solution

September 17, 2025

[29]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import minimize
```

[30]:
```
def solve_demand_ces(alpha, rho, m, p1, p2):

    def utility(x):
        x1, x2 = x
        if x1 < 0 or x2 < 0:
            return np.inf
        else:
            return -(alpha * x1 ** rho + (1 - alpha) * x2 ** rho) ** (1 / rho)

    def budget_constraint(x):
        x1, x2 = x
        return m - p1 * x1 - p2 * x2

    constraints = ({"type": "eq", "fun": budget_constraint})
    initial_guess = [1, 1]
    result = minimize(utility, initial_guess, constraints=constraints)
    x1, x2 = result.x

    return round(x1, 2), round(x2, 2)
```

[43]:
```
# Parameters
alpha = 0.5
m = 100
p1 = 5
p2 = 10

x1, x2 = solve_demand_ces(alpha, 0.99, m, p1, p2)
print(f"Good 1: {x1}", f"Good 2: {x2}")

x1, x2 = solve_demand_ces(alpha, 0.001, m, p1, p2)
print(f"Good 1: {x1}", f"Good 2: {x2}")

x1, x2 = solve_demand_ces(alpha, -200, m, p1, p2)
print(f"Good 1: {x1}", f"Good 2: {x2}")
```

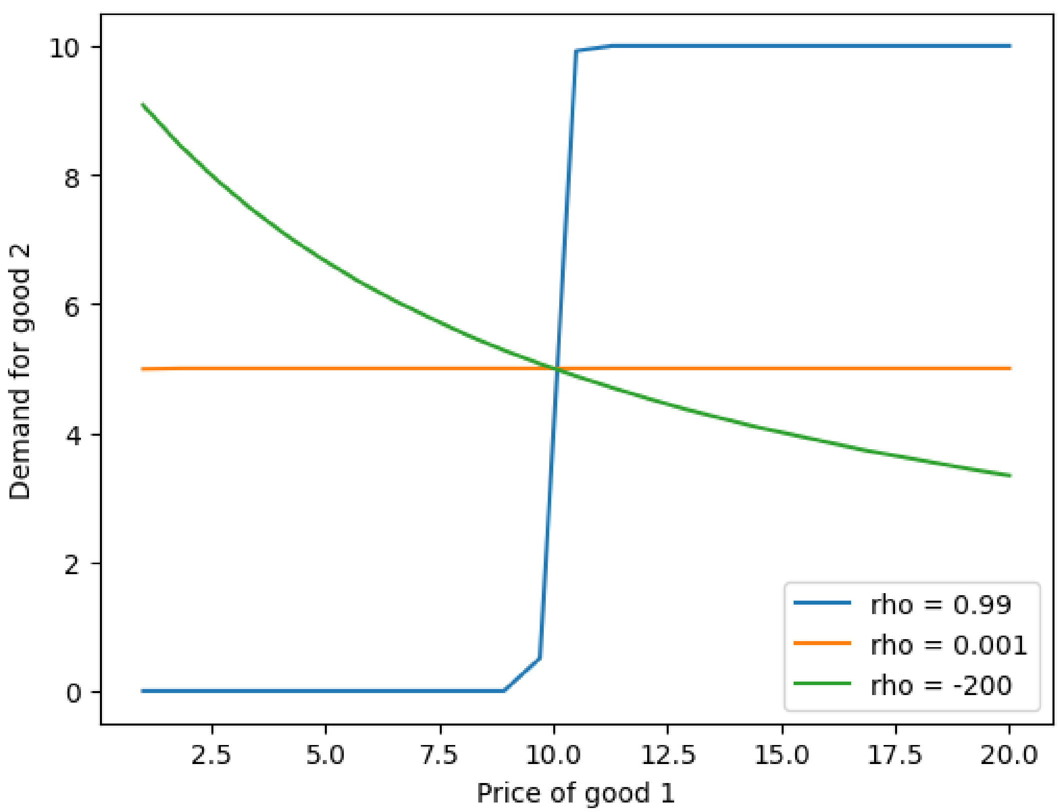
```
Good 1: 20.0 Good 2: 0.0
Good 1: 10.0 Good 2: 5.0
Good 1: 6.68 Good 2: 6.66
```

[48]:
```python
# Create a vector of 25 prices for good 1 between 1 and 10
p1_vals = np.linspace(1, 20, 25)

# Calculate demand for good 1 and good 2 at each price value
x = [solve_demand_ces(alpha, 0.99, m, p1, p2) for p1 in p1_vals]
x1_vals1, x2_vals1 = np.array(x).T
x = [solve_demand_ces(alpha, 0.001, m, p1, p2) for p1 in p1_vals]
x1_vals2, x2_vals2 = np.array(x).T
x = [solve_demand_ces(alpha, -200, m, p1, p2) for p1 in p1_vals]
x1_vals3, x2_vals3 = np.array(x).T

# Plot demand for good 1 as a function of price of good 1
plt.plot(p1_vals, x2_vals1, label='rho = 0.99')
plt.plot(p1_vals, x2_vals2, label='rho = 0.001')
plt.plot(p1_vals, x2_vals3, label='rho = -200')
plt.xlabel('Price of good 1')
plt.ylabel('Demand for good 2')
plt.legend()
```

[48]: <matplotlib.legend.Legend at 0x137dfd790>



[96]:
```
def utility(alpha, rho, x1, x2):
    return (alpha * x1 ** rho + (1 - alpha) * x2 ** rho) ** (1 / rho)

def x2(alpha, rho, x1, u):
    def objfunc(x2):
        return (utility(alpha, rho, x1, x2) - u) ** 2
    result = minimize(objfunc, 1, tol=1e-12)
    return result.x

u = 5
x1_vals = np.linspace(1, 10, 24)
x2_vals1 = [x2(0.5, 0.99, x1, u) for x1 in x1_vals]
x2_vals2 = [x2(0.5, 0.001, x1, u) for x1 in x1_vals]

plt.plot(x1_vals, x2_vals1, label='rho = 0.99')
plt.plot(x1_vals, x2_vals2, label='rho = 0.001')
plt.xlabel('Good 1')
plt.ylabel('Good 2')
plt.legend()
```

[96]:
```
<matplotlib.legend.Legend at 0x139476710>
```