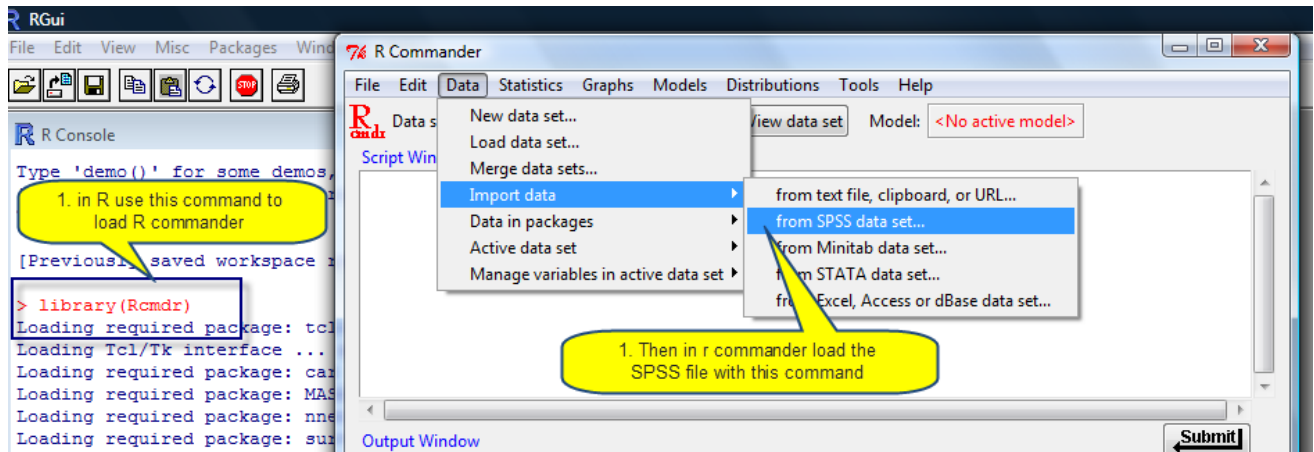
Notice I have unclicked the convert value labels to factor levels option, and given the proposed dataset the name mydata. Click OK then select the required file. Once back in R commander check to see what you have imported by clicking on the View Dataset button.

Typing into the script window names(mydata) and then highlighting it and clicking on the submit button, produces a list of the variables(i.e vectors/fields) in your current dataset.
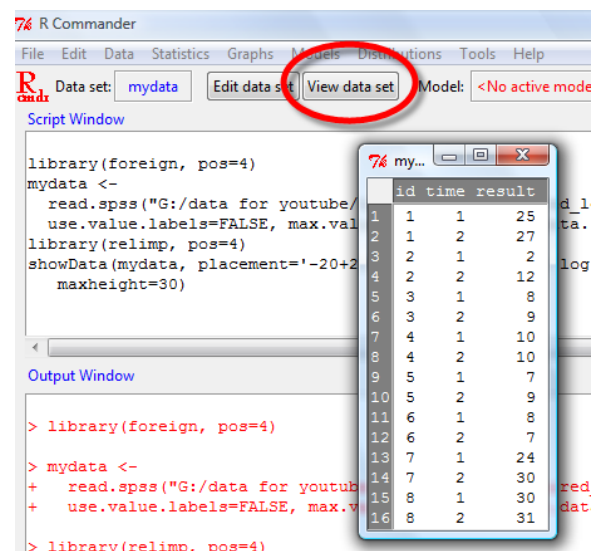
Now we have the data in R and also know the names of the variables we can request the graph we want.
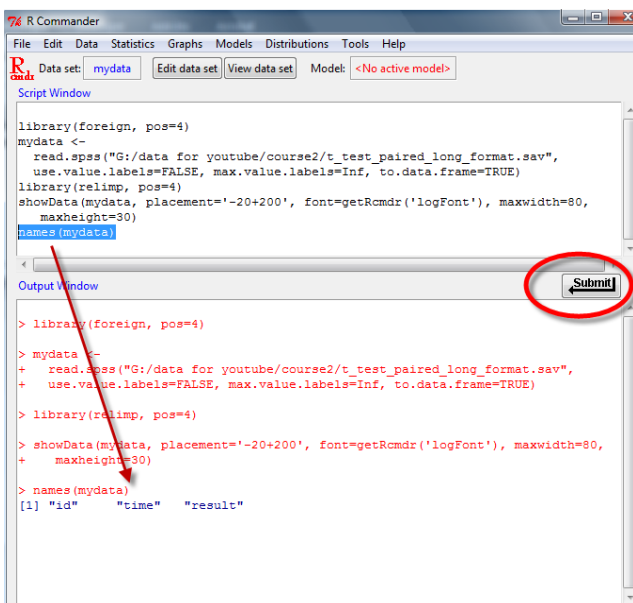
To produce the graph we will use a particular library, in R called **lattice**. I had already downloaded the library before and installed it in R (if you don't know how to do this see the getting started with R chapter) so all I need to do now is load the library remember that loading and installing a library in R are two different things.

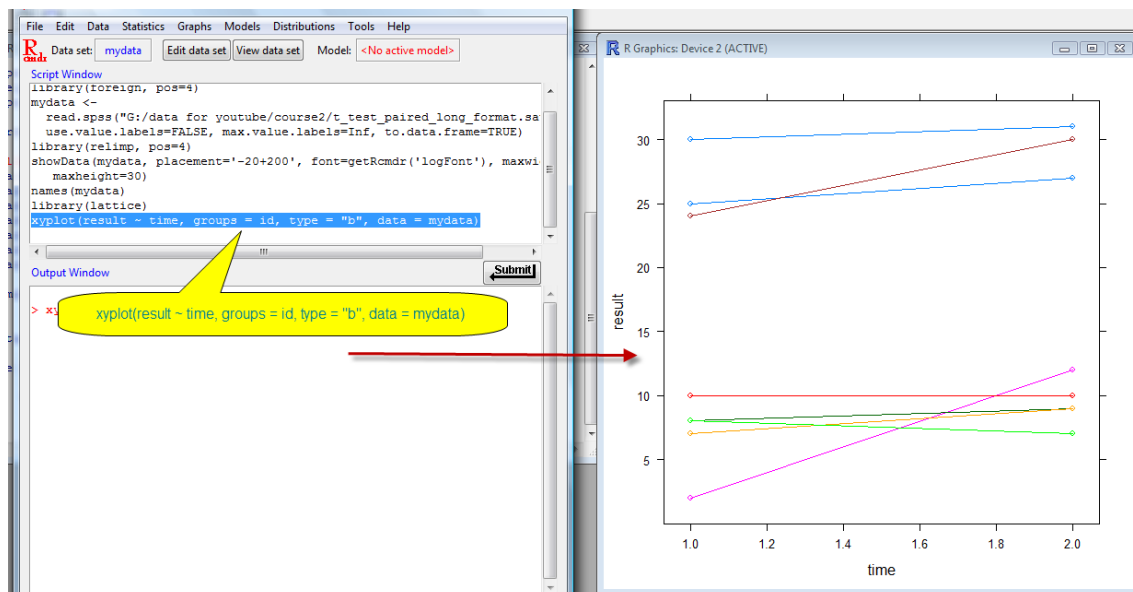We can either load the library in the R commander script window or in the R console window directly. By typing the command, `library(lattice)`.



```
xyplot(result ~ time, groups = id, type = "b", data = mydata)
```

