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A GENERALIZED IMU MODELING FRAMEWORK FOR VARYING INERTIAL SENSING TECHNOLOGIES AND PERFORMANCE GRADES

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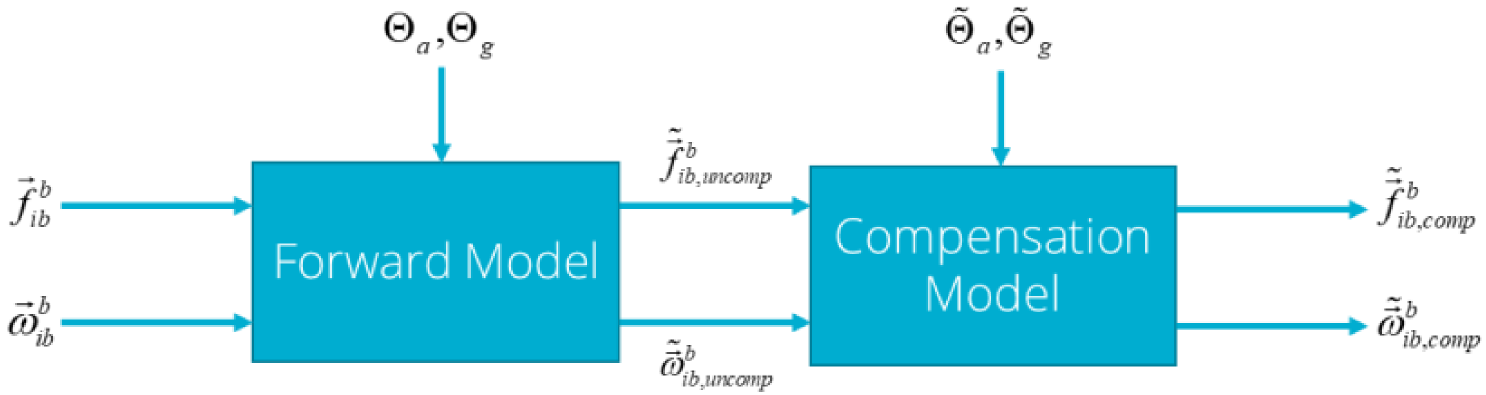
OVERVIEW

- The IMU Modeling Problem
- Basic IMU Modeling
- IEEE Standards for Inertial Sensing
- The Need for a Generalized IMU Modeling Framework
- Generalized Inertial Sensing Model Applications
- Future IMU Model Extensions

THE IMU MODELING PROBLEM

- The term “IMU Model” often has different meanings depending on the context, including:
  - Simulation of an IMU
  - IMU Calibration and Compensation
  - IMU Error Properties for Kalman Filtering Applications



IMU Modeling Questions

- What model should I use?
- What parameters belong in my model?

<u>Forward Model</u>

$$\tilde{\vec{f}}^b_{ib,uncomp} = f_a\left(\vec{f}^b_{ib};\ \Theta_a\right)$$

$$\tilde{\vec{\omega}}^b_{ib,uncomp} = f_g\left(\vec{\omega}^b_{ib};\ \Theta_g\right)$$

<u>Compensation Model</u>

$$\tilde{\vec{f}}^b_{ib,comp} = g_a\left(\tilde{\vec{f}}^b_{ib,uncomp};\ \tilde{\Theta}_a\right)$$

$$\tilde{\vec{\omega}}^b_{ib,comp} = g_g\left(\tilde{\vec{\omega}}^b_{ib,uncomp};\ \tilde{\Theta}_g\right)$$

<u>Error Model</u>

$$\Delta\vec{f}^b_{ib} = \tilde{\vec{f}}^b_{ib,comp} - \vec{f}^b_{ib}$$

$$\Delta\vec{\omega}^b_{ib} = \tilde{\vec{\omega}}^b_{ib,comp} - \vec{\omega}^b_{ib}$$

BASIC IMU MODELING

- A basic IMU Model typical consists of:
  - Fixed Bias
  - Scale Factor Error
  - Triad Misalignment

- These basic models are often sufficient for consumer-grade MEMS devices

Basic IMU Modeling Questions

- Is this model still sufficient for high-performance IMU's?
- Is this model still applicable for all inertial sensing technologies?
- Where do we look for more detailed models?

