Homework 1 Solutions

ECON 441: Introduction to Mathematical Economics   Instructor: Div Bhagia

Exercise 2.3

1. (a) $\{x \in \mathbb{R} \mid x > 34\}$ or $\{x \mid x > 34\}$

   (b) $\{x \mid 8 < x < 65\}$

2. (a), (d), (f), (g), and (h) are true.

Exercise 2.4

5. We are given the function $y = 5 + 3x$ with domain $X = \{x \mid 1 \le x \le 9\}$. Note that for this function, when $x = 1$, $y = 8$ and when $x = 9$, $y = 32$. So the range for this function is:
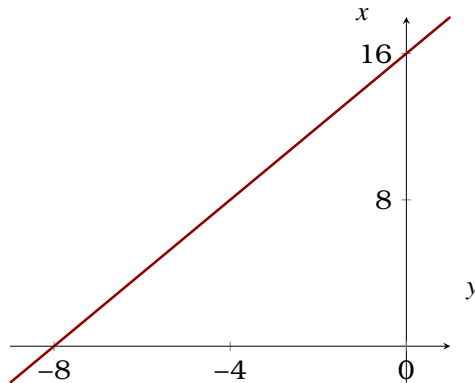$$f(X) = \{y \mid 8 \le y \le 32\}$$

   Note: It is not always the case that extreme values of the domain correspond to extreme values of the range. For example, consider $y = x^2$ with domain $\{x \mid -2 \le x \le 2\}$, the range here is $\{y \mid 0 \le y \le 4\}$.

7. (a) No, (b) Yes

8. For each output, we would want to produce at the lowest cost.

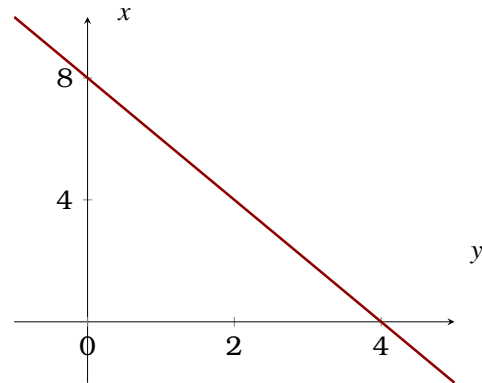Exercise 2.5

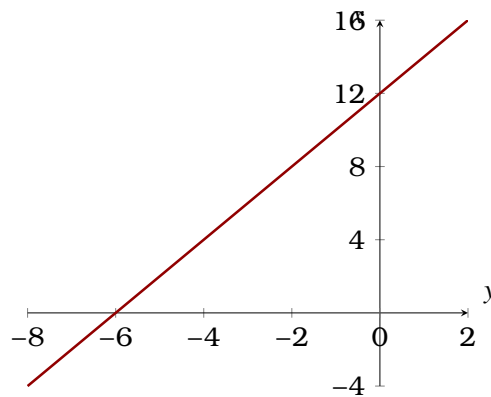1. Graph the following functions and find their inverse.

a) $y = 16 + 2x, \quad f^{-1}(y) = \dfrac{y - 16}{2}$

b) $y = 8 - 2x \quad f^{-1}(y) = \dfrac{8 - y}{2}$

c) $y = 2x + 12 \quad f^{-1}(y) = \dfrac{y - 12}{2}$

Exercise 4.2

6.

a) $x_2 + x_3 + x_4 + x_5$

b) $a_5 x_5 + a_6 x_6 + a_7 x_7 + a_8 x_8$

c) $b x_1 + b x_2 + b x_3 + b x_4$

d) $a_1 + a_2 x + a_3 x^2 + \ldots + a_n x^{n-1}$

e) $x^2 + (x + 1)^2 + (x + 2)^2 + (x + 3)^2$

8. (a)

$$\left(\sum_{i=0}^{n} x_i\right) + x_{n+1} = x_0 + x_1 + x_2 + \ldots + x_{n+1} = \sum_{i=0}^{n+1} x_i$$

(b)

$$\sum_{j=1}^{n} ab_j y_j = ab_1 y_1 + ab_2 y_2 + \ldots + ab_n y_n$$

$$= a\left(b_1 y_1 + b_2 y_2 + \ldots + b_n y_n\right)$$

$$= a\sum_{j=1}^{n} b_j y_j$$

(c)

$$\sum_{j=1}^{n}\left(x_j + y_j\right) = (x_1 + y_1) + (x_2 + y_2) + \ldots + (x_n + y_n)$$

$$= x_1 + x_2 + \ldots + x_n + x_1 + y_2 + \ldots + y_n$$

$$= \sum_{j=1}^{n} x_j + \sum_{j=1}^{n} y_j$$

Exercise 5.1

1.
a) $q \implies p$

b) $q \implies p$

c) $q \iff p$

d) $q \iff p$

e) $q \iff p$

f) $p \implies q$

g) $q \implies p$