Homework 3 (DRAFT)

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1 Problem 1

Exercise 2 in Section 3.6

1.1 Solution

Note: *My MATLAB code for this homework problem repeats all the steps for example 3.1 so that I can take on this problem. I will only cover the checkerboard test in this write-up.*

The checkerboard test using $\boldsymbol{m}_{true}$ can be reshaped to $\boldsymbol{m}_{true} \in \mathbb{R}^9$ such that

$$\boldsymbol{m}_{true} = \begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}^T$$

which allows for the creation of test data $\boldsymbol{d}_{true}$ and a recovered model $\boldsymbol{m}_{\dagger}$.

$$\boldsymbol{d}_{true} = G\boldsymbol{m}_{true}$$

$$\boldsymbol{m}_{\dagger} = G^{\dagger}\boldsymbol{d}_{true}$$

Recall from example 3.1 that $G \in \mathbb{R}^{8\times 9}$ with rank 7. Therefore the generalized pseudo-inverse of G, represented as $G^{\dagger}$, was computed using the Moore-Penrose pseudo-inverse function `pinv(G)` in MATLAB®. Figure 1 shows how the recovered model compares to the true model.

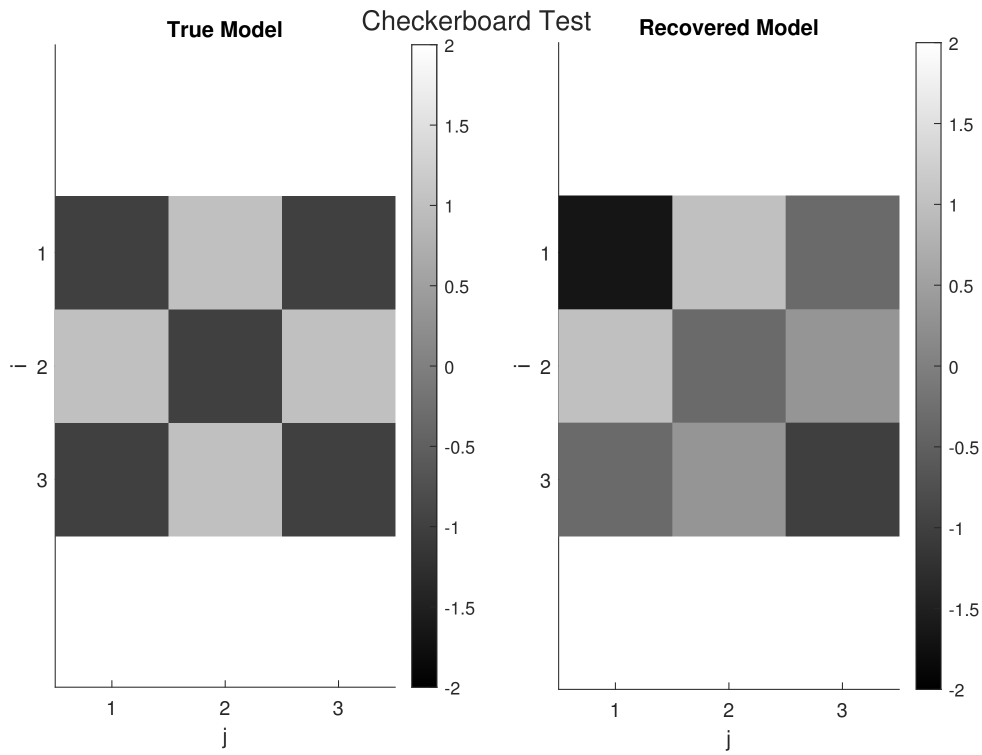

Figure 1: Checkerboard Test

Interpreting these results, only three of nine model parameters m_2, m_4, m_9 were recovered with no error. The error in the recovered model, $\boldsymbol{\Delta m} := \boldsymbol{m}_{\dagger} - \boldsymbol{m}_{true}$, is shown below.

$$\boldsymbol{\Delta m} = \begin{bmatrix} \frac{-2}{3} & 0 & \frac{2}{3} & 0 & \frac{2}{3} & \frac{-2}{3} & \frac{2}{3} & \frac{-2}{3} & 0 \end{bmatrix}^T$$

When examining the model resolution matrix R_m, it is interesting that only one diagonal value of R_m is equal to one even though three model parameters where recovered perfectly as shown in figure 2.

