Inverse Problems for Inertial Sensor Calibration

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Abstract

Inertial sensors are vital to navigation, guidance, and control (NGC) for a variety of vehicles. Inertial sensors such as accelerometers and gyroscopes measure specific and angular velocity respectively which are mechanized to produce a position, velocity, and attitude (PVA) solution [1]. All inertial sensors are subject to deterministic and stochastic error which require careful calibration and characterization before utilizing inertial sensors in a vehicle application. In industry, the current state of the art for inertial sensor calibration has made little advancement over the past couple decades, where calibration data is collected and post-processed using basic statistics to compute basic error model parameters. In the past, improved fidelity for calibration required purchasing higher and higher precision equipment, which eventually succumbs to the law of diminishing returns.

However, ongoing research addressing inertial sensor calibration seeks to break dependence on high-cost high-precision test equipment by introducing advanced estimation techniques and stochastic filtering to maintain high fidelity calibration performance [2, 3, 4, 5, 6]. While various papers only tackle small pieces of an entire calibration procedure, there is an opportunity to cast inertial sensor calibration as an inverse problem to provide a holistic methodology to the field. The forward problem is simply the process of transforming true dynamic inputs acting on the sensor to sensor outputs which is subject to error. Then, the inverse problem takes the form of calibration in which calibration parameters are estimated from collected sensor data. Using inverse problem techniques, calibration parameter estimation can advance beyond basic statistical techniques for higher fidelity performance to include covariance propagation to assign confidence to final calibration parameters. In addition, estimation may be performed in real-time during the collection of calibration data, alleviating logistical and programmatic troubles for an inertial test laboratory.

The author plans to build upon previous work [7] which homogenized multiple IEEE standards for varying accelerometer and gyroscope technologies [8, 9, 10, 11, 12, 13, 14] which provide highly detailed inertial sensor error models. Inertial sensor data will be simulated using [15] which can provide true inertial sensor measurements which can then be corrupted with error and noise. Then, inverse problem techniques may be demonstrated and contrasted with traditional methods to showcase improvements and possible limitations to using these new highly detailed models. Higher fidelity calibration yields increased sensor accuracy, which ultimately enhances inertial navigation capabilities for a wide range of vehicle applications.

1 Introduction

A key challenge for self-driving cars is autonomous navigation. Autonomous navigation fuses the inputs of multiple sensors to formulate a position, velocity, and attitude (PVA) solution for the vehicle of interest. Common navigation sensors include inertial measurement units (IMUs), global positioning system (GPS) receivers, odometers, magnetometers, radars, LiDAR, cameras, and many others. It is crucial to not only formulate a navigation solution but also to track the uncertainty of that solution, which can play an important role in decision-making and risk-assessment for autonomous self-driving vehicle applications.

IMUs generally contain three accelerometers and gyroscopes pointing in all three Cartesian axes which measure specific force and angular velocity respectively. These measurements are integrated to provide a PVA solution, but an IMU-only solution will drift away from truth unbounded due to the integration of sensor noise and other errors [1]. An inertial navigation system (INS) integrates the IMU measurements and then uses GPS to provide accurate measurements of position to fuse with the IMU solution typically via a Kalman filter thus combating any position error drift. This allows for long-term accurate navigation suitable for a self-driving car traveling long distances.

A driving need for navigation regarding self-driving cars is forming back-up modes of navigation when GPS signals are temporarily unavailable. This is especially important in urban environments where tall buildings and tunnels can cause signal blockages. Lack of GPS signals are problematic for an INS as its performance is dependent on receiving those signals, and the IMU-only solutions will drift away quickly from ground truth. Inertial-only navigation is suitable for short periods such as temporary GPS signal blockages, but the duration of a suitable inertial-only navigation is highly dependent on the quality of sensors contained within an IMU. This dilemma, among any others, motivates the need for rigorous IMU calibration and compensation.

One particular challenge with the use of commercial-grade or automotive-grade micro-electrical mechanical system (MEMS) inertial sensors is their sensitivity to temperature. Characterizing how common error sources such as bias, scale factors, and misalignments shift in response to temperature is a tough problem which requires hours of testing per unit and is often too burdensome for applications using automotive-grade devices.

