

## Homework 5 Problems

ECON 441: Introduction to Mathematical Economics

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### Exercise 6.2

2. Given the function  $y = 5x^2 - 4x$  :

- (a) Find the difference quotient as a function of  $x$  and  $\Delta x$ .
- (b) Find the derivative  $dy/dx$ . (Using the limit definition.)
- (c) Find  $f'(2)$  and  $f'(3)$ .

3. Given the function  $y = 5x - 2$  :

- (a) Find the difference quotient  $\Delta y/\Delta x$ . What type of function is it?
- (b) Since the expression  $\Delta x$  does not appear in the function  $\Delta y/\Delta x$  in part (a), does it make any difference to the value of  $\Delta y/\Delta x$  whether  $\Delta x$  is large or small? Consequently, what is the limit of the difference quotient as  $\Delta x$  approaches zero?

### Exercise 7.1

3. Find  $f'(1)$  and  $f'(2)$  for the following functions:

(tsk[af])  $f(x) = 18x$

(tsk[af])  $f(x) = cx^3$

(tsk[af])  $f(x) = -5x^{-2}$

(tsk[af])  $f(x) = \frac{3}{4}x^{4/3}$

(tsk[af])  $f(w) = 6w^{1/3}$

(tsk[af])  $f(w) = -3w^{-1/6}$

### Exercise 7.2

3. Differentiate the following by using the product rule:

(d)  $(ax - b)(cx^2)$

(e)  $(2 - 3x)(1 + x)(x + 2)$

7. Find the derivatives of:

$$(a^2 + 3)/x$$

$$(a^2 - 9)/x$$

$$(a)(x + 5)$$

$$(a^2 + b)/(cx + d)$$

8. Given the function  $f(x) = ax + b$ , find the derivatives of:

$$f(a)$$

$$f'(x)$$

$$f'(x)$$

$$f'(a)/x$$

### Exercise 7.3

1. Given  $y = u^3 + 2u$ , where  $u = 5 - x^2$ , find  $dy/dx$  by the chain rule.

2. Given  $w = ay^2$  and  $y = bx^2 + cx$ , find  $dw/dx$  by the chain rule.

3. Use the chain rule to find  $dy/dx$  for the following:

$$(a)(3x^2 - 13)^3$$

$$(a)(7x^3 - 5)^9$$

$$(a)(ax + b)^5$$

4. Given  $y = (16x + 3)^{-2}$ , use the chain rule to find  $dy/dx$ . Then rewrite the function as  $y = 1/(16x + 3)^2$  and find  $dy/dx$  by the quotient rule. Are the answers identical?

5. Given  $y = 7x + 21$ , find its inverse function. Then find  $dy/dx$  and  $dx/dy$ , and verify the inverse-function rule. Also verify that the graphs of the two functions bear a mirrorimage relationship to each other.

6. Are the following functions strictly monotonic?

$$-x^6 + 5 \quad (x > 0)$$

$$4x^5 + x^3 + 3x$$

For each strictly monotonic function, find  $dx/dy$  by the inverse-function rule.