Homework 3 **(DRAFT)**

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1 Problem 1

Exercise 2 in Section 3.6

1.1 Solution

Note: *My MATLAB code for this homework problem repeats all the steps for example 3.1 so that I can take on this problem. I will only cover the checkerboard test in this write-up.*

The checkerboard test using $\boldsymbol{m}_{true}$ can be reshaped to $\boldsymbol{m}_{true} \in \mathbb{R}^9$ such that

$$\boldsymbol{m}_{true} = \begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}^T$$

which allows for the creation of test data $\boldsymbol{d}_{true}$ and a recovered model $\boldsymbol{m}_{\dagger}$.

$$\boldsymbol{d}_{true} = G\boldsymbol{m}_{true}$$

$$\boldsymbol{m}_{\dagger} = G^{\dagger}\boldsymbol{d}_{true}$$

Recall from example 3.1 that $G \in \mathbb{R}^{8\times 9}$ with rank 7. Therefore the generalized pseudo-inverse of G, represented as $G^{\dagger}$, was computed using the Moore-Penrose pseudo-inverse function `pinv(G)` in MATLAB®. Figure 1 shows how the recovered model compares to the true model.

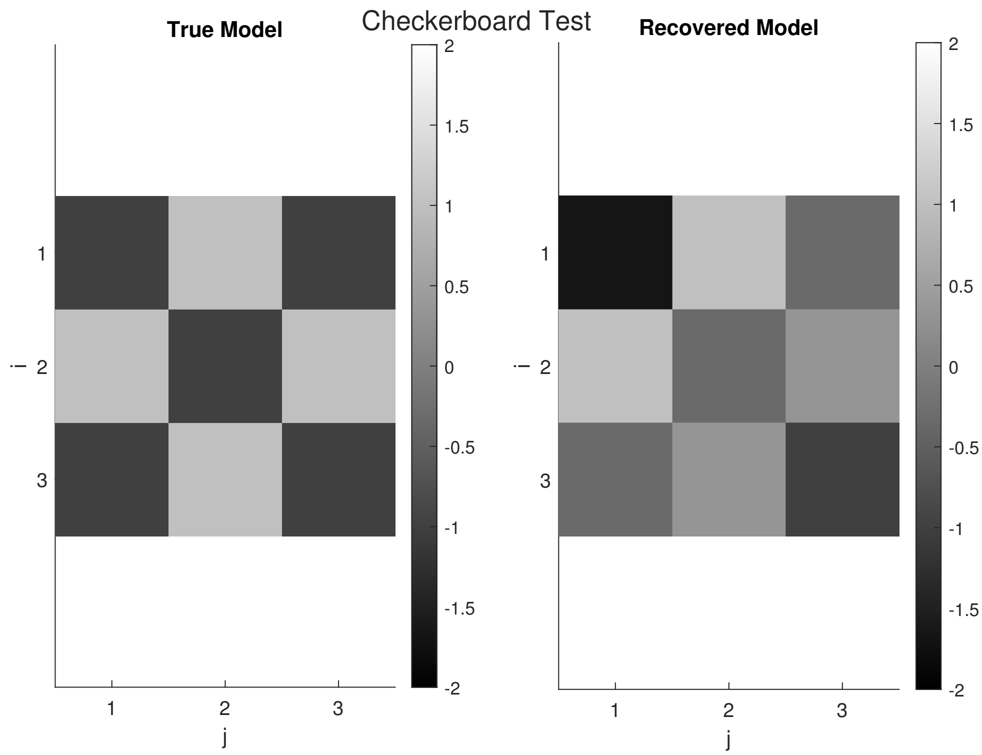

Figure 1: Checkerboard Test

Interpreting these results, only three of nine model parameters m_2, m_4, m_9 were recovered with no error. The error in the recovered model, $\boldsymbol{\Delta m} := \boldsymbol{m}_{\dagger} - \boldsymbol{m}_{true}$, is shown below.

$$\boldsymbol{\Delta m} = \begin{bmatrix} \frac{-2}{3} & 0 & \frac{2}{3} & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{2}{3} & \frac{-2}{3} & 0 \end{bmatrix}^T$$

When examining the model resolution matrix R_m, it is interesting that only one diagonal value of R_m is equal to one even though three model parameters where recovered perfectly as shown in figure 2.

