Fall 2022 Midterm Exam

ECON 441: Introduction to Mathematical Economics   Instructor: Div Bhagia

Print Name: _____

This is a closed-book test. You may not use a phone or a computer.

Time allotted: 110 minutes

Total points: 30

Please show sufficient work so that the instructor can follow your work.

I understand and will uphold the ideals of academic honesty as stated in the honor code.

Signature: _____

1. (5 pts) Answer the following questions (1 point each)

(a) The cartesian product of two sets X and Y is defined as:

$$X \times Y = \{(x, y) | x \in X, y \in Y\}$$

What is the cartesian product of $X = \{a, b\}$ and $Y = \{2, 1\}$?

(b) A matrix's inverse exists if its determinant is equal to 0.

☐ True

☐ False

(c) The function $f(x) = |x|$ is *differentiable* at $x = 0$.

☐ True

☐ False

(d) For the function $f(x) = e^x$, $f'(x) = f(x)$

☐ True

☐ False

(e) What is the derivative of $y = 3x^2 + 2$?

2. (5 pts) Given the vector x and matrix A below, show that $x'Ax$ represents a weighted sum of squares. What is the dimension of $x'Ax$?

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad A = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$$

3. (4 pts) Say I have a system of m equations with n unknowns.

$$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = b_2$$
$$\vdots \qquad\qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n = b_m$$

(a) (1 pt) What is the necessary condition for the existence of a unique solution for this system in terms of m and n?

(b) (1 pt) What is the sufficient condition for the existence of a unique solution for this system?

(c) (2 pts) How would you use the tools learned in linear algebra to solve this system of equations? (*No need to describe how you would calculate the inverse or multiply matrices, you can just say I take the inverse then multiply...*).

4. (6 pts) Find the derivative for the following functions (2 pts each):

(a) $y = \ln(x^2 + 1)$

(b) $y = \dfrac{e^x}{1 + e^x}$

(c) $y = v + v^3$ where $v = x + 1$

5. (5 pts) Given the consumption function

$$C = 200 + 0.6Y$$

where C is consumption, and Y is income.

(a) (3 pts) Find the income elasticity of consumption ε_{CY}, and determine its sign, assuming $Y > 0$.

(b) (1 pt) Show that this consumption function is inelastic at all positive income levels.

(c) (1 pt) What is the income elasticity of consumption when income is equal to $1000?

(d) (1 pt) If income increases by 1% from $1000 to $1100, by what percent does consumption increase?

6. (5 pts) Given the following function:

$$f(x, y, z) = xyz$$

(a) (2 pts) Find the partial derivatives f_x, f_y, and f_z.

(b) (1 pt) Find the gradient of f.

(c) (1 pt) Find the total differential of f. You can denote it by df.

(d) (1 pt) Find the total derivative of f with respect to x?