The xyplot function in the lattice library provides the graph we so desired in SPSS! The first expression, `result ~ time`, is just saying model result as a linear function of time (think y x axises) then divide the data into groups, specified by each value of `id` in this instance (i.e subject) and finally the `data =` expression is informing the function of what dataset to use. You can add embellishments to the basic xyplot as shown below:

xyplot(result ~ time, groups = id, type = "b", data = mydata,

 ylab = "number of curls" ,

 xlab = "before - after" ,

 main = "changes over time for experiment x" ,

 scales = list(x = list(at = c(1,2))))

Obviously we have only demonstrated this on a very small dataset with only two time points. But the same procedure would apply to a much larger dataset with possibly many time points also we have only barely scrapped the surface of the xyplot function in the lattice package, there is even a whole book about it (Sarkar 2008)!
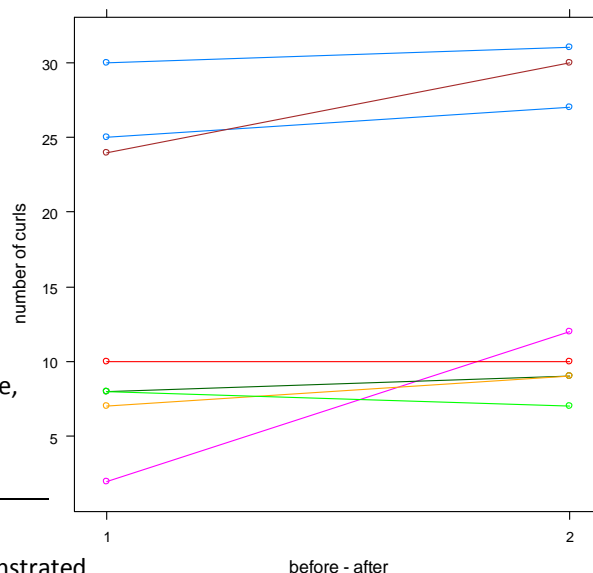
Exercise

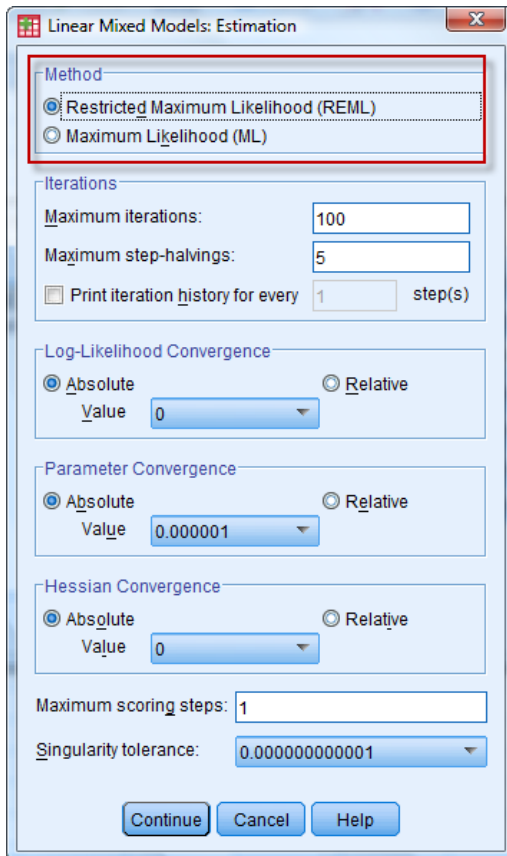Repeat the process above for SPSS and R using both the dataset demonstrated above and also the Twisk health lifestyle dataset (twisk_longitudinal_long.sav) which has four time points. How did you resolve the problem of 4 times points in SPSS?

Also try running for the Twisk data: `xyplot(health ~ time, groups = id, type="l", col= "grey", alpha = 0.25, data = mydata)` . What do you think the col and alpha parameters do?

5. Parameter Estimation

There are two methods available in SPSS for estimating the parameter values, SPSS defaults to the Restricted Maximum Likelihood (REML) method. Unfortunately there appears much controversy over which method is the best to use Twisk presents a table comparing estimates for a model using both methods and shows there is little difference (Twisk 2006 p.29), Singer & Willett (2003 p.87-90) provides a fuller description with references to simulation studies.

A major part of the analysis process involves hypothesis testing and specifically evaluating alternative models and this is where the problems arise. ML (also called the Full ML method [FML]) provides a description of the fit of the full model (both fixed and random parameters) whereas REML only takes into account the random parameters.
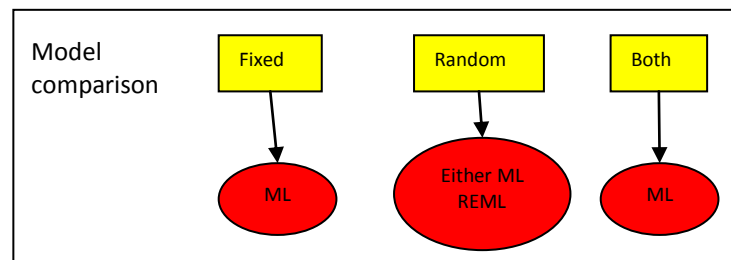
Given the above it is therefore probably best to stick with ML or at least run the analysis using both methods for comparison.

6. Getting to grips with the Mixed models dialog boxes in SPSS

The dialog boxes for carrying out repeated measures analysis using a mixed models approach are complex to say the least, therefore I am going to retrace my steps first and demonstrate how you can carry out both the independent and paired samples t test using these dialog boxes.

id	time	result
1.00	1	25.00
1.00	2	27.00
2.00	1	2.00
2.00	2	12.00
3.00	1	8.00
3.00	2	9.00
4.00	1	10.00
4.00	2	10.00
5.00	1	7.00
5.00	2	9.00
6.00	1	8.00
6.00	2	7.00
7.00	1	24.00
7.00	2	30.00
8.00	1	30.00
8.00	2	31.00

t_test_paired_long_format.sav

7. Independent t test

Referring back to the data we used to demonstrate the paired t test I have restructured it into long format opposite. Id represents the subject and time represents either a pre intervention or post intervention observation.

