Linear Regression:

5 mango & 3 apples = 50/-
3 mango & 2 apples = 32/-

1 mango
1 apple

$5M + 3A = 50$ — (i)
$3M + 2A = 32$ — (ii)

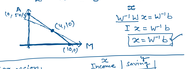
Linear system 2x2.

$$\underbrace{\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} M \\ A \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 50 \\ 32 \end{bmatrix}}_{b}$$

$Wx = b$

$W^T W x = W^T b$

$x = (W^T W)^{-1} W^T b$



Linear Regression:

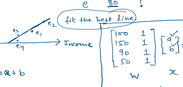
$y = f(x)$

$y = ax + b$

	Income x	Saving y
e_1	100	20
e_2	150	25
e_3	90	18
e_4	50	12

Model based.

Saving



e_1 80 ?

fit the best line

$$\underbrace{\begin{bmatrix} 100 & 1 \\ 150 & 1 \\ 90 & 1 \\ 50 & 1 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 20 \\ 25 \\ 18 \\ 12 \end{bmatrix}}_{b}$$

$80 = 100a + b$

$Wx = b$

$W^T W x = W^T b$
$2\times2 \quad 2\times1 \quad 2\times1$

$Wx = b$
$4\times2 \quad 2\times1 \quad 4\times1$

$W^T W$ 2×2 $\quad W^T b$ 2×1

$x = (W^T W)^{-1} W^T b$

Ord least square (linear regression)

$a = \frac{cov(x, y)}{\sigma_x^2}$

$b = \bar{y} - a\bar{x}$

x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4

Residuals

$y = ax + b$

$$\underset{a,b}{E} = (y_1 - (ax_1 + b))^2 + (y_2 - (ax_2 + b))^2 + (y_3 - (ax_3 + b))^2 + (y_4 - (ax_4 + b))^2$$

(a, b)

$x^2 + y^2$

$x^2 \to 0$

$y \to 0$

$\frac{\partial E}{\partial a} = 0$, $\frac{\partial E}{\partial b} = 0$

$y_1 - (ax_1 + b) \approx 0$

$y_1 = ax_1 + b$

	x_1 Salary	x_2 Peoples	x_3 Side-Income	Y Saving
e_1	x_{11}	x_{21}	x_{31}	y_1
e_2	x_{21}	x_{22}	x_{32}	y_2
e_i	x_{i1}	x_{i2}	x_{i3}	y_i

k employees

$Y = f(x_1, x_2, x_3)$ $\quad Y = f(x)$

$Y = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4$ $\quad Y = ax + b$

4 Model parameters

$$E = (y_1 - (\alpha_1 x_{11} + \alpha_2 x_{12} + \alpha_3 x_{13} + \alpha_4))^2 + (y_2 - (\alpha_1 x_{21} + \alpha_2 x_{22} + \alpha_3 x_{23} + \alpha_4))^2 + \cdots$$

$\min_{\alpha_i} E$

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} & x_{13} & 1 \\ x_{21} & x_{22} & x_{23} & 1 \\ x_{31} & x_{32} & x_{33} & 1 \\ \vdots & & & \\ x_{k1} & x_{k2} & x_{k3} & 1 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}}_{b}$$

Multiple linear Reg.

$Wx = b$

$\underset{4\times4}{W^T W} x = W^T b$

$x = (W^T W)^{-1} W^T b$

$y = ax + b$ | $a = 1$, $b = 3$

x	y
5	2 ⇒ y_1
6	3 ⇒ y_2
7	4 ⇒ y_3

$y =$

$\hat{y}_2 = 1 * x + 3$

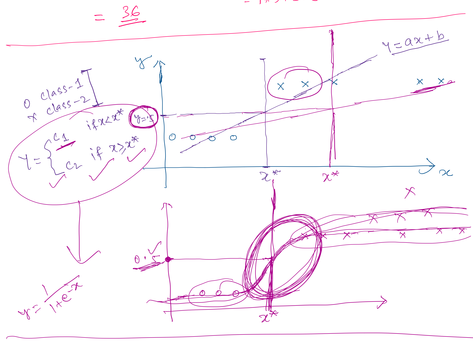
$\hat{y}_1 = 5 + 3 = 8$

$\hat{y}_2 = 6 + 3 = 9$

$\hat{y}_3 = 7 + 3 = 10$

$$MSE = \frac{(-6)^2 + (-6)^2 + (-6)^2}{3} = 36$$

$\hat{y}_1 = a\bar{x}_1 + b$
$= 1 * 5 + 3 = 8$



$$Y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_k x_k$$

$$\boxed{Y = \alpha_0 + \alpha^T x}$$ where

$\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_k]$

$x = [x_1, x_2, \ldots, x_k]$

$$y = S(\alpha_0 + \alpha^T x) = \frac{1}{1 + e^{-(\alpha_0 + \alpha^T x)}}$$

$$y = \frac{1}{1 + e^{-(\alpha_0 + \alpha_1 x_1 + \cdots + \alpha_k x_k)}}$$

$y \in [0, 1]$

+ive
-ive

$$y = \begin{cases} -\text{ive} & \text{if } y < .5 \\ +\text{ive} & \text{if } y \geq .5 \end{cases}$$

x	y
2	-ive
10	+ive
8	+ive
4	-ive
12	+ive

$y = ax + b$

$$y = \frac{1}{1 + e^{-(ax+b)}}$$

$$(*) \quad y = \begin{cases} -\text{ive} & \text{if } \frac{1}{1+e^{-(ax+b)}} < .5 \\ +\text{ive} & \text{if } \frac{1}{1+e^{-(ax+b)}} \geq .5 \end{cases}$$

Find such a and b which satisfy $(*)$ for all training pairs.

$E = MSE$

$\sim$ 'cross entropy loss function'

→ Multiple class

01 lectures [Softmax Regression. or Multinomial Regression. Pros & cons.]

01 Hour KNN [Concept + Imp.] Pros & cons.