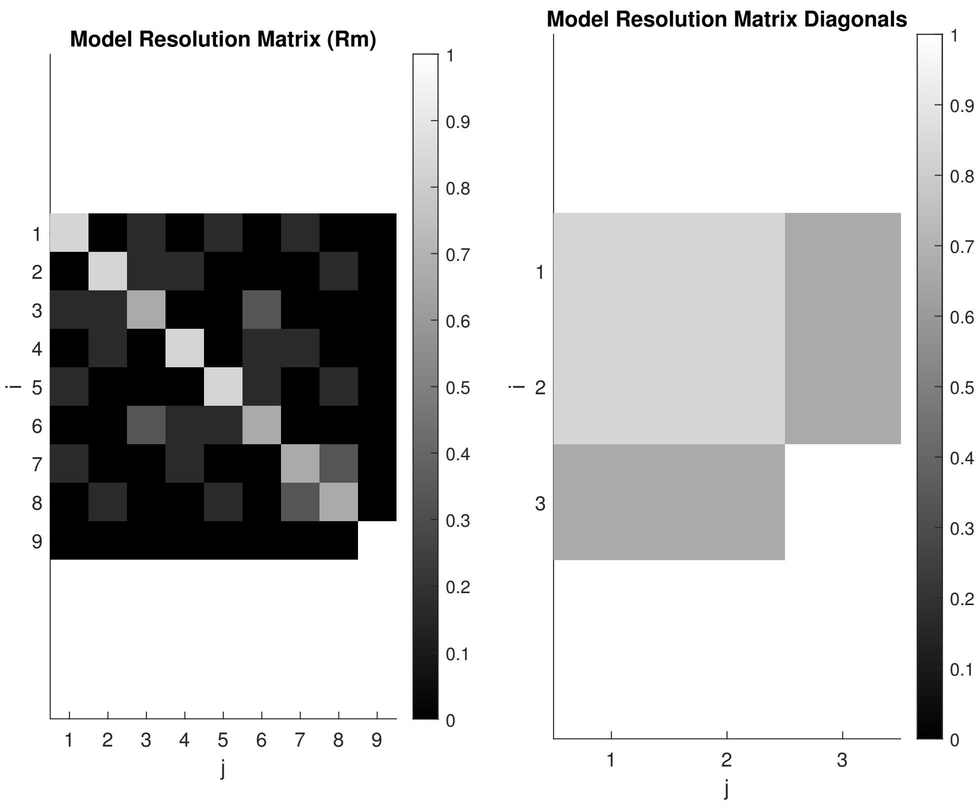

Figure 2: Checkerboard Test

There is more to analyze here, but I am a bit confused by the results right now.

2 Problem 2

Exercise 4 in Section 3.6

2.1 Solution

NOTE: *I do not have any experience in seismology, please forgive any mistakes in technicalities when I try to explain this exercise in my own words. It helps me understand what is going on so I can set up the problem correctly.*

The forward problem in this exercise allows mechanical waves to propagate through a 16×16 meter grid where each square in the grid have some slowness value $s_{x,y}$ in units of $\mathrm{s\,m^{-1}}$. Stations around the grid record time of arrivals of the mechanical waves as they pass through one or multiple grid squares. The time it takes to pass through a path of squares is formulated below.

$$t = \int_l s\left(\boldsymbol{x}\right) dl$$

$$\approx \sum_{blocks} s_{block} \Delta l$$

2.1.1 Part A - Row and Column Scans Only

16 row scans and 16 column scans are utilized in this part in the exercise as shown in figure 3.