While we know that the pre post observations are not independent we will assume they are for this demonstration, therefore while this demonstrates how to carry out an independent t test **it is an inappropriate analysis for this dataset**. We are just doing this to demonstrate how you would carry out such an analysis.

The results below are from the usual t test option in SPSS, to allow us to compare them with those produced from the Mixed models dialog boxes.

Group Statistics					
	time	N	Mean	Std. Deviation	Std. Error Mean
result	1	8	14.2500	10.40261	3.67788
	2	8	16.8750	10.46678	3.70057

Estimates								
	Levene's Test for Equality of Variances		t-test for Equality of Means					
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference
								Lower	Upper
Equal variances assumed	.026	.874	-.503	14	.623	-2.62500	5.21737	-13.81515	8.56515
Equal variances not assumed			-.503	13.999	.623	-2.62500	5.21737	-13.81519	8.56519

So now lets repeat the above analysis using the t_test_paired_long_format.sav and the Mixed models dialogues which are accessed from the menu option:
analyse -> Mixed models -> linear.

The first dialog box that appears is for specifying the various levels of data, we can ignore this for our present analysis, so just click continue.
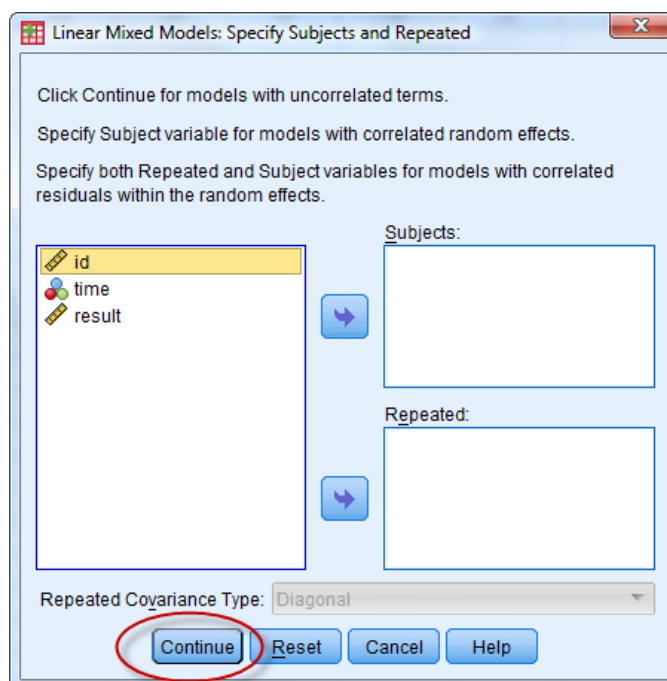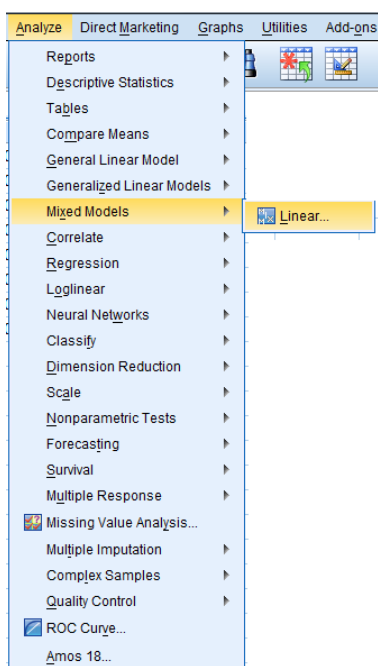
The next dialog box, the Linear Mixed Models dialog asks us to specify the input/output variables, that is the dependent and independent variables, complete it as shown below.

We also need to specify what statistics we require from the analysis, click on the statistics button and complete the Linear Mixed Models: Statistics dialog box and complete as shown below.
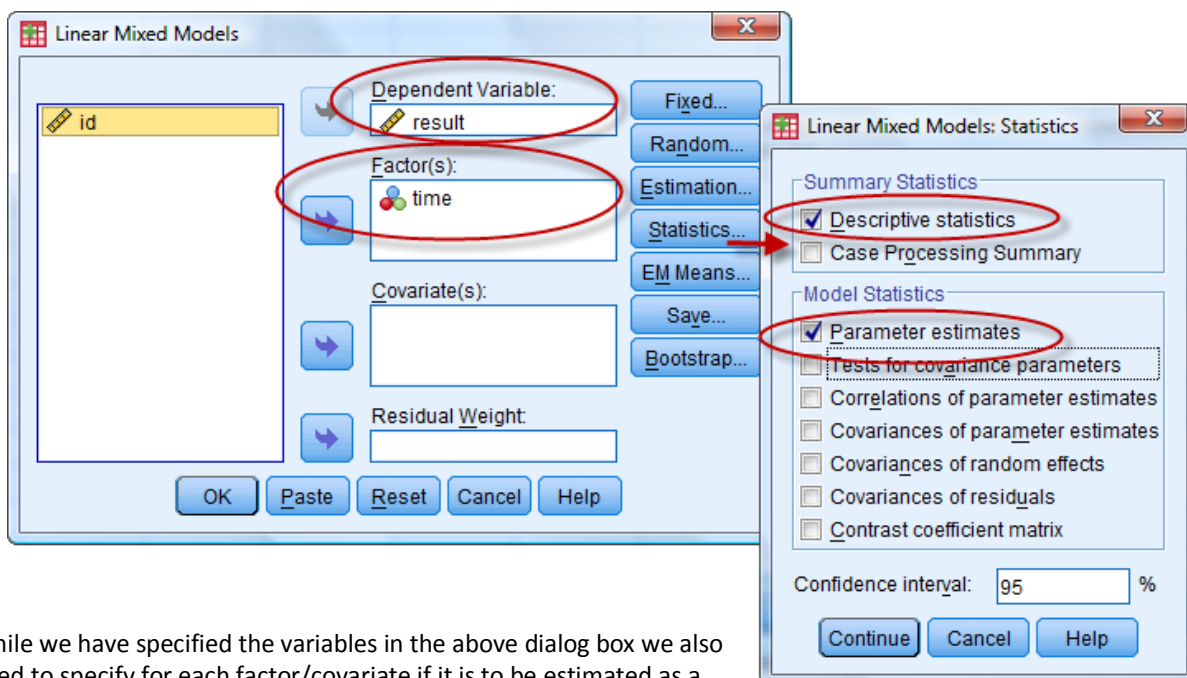
While we have specified the variables in the above dialog box we also need to specify for each factor/covariate if it is to be estimated as a fixed or random variable (i.e. effect), for this analysis we just have one variable which we wish to model as a fixed effect. Clicking on the fixed effect button above produces the dialog box below. We move the time variable to the model box by clicking on the add button.
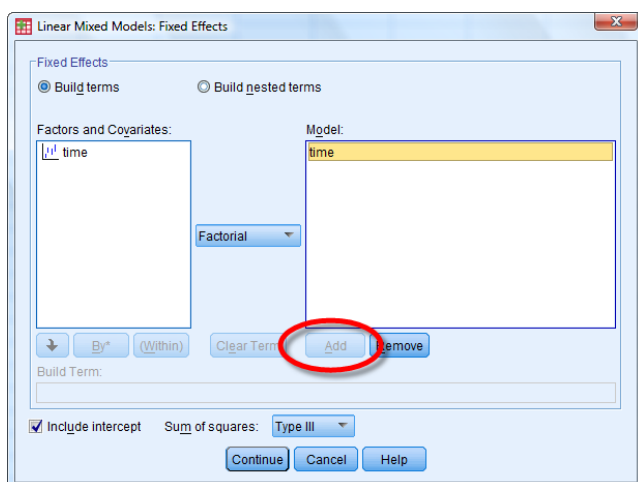
The results from this analysis are shown below.

