

Homework 2 Problems

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Exercise 4.2

1. Given $A = \begin{bmatrix} 7 & -1 \\ 6 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ 3 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 8 & 3 \\ 6 & 1 \end{bmatrix}$, find:

(a) $A + B$

(b) $C - A$

(c) $3A$

(d) $4B + 2C$

2. Given $A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$, and $C = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$:

(a) Is AB defined? Calculate AB . Can you calculate BA ? Why?

(b) Is BC defined? Calculate BC . Is CB defined? If so, calculate CB . Is it true that $BC = CB$.

4. Find the product matrices in the following (in each case, append beneath every matrix a dimension indicator):

(a) $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 5 & -1 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 5 & 2 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 5 & 0 \\ 4 & 2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(d) $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 2 \\ 1 & 4 \end{bmatrix}$

Exercise 4.4

5. (e) Find (i) $C = AB$, and (ii) $D = BA$, if

$$A = \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 6 & -2 \end{bmatrix}$$

7. If the matrix A in Example 5 had all its four elements nonzero, would $x'Ax$ still give a weighted sum of squares? Would the associative law still apply?

Exercise 4.5

1. Given $A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$, $b = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$, and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$:

Calculate: (a) AI (b) IA (c) Ix (d) $x'I$

Indicate the dimension of the identity matrix used in each case.

4. Show that the diagonal matrix

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

can be idempotent only if each diagonal element is either 1 or 0. How many different numerical idempotent diagonal matrices of dimension $n \times n$ can be constructed altogether from such a matrix?

Exercise 4.6

2. Given $A = \begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \end{bmatrix}$, verify that

(a) $(A + B)' = A' + B'$

(b) $(AC)' = C'A'$

6. Let $A = I - X(X'X)^{-1}X'$.

(a) Must A be square? Must $(X'X)$ be square? Must X be square?

(b) Show that matrix A is idempotent.