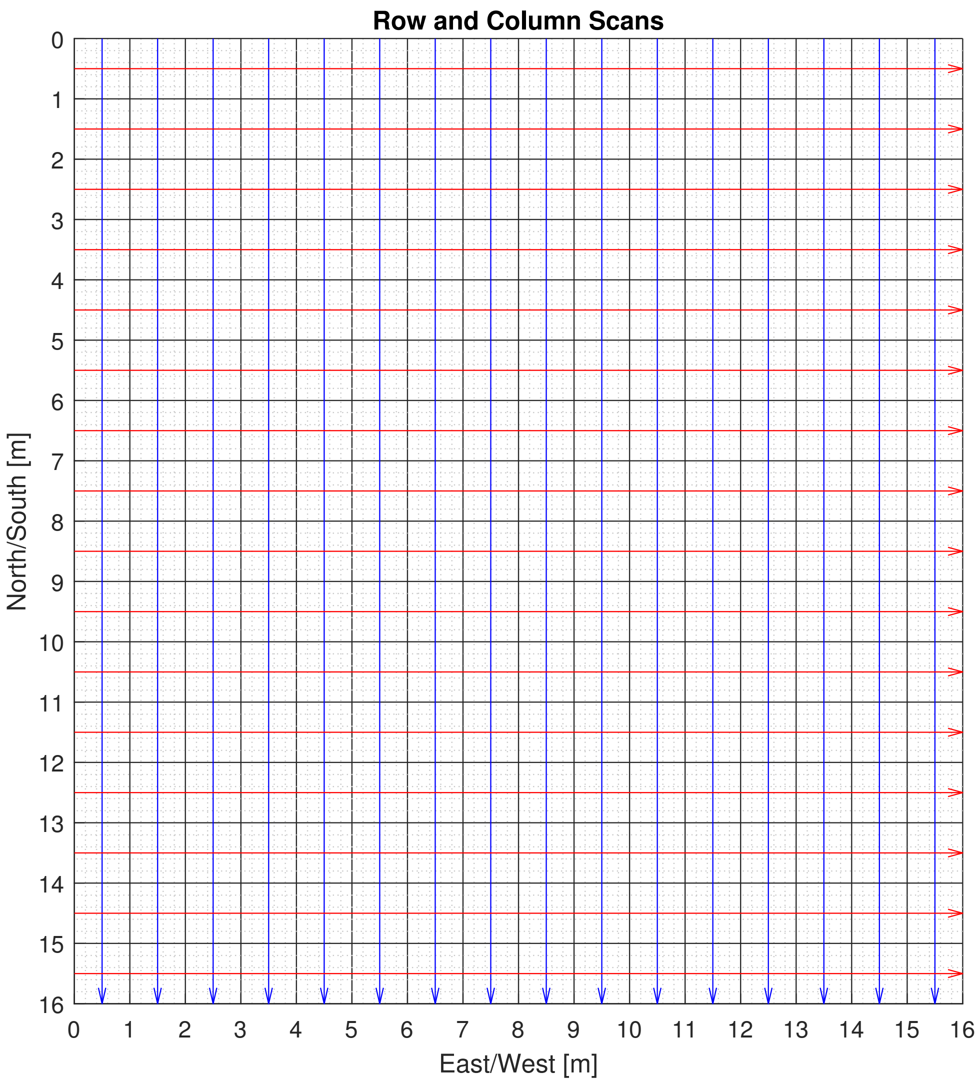

Figure 3: Row and Column Scan Visualization

This results in a total of $m = 32$ measurements. In an effort to estimate the slowness of each square in the grid, this results in a number of $n = 256$ model parameters.

$$\boldsymbol{d} \in \mathbb{R}^{32}, \quad \boldsymbol{m} \in \mathbb{R}^{256}, \quad G \in \mathbb{R}^{32\times 256}$$

The vector of measurement observations $\boldsymbol{d}$ is organized such that

$$\boldsymbol{d} = \begin{bmatrix} t_{r,1} & t_{r,2} & \dots & t_{r,16} & t_{c,1} & t_{c,2} & \dots & t_{c,16} \end{bmatrix}^T$$

where a r subscript indicates a row can and a c subscript indicates a column scan. The model parameters $\boldsymbol{m}$ are organized such that

$$\boldsymbol{m} = \begin{bmatrix} s_{1,1} & s_{1,2} & \dots & s_{1,16} & s_{2,1} & s_{2,2} & \dots & s_{16,1} & \dots & s_{16,16} \end{bmatrix}^T$$

where the first subscript indicates the row, and the second subscript indicates the column. Each row of the model operator G contain the distance traveled by the mechanical wave for each square in its path. Due to the large number of elements, a color map of the zeros and ones for this part of the problem as given instead in figure 4.

