Problem Set 2 (Due Thursday, February 2 at 11:59 pm)

Reading: Boas, Chapter 3 Sections 1–9 and Chapter 10 Section 5

Problems:

Prepare solutions to the following problems. To receive full credit, show your work completely. In the end, you will have a set of PDF files to turn in using github.

N6. **(8 pts)** Complex Numbers

Write a solution to this problem on paper or electronically; save your work as `n6.pdf`.

(a) Plot $z = -1 - \sqrt{3}i$ in the complex plane and determine its modulus and angle.

(b) Calculate the modulus of $z = \dfrac{2+i}{4i} + \dfrac{5i}{1+3i}$.

(c) Find all of the *real* values for x, y that solve $x + iy = (1-i)^2$.

(d) Put $\dfrac{(1+i)^{48}}{(\sqrt{3}-i)^{25}}$ in rectangular form.

N7. **(6 pts)** Small Angle Approximations

Do this in Mathematica; save your output as `n7.pdf`.

In your freshman-level physics course(s), you (hopefully!) learned that $\sin\theta$ can be approximated by θ for small angles.

(a) Using *Mathematica*, create a plot of θ, $\sin\theta$, and $\theta - \sin\theta$ from 0 to 2π. (All three curves should be shown on the same coordinate axes. Include axis labels and a nice legend showing which curve is which.)

(b) The fractional difference between θ and $\sin\theta$ is given by

$$f(\theta) = \frac{(\theta - \sin\theta)}{\sin\theta}.$$

For example, $f(\pi/10) \approx 0.0166$, which means that θ and $\sin\theta$ differ by about 1.7% for $\theta = \pi/10$. If you wish to keep the fractional difference less than 1.0%, what is the maximum angle θ *in degrees* for which θ is a good approximation of $\sin\theta$?

(c) What is a useful small-angle approximation for $\cos\theta$? For what values of θ is it useful?

N8. **(5 pts)** Using Euler's Formula

Write a solution to this problem on paper or electronically; save your work as `n8.pdf`.

Using Euler's formula, derive the identity,

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}\left[a\cos(bx) + b\sin(bx)\right]}{a^2+b^2}.$$

(more...)

N9. **(8 pts)** Some Trig Identifies for Complex Numbers

Write a solution to this problem on paper or electronically; save your work as `n9.pdf`.

Using the identities

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \qquad \text{and} \qquad \sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

for a complex number z, show that

(a) $\sin(x + iy) = \sin x \cosh y + i \cos x \sinh y$

(b) $\cos(x + iy) = \cos x \cosh y - i \sin x \sinh y$

(c) $|\sin z|^2 = \sin^2 x + \sinh^2 y$

(d) $|\cos z|^2 = \cos^2 x + \sinh^2 y$

This demonstrates that $|\sin z|$ and $|\cos z|$ can be greater than 1 in the complex plane!

N10. **(5 pts)** RL circuit

Write a solution to this problem on paper or electronically; save your work as `n10.pdf`.

An AC circuit consists of a voltage source $V(t) = V_0 \sin \omega t$ connected to a resistor R and an inductor L in series. Using the fact that the impedance of an inductor is $Z_L = i\omega L$, derive an expression for the voltage drop across the resistor as a function of time.

N11. **(10 pts)** Boas Chapter 2, Section 17, Problem 25 (page 81)

Write a solution to this problem on paper or electronically; save your work as `n11.pdf`.

N12. **(6 pts)** Boas Chapter 2, Section 17, Problem 28 (page 81)

Write a solution to this problem on paper or electronically; save your work as `n12.pdf`.