Descriptive Statistics				
result				
time	Count	Mean	Standard Deviation	Coefficient of Variation
1	8	14.2500	10.40261	73.0%
2	8	16.8750	10.46678	62.0%
Total	16	15.5625	10.17165	65.4%

Model Dimension[a]			Number of Levels	Number of Parameters
Fixed Effects	Intercept		1	1
	time		2	1
Residual				1
Total			3	3
a. Dependent Variable: result.				

Information Criteria[a]	
-2 Restricted Log Likelihood	109.553
Akaike's Information Criterion (AIC)	111.553
Hurvich and Tsai's Criterion (AICC)	111.886
Bozdogan's Criterion (CAIC)	113.192
Schwarz's Bayesian Criterion (BIC)	112.192
The information criteria are displayed in smaller-is-better forms.	
a. Dependent Variable: result.	

Estimates of Fixed Effects[b]							
						95% Confidence Interval	
Parameter	Estimate	Std. Error	df	t	Sig.	Lower Bound	Upper Bound
Intercept	16.87	3.68	14	4.574	.000	8.962367	24.787633
[time=1]	-2.62	5.21	14	-.503	.623	-13.815152	8.565152
[time=2]	0[a]	0	.	.	.	.	.
a. This parameter is set to zero because it is redundant.							
b. Dependent Variable: result.							

It is interesting to note that we could have extended the above to many independent groups which would have been equivalent to carrying out an ANOVA.

Comparing the about output with that from the traditional analysis we can see that they are pretty much identical, we have the same descriptive statistics with the coefficient of variation replacing the sem. The t, p value and confidence

interval values are identical. However in this output there are two additional tables, the information criteria and that for the model dimension.

The model dimension table provides details of the parameters we are requesting to be estimated, while it is not important in the above output it is vital when we want to check that we have specified a model correctly and also want to check the number and type of parameter estimates for a particular model when we wish to compare various models.

The information criteria table offers us several measures of model fit, the most basic measure being the -2log likelihood this is difficult to interpret and does not take into account the relative complexity (i.e. number of parameter estimates needed to define it), in contrast Akaikes information criteria is purported to do so, which implies we can theoretically just compare models looking at the AIC and selecting the one with the smallest value, however there is controversy about this measure and often people use both the AIC and the -2logLL to hedge their bets, we will be taking this approach when evaluating our models.

8. Taking into account dependency – Fixed and Random effects

In the above analysis we have ignored the fact that the pre – post test measures for each subject are not independent it is clear that the post test value is related to the pre test value for the individual subject.
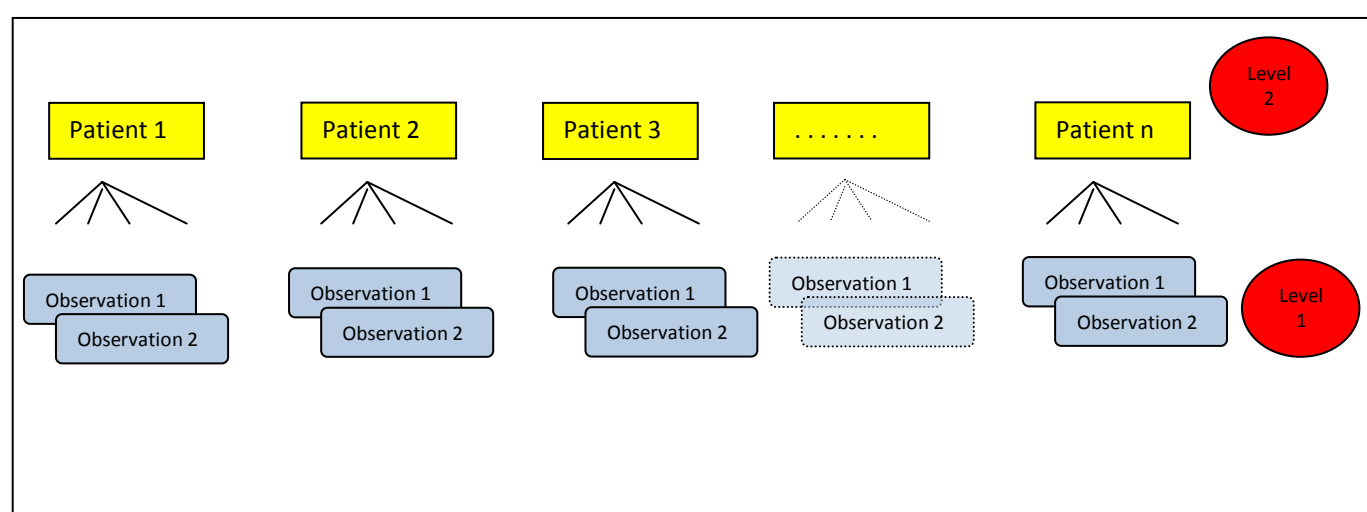
In the above analysis where we did not take into account this dependency we defined our parameter we estimated as a **fixed** effect parameter. Basically this means we only estimated a single value (a point estimate). However to define the degree of dependency between any two of the level 1 repeated measures pairs means that we would need to estimate a fixed parameter (i.e. the covariance/correlation) value for each pair, this would use up a large number of our degrees of freedom having to estimate as many parameters as half our sample size!

