EC370 APSPI

Jan-May 2015

MINI PROJECT REPORT

ON

Algebraic Reconstruction using SART and Total Variation De-noising

SUBMITTED TO:

Dr A V Narasimhadhan

Assistant Professor

SUBMITTED BY:

Mohan Rao D K	**12EC59**
Shrinidhi Anand Muddebihal	**12EC88**
Vishwajeet S B	**12EC105**

Department of Electronics and Communication Engineering

National Institute of Technology Karnataka, Surathkal

ABSTRACT: Algebraic reconstruction techniques form a major part of CT (Computerised Tomography) imaging. It consists of assuming that the cross section consists of an array of unknowns, and then setting up algebraic equations for the unknowns in terms of the measured projection data.

Here, we have implemented an algorithm for image reconstruction using SART (Simultaneous Algebraic Reconstruction Technique) and Total Variation Minimization de-noising.

Table of Contents

Introduction **2**

Image and Projection representation 2

SART (Simultaneous Algebraic Reconstruction Technique) 3

Total Variation Minimization 4

Implementation **5**

Imaging model 5

Code 5

Testing **6**

Test image 6

Projection 6

Results 7

Conclusion and further scope **8**

References **8**

Chapter 1

INTRODUCTION

1.1 Image and Projection Representation in algebraic techniques:

Fig1 shows a superimposed square grid on the image f(x, y); we will assume that in each cell the function is constant. Let f_j denote this constant value in the j^{th}cell, and let N be the total number of cell. The ray is a line of certain small width. Its line integral on the image is called "Ray-sum".

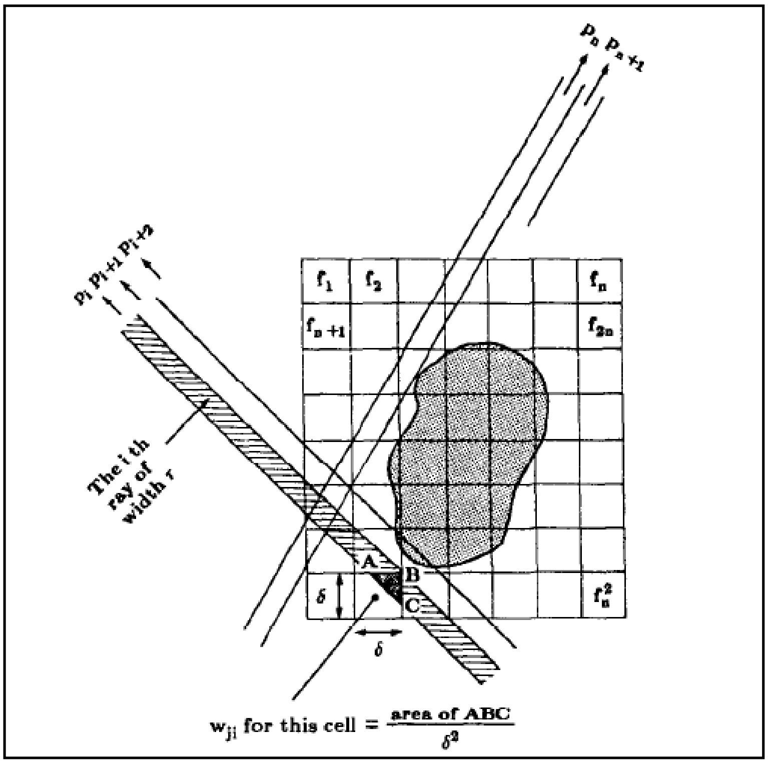


Fig .1

Let p_i be the ray-sum measured with the i^{th} ray as shown in Fig1. The relationship between the f_j's and p_i's may be expressed as

$$\sum_{j=1}^{N} \boldsymbol{w_{ij} f_j = p_i , \quad i = 1, 2, \dots, M} \qquad (1)$$

where M is the total number of rays (in all the projections) and w_{ij} is the weighting factor that represents the contribution of the j^{th} cell to the i^{th} ray integral. The factor w_{ij} is equal to the fractional area of the j^{th} image cell intercepted by the i^{th} ray as shown for one of the cells in Fig.1. Note that most of the w_{ij}'s are zero since only a small number of cells contribute to any given ray-sum. Also the M and N are very large. Eg: About 65,000 for a 256x256 image. So normal matrix inversion cannot be used and we should switch to advanced iterative techniques which can reduce the complexity. One such efficient technique is SART which is explained next.

