

## Homework 2 Problems

ECON 441: Introduction to Mathematical Economics

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### Exercise 4.2

1. Given  $A = \begin{bmatrix} 7 & -1 \\ 6 & 9 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 4 \\ 3 & -2 \end{bmatrix}$ , and  $C = \begin{bmatrix} 8 & 3 \\ 6 & 1 \end{bmatrix}$ , find:

(a)  $A + B$

(b)  $C - A$

(c)  $3A$

(d)  $4B + 2C$

2. Given  $A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$ , and  $C = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$ :

(a) Is  $AB$  defined? Calculate  $AB$ . Can you calculate  $BA$ ? Why?

(b) Is  $BC$  defined? Calculate  $BC$ . Is  $CB$  defined? If so, calculate  $CB$ . Is it true that  $BC = CB$ .

4. Find the product matrices in the following (in each case, append beneath every matrix a dimension indicator):

(a)  $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$

(b)  $\begin{bmatrix} 6 & 5 & -1 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 5 & 2 \\ 0 & 1 \end{bmatrix}$

(c)  $\begin{bmatrix} 3 & 5 & 0 \\ 4 & 2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(d)  $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 2 \\ 1 & 4 \end{bmatrix}$

### Exercise 4.4

5. (e) Find (i)  $C = AB$ , and (ii)  $D = BA$ , if

$$A = \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 6 & -2 \end{bmatrix}$$

7. If the matrix  $A$  in Example 5 had all its four elements nonzero, would  $x'Ax$  still give a weighted sum of squares? Would the associative law still apply?

### Exercise 4.5

1. Given  $A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$ ,  $b = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$ , and  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ :

Calculate: (a)  $AI$  (b)  $IA$  (c)  $Ix$  (d)  $x'I$

Indicate the dimension of the identity matrix used in each case.

4. Show that the diagonal matrix

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

can be idempotent only if each diagonal element is either 1 or 0. How many different numerical idempotent diagonal matrices of dimension  $n \times n$  can be constructed altogether from such a matrix?

### Exercise 4.6

2. Given  $A = \begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}$ , and  $C = \begin{bmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \end{bmatrix}$ , verify that

(a)  $(A + B)' = A' + B'$

(b)  $(AC)' = C'A'$

6. Let  $A = I - X(X'X)^{-1}X'$ .

(a) Must  $A$  be square? Must  $(X'X)$  be square? Must  $X$  be square?

(b) Show that matrix  $A$  is idempotent.