Basic Accelerometer Triad Model

$$f_{ib,uncomp}^{b} = f_a\left(f_{ib}^{b};\Theta_a\right) = (I_3 + M_a) f_{ib}^{b} + b_{a,FB} = \begin{bmatrix} 1+s_{a,x} & m_{a,xy} & m_{a,xz} \\ m_{a,yx} & 1+s_{a,y} & m_{a,yz} \\ m_{a,zx} & m_{a,zy} & 1+s_{a,z} \end{bmatrix}\begin{bmatrix} f_{ib,x}^{b} \\ f_{ib,y}^{b} \\ f_{ib,z}^{b} \end{bmatrix} + \begin{bmatrix} b_{a_x,FB} \\ b_{a_y,FB} \\ b_{a_z,FB} \end{bmatrix}$$

$$f_{ib,comp}^{b} = g_a\left(f_{ib,uncomp}^{b};\Theta_a\right) = (I_3 + M_a)^{-1}\left(f_{ib,uncomp}^{b} - b_{a,FB}\right)$$

$$\Theta_a \in b_{a,FB}, M_a$$

Basic Gyroscope Triad Model

$$\omega_{ib,uncomp}^{b} = f_g\left(\omega_{ib}^{b};\Theta_g\right) = (I_3 + M_g)\omega_{ib}^{b} + b_{g,FB} = \begin{bmatrix} 1+s_{g,x} & m_{g,xy} & m_{g,xz} \\ m_{g,yx} & 1+s_{g,y} & m_{g,yz} \\ m_{g,zx} & m_{g,zy} & 1+s_{g,z} \end{bmatrix}\begin{bmatrix} \omega_{ib,x}^{b} \\ \omega_{ib,y}^{b} \\ \omega_{ib,z}^{b} \end{bmatrix} + \begin{bmatrix} b_{g_x,FB} \\ b_{g_y,FB} \\ b_{g_z,FB} \end{bmatrix}$$

$$\omega_{ib,comp}^{b} = g_g\left(\omega_{ib,uncomp}^{b};\Theta_g\right) = (I_3 + M_g)^{-1}\left(\omega_{ib,uncomp}^{b} - b_{g,FB}\right)$$

$$\Theta_g \in b_{g,FB}, M_g$$

IEEE STANDARDS FOR INERTIAL SENSING

The IEEE Gyroscope and Accelerometer (GAP) Panel maintains an international standard of specifying and testing inertial sensing components.

Accelerometer Components

- IEEE Std 1293-2018 – Linear Single Axis Nongyroscopic Accelerometers

Gyroscope Components

- IEEE Std 292-1969 – Single DOF Spring-Restrained Gyroscopes
- IEEE Std 517-1974 – Single DOF Rate-Integrating Gyroscopes
- IEEE Std 813-1988 – Two DOF Dynamically Tuned Gyroscopes
- IEEE Std 1431-2004 – Coriolis Vibratory Gyroscopes
- IEEE Std 647-2006 – Single Axis Laser Gyroscope
- IEEE Std 952-2020 – Single Axis Interferometric Fiber Optic Gyro

Inertial Measurement Units

- IEEE Std 1780-2022 – Specifying Inertial Measurement Units

Accelerometer Component Model from IEEE Std 1293-2018

$$E = K_1\{K_0 + \frac{K_{0'}}{2}\,\mathrm{sign}(\mathrm{a_i}) + \left(1 + \frac{K_{1'}}{2}\,\mathrm{sign}(a_i)\right)a_i + K_{oq}a_i\left|a_i\right| + K_2 a_i^2 + K_3 a_i^3 + \sum_{n\geq 4} K_n a_i^4$$
$$+ \delta_o a_p - \delta_p a_o + K_{ip}a_i a_p + K_{io}a_i a_o + K_{po}a_p a_p + K_{pp}a_p^2 + K_{oo}a_o^2$$
$$+ K_{\mathrm{spin}}\omega_i\omega_p + K_{\mathrm{ang,accel}}\dot{\omega}_o + \varepsilon\}$$