1.1 Defining Sensor Error

Accelerometers and gyroscopes are subject to a variety of error sources such as biases, scale factor imperfections, axis misalignments, noise, and many others. IMU calibration is the process of characterizing these error sources, while compensation is the process of correcting measurements in real-time to provide the most accurate and precise measurements possible. The better these error sources are characterized, the slower that the IMU-only navigation solution will drift away from truth.

Inertial sensor calibration can be treated as an inverse problem. Consider an accelerometer as an example where the ideal forward model is quite straight forward; specific force in equals specific force out. Unfortunately in practice, we are not quite so lucky. Any error in the forward model is defined as

$$\Delta f := \tilde{f} - f$$

where Δf is the measurement error, $\tilde{f}$ is the specific force measured by the sensor which is subject to error, and f is the true specific force acting on the accelerometer. The same applies to gyroscope measurements.

$$\Delta\omega := \tilde{\omega} - \omega$$

Inertial sensor errors Δf and $\Delta\omega$ are subject to both deterministic and stochastic error and any number of contributing factors can make up these terms.

There are many procedures available to calibrate inertial sensors such as [2] which require high-precision equipment to point and spin inertial sensors at known directions or quantities to separate measurement truth from measurement error.

1.1.1 Basic IMU Error Sources and Error Models

Consider an IMU which contains three accelerometers and three gyroscopes arranged to point in a standard Cartesian right-handed coordinate frame. A basic forward model framework appropriate for commercial- and automotive-grade inertial sensors is provided in [1].

$$\begin{aligned}\tilde{\boldsymbol{f}} &= [I + M_a]\,\boldsymbol{f} + \boldsymbol{b}_a + \boldsymbol{w}_a, \quad \boldsymbol{w}_a \sim N\left(0,\, \sigma^2\right) \\ \tilde{\boldsymbol{\omega}} &= [I + M_g]\,\boldsymbol{\omega} + \boldsymbol{b}_g + \boldsymbol{w}_g \quad \boldsymbol{w}_g \sim N\left(0,\, \sigma^2\right)\end{aligned} \tag{1}$$

In these models are bias terms $\boldsymbol{b}_a$, $\boldsymbol{b}_g \in \mathbb{R}^3$ for the accelerometers and gyroscopes respectively. In addition are the misalignment matrices M_a, $M_g \in \mathbb{R}^{3\times3}$, whose diagonal elements are scale factor error terms. The off-diagonal elements are misalignment terms which account for imperfections in the alignment of the sensing axes. All inertial sensors are also subject to noise, which is captured by terms $\boldsymbol{w}_a$, $\boldsymbol{w}_g$ which are additive zero-mean white Gaussian noise. Equation 2 restates equation 1 in terms of all its various elements.

$$\begin{aligned}\begin{bmatrix}\tilde{f}_x \\ \tilde{f}_y \\ \tilde{f}_z\end{bmatrix} &= \begin{bmatrix}1+s_{a,x} & m_{a,xy} & m_{a,xz} \\ m_{a,yx} & 1+s_{a,y} & m_{a,yz} \\ m_{a,zx} & m_{a,zy} & 1+s_{a,z}\end{bmatrix}\begin{bmatrix}f_x \\ f_y \\ f_z\end{bmatrix} + \begin{bmatrix}b_{a,x} \\ b_{a,y} \\ b_{a,z}\end{bmatrix} + \begin{bmatrix}w_{a,x} \\ w_{a,y} \\ w_{a,z}\end{bmatrix} \\ \begin{bmatrix}\tilde{\omega}_x \\ \tilde{\omega}_y \\ \tilde{\omega}_z\end{bmatrix} &= \begin{bmatrix}1+s_{g,x} & m_{g,xy} & m_{g,xz} \\ m_{g,yx} & 1+s_{g,y} & m_{g,yz} \\ m_{g,zx} & m_{g,zy} & 1+s_{g,z}\end{bmatrix}\begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix} + \begin{bmatrix}b_{g,x} \\ b_{g,y} \\ b_{g,z}\end{bmatrix} + \begin{bmatrix}w_{g,x} \\ w_{g,y} \\ w_{g,z}\end{bmatrix}\end{aligned} \tag{2}$$

The goal of IMU calibration is characterize to these IMU error sources, resulting in estimates $\hat{\boldsymbol{b}}_a$, $\hat{M}_a$, $\hat{\boldsymbol{b}}_g$, $\hat{M}_g$ which then allows for the correction of inertial sensor measurements in real-time, which is referred to as compensation. In essence, this can be treated as the inverse model with estimated parameters. Using the estimated model parameters, inertial sensor measurements $\tilde{\boldsymbol{f}}$, $\tilde{\boldsymbol{\omega}}$ are compensated to provide new measurements $\hat{\boldsymbol{f}}$, $\hat{\boldsymbol{\omega}}$ per equation 3.

