Chapter 3 - Vectors

(A purely mathematical topic that we will apply to physics in upcoming chapters)

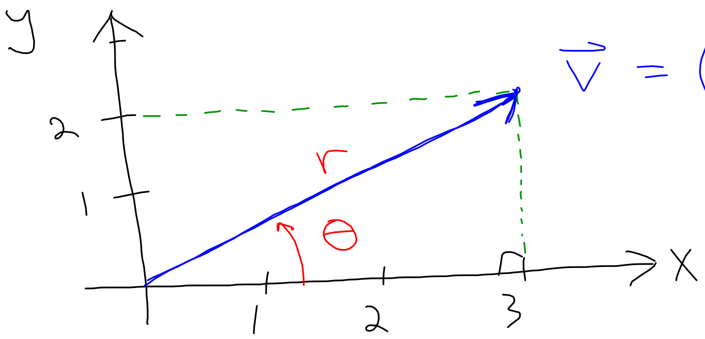
Optional: Look up the formal definition of a vector space (has nothing to do with magnitude and direction)

Working definition of a vector for physics I:
something with magnitude and direction.

mag = 32 cm
dir = up & left

mag = 13 cm
dir = right

mag = 88 cm
dir = down & left



$\vec{V} = (3, 2)$ or $\vec{V}_1 = 3\hat{i} + 2\hat{j}$

y-component of $\vec{V}$ (V_y) ← scalar

x-component of $\vec{V}$ (V_x) ← scalar

r = magnitude ("length")

θ = angle w.r.t. x-axis (direction)

$$r = \sqrt{x^2 + y^2}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\underbrace{\theta = \arctan\left(\frac{y}{x}\right)}_{\text{* be careful}}$$ (see example)

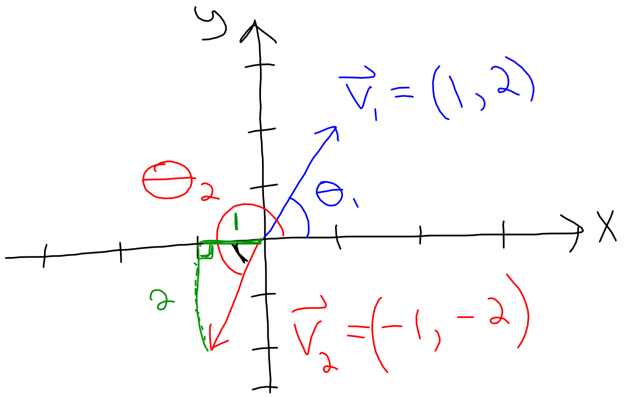
$\tan^{-1} = \arctan$
$\neq \frac{1}{\tan}$

$(2)^{-1} = \frac{1}{2}$

$z^{-1} = \frac{1}{z}$

$$\arctan(\tan(\theta)) = \theta$$

Example:



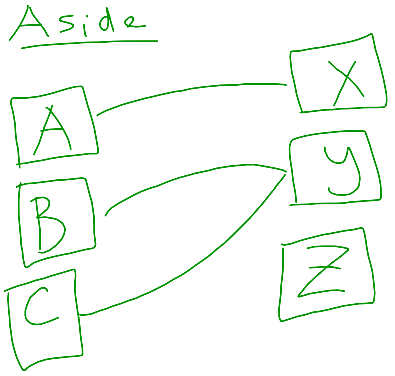
$$\theta_1 = \arctan\left(\frac{2}{1}\right) = 63.435^\circ$$

$$\theta_2 = \arctan\left(\frac{-2}{-1}\right)$$

$$= \cancel{63.435^\circ}$$

$$\theta_2 = 180^\circ + 63.435^\circ$$

Aside



tip: in SageMath, use

arctan2(−2, −1)

3.3 - (some) properties of vectors

- If $\vec{V} = (x, y)$ the $a\vec{V} = (ax, ay)$
- If $\vec{V_1} = (x_1, y_1)$ and $\vec{V_2} = (x_2, y_2)$ then

$$\vec{V_1} + \vec{V_2} = (x_1 + x_2,\ y_1 + y_2)$$

Example

$\vec{A} = 75\hat{j}$

$\vec{B} = 250\hat{i}$

$\vec{C} = 125\cos(30°)\hat{i} + 125\sin(30°)\hat{j}$

$\vec{D} = -150\hat{j}$

1. write each vector in (x, y) notation
2. find mag. (r) and direction (θ) of each
3. calculate $2\vec{A} + \vec{B} + \vec{C} + \vec{D}$