Gyroscope Component Model from IEEE Std 952-2020

$$S_0\left(\Delta N/\Delta t\right) = \frac{I + E + D}{1 + 10^{-6}\varepsilon_K}$$

$$I = \omega_{IRA}\sqrt{1 - \left[\left(\frac{\sin(\alpha)}{\alpha}\right)\alpha_x\right]^2 - \left[\left(\frac{\sin(\alpha)}{\alpha}\right)\alpha_y\right]^2} + \omega_{XRA}\left[\left(\frac{\sin(\alpha)}{\alpha}\right)\alpha_y\right] - \omega_{YRA}\left[\left(\frac{\sin(\alpha)}{\alpha}\right)\alpha_x\right]$$

$$E = D_T\Delta T + D_{\bar{T}}\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) + \bar{D}_{\nabla\bar{T}} \times \frac{\mathrm{d}\nabla\bar{T}}{\mathrm{d}t}$$

$$D = D_F + D_R + \mathrm{D}_Q$$

THE NEED FOR A GENERALIZED FRAMEWORK

A review of all IEEE Standards published by the GAP provide the following common inertial sensing component errors:

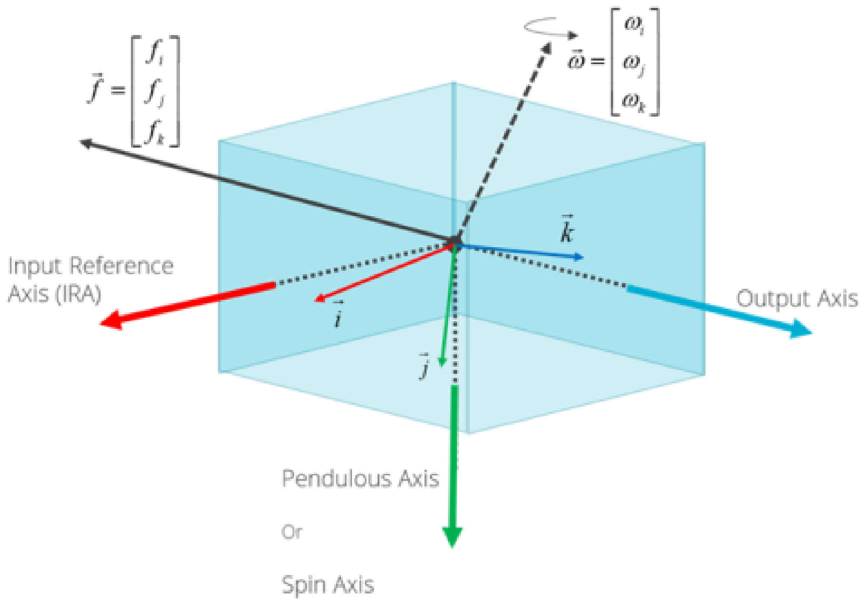
- Bias, Scale Factor, and Higher Order Terms
- Asymmetries of Bias, Scale Factor, and Higher Order Terms
- Component-Level Misalignment
- Cross Coupling and Cross Axis Nonlinearities
- Sensitivity to the Opposite Phenomenon

A new generalized component model should provide:

- Consistent interpretations and usage of the above common error sources
- Expandable and adaptable terms for higher order models
- Express the model in a sum-of-products form for simple implementation

GENERALIZED INERTIAL SENSING COMPONENT MODELS

Inertial Sensing Component Diagram



- This inertial sensing diagram contains:
  - The internal inertial sensing axis (e.g., i, j, k)
  - The external reference cases
  - The specific forces and angular rates applied to the center of percussion