A solution to this problem is to estimate a **Random** Effect parameter instead. A random effect parameter specifies a normal distribution instead of a single value and as we know a normal distribution is defined by its mean and variance (standard deviation). Ignoring the mean value, this results in us now being able to estimate the (covariance/correlation) between each of the repeated measures pairs as a normal distribution (assuming they are in reality normally distributed).

Remembering that we need an estimate of the variance for each of the observations to calculate the covariance/correlation, we have now reduced the number of parameters we need to estimate from half the sample size down to just three, and thereby increasing the degrees of freedom substantially.

Another advantage of defining the relationship between the repeated measures as a Random effect parameter is that we are not now just considering our observed repeated measures pairings but a population of repeated measures from which they come clearly giving us more generalisability (assuming this is a valid assumption!).

In the following chapters we will look in much more depth at the concept of Random effects.

We will now consider the paired samples t test which does in effect take into account the dependency between the pre and post test score for each subject.

9. Paired samples t test

First of all we carry out the paired samples t test using the usual dialog boxes in SPSS, therefore we have the data in wide format to allow that.

pre	post	id
25.00	27.00	1.00
2.00	12.00	2.00
8.00	9.00	3.00
10.00	10.00	4.00
7.00	9.00	5.00
8.00	7.00	6.00
24.00	30.00	7.00
30.00	31.00	8.00

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	pre	14.2500	8	10.40261	3.67788
	post	16.8750	8	10.46678	3.70057

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	pre & post	8	.940	.001

Paired Samples Test

Pair 1	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
pre - post	-2.62500	3.62284	1.28087	-5.65377	.40377	-2.049	7	.080

Note from the above that when you square the standard deviations for the pre and post test scores you obtain the variances of; 108.2 and 109.53 also remember that the correlation is just the standardised covariance, that is by dividing the covariance by the product of the two standard deviations we obtain the correlation.

So: $correlation_{a.b}$ = covariance/(sd_a x sd_b)   = covariance /($\sqrt{variance}$ $_a$ x $\sqrt{variance}$ $_b$) = .94

To carry out the equivalent analysis using the Linear mixed models dialog boxes you need the data in log format using the t_test_paired_long_format.sav file. Within the Linear mixed models dialog boxes you specify the repeated measures, by using the first dialog box specifying the subjects and repeated measures variables as shown below.
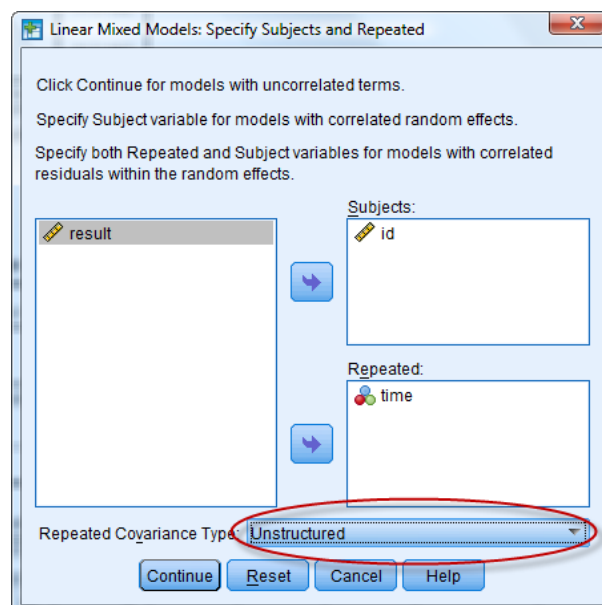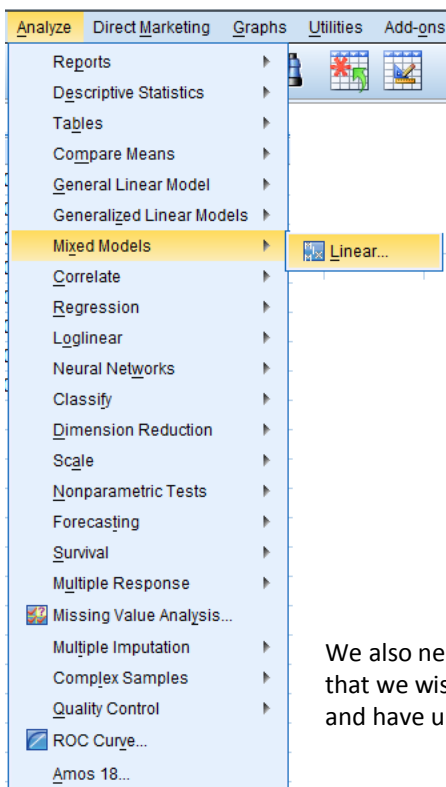
We also need to change the repeated covariance type to unstructured which informs SPSS that we wish to allow the pre post measures (i.e. the repeated measures) to be correlated and have unequal variances in the model.

As before we setup the other dialog boxes, ensuring we use Restricted Maximum likelihood estimation (usually we would not select this option):
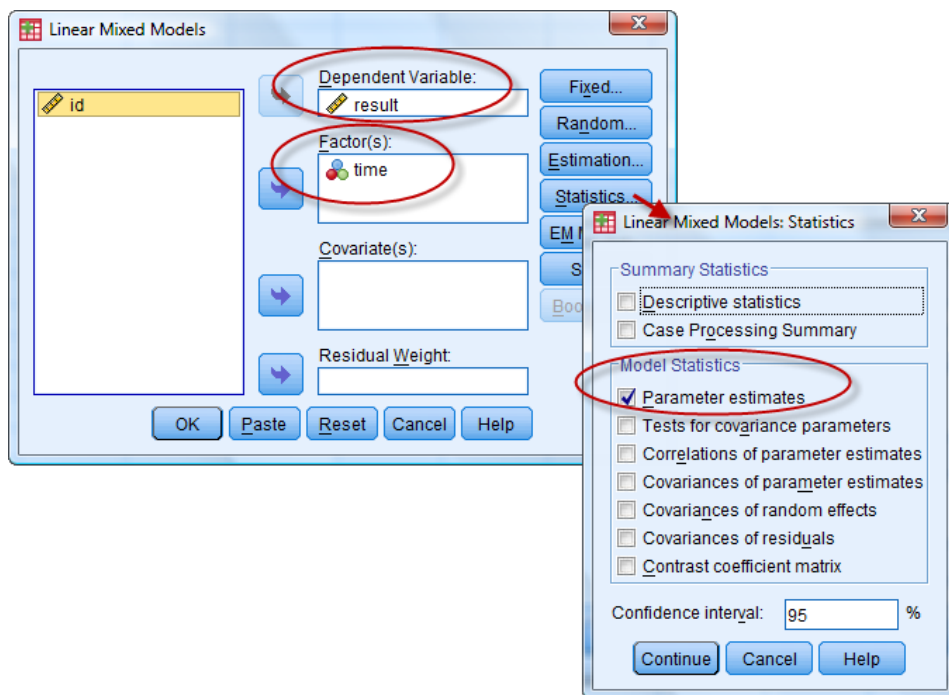
The results are shown below.