$$\hat{\boldsymbol{f}} = \left[I + \hat{M}_a\right]^{-1}\left(\tilde{\boldsymbol{f}} - \hat{\boldsymbol{b}}_a\right)$$

$$\hat{\boldsymbol{\omega}} = \left[I + \hat{M}_g\right]^{-1} \left(\tilde{\boldsymbol{\omega}} - \hat{\boldsymbol{b}}_g\right) \tag{3}$$

1.1.2 Traditional Inertial Sensor Calibration

IMUs are traditionally calibrated on some sort of a rotational test bed. These rotation test beds are able to point and spin inertial sensors at known quantities with sharp precision. To calibrate accelerometers, the IMU is aligned in various poses along the local gravity vector at that location, therefore a gravity survey within an inertial test laboratory is critical. To calibrate gyroscopes, the IMU is spun at known speeds.

For example hardware, consider the 2103C Series Three-Axis Position and Rate Table System from Ideal Aerosmith shown in figure 1.

Figure 1: 2103C Series Three Axis Position and Rate Table System

Traditional calibration comprises of twelve tests with six for the accelerometers and six for the gyroscopes. In each test, the IMU is stationary for accelerometer tests and spinning at a constant speed for gyroscope tests. Each test is completed for a set amount of time, usually 60 seconds, and the measurements are averaged. Table 1 lists the six tests as well as the quantity for each averaged test result.

Accelerometer Tests	**Gyroscope Tests**
• Point X-Axis Up $\left(\bar{\boldsymbol{f}}^{+\mathrm{x}}\right)$ • Point X-Axis Down $\left(\bar{\boldsymbol{f}}^{-\mathrm{x}}\right)$ • Point Y-Axis Up $\left(\bar{\boldsymbol{f}}^{+\mathrm{y}}\right)$ • Point Y-Axis Down $\left(\bar{\boldsymbol{f}}^{-\mathrm{y}}\right)$ • Point Z-Axis Up $\left(\bar{\boldsymbol{f}}^{+\mathrm{z}}\right)$ • Point Z-Axis Down $\left(\bar{\boldsymbol{f}}^{-\mathrm{z}}\right)$	• Positive Spin about X-Axis $(\bar{\boldsymbol{\omega}}^{+\mathrm{x}})$ • Negative Spin about X-Axis $(\bar{\boldsymbol{\omega}}^{-\mathrm{x}})$ • Positive Spin about Y-Axis $(\bar{\boldsymbol{\omega}}^{+\mathrm{y}})$ • Negative Spin about Y-Axis $(\bar{\boldsymbol{\omega}}^{-\mathrm{y}})$ • Positive Spin about Z-Axis $(\bar{\boldsymbol{\omega}}^{+\mathrm{z}})$ • Negative Spin about Z-Axis $(\bar{\boldsymbol{\omega}}^{-\mathrm{z}})$

Table 1: Traditional IMU Calibration Tests

As an example from equation 2, the accelerometer bias $\boldsymbol{b}_a$ can be solved from the collected test data.

$$\boldsymbol{b}_a = \begin{bmatrix} b_{a,x} \\ b_{a,y} \\ b_{a,z} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \bar{f}_x^{+\mathrm{x}} + \bar{f}_x^{-\mathrm{x}} \\ \bar{f}_y^{+\mathrm{y}} + \bar{f}_y^{-\mathrm{y}} \\ \bar{f}_z^{+\mathrm{z}} + \bar{f}_z^{-\mathrm{z}} \end{bmatrix} \tag{4}$$

All model in parameters in equation 2 can be solved in a similar manner, which have all been listed in section 3.1.

1.2 Beyond Traditional IMU Calibration

While traditional calibration methods have proven successful over the past several decades, there is a push within the navigation community to move to systematic calibration methods [16]. It is difficult to estimate more model parameters with the same limited dynamics, and the piece-meal approach to post-processing IMU calibration data is cumbersome and complicated. Additionally, traditional means of calibration do not provide any sort of measure of uncertainty which many navigation algorithms, i.e., Kalman filters, require for any sort of state estimation.

