Ch. 1

• (some) dimensions & units (for physics I)

The words "dimensions" and "units" are often used interchangeably, but have a subtle difference.

fundamental

Dimensions	Units
time	seconds, minutes, hours, ...
mass	grams, kg, ... (not pounds)
	unit of force
length	inches, feet,
⋮	meters, ...

abstract specific

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non-fundamental

Dimensions	Units
velocity	mph, m/s, km/h
area	in^2, ft^2, m^2
volume	in^3, ft^3, m^3
acceleration	m/s^2, mph^2
jerk	m/s^3
snap	m/s^4
crackle	m/s^5
pop	m/s^6

* "dimensions" can sometimes mean "size"

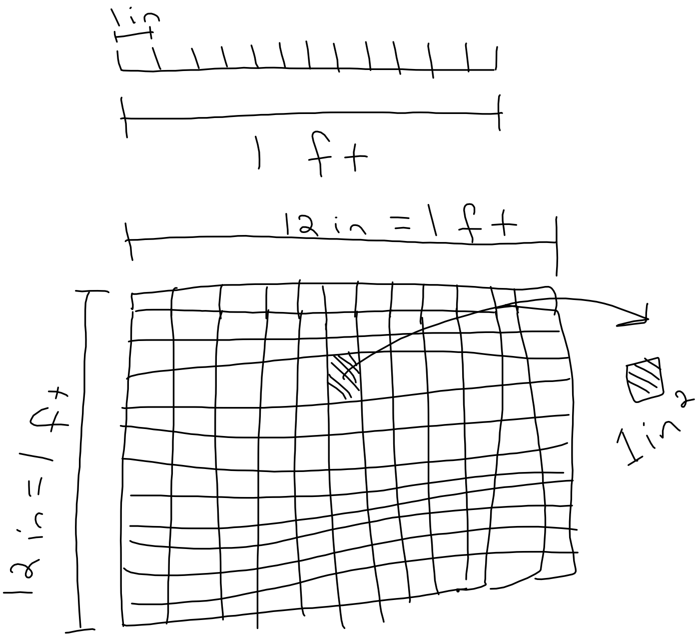
e.g. dimensions of a package: 8 in x 10 in x 2 in

Unit conversion

1 yr = 365 days

1 day = 24 hrs

1 ft = 12 in



$1\,ft^2 = 144\,in^2$

Example:

convert $3\,kg/m^3$ to g/cm^3

trick: multiply by 1

$$\left(\frac{3\,\cancel{kg}}{\cancel{m^3}}\right)\cdot\underbrace{\left(\frac{1000\,g}{1\,\cancel{kg}}\right)}_{1}\cdot\underbrace{\left(\frac{1\,\cancel{m}}{100\,cm}\right)}_{1}\cdot\underbrace{\left(\frac{1\,\cancel{m}}{100\,cm}\right)}_{1}\cdot\underbrace{\left(\frac{1\,\cancel{m}}{100\,cm}\right)}_{1}$$

$$\frac{3\cancel{000}\,g}{1000\cancel{000}\,cm^3} = \boxed{0.003\ g/cm^3}$$

Dimensional Analysis

aka units/dimensions must make sense

- How many seconds in a gallon?
  ↳ doesn't make sense!

- Example - We'll see later that

$$F_g = G\frac{m_1 m_2}{r^2}, \text{ also } [F] = \frac{kg \cdot m}{s^2}$$

what are the units of G?

i.e. $[G] = ?$

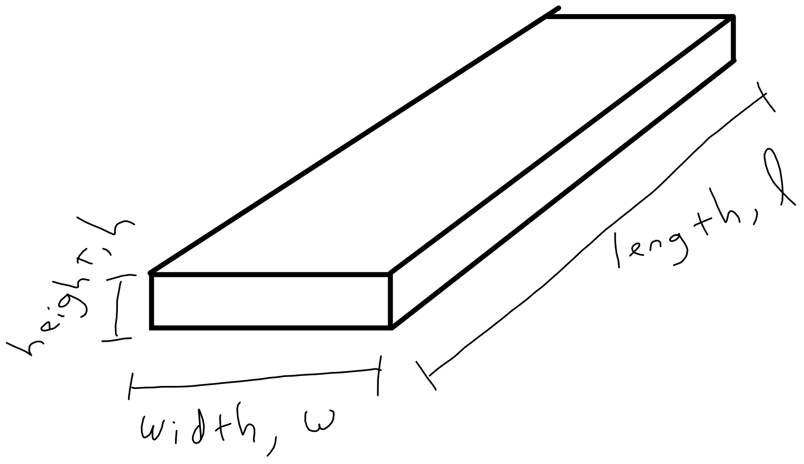
Solution -

$$\frac{\cancel{kg} \cdot m}{s^2} = [G]\frac{\cancel{kg} \cdot kg}{m^2}$$

$$\boxed{[G] = \frac{m^3}{kg \cdot s^2}}$$

Surface Area & Volume

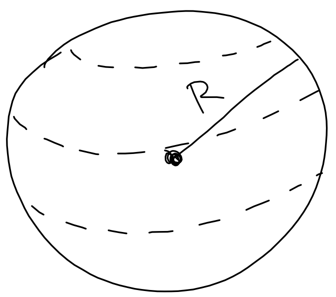
Rectangular prism:



$SA = 2(w \times l + l \times h + w \times h)$

$V = l \times w \times h$

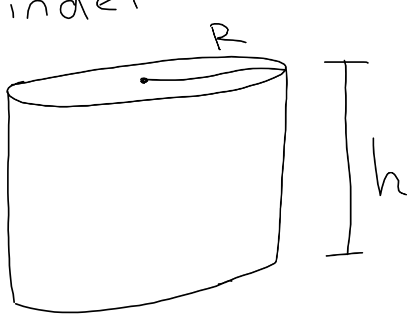
Sphere -



$SA = 4\pi R^2$

$V = \frac{4}{3}\pi R^3$

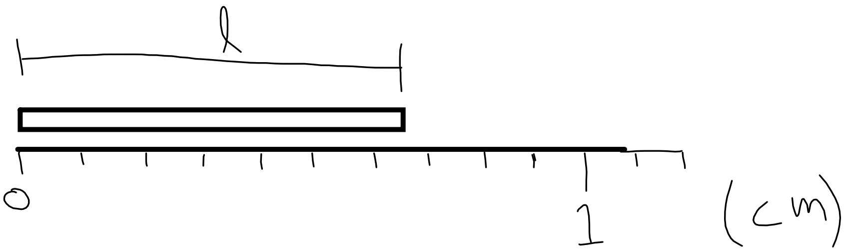
cylinder -



$V = \pi R^2 h$

$SA = 2\pi R^2 + 2\pi R h$

Error and Error Propagation (aka uncertainty)



mm is smallest reading (0.1 cm)

$\Rightarrow \delta l = 0.05$ cm

(half of 0.1 cm)

$$l = 0.66 \pm 0.05 \text{ cm}$$

Example 2:

$$\text{mass} = 16.1 \pm \underbrace{0.05}_{\text{half of smallest reading}} \text{ g}$$

When calculating a quantity from measured quantities (e.g. volume of a rectangular prism from measured w, L, h, you must propagate the errors:

suppose $Z = Z(x, y)$

and x, y are measured:

$$x = x_0 \pm \delta x$$

$$y = y_0 \pm \delta y$$

then $z_0 = Z(x_0, y_0)$ and

$$\delta z = \sqrt{\delta z_x^2 + \delta z_y^2}$$

← Adding in quadrature

where

$$\delta z_x = \left.\frac{dz(x, y_0)}{dx}\right|_{x_0} \cdot \delta x$$

$$\delta z_y = \left.\frac{dz(x_0, y)}{dy}\right|_{y_0} \cdot \delta y$$

see uncertaintyExample.pdf

$$V_0 = \frac{1}{3}\pi\left(\frac{2.62}{2}\right)^2(2.44) = 4.38 \text{ cm}^3$$

δV: $V = \frac{1}{12}\pi d^2 h$

$$\delta V_d = \frac{dV(d, 2.44)}{dd}\bigg|_{d=2.62} \cdot \delta d$$

$$= \frac{d}{dd}\left(\frac{1}{12}\pi d^2 (2.44)\right)\bigg|_{d=2.62} \cdot (0.05)$$

$$= \frac{2.44\pi}{12} \cdot 2d\bigg|_{d=2.62} \cdot (0.05)$$

$$= 0.167 \text{ cm}^3$$

δV_h: $V = \frac{1}{12}\pi d^2 h$

$$\frac{dV(h, d=2.62)}{dh} = \frac{d}{dh}\left(\frac{1}{12}\pi(2.62)^2 h\right)$$

$$= 1.797\bigg|_{h=2.44} = 1.797$$

$$\delta V_h = 1.797 \cdot \delta h = 1.797 \cdot (0.1)$$

$$= 0.1797 \text{ cm}^3$$

Finally,

$$\delta V = \sqrt{0.167^2 + 0.1797^2}$$

$$= 0.245 \text{ cm}^3$$

Finally (again),

$$\boxed{V = 4.38 \pm 0.245 \text{ cm}^3}$$