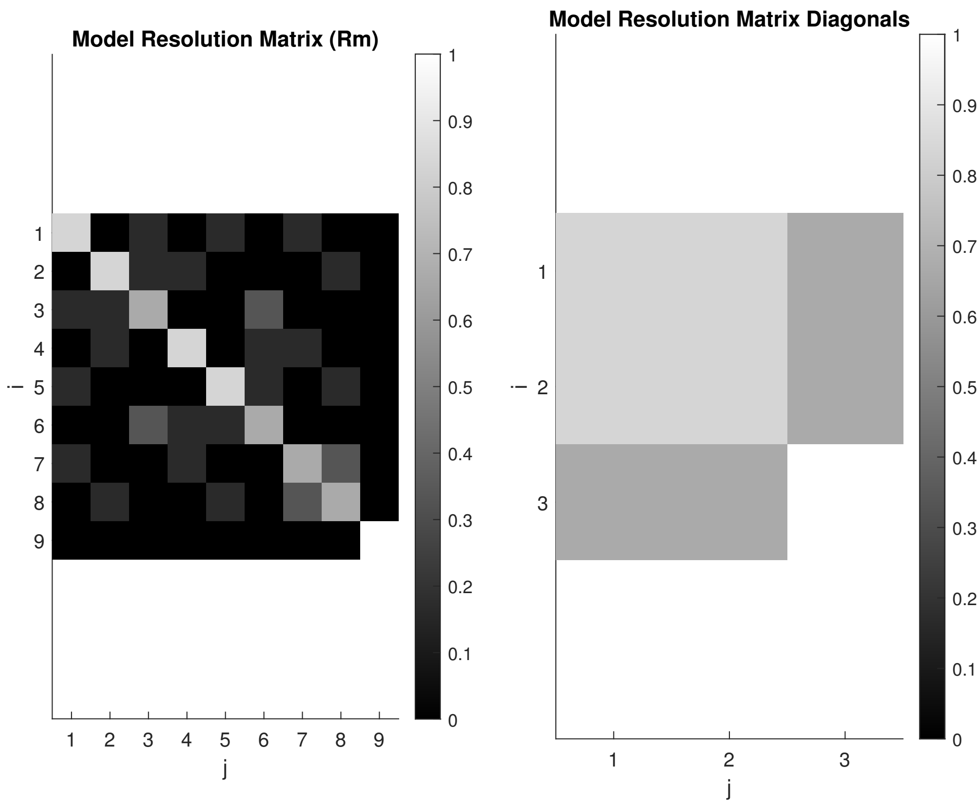

Figure 2: Checkerboard Test

There is more to analyze here, but I am a bit confused by the results right now.

2 Problem 2

Exercise 4 in Section 3.6

2.1 Solution

NOTE: *I do not have any experience in seismology, please forgive any mistakes in technicalities when I try to explain this exercise in my own words. It helps me understand what is going on so I can set up the problem correctly.*

The forward problem in this exercise allows mechanical waves to propagate through a 16×16 meter grid where each square in the grid have some slowness value $s_{x,y}$ in units of $\mathrm{s\,m^{-1}}$. Stations around the grid record time of arrivals of the mechanical waves as they pass through one or multiple grid squares. The time it takes to pass through a path of squares is formulated below.

$$t = \int_l s\left(\boldsymbol{x}\right) dl$$

$$\approx \sum_{blocks} s_{block} \Delta l$$

2.1.1 Part A - Row and Column Scans Only

16 row scans and 16 column scans are utilized in this part in the exercise as shown in figure 3.