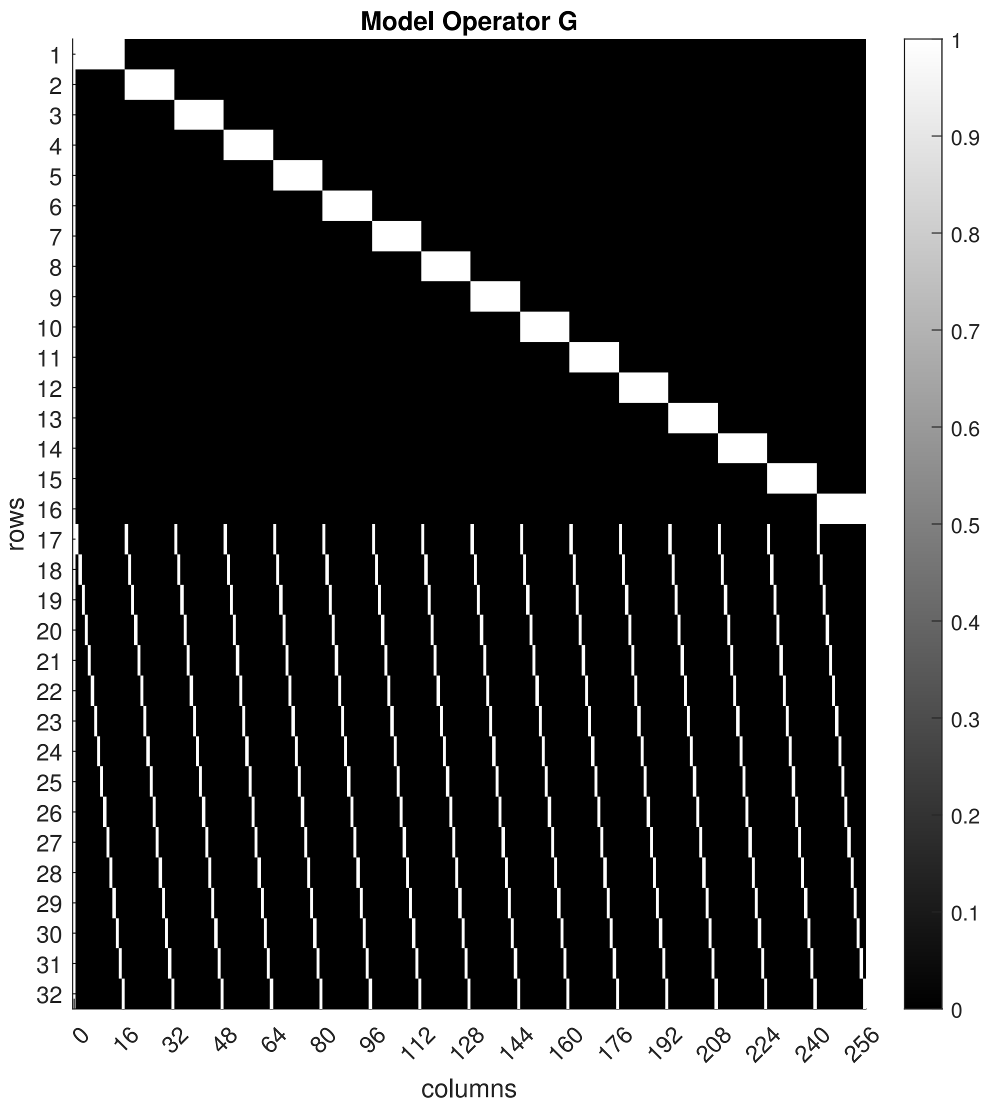

Figure 4: Model Operator G

Subpart A

Per MATLAB®, the rank of G is 31.

Subpart B

This is the environment of "$p < m$ and $p < n$", in which both the data null space and model null space are nontrivial. For this problem of $\boldsymbol{Gm} = \boldsymbol{d}$, the solution $\boldsymbol{m}^\dagger$ is the minimum length least squares solution.

$$m_\dagger = V_p S_p^{-1} U_p^T$$

I do not understand what it means to "plot and interpret an element from each space and contour or otherwise display a non-zero model that fits the trivial data $Gm = d = 0$"

The model resolution matrix is provided in figure 5.