Model Dimension[a]						
		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	time	2		1		
Repeated Effects	time	2	Unstructured	3	id	8
Total		5		5		
a. Dependent Variable: result.						

Information Criteria[a]	
-2 Restricted Log Likelihood	94.526
Akaike's Information Criterion (AIC)	100.526
Hurvich and Tsai's Criterion (AICC)	102.926
Bozdogan's Criterion (CAIC)	105.443
Schwarz's Bayesian Criterion (BIC)	102.443
The information criteria are displayed in smaller-is-better forms.	
a. Dependent Variable: result.	

Type III Tests of Fixed Effects[a]				
Source	Numerator df	Denominator df	F	Sig.
Intercept	1	7	18.347	.004
time	1	7	4.200	.080
a. Dependent Variable: result.				

Estimates of Fixed Effects[b]							
Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	16.87	3.70	7	4.560	.003	8.12	25.62
[time=1]	-2.62	1.28	7	-2.049	.080	-5.65	.403
[time=2]	0[a]	0	.	.	.	.	.
a. This parameter is set to zero because it is redundant.							
b. Dependent Variable: result.							

Estimates of Covariance Parameters[a]			
Parameter		Estimate	Std. Error
Repeated Measures	UN (1,1)	108.214286	57.842969
	UN (2,1)	102.321429	56.473681
	UN (2,2)	109.553571	58.558847
a. Dependent Variable: result. (using restricted ML if we had used ML we would have got smaller values)			

Ignoring for now much of the above you will notice that we obtained exactly the same *t* and *p* values as that obtained by the traditional paired *t* test.

Referring back to the original SPSS paired t test output and noting that the variances are; 108.2 and 109.53 and also the correlation we have equivalent values in the estimates of covariance table.

Diagonals = Variance = sd²;   **Off diagonals = Covariance**

	T1 = pre	T2 = post
T1 = pre	108.214286	
T2 = post	102.321429	109.553571

Remembering that the correlation is just the standardised covariance, that is if we divide the covariance by the product of the two standard deviations (= square roots of the variances) we obtain the correlation coefficient.

$correlation_{a.b} = covariance/(sd_a \times sd_b) = covariance /(\sqrt{variance}_a \times \sqrt{variance}_b)$

$correlation_{pre.post} = 102.3214 / (\sqrt{108.214} \times \sqrt{109.5535}) = .939$

10. Comparing models

Information Criteria[a] using ML estimation	
-2 Log Likelihood	101.140
Akaike's Information Criterion (AIC)	111.141
Hurvich and Tsai's Criterion (AICC)	117.141
Bozdogan's Criterion (CAIC)	120.004
Schwarz's Bayesian Criterion (BIC)	115.004
The information criteria are displayed in smaller-is-better forms.	
a. Dependent Variable: result.	

Estimates of Covariance Parameters[a] using ML estimation			
Parameter		Estimate	Std. Error
Repeated Measures	UN (1,1)	94.687500	47.343750
	UN (2,1)	89.531250	46.223005
	UN (2,2)	95.859375	47.929687
a. Dependent Variable: result.			

From the above analysis you may have noticed that I used the restricted maximum likelihood estimation option if I have used (full) maximum likelihood I would have obtained the table opposite. Interestingly while the fixed parameter (and p value) estimate would have been identical the estimate of the covariance parameters table would have had contained smaller values and I would not have been able to carry out the comparison between the traditional and Linear mixed models variances and correlation.

Regardless of the estimation method, you may have noticed that the variances of both repeated measures, that is the pre and post test scores are almost identical. Additionally from the dimension table you notice that we used up three degrees of freedom to estimate the unstructured covariance matrix for the repeated measures, one for the pre test variance, one for the post test variance and one for the covariance, because we specified a unstructured covariance matrix.
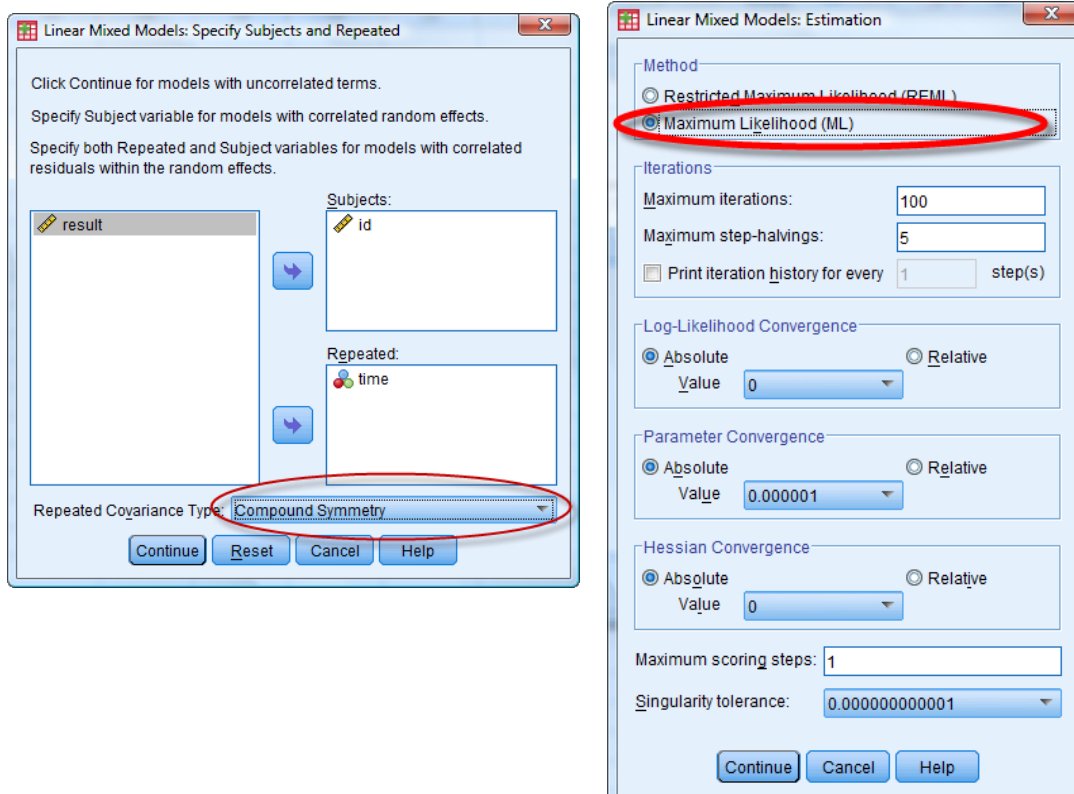
One of the basic ideas concerning model development is to find the model that uses the least number of parameters (freeing up the largest number of data items = degrees of freedom) along with the best fit.