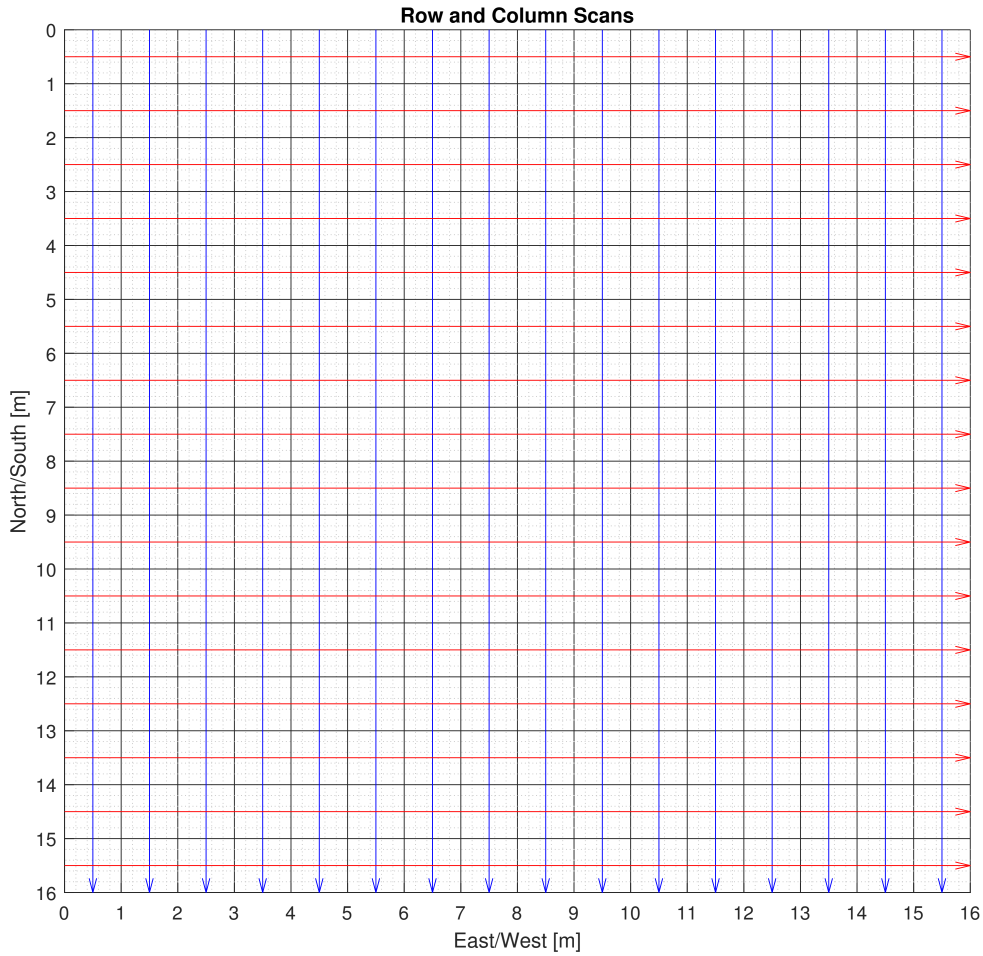

Figure 3: Row and Column Scan Visualization

This results in a total of $m = 32$ measurements. In an effort to estimate the slowness of each square in the grid, this results in a number of $n = 256$ model parameters.

$$\boldsymbol{d} \in \mathbb{R}^{32}, \quad \boldsymbol{m} \in \mathbb{R}^{256}, \quad G \in \mathbb{R}^{32\times 256}$$