1.2 SART (Simultaneous Algebraic Reconstruction Technique) [1]

This technique yields reconstructions of good quality and numerical accuracy in only one iteration. The main features of SART: First, to reduce errors in the approximation of ray integrals of a smooth image by finite sums, the traditional pixel basis is abandoned in favour of bilinear elements. To further reduce the noise resulting from the unavoidable but now presumably considerably smaller inconsistencies with real projection data, the correction terms are simultaneously applied for all the rays in one projection; this is in contrast with the ray-by-ray updates in ART. In addition, a heuristic procedure is used to improve the quality of reconstructions: a longitudinal Hamming window is used to emphasize the corrections applied near the middle of a ray relative to those applied near its ends.

Now suppose we assume that in an expansion for the image f(x, y), we use basis functions b_j(x, y) and that a good approximation to f(x, y) is obtained by using N of them.

$$f(x, y) \approx \hat{f}(x, y) \equiv \sum_{j=1}^{N} g_j b_j(x, y)$$

where g_j's are the coefficients of expansion. They form a finite set of numbers which describe the image f(x, y) relative to the chosen basis set b_j(x, y). Now the forward process can be written as,

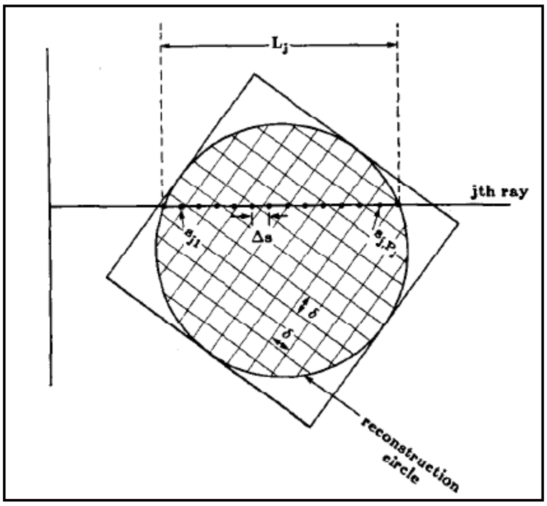
$$p_i = R_i f(x, y) \approx R_i \hat{f}(x, y) = \sum_{j=1}^{N} g_j R_i b_j(x, y) = \sum_{j=1}^{N} g_j a_{ij}$$

where a, represents the line integral of bj(X, y) along the ith ray. This equation has the same basic form as (1), yet it is more general in the sense that g_i's aren't constrained to be image gray level values over an array of points.

$$b_j(x, y) = \begin{cases} 1 & \text{inside the } j\text{th pixel} \\ 0 & \text{everywhere else.} \end{cases}$$

Rather than try to find separately the individual coefficients aij for a particular ray, we approximate the overall ray integral $R_i \hat{f}$(x, y) by a finite sum involving a set of $M_i$equidistant points {$\hat{f}$ (s_{im})}, for $1 \leq m \leq M_i$.

$$p_i \approx \sum_{m=1}^{M_i} \hat{f}(s_{im}) \Delta s.$$



$$\hat{f}(s_{im}) = \sum_{j=1}^{N} d_{ijm} g_j \qquad \text{for } m = 1, 2, \cdots, M_i.$$

So now the projections can be written as,

$$\begin{aligned} p_i &= \sum_{m=1}^{M_i} \sum_{j=1}^{N} d_{ijm} g_j \Delta s \\ &= \sum_{j=1}^{N} \sum_{m=1}^{M_i} d_{ijm} g_j \Delta s \qquad \text{for } 1 \le i \le J \\ &= \sum_{j=1}^{N} a_{ij} g_j \end{aligned}$$

where the coefficients au represent the net effect of the linear transformations. They are determined as the sum of the contributions from different points along the ray:

