Inverse Problems David L. Olson

Homework 3 (DRAFT)

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1 Problem 1

Exercise 2 in Section 3.6

1.1 Solution

Note: My MATLAB code for this homework problem repeats all the steps for example 3.1 so that I can take on this problem. I will only cover the checkerboard test in this write-up.

The checkerboard test using m_{true} can be reshaped to $m_{true} \in \mathbb{R}^9$ such that

$$oldsymbol{m}_{true} = egin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}^T$$

which allows for the creation of test data d_{true} and a recovered model m_{\dagger} .

$$d_{true} = Gm_t rue$$

$$m_{\dagger} = G^{\dagger} d_{true}$$

Recall from example 3.1 that $G \in \mathbb{R}^{8 \times 9}$ with rank 7. Therefore the generalized pseudo-inverse of G, represented as G^{\dagger} , was computed using the Moore-Penrose pseudo-inverse function pinv(G) in MATLAB ®. Figure 1 shows how the recovered model compares to the true model.

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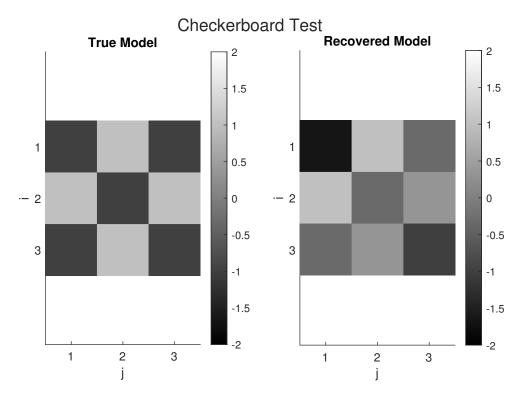


Figure 1: Checkerboard Test

Interpreting these results, only three of nine model parameters m_2, m_4, m_9 were recovered with no error. The error in the recovered model, $\Delta \boldsymbol{m} := \boldsymbol{m}_{\dagger} - \boldsymbol{m}_{true}$, is shown below.

$$\Delta \boldsymbol{m} = \begin{bmatrix} \frac{-2}{3} & 0 & \frac{2}{3} & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{2}{3} & \frac{-2}{3} & 0 \end{bmatrix}^T$$

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