Therefore, this paper aims to investigate systematic IMU calibration through the lens of inverse problem techniques. Come back here and tell everyone what you ended up doing.

2 Methods

Consider an IMU under test strapped down to a three axis rotational test bed. The rotational test bed provides measurements of angular position and angular rate along each test axis. As exemplar hardware, the unit under test will be a STIM 300 IMU from Safran and the 210C Series Three Axis Position and Rate Table System from Ideal Aerosmith.

2.1 STIM 300 IMU Specifications

The STIM 300 IMU, henceforth referred to as the unit under test (UUT), provides measurements of specific force and angular velocity. While the true forward model is unknown, it will be assumed that the basic forward model in equation 1 will sufficiently model the error of the device. Assuming the UUT uses the "10g" variant of accelerometers, bounds for the accelerometer-related model parameters are provided in table 5-5 of the specification sheet [17].

- Bias: $|b_a| \leq 7.5 \times 10^{-3} g$
- Scale Factor Error : $|s_a| \leq 200\,\mathrm{ppm}$
- Misalignment: $|m_a| \leq 1 \times 10^{-3}\,\mathrm{rad}$

The accelerometers are also subject to zero-mean Gaussian noise with a velocity random walk (VRW) value of $0.07\mathrm{m/s}/\sqrt{\mathrm{Hz}}$, which translates to a $\sigma_a = \frac{0.07}{60} = 0.0012\mathrm{m\,s^{-1}}$. Likewise, [17] provides bounds for the gyroscopes in table 5-3.

- Bias: $|b_g| \leq 250^\circ\,\mathrm{h}^{-1}$
- Scale Factor Error : $|s_g| \leq 500\,\mathrm{ppm}$
- Misalignment: $|m_g| \leq 1 \times 10^{-3}\,\mathrm{rad}$

The gyroscopes are also subject to zero-mean Gaussian noise with an angle random walk (ARW) value of $0.15^\circ/\sqrt{\mathrm{h}}$ which translates to a $\sigma_g = \frac{0.15}{60}\frac{\pi}{180} = 4.3633 \times 10^{-5}\mathrm{rad\,s^{-1}}$.

For simulation purposes, true model parameters will be selected within these bounds in later sections.

2.2 Formulating IMU Calibration as a Discrete Linear Inverse Problem

Recall the system of equations from equation 2 and consider the x-axis accelerometer measurements.

$$\tilde{f}_x = (1 + s_{a,x})\, f_x + m_{a,xy} f_y + m_{a,xz} f_z + b_{a,x}$$

The equation above can be re-arranged to express the model parameters as a function of the accelerometer error $\Delta f = \tilde{f} - f$.

$$\tilde{f}_x = (1 + s_{a,x})\, f_x + m_{a,xy} f_y + m_{a,xz} f_z + b_{a,x}$$

$$\tilde{f}_x - f_x = b_{a,x} + s_{a,x} f_x + m_{a,xy} f_y + m_{a,xz} f_z$$

$$\Delta f_x = b_{a,x} + s_{a,x} f_x + m_{a,xy} f_y + m_{a,xz} f_z$$

Assuming that both the UUT and test bed are able to provide synchronized measurements at the same sampling frequency, a series of measurements from these devices can be organized into another system of equations.

$$\begin{bmatrix} 1 & f_x[1] & f_y[1] & f_z[1] \\ 1 & f_x[2] & f_y[2] & f_z[2] \\ \vdots & \vdots & \vdots & \vdots \\ 1 & f_x[m] & f_y[m] & f_z[m] \end{bmatrix} \begin{bmatrix} b_{a,x} \\ s_{a,x} \\ m_{a,xy} \\ m_{a,xz} \end{bmatrix} = \begin{bmatrix} \Delta f_x[1] \\ \Delta f_x[2] \\ \vdots \\ \Delta f_x[m] \end{bmatrix} \tag{5}$$

This system of equations can be expressed in the form $\boldsymbol{Gm} = \boldsymbol{d}$. In this expression, "true" measurements $f_x[n]$, $f_y[n]$, $f_z[n]$ within the model operator are computed from measurements provided by the test bed, and the model parameters are various calibration factors from the forward model. Elements of the data vector $\boldsymbol{d}$ are the difference of the UUT output and test bed output such that $d[n] = \tilde{f}_x[n] - f_x[n]$.

