

Homework 5 Problems

ECON 441: Introduction to Mathematical Economics

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Exercise 6.2

2. Given the function $y = 5x^2 - 4x$:

- (a) Find the difference quotient as a function of x and Δx .
- (b) Find the derivative dy/dx . (Using the limit definition.)
- (c) Find $f'(2)$ and $f'(3)$.

3. Given the function $y = 5x - 2$:

- (a) Find the difference quotient $\Delta y/\Delta x$. What type of function is it?
- (b) Since the expression Δx does not appear in the function $\Delta y/\Delta x$ in part (a), does it make any difference to the value of $\Delta y/\Delta x$ whether Δx is large or small? Consequently, what is the limit of the difference quotient as Δx approaches zero?

Exercise 7.1

3. Find $f'(1)$ and $f'(2)$ for the following functions:

a) $y = f(x) = 18x$

b) $y = f(x) = cx^3$

c) $f(x) = -5x^{-2}$

d) $f(x) = \frac{3}{4}x^{4/3}$

e) $f(w) = 6w^{1/3}$

f) $f(w) = -3w^{-1/6}$

Exercise 7.2

3. Differentiate the following by using the product rule:

(d) $(ax - b)(cx^2)$

(e) $(2 - 3x)(1 + x)(x + 2)$

7. Find the derivatives of:

a) $(x^2 + 3)/x$

b) $(x + 9)/x$

c) $6x/(x + 5)$

d) $(ax^2 + b)/(cx + d)$

8. Given the function $f(x) = ax + b$, find the derivatives of:

a) $f(x)$

b) $xf(x)$

c) $1/f(x)$

d) $f(x)/x$

Exercise 7.3

1. Given $y = u^3 + 2u$, where $u = 5 - x^2$, find dy/dx by the chain rule.

2. Given $w = ay^2$ and $y = bx^2 + cx$, find dw/dx by the chain rule.

3. Use the chain rule to find dy/dx for the following:

a) $y = (3x^2 - 13)^3$

b) $y = (7x^3 - 5)^9$

c) $y = (ax + b)^5$

4. Given $y = (16x + 3)^{-2}$, use the chain rule to find dy/dx . Then rewrite the function as $y = 1/(16x + 3)^2$ and find dy/dx by the quotient rule. Are the answers identical?

5. Given $y = 7x + 21$, find its inverse function. Then find dy/dx and dx/dy , and verify the inverse-function rule. Also verify that the graphs of the two functions bear a mirrorimage relationship to each other.

6. Are the following functions strictly monotonic?

a) $y = -x^6 + 5 \quad (x > 0)$

b) $y = 4x^5 + x^3 + 3x$

For each strictly monotonic function, find dx/dy by the inverse-function rule.