Is this instance we can reduce the number of parameters needed to be estimated by using the Compound symmetry covariance matrix which assumes constant variance and co variation so in this instance means the same variance for the repeated measures (i.e. pre post test scores).

As mentioned previously we can compare the models by taking the -2LL and the carrying out a Likelihood ratio test. As well as comparing the various other information criteria measures.

The important thing to realise here is that you usually need to use **ML estimation of the -2LL value if you plan to compare models rather than the restricted maximum likelihood estimate.**

To re-run the analysis we just need to edit the first box, and make sure we have the estimation method to be maximum likelihood.

Model Dimension[a]		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	time	2		1		
Repeated Effects	time	2	Compound Symmetry	2	id	8
Total		5		4		
a. Dependent Variable: result.						

Information Criteria[a]	
-2 Log Likelihood	101.143
Akaike's Information Criterion (AIC)	109.143
Hurvich and Tsai's Criterion (AICC)	112.780
Bozdogan's Criterion (CAIC)	116.234
Schwarz's Bayesian Criterion (BIC)	112.234
The information criteria are displayed in smaller-is-better forms.	
a. Dependent Variable: result.	

Estimates of Fixed Effects[b]						95% Confidence Interval	
Parameter	Estimate	Std. Error	df	t	Sig.	Lower Bound	Upper Bound
Intercept	16.87	3.45	8.497	4.890	.001	8.99	24.75
[time=1]	-2.62	1.19	8	-2.191	.060	-5.38	.137
[time=2]	0[a]	0	.	.	.	.	.
a. This parameter is set to zero because it is redundant.							
b. Dependent Variable: result.							

| Estimates of Covariance Parameters[a]		Estimate	Std. Error
Repeated Measures	CS diagonal offset	5.742187	2.871094
	CS covariance	89.531250	46.223469
a. Dependent Variable: result.			

Comparing the two models we have a -2 LL of 101.140 (corrected to 3 d.p) (df=5) for the more complex model and 101.143 (df=4) for the less complex one. Subtracting the value of the less complex model from that of the other gives a value of 0.003 which if both models as equivalent, that is the difference is due to random sampling, then the value follows a chi square distribution with degrees of freedom equal to the difference in degrees of freedom between the models. Looking at the model dimension tables we see that the df difference is 5-4=1 and using r commander in r, we can obtain a graphical interpretation of the P value this being the area under the curve to right of the chi square value. Finally in R we can use the 1-pchisq(0.003,1) expression to obtain the numerical p value, that is the proportion of the area under the curve highlighted below.

```
> abline(v=0.003)
> 1-pchisq(0.003,1)
[1] 0.9563199
> text(6,.6,"chisq of 0.003 p=.956")
> |
```
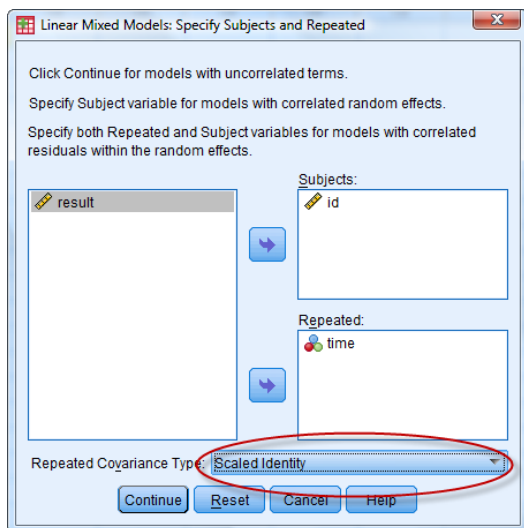
We can conclude that we would obtain two models displaying a similar degree of fit or one less similar 96 times in a hundred given that they come from the same population. Considering the hypothesis testing approach as this is greater than the traditional level of 1 in 20 (0.05) we accept it and conclude there is no difference in the degree of fit between the two models,

However because one is less complex i.e. uses less parameters we would probably use the simpler one.

Chi-Squared Distribution: df = 1

chisq of 0.003 (df=1) p=.956
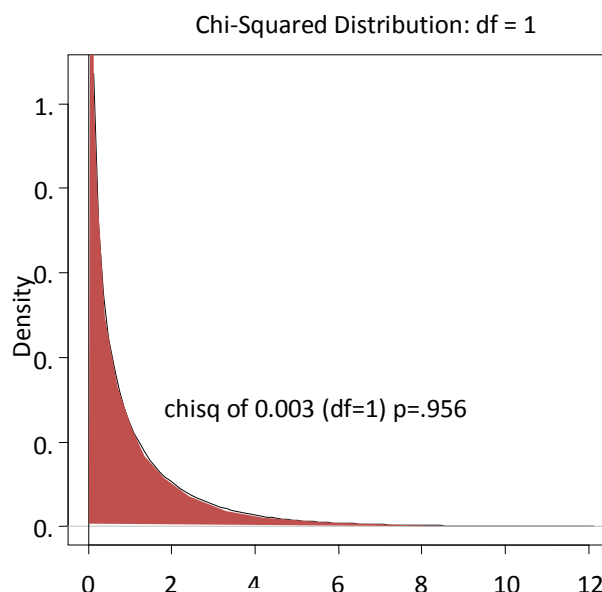
11.   Model Covariance matrixes for repeated measures (level one)

The first Dialog box in the Linear Mixed Models menu provides an option for specifying how you want the repeated measures to be modelled, we have used two options so far, unstructured and compound symmetry. However there are several more and a particularly interesting one is scaled identity. Scaled Identity covariance structure matrix allows us to model a set of repeated measures that we feel may be independent and all of equal variance. If we repeat the analysis described in the previous page with this covariance structure we would have got back to square one, that is the independent *t* test? So within these dialog boxes there are many different ways of obtaining the same model and one needs to be very specific about specifying the model and understanding what the results mean.