Bias, Scale Factor, and Higher Order Terms

Asymmetries of Bias, Scale Factor, and Higher Order Terms

Component Level Misalignment

Cross Coupling and Cross Axis Nonlinearities

Sensitivity to the Opposite Phenomenon

$$f_{IRA} = f_a\left(\vec{f}, \vec{\omega}, \dot{\vec{\omega}}\right) = \sum_{n=0}^{N_a} b_{a,n} f_i^n + \sum_{n'=0}^{N_{a'}} b_{a,n'} \mathrm{sign}(f_i) f_i^{n'} + \vec{m}_a^T \vec{f} + \vec{f}^T C_a \vec{f} + \vec{\omega}^T S_\omega \vec{\omega} + \vec{s}_{\dot{\omega}}^T \dot{\vec{\omega}}$$

$$\omega_{IRA} = f_g\left(\vec{\omega}, \vec{f}\right) = \sum_{n=0}^{N_g} b_{g,n} \omega_i^n + \sum_{n'=0}^{N_{g'}} b_{g,n'} \mathrm{sign}(\omega_i) \omega_i^{n'} + \vec{m}_g^T \vec{\omega} + \vec{\omega}^T C_g \vec{\omega} + \vec{f}^T S_f \vec{f}$$

A detailed explanation of these equations is available in the back-up slides!

GENERALIZED INERTIAL SENSING MODEL APPLICATION

Why use these new generalized models?

- These models provide coverage for:
  - Varying inertial sensing technologies
  - Varying performance grades of devices
- Rather than starting with a basic model and adding parameters, instead consider all parameters and eliminate the terms that are not relevant to your use case

How can I apply these models to my use case?

- In general, model the parameters which you can observe!
- Include the terms you can reasonably measure or estimate, and remove the terms you can not

Accelerometer Parameters	Symbol(s)	IEEE 1293 Req. Clause	IEEE 1293 Test Clause
Bias			
Absolute Value	K_0	5.3.7.1	12.3.4
Asymmetry	—	5.3.7.2	12.3.15
Scale Factor			
Absolute Value	K_1	5.3.5.1	12.3.4
Asymmetry	—	5.3.5.2	12.3.15
Nonlinearity	K_2, K_3	5.3.6	12.3.15
IA Misalignment	δ_p, δ_o	5.3.8	12.3.4
Cross-axis Nonlinearity	K_{pp}, K_{oo}	5.3.10	12.3.15
Cross Coupling	K_{ip}, K_{io}, K_{po}	5.3.11	12.3.15

Gyroscope Parameters	Symbol(s)	IEEE 952 Req. Clause	IEEE 952 Test Clause
Bias	D_F	5.3.3.1.1	12.11
Scale Factor			
Absolute Value	S	5.3.2	12.9
Asymmetry	—	5.3.2.1.2	12.9
IA Misalignment	α	5.3.4.1	12.12

$$\tilde{f}_{IRA} = f_a\left(\vec{f}, \vec{\omega}, \dot{\vec{\omega}}\right) = \sum_{n=0}^{N_a} b_{a,n} f_i^n + \sum_{n'=0}^{N_{a'}} b_{a,n'} \mathrm{sign}(f_i) f_i^{n'} + \vec{m}_a{}^T \vec{f} + \vec{f}^T C_a \vec{f} + \vec{\omega}^T S_{\vec{\omega}} \vec{\omega} + \vec{s}_{\dot{\vec{\omega}}}^T \dot{\vec{\omega}}$$

$$\tilde{\omega}_{IRA} = f_g\left(\vec{\omega}, \vec{f}\right) = \sum_{n=0}^{N_g} b_{g,n} \omega_i^n + \sum_{n'=0}^{N_{g'}} b_{g,n'} \mathrm{sign}(\omega_i) \omega_i^{n'} + \vec{m}_g{}^T \vec{\omega} + \vec{\omega}^T C_g \vec{\omega} + \vec{f}^T S_{\vec{f}} \vec{f}$$

