

Homework 7 Solutions

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Exercise 8.5

1. For each $F(x, y) = 0$, find dy/dx for each of the following:

(tsk[a]) $6x + 7 = 0$

$$\frac{dy}{dx} - 6 = 0 \rightarrow \frac{dy}{dx} = 6$$

(tsk[a]) $12x + 17 = 0$

$$3\frac{dy}{dx} + 12 = 0 \rightarrow \frac{dy}{dx} = \frac{-12}{3} = -4$$

(tsk[a]) $6x - 13 - y = 0$

$$2x + 6 - \frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = 2x + 6$$

2. (d) $F(x, y) = 6x^3 - 3y = 0$. By implicit function theorem:

$$\frac{dy}{dx} = \frac{-F_x}{F_y}$$

$$F_x = 18x^2, \quad F_y = -3$$

$$\frac{dy}{dx} = \frac{-18x^2}{-3} = 6x^2$$

3. (a) $F(x, y, z) = x^2y^3 + z^2 + xyz = 0$. By implicit function theorem:

$$\frac{\partial y}{\partial x} = \frac{-F_x}{F_y} = \frac{-(2xy^3 + yz)}{3x^2y^2 + xz}$$

$$\frac{\partial y}{\partial z} = \frac{-F_z}{F_y} = \frac{-(2z + xy)}{3x^2y^2 + xz}$$

Exercise 14.2

1. Find the following:

$$(a) \int 16x^{-3} dx = \frac{16x^{-2}}{-2} + c = -8x^{-2} + c \quad (x \neq 0)$$

$$(c) \int (x^5 - 3x) dx = \int x^5 dx - 3 \int x dx = \frac{x^6}{6} - \frac{3x^2}{2} + c$$

$$(d) \int 2e^{-2x} dx = 2 \int e^{-2x} dx = 2 \frac{e^{-2x}}{-2} + c = -e^{-2x} + c$$

Exercise 14.3

1. Evaluate the following:

$$(a) \int_1^3 \frac{1}{2}x^2 dx = \left[\frac{x^3}{6} \right]_1^3 = \frac{3^3 - 1^3}{6} = \frac{26}{6}$$

$$(e) \left[\frac{ax^3}{3} + \frac{bx^2}{2} + cx \right]_{-1}^1 = \left(\frac{a}{3} + \frac{b}{2} + c \right) - \left(-\frac{a}{3} + \frac{b}{2} - c \right) = 2 \left(\frac{a}{3} + c \right)$$

2. Evaluate the following:

$$(a) \int_1^2 e^{-2x} dx = \left[\frac{-e^{-2x}}{2} \right]_1^2 = -\frac{1}{2} (e^{-4} - e^{-2}) = \frac{1}{2} (e^{-2} - e^{-4})$$

$$(d) [\ln x + \ln(1+x)]_e^6 = \ln 6 + \ln 7 - \ln e - \ln(1+e) = \ln 42 - 1 - \ln(1+e)$$

5. Verify that a constant c can be equivalently expressed as a definite integral:

$$(tsk[a]) \int_0^b \frac{c}{b} dx = \left[\frac{cx}{b} \right]_0^b = \frac{cb}{b} - \frac{c \cdot 0}{b} = c$$

$$(tsk[a]) \int_0^c 1 dt = [t]_0^c = c - 0 = c$$