Exercise 6.2

2. $y = 5x^2 - 4x$

(a)
$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \frac{5(x + \Delta x)^2 - 4(x + \Delta x) - (5x^2 - 4x)}{\Delta x}$$
$$= \frac{5(x^2 + \Delta x^2 + 2x\Delta x) - 4x - 4\Delta x - 5x^2 + 4x}{\Delta x}$$
$$= \frac{5\Delta x^2 + 10x\Delta x - 4\Delta x}{\Delta x} = 5\Delta x + 10x - 4$$

(b)
$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 5\Delta x + 10x - 4 = 10x - 4$$

(c)
$$f'(2) = 10 \times 2 - 4 = 16$$
$$f'(3) = 10 \times 3 - 4 = 26$$

3. $y = 5x - 2$

(a)
$$\frac{\Delta y}{\Delta x} = \frac{5(\Delta x + x) - 2 - (5x - 2)}{\Delta x}$$
$$= \frac{5\Delta x}{\Delta x} = 5$$

It is a constant function.

(b) No it doesn't matter whether Δx is small or large. $\frac{dy}{dx} = 5$

Exercise 7.1

3. (a) $f'(x) = 18$
    $f'(1) = 18$ and $f'(2) = 18$.

    (b) $f'(x) = 3cx^2$
    $f'(1) = 3c$ and $f'(2) = 12c$.

    (c) $f'(x) = 10x^{-3}$
    $f'(1) = 10$ and $f'(2) = \frac{10}{8} = \frac{5}{4}$

    (d) $f'(x) = x^{1/3} = \sqrt[3]{x}$
    $f'(1) = 1$ and $f'(2) = \sqrt[3]{2}$

    (e) $f'(w) = 2w^{-2/3}$
    $f'(1) = 2$ and $f'(2) = 2 \cdot 2^{-2/3} = 2^{1/3}$

    (f) $f'(w) = \frac{1}{2}w^{-7/6}$
    $f'(1) = \frac{1}{2}$ and $f'(2) = \frac{1}{2}\left(2^{-7/6}\right) = 2^{-1} \cdot 2^{-7/6}$

Exercise 7.2

3. (d) $\underbrace{(ax - b)}_{f(x)} \underbrace{\left(cx^2\right)}_{g(x)}$

    $f'(x) = a$
    $g'(x) = 2cx$

    $$\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$
    $$= acx^2 + (ax - b)2cx$$
    $$= acx^2 + 2acx^2 - 2bcx$$
    $$= 3acx^2 - 2bcx$$

(e) $\underbrace{(2-3x)(1+x)}_{f(x)} \underbrace{(x+2)}_{g(x)}$

$g'(x) = 1$

$f(x) = \underbrace{(2-3x)}_{h(x)} \underbrace{1+x)}_{p(x)}$

$f'(x) = h'(x)p(x) + h(x)p'(x)$

$\quad = -3(1+x) + (2-3x)$

$\quad = -(6x+1)$

$\dfrac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$

$\qquad = -(6x+1)(x+2) + (2-3x)(1+x)$

$\qquad = -6x^2 - x - 12x - 2 + 2 - 3x + 2x - 3x^2$

$\qquad = -9x^2 - 14x$

$\qquad = -x(9x+14)$

7. (a)

$$\frac{x^2+3}{x}$$

$$f(x) = x^2 + 3 \quad \rightarrow \quad f'(x) = 2x$$
$$g(x) = x \qquad\quad \rightarrow g'(x) = 1$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$= \frac{(2x)x - (x^2+3)\cdot 1}{x^2}$$

$$= \frac{2x^2 - x^2 - 3}{x^2} = \frac{x^2-3}{x^2}$$

(b)

$$f(x) = \frac{x+9}{x}$$

$$f'(x) = \frac{1 \cdot x - 1 \cdot (x+9)}{x^2} = \frac{-9}{x^2}$$

(c)

$$f(x) = \frac{6x}{x+5}$$

$$f'(x) = \frac{6 \cdot (x+5) - 1.6x}{(x+5)^2} = \frac{30}{(x+5)^2}$$

(d)

$$f(x) = \frac{ax^2 + b}{cx + d}$$

$$f'(x) = \frac{2ax(cx+d) - (ax^2 + b)c}{(cx+d)^2}$$
$$= \frac{2acx^2 + 2adx - cax^2 - bc}{(cx+d)^2}$$
$$= \frac{acx^2 + 2adx - bc}{(cx+d)^2}$$

8. $f(x) = ax + b$

(a) $f'(x) = a$

(b) $\frac{d}{dx} ax^2 + bx = 2ax + b$

(c)

$$\frac{d}{dx} \frac{1}{ax+b} = \frac{0(ax+b) - (a) \cdot 1}{(ax+b)^2} = \frac{-a}{(ax+b)^2}$$

(d)

$$\frac{d}{dx} \frac{ax+b}{x} = \frac{ax - (ax+b) \cdot 1}{x^2} = \frac{-b}{x^2}$$

Exercise 7.3

1.
$$y = u^3 + 2u$$
$$u = 5 - x^2$$

$$\frac{dy}{du} = 3u^2 + 2$$
$$\frac{du}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{dy}{dy} \cdot \frac{dy}{dx} = \left(3y^2 + 2\right)(-2x)$$
$$= \left(3\left(5 - x^2\right)^2 + 2\right)(-2x)$$

2.
$$w = ay^2$$
$$y = bx^2 + cx$$

$$\frac{dw}{dx} = \frac{dw}{dy} \cdot \frac{dy}{dx}$$
$$= 2ay(2bx + c)$$
$$= 2a\left(bx^2 + cx\right)(2bx + c)$$

3.   (a) $y = \left(3x^2 - 13\right)^3$

     Denote, $z = 3x^2 - 13$

     Then, $y = z^3$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = 3z^2(6x) = 18x\left(3x^2 - 13\right)^2$$

(b)

$$y = \left(7x^3 - 5\right)^9$$

$$z = 7x^3 - 5 \rightarrow y = z^9$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = 9z^8 \left(21x^2\right) = 189x^2 \left(7x^3 - 5\right)^8$$

(c)

$$y = (ax + b)^5$$

$$z = ax + b \rightarrow y = z^5$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = 5z^4(a) = 5a(ax + b)^4$$

4.

$$y = (16x + 3)^{-2}$$

$$z = 16x + 3$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = -32(16x + 3)^{-3}$$

Using the quotient rule:

$$y = \frac{1}{(16x + 3)^2}$$

$$\frac{dy}{dx} = \frac{0 \cdot (16x + 3)^2 - 32(16x + 3)}{(16x + 3)^4}$$

$$= \frac{-32}{(16x + 3)^{-3}}$$

5.

$$y = 7x + 21 \rightarrow \frac{dy}{dx} = 7$$

$$x = \frac{y - 21}{7} \rightarrow \frac{dx}{dy} = \frac{1}{7}$$

6. (a)

$$\frac{dy}{dx} = -6x^5 < 0$$

For $x > 0$, this function is strictly decreasing as its derivative is negative.

$$\frac{dx}{dy} = \frac{1}{dy/dx} = \frac{-1}{6x^5}$$

(b)

$$y = 4x^5 + x^3 + 3x$$

$$\frac{dy}{dx} = 20x^4 + 3x^2 + 3 > 0$$

This function is strictly increasing as $\frac{dy}{dx} > 0$ for all x.

$$\frac{dx}{dy} = \frac{1}{20x^4 + 3x^2 + 3}$$