Homework 6 Problems

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Exercise 10.5

1. Find the derivatives of:

   a) $y = e^{2t+4}$ b) $y = e^{1-9t}$ c) $y = e^{t^2+1}$

   d) $y = 5e^{2-t^2}$ e) $y = e^{ax^2+bx+c}$ f) $y = xe^x$

   g) $y = x^2e^{2x}$ h) $y = axe^{bx+c}$

3. Find the derivatives of:

   a) $y = \ln\left(7t^5\right)$ b) $y = \ln\left(at^{\circ}\right)$ c) $y = \ln(t+19)$

   d) $y = 5\ln(t+1)^2$ e) $y = \ln x - \ln(1+x)$ f) $y = \ln\left[x(1-x)^8\right]$

   g) $y = \ln\left(\frac{2x}{1+x}\right)$ h) $y = 5x^4\ln x^2$

7. Find the derivatives of the following by first taking the natural log of both sides:

   (a) $y = \dfrac{3x}{(x+2)(x+4)}$

   (b) $y = \left(x^2+3\right)e^{x^2+1}$

Exercise 7.4

1. Find $\partial y/\partial x_1$ and $\partial y/\partial x_2$ for each of the following functions:

   (a) $y = 2x_1^3 - 11x_1^2x_2 + 3x_2^2$ (d) $y = \dfrac{5x_1+3}{x_2-2}$

2. Find f_x and f_y from the following:

   (a) $f(x,y) = x^2 + 5xy - y^3$ (b) $f(x,y) = \left(x^2-3y\right)(x-2)$

3. From the answers to Prob. 2 , find $f_x(1,2)$, the value of the partial derivative f_x when $x = 1$ and $y = 2$, for each function.

5. If the utility function of an individual takes the form

$$U = U(x_1, x_2) = (x_1 + 2)^2 (x_2 + 3)^3$$

where U is total utility, and x_1 and x_2 are the quantities of two commodities consumed:

(a) Find the marginal-utility function of each of the two commodities.

(b) Find the value of the marginal utility of the first commodity when 3 units of each commodity are consumed.

7. Write the gradients of the following functions:

(a) $f(x, y, z) = x^2 + y^3 + z^4$

(b) $f(x, y, z) = xyz$

Exercise 8.1

1. Find the differential dy, given:

(a) $y = -x\left(x^2 + 3\right)$

4. Find the point elasticity of demand, given $Q = k/P^n$, where k and n are positive constants.

(a) Does the elasticity depend on the price in this case?

(b) In the special case where $n = 1$, what is the shape of the demand curve? What is the point elasticity of demand?

6. Given $Q = 100 - 2P + 0.02Y$, where Q is quantity demanded, P is price, and Y is income, and given $P = 20$ and $Y = 5,000$, find the

(a) Price elasticity of demand.

(b) Income elasticity of demand.

Exercise 8.2

3. Find the total differential, given

   (a) $y = \dfrac{x_1}{x_1 + x_2}$

4. The supply function of a certain commodity is

$$Q = a + bP^2 + R^{1/2} \quad (a < 0, b > 0) \quad \text{[R: rainfall]}$$

   Find the price elasticity of supply ε_{QP}, and the rainfall elasticity of supply ε_{QR}.

5. How do the two partial elasticities in Prob. 4 vary with P and R? In a strictly monotonic fashion (assuming positive P and R)?

6. The foreign demand for our exports X depends on the foreign income Y_f and our price level P : $X = Y_f^{1/2} + P^{-2}$. Find the partial elasticity of foreign demand for our exports with respect to our price level.

7. Find the total differential for each of the following functions:

   (b) $U = 7x^2y^3$

   (f) $U = (x - 3y)^3$

Exercise 8.4

2. Find the total derivative dz/dt, given

   (a) $z = x^2 - 8xy - y^3$, where $x = 3t$ and $y = 1 - t$

   (b) $z = 7u + vt$, where $u = 2t^2$ and $v = t + 1$

   (c) $z = f(x, y, t)$, where $x = a + bt$ and $y = c + kt$

4. Find the partial total derivatives $\S W/\xi u$ and $\S W/\xi v$ if

   (a) $W = ax^2 + bxy + cu$, where $x = \alpha u + \beta v$ and $y = \gamma u$

   (b) $W = f(x_1, x_2)$, where $x_1 = 5u^2 + 3v$ and $x_2 = u - 4v^3$