Let F be the model operator demonstrated by equation 5.

$$F := \begin{bmatrix} 1 & f_x[1] & f_y[1] & f_z[1] \\ 1 & f_x[2] & f_y[2] & f_z[2] \\ \vdots & \vdots & \vdots & \vdots \\ 1 & f_x[m] & f_y[m] & f_z[m] \end{bmatrix}, \quad F \in \mathbb{R}^{m \times 4} \tag{6}$$

Then, the discrete linear inverse problem for all accelerometer calibration parameters given in equation 2 can defined below.

$$G_a \boldsymbol{m}_a = \boldsymbol{d}_a$$

$$\begin{bmatrix} F & 0_{m\times 4} & 0_{m\times 4} \\ 0_{m\times 4} & F & 0_{m\times 4} \\ 0_{m\times 4} & 0_{m\times 4} & F \end{bmatrix} \begin{bmatrix} b_{a,x} \\ s_{a,x} \\ m_{a,xy} \\ m_{a,xz} \\ b_{a,y} \\ m_{a,yz} \\ s_{a,y} \\ m_{a,yz} \\ b_{a,z} \\ m_{a,zx} \\ m_{a,zy} \\ s_{a,z} \end{bmatrix} = \begin{bmatrix} \Delta f_x[1] \\ \vdots \\ \Delta f_x[m] \\ \\ \Delta f_y[1] \\ \vdots \\ \Delta f_y[m] \\ \\ \Delta f_z[1] \\ \vdots \\ \Delta f_z[m] \end{bmatrix} \tag{7}$$

Likewise, let Ω be the model operator for all gyroscope measurements specific to one sensor.

$$\Omega := \begin{bmatrix} 1 & \omega_x[1] & \omega_y[1] & \omega_z[1] \\ 1 & \omega_x[2] & \omega_y[2] & \omega_z[2] \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \omega_x[m] & \omega_y[m] & \omega_z[m] \end{bmatrix}, \ \Omega \in \mathbb{R}^{m\times 4} \tag{8}$$

Then, the discrete linear inverse problem for all gyroscope calibration parameters given in equation 2 can defined below.

$$\begin{gathered} G_g \boldsymbol{m}_g = \boldsymbol{d}_g \\ \begin{bmatrix} \Omega & 0_{m\times 4} & 0_{m\times 4} \\ 0_{m\times 4} & \Omega & 0_{m\times 4} \\ 0_{m\times 4} & 0_{m\times 4} & \Omega \end{bmatrix} \begin{bmatrix} b_{g,x} \\ s_{g,x} \\ m_{g,xy} \\ m_{g,xz} \\ b_{g,y} \\ m_{g,yz} \\ s_{g,y} \\ m_{g,yz} \\ b_{g,z} \\ m_{g,zx} \\ m_{g,zy} \\ s_{g,z} \end{bmatrix} = \begin{bmatrix} \Delta\omega_x[1] \\ \vdots \\ \Delta\omega_x[m] \\ \\ \Delta\omega_y[1] \\ \vdots \\ \Delta\omega_y[m] \\ \\ \Delta\omega_z[1] \\ \vdots \\ \Delta\omega_z[m] \end{bmatrix} \end{gathered} \tag{9}$$

2.3 Model Covariance and Weighted Least Squares

As previously mentioned, one major benefit of systematic calibration vs. traditional calibration is the ability to produce some measure of uncertainty with respect to the estimated calibration model parameters. Given that inertial sensors are provided some terms within their specification sheets to determine a zero-mean Gaussian distribution for sensor noise, each discrete linear inverse problems formulated in the previous section can be reweighted to form weighted least squares problems. This provides an opportunity to examine how well the calibration model parameters fit the collected data by performing a χ^2 test and examining the resulting p-values.

