Echelon Form

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

We can check for linear independence and find the rank of a matrix by converting the matrix to its *echelon* form.

How to convert a 3×3 matrix to its echelon form?

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Step 1: Try to make $a_{31} = 0$

$$A_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$$

Step 2: Try to make $a_{21} = 0$

$$A_2 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$$

Step 3: Try to make $a_{32} = 0$

$$A_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Valid operations to convert to echelon form:

- Interchange any two rows
- Multiplication (or division) of a row by a scalar $k \neq 0$
- Addition of a (or k times of a) row to another

Example.

$$A = \begin{bmatrix} 1 & 3 & -4 \\ 0 & 1 & 4 \\ -1 & 2 & 0 \end{bmatrix}$$