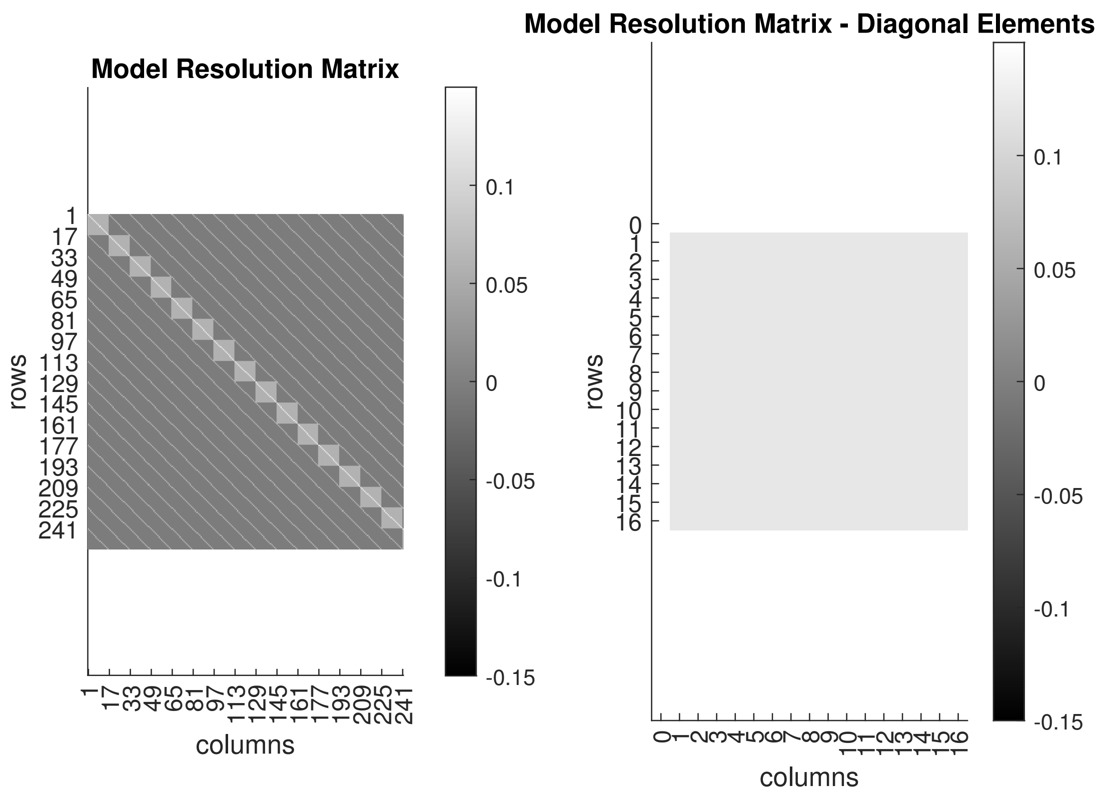

Figure 5: Model Resolution Matrix R_m

Note that there are no diagonal elements which provide perfect model resolution.

Subpart C

Model parameters are computed using the `pinv()` function in MATLAB®, and results are provided in figure 6.

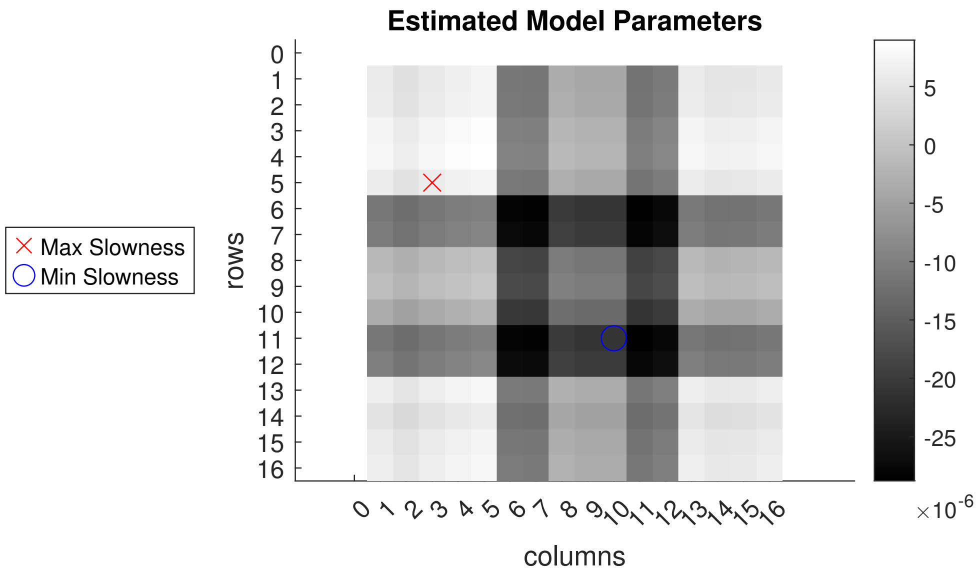

Figure 6: Estimated Model Parameters

The maximum and minimum estimated slowness values are

$$s_{max} = 12.345\,\mathrm{s\,m^{-1}}$$
$$s_{min} = 12.345\,\mathrm{s\,m^{-1}}$$

which imply velocities of

$$v_{max} = 12.345\,\mathrm{m\,s^{-1}}$$
$$v_{min} = 12.345\,\mathrm{m\,s^{-1}}$$

throughout the various square grids. The maximum velocity square is in the i^{th} row and j^{th}. *Again, I do not study seismology*, but assuming that the dinosaur bones are related to the highest velocity we expect to find in our search space, then they must be located in the square with the highest velocity value.

The values above are dummy values. Right now I have negative slowness values which do not make physical sense unless the mechanical waves reflect off of other surfaces in the ground?

Subpart D

Right now I have no idea what is going on with this question. I suspect it might make sense once I understand part B.