FUTURE IMU MODEL EXTENSIONS

- The generalized model presented today solely focuses on analytic parameters
- Future model extensions include:
  - Build upon IEEE Standards to develop standardized calibration procedures
  - Expanding model parameters as functions of the environment, (i.e., temperature)
  - Vectorizing the component level equations into a fully-realized IMU model
  - Identify and express stochastic error models available for characterization while the unit is under test

SUMMARY

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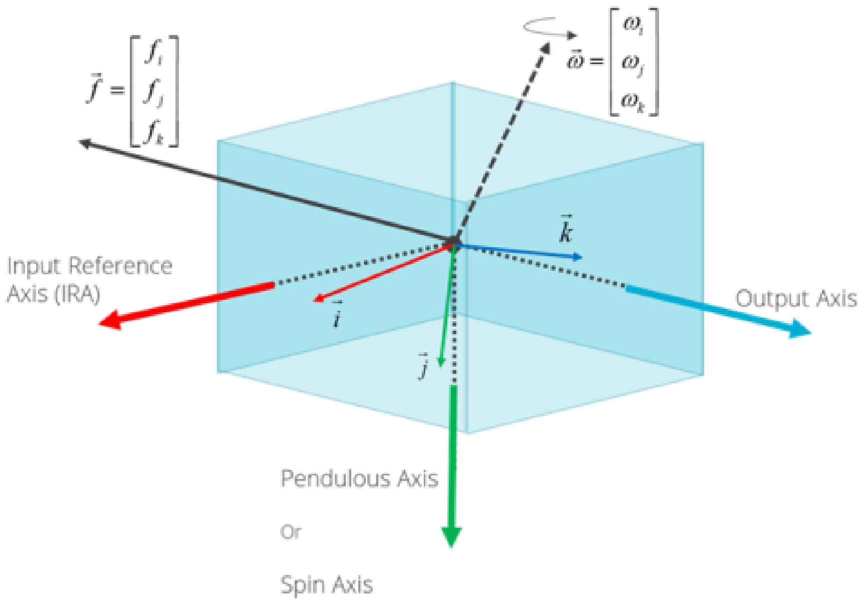
- IEEE Standards for Inertial Sensing
- The Need for a Generalized IMU Modeling Framework
- Generalized Inertial Sensing Model Applications
- Future IMU Model Extensions

Inertial Sensing Component Diagram



Bias, Scale Factor, and Higher Order Terms

Asymmetries of Bias, Scale Factor, and Higher Order Terms

Component Level Misalignment

Cross Coupling and Cross Axis Nonlinearities

Sensitivity to the Opposite Phenomenon

$$\tilde{f}_{IRA} = f_a\left(\vec{f},\vec{\omega},\dot{\vec{\omega}}\right) = \sum_{n=0}^{N_a} b_{a,n} f_i^{n} + \sum_{n'=0}^{N_{a'}} b_{a,n'}\mathrm{sign}(f_i) f_i^{n'} + \vec{m}_a{}^T \vec{f} + \vec{f}^T C_a \vec{f} + \vec{\omega}^T S_{\vec{\omega}} \vec{\omega} + \vec{s}_{\dot{\vec{\omega}}}^T \dot{\vec{\omega}}$$

$$\tilde{\omega}_{IRA} = f_g\left(\vec{\omega},\vec{f}\right) = \sum_{n=0}^{N_g} b_{g,n}\,\omega_i^{n} + \sum_{n'=0}^{N_{g'}} b_{g,n'}\mathrm{sign}(\omega_i)\,\omega_i^{n'} + \vec{m}_g{}^T \vec{\omega} + \vec{\omega}^T C_g \vec{\omega} + \vec{f}^T S_{\vec{f}} \vec{f}$$