For example, consider the system of equations for the accelerometer calibration parameters defined in equation 7. Recall, the standard deviation for accelerometer sensor noise is provided in section 2.1. This system of equations can be reweighted accordingly below.

$$\begin{gathered} W = \sigma_a I \\ W G_a \boldsymbol{m}_a = W \boldsymbol{d}_a \end{gathered} \tag{10}$$

$$G_{a,w}\boldsymbol{m}_a = \boldsymbol{d}_{a,w}$$

The model covariance matrix for the accelerometer parameters is given in equation 11.

$$\text{Cov}\left(\boldsymbol{m}_{L2}\right) = \left(G_{a,w}^T G_{a,w}\right)^{-1} \tag{11}$$

From this, the 95% confidence bounds for the L2 regression solution $\boldsymbol{m}_{L2}$ are given below.

$$\boldsymbol{m}_{L2} \pm 1.96\text{diag}\left(\text{Cov}\left(\boldsymbol{m}_{L2}\right)\right)^{\frac{1}{2}} \tag{12}$$

Additionally, the χ^2 test may be formed to compute a p-value, allowing for another quantitative measure to determine how well the model fits the provided data. The χ^2 statistic is formed according to equation 13.

$$\chi^2_{\text{obs}} = \sum_i^m \frac{\left(d_i - \left(G\boldsymbol{m}_{L2}\right)_i\right)^2}{\sigma_i^2} \tag{13}$$

2.4 Simulating Calibration Data and Model Parameters

Rotational motion on the test bed will be simulated by developing a true sequence of angular velocities $\boldsymbol{\omega}$ across time t experienced by the UUT. The attitude of the UUT, represented by the direction cosine matrix $C\left(t\right)$, will be integrated in discrete steps such that the k^{th} step in the sequence is

$$C\left(t_k\right) = C_0 \prod_{i=1}^{k} e^{\left[\boldsymbol{\omega}\left(t_i\right)\times\right]\Delta t} \tag{14}$$

where $\left[\boldsymbol{\omega}\left(t_i\right)\times\right]$ is a skew-symmetric matrix and Δt is the time step. While the test bed rotates, the accelerometers will each be measuring components of the normal force from the test bed resulting from the acceleration due to gravity g. For each time step k, the true specific force quantity is given below.

$$\boldsymbol{f}\left(t_k\right) = C\left(t_k\right)\begin{bmatrix}0\\0\\g\end{bmatrix} \tag{15}$$

In the simulation, it will be assumed that quantities $\boldsymbol{\omega}$ and $\boldsymbol{f}$ will be available through measurement outputs of the test bed itself. Measurements of these quantities from the UUT will be corrupted with measurement error according to equations 2. The values of the model parameters are recorded in table 2 and are selected with reasonable orders of magnitude as discussed in section 2.1.

Model Parameter	Accel Value	Gyro Value
Fixed Bias X	$0\,\mathrm{m\,s^{-2}}$	$0\,\mathrm{rad\,s^{-1}}$
Fixed Bias Y	$0\,\mathrm{m\,s^{-2}}$	$0\,\mathrm{rad\,s^{-1}}$
Fixed Bias Z	$0\,\mathrm{m\,s^{-2}}$	$0\,\mathrm{rad\,s^{-1}}$
Scale Factor Error X	0 ppm	0 ppm
Scale Factor Error Y	0 ppm	0 ppm
Scale Factor Error Z	0 ppm	0 ppm
Misalignment XY	0 mrad	0 mrad
Misalignment XZ	0 mrad	0 mrad
Misalignment YX	0 mrad	0 mrad
Misalignment YZ	0 mrad	0 mrad
Misalignment ZX	0 mrad	0 mrad
Misalignment ZY	0 mrad	0 mrad

Table 2: UUT Simulated Model Parameters

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3 Appendix

3.1 Traditional IMU Calibration Post-Processing

From equation 2 and collected test data listed in table 1, the accelerometer bias $\boldsymbol{b}_a$ can be solved from the collected test data.

$$\boldsymbol{b}_a = \begin{bmatrix} b_{a,x} \\ b_{a,y} \\ b_{a,z} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \bar{f}_x^{+\mathrm{x}} + \bar{f}_x^{-\mathrm{x}} \\ \bar{f}_y^{+\mathrm{y}} + \bar{f}_y^{-\mathrm{y}} \\ \bar{f}_z^{+\mathrm{z}} + \bar{f}_z^{-\mathrm{z}} \end{bmatrix} \tag{16}$$

Likewise, accelerometer scale factor terms within the quantity M_a can also be computed, where g is the magnitude of the accelerometer due to gravity at that specific location at the inertial test laboratory.

$$\boldsymbol{s}_a = \begin{bmatrix} s_{a,x} \\ s_{a,y} \\ s_{a,z} \end{bmatrix} = \frac{1}{2g} \begin{bmatrix} \bar{f}_x^{+\mathrm{x}} - \bar{f}_x^{-\mathrm{x}} \\ \bar{f}_y^{+\mathrm{y}} - \bar{f}_y^{-\mathrm{y}} \\ \bar{f}_z^{+\mathrm{z}} - \bar{f}_z^{-\mathrm{z}} \end{bmatrix} - 1 \tag{17}$$

