ECON 340
Economic Research Methods

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Lecture 14: Hypothesis Testing & p-Values
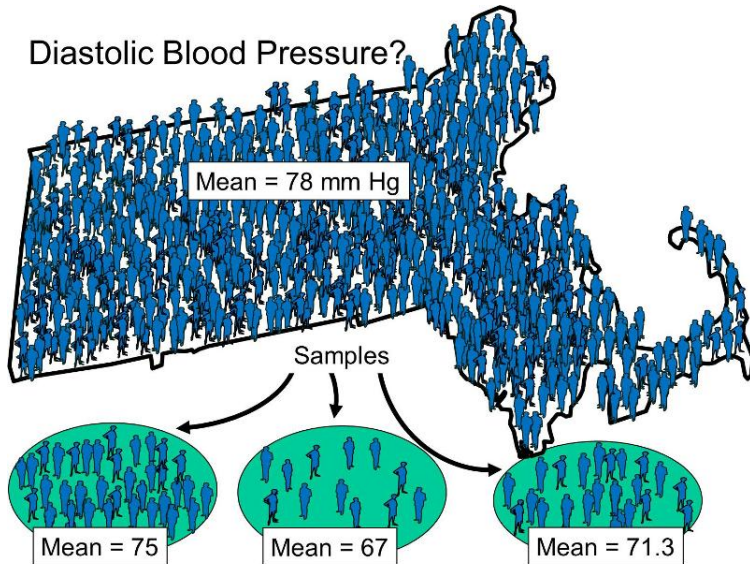
Sample Mean Distribution

Let $X_1, X_2, ..., X_n$ denote independent random draws (random sample) from a population with mean μ and variance σ^2.

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

The sample mean $\bar{X}$ is normally distributed when the underlying population is normal *or* sample size is large, say $n \geqslant 100$ (CLT).

Blood Pressure in Massachusetts

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We found that the average diastolic blood pressure in our sample was 75 i.e. $\bar{x} = 75$

Let's say our prior was $\mu = 78$, but we found $\bar{x} = 75$. Can we conclude our prior was wrong?

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Not yet! It could be that the true mean was 78, but just by chance we got 75. We can formally test our "hypothesis."

Hypothesis Testing

- Note that $n = 100$. For now, assume we know $\sigma^2 = 552.25$.

- So if our initial hypothesis ($\mu = 78$) is correct, then

$$\bar{X} \sim N(78, 5.52)$$

- To test our hypothesis, say at 10% level of significance:

  - Find the 10% most surprising outcomes, assuming our initial hypothesis is true

  - If the obtained sample mean $\bar{x} = 75$ is in the 10% most surprising outcomes, agree that we were wrong

Hypothesis Testing

- If 75 is not in the 10% most surprising outcome.

- Then it is quite likely that with a true population mean of 78, we actually got a sample mean of 75.

- So we cannot conclude that our initial hypothesis was wrong at least under this criterion.

- To make this easy on ourselves, we will find the most surprising outcomes in terms of the standard normal distribution

Hypothesis Testing: Recipe

1. We will call our initial hypothesis as the **null hypothesis**.

$$H_0 : \mu = 78$$

   We will test this against an **alternative hypothesis**. Natural alternative here is

$$H_1 : \mu \neq 78$$

Hypothesis Testing: Recipe

2. Given that our null is true, in this example we know that

$$\bar{X} \sim N(78, 5.52)$$

Test statistic under the null:

