Confidence Intervals and Hypothesis Testing

ECON 340: Economic Research Methods — Instructor: Div Bhagia

Confidence Intervals

Known population variance:
$1-\alpha$ confidence interval for the population mean μ:

$$\bar{X} \pm \underbrace{z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}_{\text{Margin of Error}}$$

where $z_{\alpha/2}$ is the z-value that leaves area $\alpha/2$ in the upper tail of the standard normal distribution.

Unknown population variance:
$1-\alpha$ confidence interval for the population mean μ:

$$\bar{X} \pm t_{n-1,\alpha/2}\frac{S}{\sqrt{n}}$$

where $t_{n-1,\alpha/2}$ is the t-value that leaves area $\alpha/2$ in the upper tail of the t-distribution. $n-1$ is the degrees of freedom. Since t distribution looks just like the standard normal for large n, for $n \geq 100$ continue using the standard normal table.

Exercise. A car manufacturer wants to estimate the mean CO2 emissions of a new model of car. A sample of 196 cars is randomly selected and their CO2 emissions are measured. The sample mean and standard deviation are 120 g/km and 20 g/km, respectively. Construct a 95% confidence interval for the true mean CO2 emissions of this car model.

Hypothesis Testing

Test null hypothesis $H_0 : \mu = \mu_0$ against alternative hypothesis $H_1 : \mu \neq \mu_0$. Construct test statistic Z if true population variance is known, else use T-statistic.

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \quad \text{and} \quad t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

Under the null $\bar{X} \sim N(\mu_0, \sigma^2/n)$, then $Z \sim N(0,1)$ and $T \sim t_{n-1}$. In case of known population variance, reject the null if $|z_0| > z_{\alpha/2}$. In the case of unknown population variance, reject the null if $|t_0| > t_{n-1,\alpha/2}$. When $n \geq 100$ you can reject the null if $|t_0| > z_{\alpha/2}$.

p-value:

Known variance: $p = 2Pr(Z > |z_0|)$
Unknown variance, $n < 100$: $p = 2Pr(T > |t_0|)$
Unknown variance, $n \geq 100$: $p = 2Pr(Z > |t_0|)$

Exercise. The car manufacturer had initially claimed that the average CO2 emissions of this model would be 115 g/km. Test the manufacturer's claim at a 5% level of significance. What is the p-value associated with your test statistic?