The vector of measurement observations $\boldsymbol{d}$ is organized such that

$$\boldsymbol{d} = \begin{bmatrix} t_{r,1} & t_{r,2} & \dots & t_{r,16} & t_{c,1} & t_{c,2} & \dots & t_{c,16} \end{bmatrix}^T$$

where a r subscript indicates a row can and a c subscript indicates a column scan. The model parameters $\boldsymbol{m}$ are organized such that

$$\boldsymbol{m} = \begin{bmatrix} s_{1,1} & s_{1,2} & \dots & s_{1,16} & s_{2,1} & s_{2,2} & \dots & s_{16,1} & \dots & s_{16,16} \end{bmatrix}^T$$

where the first subscript indicates the row, and the second subscript indicates the column. Each row of the model operator G contain the distance traveled by the mechanical wave for each square in its path. Due to the large number of elements, a color map of the zeros and ones for this part of the problem as given instead in figure 4.

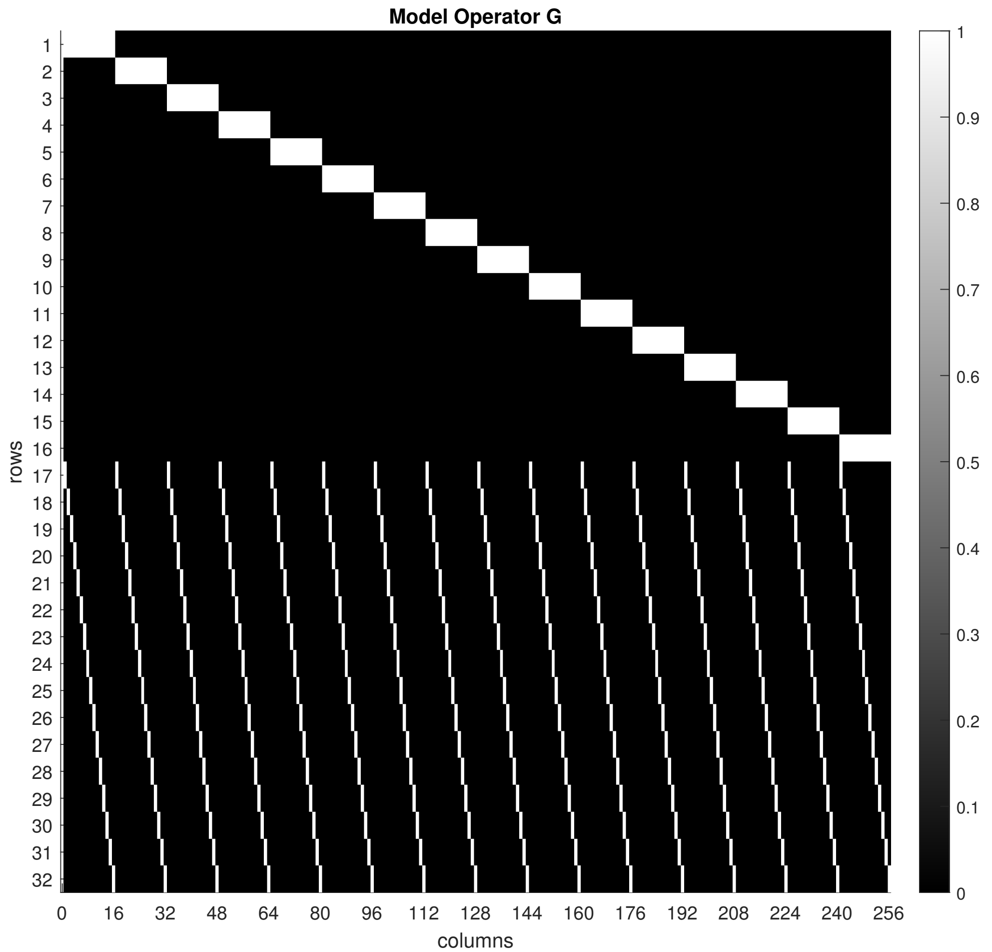

Figure 4: Row and Column Scan Visualization

Subpart A
Per MATLAB®, the rank of G is 31.
Subpart B