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GENERALIZED INERTIAL SENSING COMPONENT BUILD-UP PT.1

Bias, Scale Factor, and Higher Order Terms

These terms follow a common polynomial expansion, for example:

$$f_{IRA} = b_{a,0} + b_{a,1} f_i + b_{a,2} f_i^2 + \ldots$$

$$\omega_{IRA} = b_{g,0} + b_{g,1} \omega_i + b_{g,2} \omega_i^2 + \ldots$$

This polynomial expansion can be re-expressed as the following summation:

$$f_{IRA} = \sum_{n=0}^{N_a} b_{a,n} f_i^n$$

$$\omega_{IRA} = \sum_{n=0}^{N_g} b_{g,n} \omega_i^n$$

Asymmetries in Bias, Scale Factor, and Higher Order Terms

Accompanying each term to the left is the possibility of an asymmetry, which is depending on the *sign* of the input quantity

$$f_{IRA} = \left(b_{a,0} + b_{a,0'}\mathrm{sign}\left(f_i\right)\right) + \left(b_{a,1} + b_{a,1'}\mathrm{sign}\left(f_i\right)\right) f_i + \left(b_{a,2} + b_{a,2'}\mathrm{sign}\left(f_i\right)\right) f_i^2 + \ldots$$

$$\omega_{IRA} = \left(b_{g,0} + b_{g,0'}\mathrm{sign}\left(\omega_i\right)\right) + \left(b_{g,1} + b_{g,1'}\mathrm{sign}\left(\omega_i\right)\right) \omega_i + \left(b_{g,2} + b_{g,2'}\mathrm{sign}\left(\omega_i\right)\right) \omega_i^2 + \ldots$$

Re-stating this expansion into summations provide:

$$f_{IRA} = \sum_{n=0}^{N_a} b_{a,n} f_i^n + \sum_{n'=0}^{N_a'} b_{a,n'} \mathrm{sign}\left(f_i\right) f_i^n$$

$$\omega_{IRA} = \sum_{n=0}^{N_g} b_{g,n} \omega_i^n + \sum_{n'=0}^{N_g'} b_{g,n'} \mathrm{sign}\left(\omega_i\right) \omega_i^n$$

GENERALIZED INERTIAL SENSING COMPONENT BUILD-UP PT.2

Component Level Misalignment

- Component level misalignment differs from a traditional misalignment matrix used for a triad of sensors. Instead, component level misalignment is only concerned out input axis deflection in the pendulous/spin and output axes.

$$m_a = \begin{bmatrix} \cos(\alpha_a) \\ \frac{\sin(\alpha_a)}{\alpha_a}\alpha_{a,k} \\ \frac{\sin(\alpha_a)}{\alpha_a}\alpha_{a,j} \end{bmatrix}, \quad m_g = \begin{bmatrix} \cos(\alpha_g) \\ \frac{\sin(\alpha_g)}{\alpha_g}\alpha_{g,k} \\ \frac{\sin(\alpha_g)}{\alpha_g}\alpha_{g,j} \end{bmatrix}, \quad \alpha_{a/g} = \alpha_j^2 + \alpha_k^2$$

- Component level misalignment can be characterized will angles α_k and α_o , which describe the amount of angle deflection into its respective axis.

- To account for this misalignment in the IMU model, the deflection angles are re-expressed as:

- Then, the dot product of these misalignment terms and its respective quantity provides:

$$f_{IRA} = \sum_{n=0}^{N_a} b_{a,n} f_i^n + \sum_{n'=0}^{N_a'} b_{a,n'} \mathrm{sign}(f_i) f_i^n + m_a^T f$$

$$\omega_{IRA} = \sum_{n=0}^{N_g} b_{g,n} \omega_i^n + \sum_{n'=0}^{N_g'} b_{g,n'} \mathrm{sign}(\omega_i) \omega_i^n + m_g^T \omega$$

GENERALIZED INERTIAL SENSING COMPONENT BUILD-UP PT.3

Cross Coupling and Cross Axis Nonlinearities

- Cross coupling effects are proportional to the product of the sensed quantity of each axis.

$$\Delta f_{CA} = \vec{f}^T C_a \vec{f} = \begin{bmatrix} f_i & f_j & f_k \end{bmatrix} \begin{bmatrix} 0 & c_{a,ij} & c_{a,ik} \\ 0 & c_{a,jj} & c_{a,jk} \\ 0 & 0 & c_{a,kk} \end{bmatrix} \begin{bmatrix} f_i \\ f_j \\ f_k \end{bmatrix}$$

