Constrained Optimization

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Given an objective function

$$y = f(x_1, x_2)$$

subject to the constraint

$$g(x_1, x_2) = c$$

where c is a constant. We can write the Lagrange function as

$$L(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda[c - g(x_1, x_2)]$$

First-order conditions:

$$\frac{\partial L}{\partial x_1} = 0, \quad \frac{\partial L}{\partial x_2} = 0, \quad \frac{\partial L}{\partial \lambda} = 0$$

For each of the problems below, set up the Lagrange function, write down the first-order conditions, and find the critical points.

Example I:

$$\max_{\{x_1, x_2\}} \quad x_1^2 x_2 \quad s.t. \quad 2x_1^2 + x_2^2 = 3$$

Example II:

$$\max_{\{x,y\}} \quad x^a y^b \quad s.t. \quad x + y = 10$$

Example III:

$$\max_{\{x_1,x_2\}} \quad x_1^{1/3} x_2^{2/3} \quad s.t. \quad x_1 + 4x_2 = 30$$

Example IV:

$$\max_{\{x,y,z\}} \quad yz + xz \quad s.t. \quad y^2 + z^2 = 1 \quad and \quad xz = 3$$