

Summation Notation

ECON 340: Economic Research Methods

Instructor: Div Bhagia

The capital sigma (Σ) stands for summing everything on the right.

$$\sum_{i=1}^N X_i = X_1 + X_2 + \dots + X_N$$

When we have sets, the index i denotes the i -th position in the set.

Example: For $X = \{1, 3, 5, 1\}$, we have $\sum_{i=1}^3 X_i = X_1 + X_2 + X_3 = 1 + 3 + 5 = 9$

Note: Another way of using a summation sign is to write $\sum_{x \in A} x$, which refers to summing up all elements in A . Similarly, to sum up x for all possible values x , we can simply write $\sum_x x$.

Things you CAN do to summations:

1. Pull constants out of them or into them.

$$\sum_{i=1}^N bX_i = b \sum_{i=1}^N X_i$$

Example: $\sum_{i=1}^2 bX_i = bX_1 + bX_2 = b(X_1 + X_2) = b \sum_{i=1}^2 X_i$

2. Split apart (or combine) sums (addition) or differences (subtraction)

$$\sum_{i=1}^N (bX_i + cY_i) = b \sum_{i=1}^N X_i + c \sum_{i=1}^N Y_i$$

Example: $\sum_{i=1}^2 (X_i - 2Y_i) = (X_1 - 2Y_1) + (X_2 - 2Y_2) = X_1 + X_2 - 2(Y_1 + Y_2)$. So we can write

$$\sum_{i=1}^2 (X_i - 2Y_i) = \sum_{i=1}^2 X_i - 2 \sum_{i=1}^2 Y_i$$

3. Multiply through constants by the number of terms in the summation

$$\sum_{i=1}^N (a + bX_i) = aN + b \sum_{i=1}^N X_i$$

Example: $\sum_{i=1}^3 a = a + a + a = 3a$.

Things you CAN NOT do to summations:

1. Split apart (or combine) products (multiplication) or quotients (division).

$$\sum_{i=1}^N X_i Y_i \neq \sum_{i=1}^N X_i \times \sum_{i=1}^N Y_i$$

Example: Note that $\sum_{i=1}^2 X_i Y_i = X_1 Y_1 + X_2 Y_2$, while $(\sum_{i=1}^2 X_i) \cdot (\sum_{i=1}^2 Y_i) = (X_1 + X_2)(Y_1 + Y_2) = X_1 Y_1 + X_2 Y_2 + X_1 Y_2 + X_2 Y_1$.

2. Move the exponent out of or into the summation.

$$\sum_{i=1}^N X_i^a \neq \left(\sum_{i=1}^N X_i \right)^a$$

Example: Note that $\sum_{i=1}^2 X_i^2 = X_1^2 + X_2^2$, while $\left(\sum_{i=1}^2 X_i \right)^2 = (X_1 + X_2)^2 = X_1^2 + X_2^2 + 2X_1 X_2$.