- Cross axis nonlinearity effects are proportional to the square of the sensed quantity on the pendulous/spin and output axes.

$$\Delta \omega_{CA} = \vec{\omega}^T C_g \vec{\omega} = \begin{bmatrix} \omega_i & \omega_j & \omega_k \end{bmatrix} \begin{bmatrix} 0 & c_{g,ij} & c_{g,ik} \\ 0 & c_{g,jj} & c_{g,jk} \\ 0 & 0 & c_{g,kk} \end{bmatrix} \begin{bmatrix} \omega_i \\ \omega_j \\ \omega_k \end{bmatrix}$$

- Both sets of terms can be expressed within the same matrix such that:

$$\vec{f}_{IRA} = \sum_{n=0}^{N_a} b_{a,n} f_i^n + \sum_{n'=0}^{N_a'} b_{a,n'} \mathrm{sign}(f_i) f_i^n + \vec{m}_a^T \vec{f} + \vec{f}^T C_a \vec{f}$$

$$\vec{\omega}_{IRA} = \sum_{n=0}^{N_g} b_{g,n} \omega_i^n + \sum_{n'=0}^{N_g'} b_{g,n'} \mathrm{sign}(\omega_i) \omega_i^n + \vec{m}_g^T \vec{\omega} + \vec{\omega}^T C_g \vec{\omega}$$

GENERALIZED INERTIAL SENSING COMPONENT BUILD-UP PT.3

Sensitivity to the Opposite Phenomenon

- It is not uncommon for accelerometers to be sensitive to angular velocities or angular accelerations.

$$\Delta f_{\vec{\omega},\dot{\vec{\omega}}} = \vec{\omega}^T S_{\vec{\omega}} \vec{\omega} + \vec{s}_{\dot{\vec{\omega}}}^T \dot{\vec{\omega}} = \begin{bmatrix} \omega_i & \omega_j & \omega_k \end{bmatrix} \begin{bmatrix} 0 & s_{\omega,ij} & s_{\omega,ik} \\ 0 & 0 & s_{\omega,jk} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_i \\ \omega_j \\ \omega_k \end{bmatrix} + \begin{bmatrix} s_{\dot{\omega},i} & s_{\dot{\omega},j} & s_{\dot{\omega},k} \end{bmatrix} \begin{bmatrix} \dot{\omega}_i \\ \dot{\omega}_j \\ \dot{\omega}_k \end{bmatrix}$$

- It is also not uncommon for gyroscopes to be sensitive to specific force.

$$\Delta \omega_{\vec{f}} = \vec{f}^T S_{\vec{f}} \vec{f} = \begin{bmatrix} f_i & f_j & f_k \end{bmatrix} \begin{bmatrix} 0 & s_{f,ij} & s_{f,ik} \\ 0 & 0 & s_{f,jk} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_i \\ f_j \\ f_k \end{bmatrix}$$

- These sensitivities are more common in mechanical designs such as MEMS technologies others that contain a physical proof mass.

- These sensitivities are model such that:

$$\vec{f}_{IRA} = \sum_{n=0}^{N_a} b_{a,n} f_i^n + \sum_{n'=0}^{N_a'} b_{a,n'} \operatorname{sign}(f_i) f_i^{n'} + \vec{m}_a^T \vec{f} + \vec{f}^T C_a \vec{f} + \vec{\omega}^T S_{\vec{\omega}} \vec{\omega} + \vec{s}_{\dot{\vec{\omega}}}^T \dot{\vec{\omega}}$$

$$\vec{\omega}_{IRA} = \sum_{n=0}^{N_g} b_{g,n} \omega_i^n + \sum_{n'=0}^{N_g'} b_{g,n'} \operatorname{sign}(\omega_i) \omega_i^{n'} + \vec{m}_g^T \vec{\omega} + \vec{\omega}^T C_g \vec{\omega} + \vec{f}^T S_{\vec{f}} \vec{f}$$