$$a_{ij} = \sum_{m=1}^{M_i} d_{ijm} \Delta s.$$

1.3 Total Variation Minimization De-noising [2]

Total variation has been widely used in medical image denoising. It is based on minimizing variation between spurious data enclosed as pixel in the image, by gradient descent algorithm. The advantage of TV regularization is it improves recovered image quality by preserving edge and boundaries. It amounts to minimize the energy functional of the form:

$$\min_u \int |\nabla u| + \mu \|Ku - v\|_2^2$$

where $||K_u - v||$ is the L2 fidelity term forced by noise assumption and 11 is a parameter which used to balance two minimization parts. To make the above equation converge quickly, Bregman iteration term is added. Bregman iteration is originated to find extrema of convex functionals. It also avoids the problems of numerical instabilities that occurs as $\mu \rightarrow \infty$, when using continuation methods.

Chapter 2

IMPLEMENTATION

2.1 Imaging model

The obtained data v and reconstructed image u can be interpreted as a discrete linear system v=Ku where K is an M x N matrix which transform the original image to measurements.

SART uses the following iterative updating rule to obtain the image u, from v [1] [2]:

$$u_i^{p+1} = u_i^p + \frac{\sum_j \left[k_{ij} \frac{v_j - \vec{k}_j^T \vec{u}^p}{\sum_{i=1}^{N} k_{ij}} \right]}{\sum_j k_{ij}}$$

In each step, first the image function is reconstructed using SART and then TV minimization denoising is done on this grey values to refine the image. This procedure is done for several iterations to get a good reconstructed image.

This algorithm is implemented and tested using MATLAB simulation and the results are presented in the next section.

2.2 Code

The snippet of the code which reveals the operation done in each iteration explained above is given below.

```
for iteration_number = 1 : NO_ITERATIONS
    iteration_number
    for sample_index = 1 : NO_PROJECTION_SAMPLES
        theta_index = 1;
        for k = 1 : NO_ANGLES
            ray_number = (theta_index - 1) * NO_PROJECTION_SAMPLES + sample_index;
            if ((a (: , ray_number)' * a (: , ray_number)) ~= 0)
                g_aux = g_aux + (t (: , ray_number) * (P (sample_index, theta_index) - a (: , ray_number)' * g) / (a (: , ray_number)' * a (: , ray_number))) ...
                                .* precomputed_thing *  (1./(1+log(iteration_number))).^0.05;
            end
            theta_index = mod (theta_index + angle_increment, NO_ANGLES) + 1;
        end
    end
    g = g + g_aux; % comment this line out for sequential update
    figure;imshow (vec2mat (g, size_f_x)', []);
    if(iteration_number == NO_ITERATIONS )
    continue;
    end
    g = tvtest(vec2mat (g, size_f_x)');
    figure;imshow (vec2mat (g, size_f_x)', []);
end
```

Chapter 3

TESTING

3.1 *Test Image [1]*

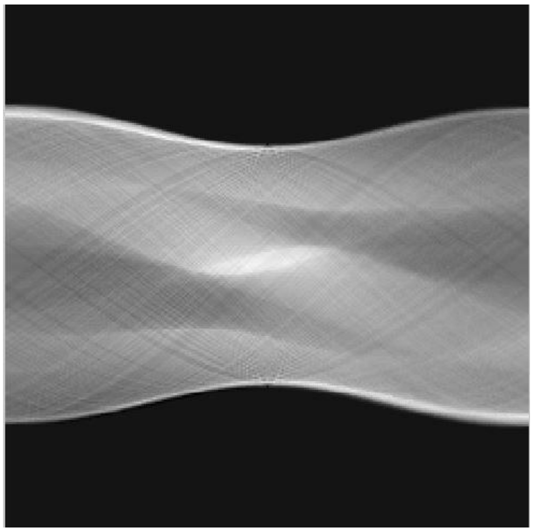
All the computer simulation results will be shown for the image in Fig. 3. This is the well-known Shepp and Logan "head phantom", so called because of its use in testing the accuracy of is collected if all the rays meet in one location.

Fig. 3

Reconstruction algorithms for their ability to reconstruct cross sections of the human head with x-ray tomography. (The human head is believed to place the greatest demands on the numerical accuracy and the freedom from artifacts of a reconstruction method.)

