

# Summation Notation

ECON 340: Economic Research Methods

Instructor: Div Bhagia

The capital sigma ( $\Sigma$ ) stands for summing everything on the right.

$$\sum_{i=1}^N X_i = X_1 + X_2 + \dots + X_N$$

When we have sets, the index  $i$  denotes the  $i$ -th position in the set.

*Example:* For  $X = \{1, 3, 5, 1\}$ , we have  $\sum_{i=1}^3 X_i = X_1 + X_2 + X_3 = 1 + 3 + 5 = 9$

*Note:* Another way of using a summation sign is to write  $\sum_{x \in A} x$ , which refers to summing up all elements in  $A$ . Similarly, to sum up  $x$  for all possible values  $x$ , we can simply write  $\sum_x x$ .

Things you CAN do to summations:

1. Pull constants out of them or into them.

$$\sum_{i=1}^N bX_i = b \sum_{i=1}^N X_i$$

*Example:*  $\sum_{i=1}^2 bX_i = bX_1 + bX_2 = b(X_1 + X_2) = b \sum_{i=1}^2 X_i$

2. Split apart (or combine) sums (addition) or differences (subtraction)

$$\sum_{i=1}^N (bX_i + cY_i) = b \sum_{i=1}^N X_i + c \sum_{i=1}^N Y_i$$

*Example:*  $\sum_{i=1}^2 (X_i - 2Y_i) = (X_1 - 2Y_1) + (X_2 - 2Y_2) = X_1 + X_2 - 2(Y_1 + Y_2)$ . So we can write

$$\sum_{i=1}^2 (X_i - 2Y_i) = \sum_{i=1}^2 X_i - 2 \sum_{i=1}^2 Y_i$$

3. Multiply through constants by the number of terms in the summation

$$\sum_{i=1}^N (a + bX_i) = aN + b \sum_{i=1}^N X_i$$

*Example:*  $\sum_{i=1}^3 a = a + a + a = 3a$ .

Things you CAN NOT do to summations:

1. Split apart (or combine) products (multiplication) or quotients (division).

$$\sum_{i=1}^N X_i Y_i \neq \sum_{i=1}^N X_i \times \sum_{i=1}^N Y_i$$

*Example:* Note that  $\sum_{i=1}^2 X_i Y_i = X_1 Y_1 + X_2 Y_2$ , while  $(\sum_{i=1}^2 X_i) \cdot (\sum_{i=1}^2 Y_i) = (X_1 + X_2)(Y_1 + Y_2) = X_1 Y_1 + X_2 Y_2 + X_1 Y_2 + X_2 Y_1$ .

2. Move the exponent out of or into the summation.

$$\sum_{i=1}^N X_i^a \neq \left( \sum_{i=1}^N X_i \right)^a$$

*Example:* Note that  $\sum_{i=1}^2 X_i^2 = X_1^2 + X_2^2$ , while  $\left( \sum_{i=1}^2 X_i \right)^2 = (X_1 + X_2)^2 = X_1^2 + X_2^2 + 2X_1 X_2$ .