Estimates of Fixed Effects[b]							
						95% Confidence Interval	
Parameter	Estimate	Std. Error	df	t	Sig.	Lower Bound	Upper Bound
Intercept	16.87	3.68	14	4.574	.000	8.96	24.78
[time=1]	-2.62	5.21	14	-.503	.623	-13.81	8.56
[time=2]	0[a]	0	.	.	.	.	.
a. This parameter is set to zero because it is redundant.							
b. Dependent Variable: result.							

I have produced a table below of the common covariance matrixes along with a screen dump of the options available from SPSS for the repeated measures covariance types.
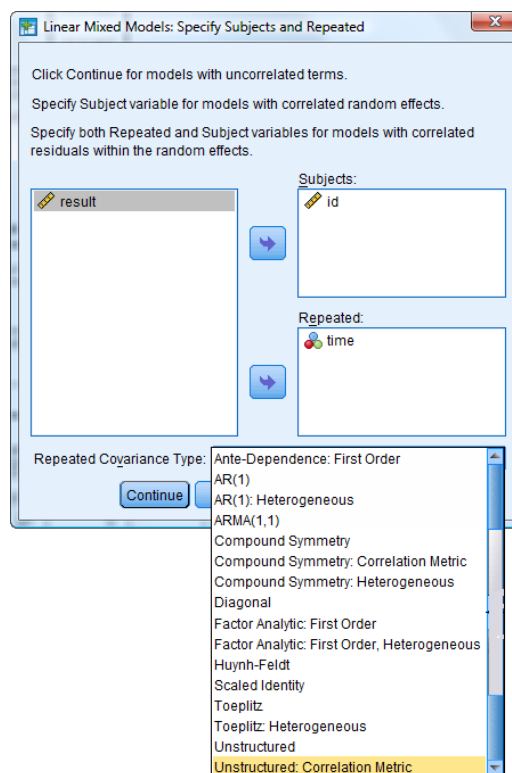
name	Variance at each time point	Correlation (covariance) between measurement times
Scaled identity	constant	No correlation (i.e. independent)
Compound symmetry	constant	constant
Diagonal	different at each time	No correlation (i.e. independent)
Unstructured	different at each time	Different for each time pairing
AR(1) =Autoregressive	constant	Correlation gets less as time points get further apart (i.e. $t_1, t_2 = \rho$ but $t_1, t_3 = \rho^2$)

12.   Summary

This chapter has provided a short introduction into the Linear Mixed models dialog boxes in SPSS and has demonstrated how you can obtain the equivalent to an independent sample t test (one way ANOVA) and also a paired t test. The concepts of Maximum likelihood estimation and partial maximum likelihood estimation were introduced as well as Fixed and Random effects. The ability to compare models was demonstrated by looking at the difference in the -2LL and degrees of freedom between them where the value follows a chi square distribution (df= difference in the df's) and also mention was make of Akaikes Information Criterion .

I have tried to emphasis the importance of visualising the data and this aspect, although can be difficult is an important first step in the analysis.

This was very much an introduction which we will develop in the next few chapters.

I have deliberately not considered the R equivalents here – they are so much easier than the baroque dialog boxes of SPSS. A YouTube video demonstrates this.

13.   References

Bickel R 2007 Multilevel Analysis for Applied Research: It's Just Regression! The Guilford Press.

Bollen K A, Curran P J. 2006 Latent Curve Models: A Structural Equation Perspective. Wiley

Bollen K. A. 1989 Structural Equations with Latent Variables. Wiley.

Brown H, Prescott R 2006 (2nd ed.) Applied Mixed models in Medicine. Wiley.

Campbell 2006 (2nd ed.) Statistics at square 2. BMJ books.

Crawley M J 2005 Statistics: An introduction using R. Wiley

Crowder M J.Hand D J. 1990 Analysis of repeated measures. London: Chapman and Hall.

Diggle P J Hegaerty P Liang K-Y. Zeger S. 2002 (2nd ed.) Analysis of Longitudinal Data. Oxford Science publications.

Duncan T E, Duncan S C, Strycker L A. 2006 (2nd ed.) An Introduction to Latent Variable Growth Curve Modeling: Concepts, Issues, and Applications: Lawrence Erlbaum.

Everitt B S, Hothorn T 2006 (1st ed) A handbook of statistical analyses using R. Chapman & Hall.

Everitt B S, Hothorn T 2010 (2st ed) A handbook of statistical analyses using R. Chapman & Hall.

Field A 2009 Discovering Statistics Using SPSS. Sage [Chapter nineteen describes multilevel modelling including restructuring data].

Fitzmaurice G M, Laird N M, Ware J M, 2004 Applied Longitudinal Analysis. Wiley

Goldstein H. 2011 (4th ed.) Multilevel statistical models. Wiley.

Hand D, Crowder M 1996 Practical Longitudinal data analysis. Chapman & hall

Heck R H, Thomas S L & Tabata L N. 2010 Multilevel and Longitudinal modelling with IBM SPSS. Routledge.

Hedeker, D., and R. D. Gibbons. 2006. Longitudinal Data Analysis. Wiley. Website: tigger.uic.edu/~hedeker/ml.html

Hox J 2010 (2nd ed) Multilevel analysis: Techniques and applications. Lawrence Erlbaum

Landau S, Everitt B S 2004 A handbook of statistical analyses using SPSS. Chapman & Hall.

Leeuw J D,·Meijer E 2008 (eds.) Handbook of Multilevel Analysis. Springer.

Matthews J N S, Altman D G, Campbell M J, Royston J P 1990 Analysis of serial measurements in medical research. B M J 300 230 – 235

Norman G R, Streiner D L. 2008 (3rd ed) Biostatistics: The bare Essentials.

Sarkar D 2008 Lattice: Multivariate Data Visualization with R. Springer.

Sheather S J 2009 A Modern approach to regression with R. Springer. Website, includes R tutorials and datasets: http://www.stat.tamu.edu/~sheather/book/

Singer, J. D., and J. B. Willett. 2003. Applied Longitudinal Data Analysis: Modeling Change and Event Occurrence. Oxford University Press Inc, USA. gseacademic.harvard.edu/alda/

Twisk J W R 2003 Applied Longitudinal analysis for Epidemiology: A practical Approach. Cambridge University Press.

Twisk J W R 2006 Applied multilevel analysis. Cambridge University Press.

Weiss R 2005 Modeling Longitudinal Data. Springer.

West B T, Welch K B, Galecki A T. 2007 Linear Mixed Models: A Practical Guide Using Statistical Software. Chapman & Hall. Book website: http://www.umich.edu/~bwest/almmussp.html

Wu L 2010 Mixed Effects Models for Complex Data. Chapman & Hall.

Zuur A F, ·Ieno E N, · Walker N J, Saveliev A A·Smith G M 2009 Mixed Effects Models and Extensions in Ecology with R. Springer