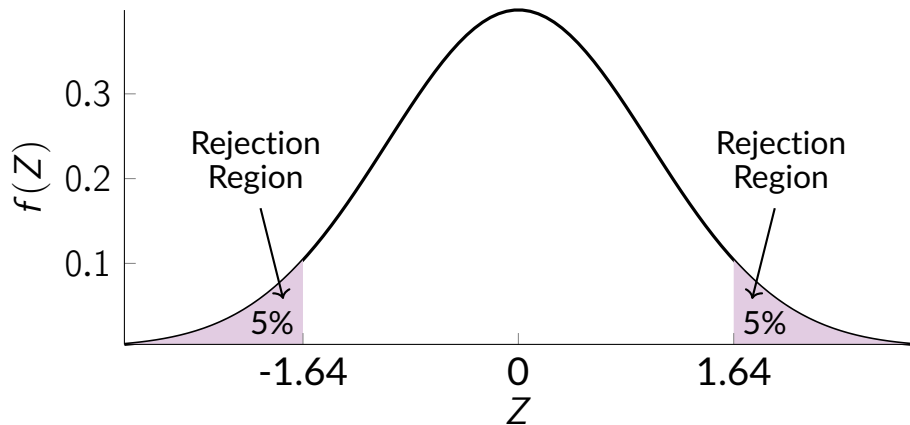$$Z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{x}}} \sim N(0, 1)$$

Hypothesis Testing: Recipe

3. **Significance level** α: determines how surprised do you have to be before you reject the null hypothesis.

   So our **rejection region** is $\alpha\%$ of outcomes that are most surprising given the null.

Rejection Region

At 10% level of significance:

Hypothesis Testing: Recipe

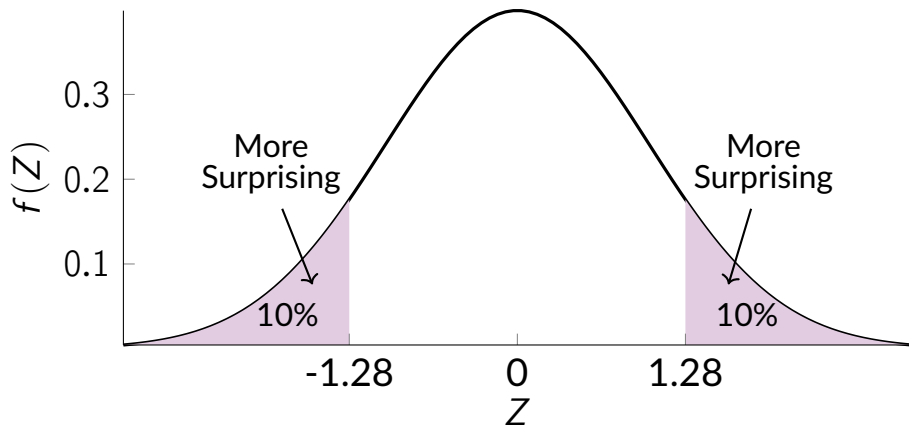Here $z_{0.05} = 1.64$ is the **critical value**.

So if our test statistic

$$|z| = \left| \frac{\bar{x} - 78}{\sigma_{\bar{x}}} \right| > 1.64 \rightarrow \text{Reject the null}$$

Otherwise, do not reject the null.

In our example, we cannot reject the null at 10% level of significance as $|z| = \left| \frac{75-78}{\sqrt{5.52}} \right| = 1.28 < 1.64$.

p-Value

p-value is defined as the probability of randomly drawing an outcome as surprising or more surprising given the null.

p-Value: Recipe

$$\text{p-value} = 2P\left(Z > \left|\frac{\bar{x} - 78}{\sigma_{\bar{x}}}\right| \,\Big|\, H_0 : \mu = 78\right)$$
$$= 2P(Z > 1.28) = 0.2$$

If p-value $< \alpha$, we can reject the null at α level of significance.

What if we don't know σ^2

- Proceed as before and create a test statistic using S:

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$$

- Reject the null if $|t| > t_{n-1,\alpha/2}$

- For large n, say $n \geqslant 100$, standard normal distribution approximates the t-distribution well

- So with large samples, we calculate a t-statistic but can just look at the standard normal table for critical values

Next up

- Next class: Review for the midterm

- Midterm exam on Thurs (10/12). Study guide and sample exam on Canvas.

- Research project meetings next to next week on Tues (10/17). Online or in-person, 15-20 minutes slots for each group. I will send details on how to sign up on Canvas.