Homework 1

David L. Olson

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1 Problem 1

1. Exercise Consider the Fredholm integral equation of the first kind,

$$\int_0^1 g(x,z)m(z)dz = d(x)$$

with $g(x,z) = 5\sin(xz)$, and $d(x) = 50\sin(x) - 50\sin(x)\cos(x)$, $0 \leq x \leq 1$. The exact solution to this equation is $m(x) = 10x\sin(x)$ as can easily be verified by substituting it into the equation.

   (a) Using MATLAB, discretize this integral equation using midpoints of $n = 20$ equally spaced intervals of width 0.05. Your discretized model should be of the form $Gm = d$. Output G and d. Use the MATLAB backslash command to solve for m and output your inverse model.

   (b) What is the condition number of G?

   (c) Plot your solution and the exact solution.

   (d) Why is the solution to the discretized model so poor?

1.1 Solution

First, let's verify the solution of $m(x)$ via substitution. (This helped me understand the problem immensely, so I will include it here)

$$\int_0^1 g(x,z)m(z)dz = d(x)$$

$$\int_0^1 5\sin(xz)m(z)dz = 50\sin(x) - 50\sin(x)\cos(x)$$

$$\int_0^1 5\sin(xz)m(z)dz = \int_0^1 5\sin(xz)m(x)dz$$

$$= \int_0^1 5\sin(xz)\left(10x\sin(x)\right)dz$$

$$= 50x\sin(x)\int_0^1 \sin(xz)dz$$

$$= -\frac{50x\sin(x)}{x}\left[\cos(xz)\right]|_0^1$$

$$= -\frac{50x\sin(x)}{x}\left(\cos(x)-\cos(0)\right)$$

$$= 50\sin(x)\left(1-\cos(x)\right)$$

$$= 50\sin(x) - 50\sin(x)\cos(x) = d(x) \quad \checkmark$$

Put answers here

1.1.1 Part A

If subpart to the question exist