Homework 3 Problems

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Exercise 5.1

3. Are the rows linearly independent in each of the following?

   (tsk[a]) $\begin{bmatrix} 24 & 8 \\ 9 & -3 \end{bmatrix}$ (tsk[a]) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

   (tsk[a]) $\begin{bmatrix} 0 & 4 \\ 3 & 2 \end{bmatrix}$ (tsk[a]) $\begin{bmatrix} -1 & 5 \\ 2 & -10 \end{bmatrix}$

4. Check whether the columns of each matrix in Prob. 3 are also linearly independent. Do you get the same answer as for row independence?

5. Find the rank of each of the following matrices from its echelon matrix and comment on the question of nonsingularity.

   (tsk[a]) $A = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 3 & 9 \\ -1 & 0 & 0 \end{bmatrix}$ (tsk[a]) $B = \begin{bmatrix} 0 & -1 & -4 \\ 3 & 1 & 2 \\ 6 & 1 & 0 \end{bmatrix}$

   (tsk[a]) $C = \begin{bmatrix} 7 & 6 & 3 & 3 \\ 0 & 1 & 2 & 1 \\ 8 & 0 & 0 & 8 \end{bmatrix}$ (tsk[a]) $D = \begin{bmatrix} 2 & 7 & 9 & -1 \\ 1 & 1 & 0 & 1 \\ 0 & 5 & 9 & -3 \end{bmatrix}$

6. By definition of linear dependence among rows of a matrix, one or more rows can be expressed as a linear combination of some other rows. In the echelon matrix, linear dependence is signified by the presence of one or more zero rows. What provides the link between the presence of a linear combination of rows in a given matrix and the presence of zero rows in the echelon matrix?

Exercise 5.2

1. Evaluate the following determinants:

(c) $\begin{vmatrix} 4 & 0 & 2 \\ 6 & 0 & 3 \\ 8 & 2 & 3 \end{vmatrix}$ (e) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (f) $\begin{vmatrix} x & 5 & 0 \\ 3 & y & 2 \\ 9 & -1 & 8 \end{vmatrix}$

2. Determine the signs to be attached to the relevant minors in order to get the following cofactors of a determinant: $|C_{13}|$, $|C_{23}|$, $|C_{33}|$, $|C_{41}|$, and $|C_{34}|$.

3. Given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$, find the minors and cofactors of the elements a, b and f.

6. Find the minors and cofactors of the third row, given

$$A = \begin{bmatrix} 9 & 11 & 4 \\ 3 & 2 & 7 \\ 6 & 10 & 4 \end{bmatrix}$$