3.2 *Projection*

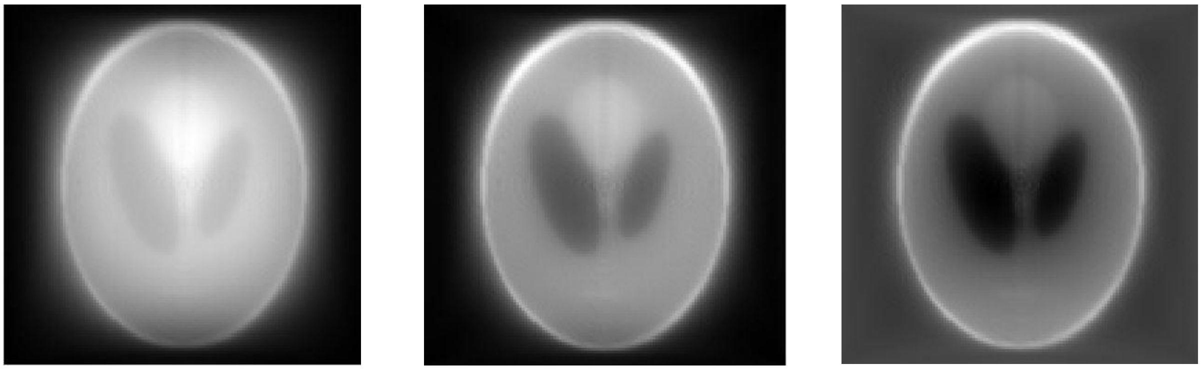
The projection data for the head phantom is obtained using forward modelling described in section 1.2. The sinogram of this is shown below.



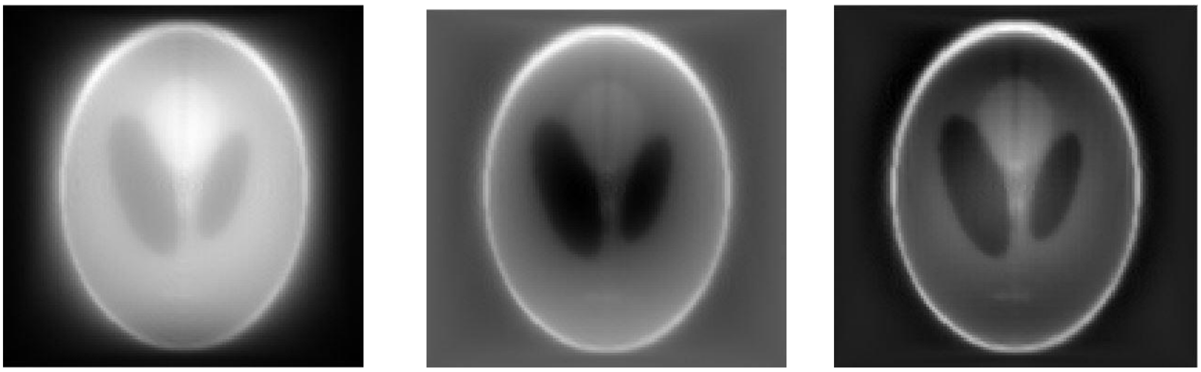
3.3 ***Results***

The image reconstructed from the projection data for 12 iterations and the results are given below.

Pure SART algorithm after 4, 8 and 12 iterations respectively.



Modified SART algorithm with Total Variation Minimization de-noising after 4, 8 and 12 iterations respectively.



It is seen that there is a good improvement in the quality of reconstructed image.

Conclusion

In this project the projection data is obtained from the head phantom image. Then using this projection the image was reconstructed using pure SART and a modified technique of combining SART with Total Variation Minimization which showed good improvement in results.

Further scope

This reconstruction algorithm can be further improved by combining with other advanced nonlinear iterating techniques and image processing techniques mentioned in [3].

References

1. "*Simultaneous Algebraic Reconstruction Technique (SART): A superior implementation of the ART algorithm*", Anderson, Kak; 1984
2. "*Regularization based CT image reconstruction using algebraic techniques*", Varun, Fayiz, Palanisamy; 2014
3. "*Improving algebraic reconstruction techniques with nonlinear iterating algorithms*", Zunying Li, Yizhong Song; 2009