Accelerometer misalignment uses off-axis terms from each collected test where each misalignment term is computed separately.

$$\begin{aligned}
m_{a,xy} &= \frac{1}{2g}\left(\bar{f}_x^{+\mathrm{y}} - \bar{f}_x^{-\mathrm{y}}\right) \\
m_{a,xz} &= \frac{1}{2g}\left(\bar{f}_x^{+\mathrm{z}} - \bar{f}_x^{-\mathrm{z}}\right) \\
m_{a,yx} &= \frac{1}{2g}\left(\bar{f}_y^{+\mathrm{x}} - \bar{f}_y^{-\mathrm{x}}\right) \\
m_{a,yz} &= \frac{1}{2g}\left(\bar{f}_y^{+\mathrm{z}} - \bar{f}_y^{-\mathrm{z}}\right) \\
m_{a,zx} &= \frac{1}{2g}\left(\bar{f}_z^{+\mathrm{x}} - \bar{f}_z^{-\mathrm{x}}\right) \\
m_{a,zy} &= \frac{1}{2g}\left(\bar{f}_z^{+\mathrm{y}} - \bar{f}_z^{-\mathrm{y}}\right)
\end{aligned} \tag{18}$$

Also from equation 2, the gyroscope bias $\boldsymbol{b}_g$ can be solved from the collected test data.

$$\boldsymbol{b}_g = \begin{bmatrix} b_{a,x} \\ b_{a,y} \\ b_{a,z} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \bar{\omega}_x^{+\mathrm{x}} + \bar{\omega}_x^{-\mathrm{x}} \\ \bar{\omega}_y^{+\mathrm{y}} + \bar{\omega}_y^{-\mathrm{y}} \\ \bar{\omega}_z^{+\mathrm{z}} + \bar{\omega}_z^{-\mathrm{z}} \end{bmatrix} \tag{19}$$

Gyroscope scale factor terms within the quantity M_g can also be computed, where ω_{test} is the magnitude of the accelerometer due to gravity at that specific location at the inertial test laboratory.

$$\boldsymbol{s}_g = \begin{bmatrix} s_{g,x} \\ s_{g,y} \\ s_{g,z} \end{bmatrix} = \frac{1}{2\omega_{\text{test}}} \begin{bmatrix} \bar{\omega}_x^{+\mathrm{x}} - \bar{\omega}_x^{-\mathrm{x}} \\ \bar{\omega}_y^{+\mathrm{y}} - \bar{\omega}_y^{-\mathrm{y}} \\ \bar{\omega}_z^{+\mathrm{z}} - \bar{\omega}_z^{-\mathrm{z}} \end{bmatrix} - 1 \tag{20}$$

Gyroscope misalignment uses off-axis terms from each collected test where each misalignment term is computed separately.

$$\begin{aligned}
m_{g,xy} &= \frac{1}{2\omega_{\text{test}}} \left(\bar{\omega}_x^{+\mathrm{y}} - \bar{\omega}_x^{-\mathrm{y}} \right) \\
m_{g,xz} &= \frac{1}{2\omega_{\text{test}}} \left(\bar{\omega}_x^{+\mathrm{z}} - \bar{\omega}_x^{-\mathrm{z}} \right) \\
m_{g,yx} &= \frac{1}{2\omega_{\text{test}}} \left(\bar{\omega}_y^{+\mathrm{x}} - \bar{\omega}_y^{-\mathrm{x}} \right) \\
m_{g,yz} &= \frac{1}{2\omega_{\text{test}}} \left(\bar{\omega}_y^{+\mathrm{z}} - \bar{\omega}_y^{-\mathrm{z}} \right) \\
m_{g,zx} &= \frac{1}{2\omega_{\text{test}}} \left(\bar{\omega}_z^{+\mathrm{x}} - \bar{\omega}_z^{-\mathrm{x}} \right) \\
m_{g,zy} &= \frac{1}{2\omega_{\text{test}}} \left(\bar{\omega}_z^{+\mathrm{y}} - \bar{\omega}_z^{-\mathrm{y}} \right)
\end{aligned} \tag{21}$$