# Exercise

The utility function used in Tutorial 2 is called a Cobb-Douglas function. In this exercise, we will consider utility maximization with a CES utility function. In particular, the CES utility function is given by:

$$U(x_1, x_2) = (\alpha x_1^{\rho} + (1-\alpha)x_2^{\rho})^{1/\rho}$$

Here,

- $\alpha$ is a parameter between 0 and 1 that reflects the relative weight or importance of the goods.
- $\rho$ controls how easily the consumer can substitute between the two goods.
  - When $\rho \to 1$, the goods are perfect substitutes.
  - When $\rho \to -\infty$, the goods are perfect complements.

Also, note that the Cobb-Douglas utility function is a special case of the CES utility function with $\rho \to 0$.

Since this exercise builds on Tutorial 2, here is the link to a Google Colab notebook containing the essential code from that tutorial.

Each part is worth 2.5 points.

1. Modify the `solve_demand` function from Tutorial 2 to create a new function, `solve_demand_ces`, which solves for the optimal quantities of good 1 and good 2 given the CES utility function. (This new function will include an additional parameter, $\rho$.)
2. With the values of the other parameters set to:

   $$\alpha = 0.5,\ m = 100,\ p_1 = 5,\ p_2 = 10$$

   Find the optimal values of $x_1$ and $x_2$ in three cases:

   (a) $\rho = 0.99$
   (b) $\rho = 0.001$

(c) $\rho = -200$

Print the results for each of the three cases and intuitively explain why the results make sense.

3. Plot the demand for *good 2* as a function of the price of *good 1* for the three cases in part 2. Let other parameters be the same as in part 2 and use the range of 1 to 20 for the price of good 1. Superimpose the three demand curves on the same plot. Make sure to include a legend and label the axes. Intuitively explain what the demand curves are telling us about the relationship between the two goods in each case.

4. An indifference curve shows combinations of two goods that give a consumer equal utility. Plot the indifference curves for the *first two* cases specified in part 2 when utility $U = 5$. Continue to assume $\alpha = 0.5$. As before, superimpose the two indifference curves on the same plot, include a legend, and label the axes.

Write your code and explanations in a Google Colab notebook. Save the notebook as `firstname_lastname.ipynb`, download it on your local machine, and upload it on Canvas.