0701EE191025 EE 3301

UJJAIN ENGINEERING COLLEGE, UJJAIN

(Affiliated to RGPV and declared autonomous by the M.P. State Government)
Indore Rd. Ujjain, Madhya Pradesh

END SEM EXAMINATION - DECEMBER, 2020

(For Offline Open Book Examination only)

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Enrollment No: 0701EE191025

Date of Examination: 16 feb 2021 Time of Examination: 10:00 Am to 1:00 Pm

Signature of the Candidate (Must be as per Institute Record) : Divesh.k

Date of Birth (Candidate need to fill) :

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Q.6

Solution The given waveform has odd symmetry, half wave symmetry and quarter wave symmetry

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$\therefore\ a_0 = 0,\ a_n = 0,\ b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin n\Omega_0 t\,dt$

The mathematical equation of the given waveform is

$x(t) = A\ ;$ for $t = 0$ to $\frac{T}{2}$

$= -A\ ;$ for $t = \frac{T}{2}$ to T

for b_n

$$b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin n\Omega_0 t\,dt$$

$$= \frac{4}{T}\int_0^{T/2} A\sin n\Omega_0 t\,dt = \frac{4A}{T}\left[\frac{-\cos n\Omega_0 t}{n\Omega_0}\right]_0^{T/2}$$

$$= \frac{4A}{T}\left[\frac{-\cos n\frac{2\pi}{T}t}{n\frac{2\pi}{T}}\right]_0^{T/2} = \frac{4A}{T}\left[\frac{-\cos n\frac{2\pi}{T}\cdot\frac{T}{2}}{n\frac{2\pi}{T}} + \frac{\cos 0}{n\frac{2\pi}{T}}\right]$$

$$= \frac{4A}{T}\left[-\frac{T}{2n\pi}\cos n\pi + \frac{T}{2n\pi}\right]$$

$\cos n\pi = -1$ for $n = \text{odd}$

$\cos n\pi = +1$ for $n = \text{even}$

$\therefore\ b_n = 0$ for even values of n

$= \frac{4A}{T}\left[\frac{T}{2n\pi} + \frac{T}{2n\pi}\right] = \frac{4A}{n\pi}$; for odd values of n

$\therefore\ b_1 = \frac{4A}{\pi}\ ;\ b_3 = \frac{4A}{3\pi}\ ;\ b_5 = \frac{4A}{5\pi}$ and so on

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fourier Series

The trignometric form of fourier series of $x(t)$ is

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\Omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\Omega_0 t$$

Here $a_0 = 0$, $a_n = 0$ and b_n exists only for odd values of n

$$x(t) = \sum_{n=odd} b_n \sin n\Omega_0 t$$

$$= b_1 \sin \Omega_0 t + b_3 \sin 3\Omega_0 t + b_5 \sin 5\Omega_0 t + \dots$$

$$= \frac{4A}{\pi} \sin \Omega_0 t + \frac{4A}{3\pi} \sin 3\Omega_0 t + \frac{4A}{5\Omega} \sin 5\Omega_0 t + \dots$$

$$= \frac{4A}{\pi} \left[\sin \Omega_0 t + \frac{\sin 3\Omega_0 t}{3} + \frac{\sin 5\Omega_0 t}{5} \right.$$

Q.2
(a)

Properties of ROC of z transform

Property ① The region of convergence (ROC) of $X(z)$ consists of a ring or circle in the z plane centred about the origin

Property ② The region of convergence (ROC) does not contain any pole

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Property ③ The z transform X(z) Converges uniformly if and only if the region of convergence (ROC) of the z-transform X(z) of the given discrete time signal includes the unit circle. The ROC of X(z) consists of a ring in the z plane centred about the origin. This implies that the ROC of the z transform of x(n) has values of z for which $(x(n) r^{-n})$ is always absolutely summable

$$\sum_{n=-\infty}^{\infty} |x(n) r^{-n}| < \infty$$

Property ④ If the discrete time signal x(n) is of finite duration, then the region of convergence (ROC) will be entire z-plane except z=0 and z=∞

Property ⑤ If the discrete time signal x(n) is right sided sequence, then the region of convergence (ROC) will not include ~~i~~ infinity (∞)

Property ⑥ If the discrete time signal x(n) is left sided sequence then the region of convergence will not include z=0. But in a case if x(n) = 0 for all n>0 then the ROC will include z=0

Property 7 If the discrete time signal x(n) is a two sided sequence, and if the circle $|z| = r_0$ is in the ROC then the ROC will consist of a ring in the z plane that includes the circle

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$|z| = r_0$ This means that the ROC will be ~~bounded~~ include the intersection of the ROC's of the component

Property (8) If the z-transform $X(z)$ is rational, then the ROC will extend to infinity. This means that the ROC will be bounded by poles.

Property (9) If the discrete time signal $x(n)$ is casual then the ROC will include $z = \infty$

Property (10) If the discrete time signal $x(n)$ is anti-casual then the ROC will include $z = 0$

Q.2 (b) Time shifting property :-

Time shifting property states that a shift in the ~~dot~~ time-domain by an amount b is equivalent to multiplication by $e^{-j\omega b}$ in the frequency domain. This means that magnitude spectrum $|X(\omega)|$ remains unchanged but phase spectrum $\theta(\omega)$ is changed by $-\omega b$. A shift in time domain does not change the magnitude of a frequency component. A shift of b at a frequency ω is equivalent to a phase shift of ωb.

Mathematically .

if $x(t) \longleftrightarrow x(\omega)$

$x(t-b) \longleftrightarrow x(\omega) \cdot e^{-j\omega b}$

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Proof

The general expression for fourier transform is

$$x(\omega) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

And $$F[x(t-b)] = \int_{-\infty}^{\infty} x(t-b)\, e^{-j\omega t}\, dt$$

putting $t-b = y$, $dt = dy$

$$F[x(t-b)] = \int_{-\infty}^{\infty} x(y)\, e^{-j\omega(b+y)}\, dy$$

$$= \int_{-\infty}^{\infty} x(y)\, e^{-j\omega b} \cdot e^{-j\omega y}\, dy$$

or $$F[x(t-b)] = e^{-j\omega b} \int_{-\infty}^{\infty} x(y)\, e^{-j\omega y}\, dy$$

$$F[x(t-b)] = e^{-j\omega b} \cdot X(\omega)$$

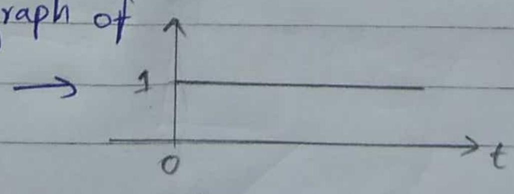
$$= X(\omega) = e^{-j\omega b}$$

$$x(t-b) \longleftrightarrow x(\omega) \cdot e^{-j\omega b}$$

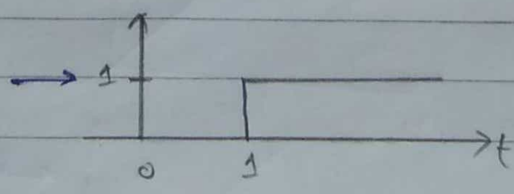
Q. 1(a)

1) $-2u(t-1)$

we know that graph of $u(t)$



$u(t-1)$



by using right shifting

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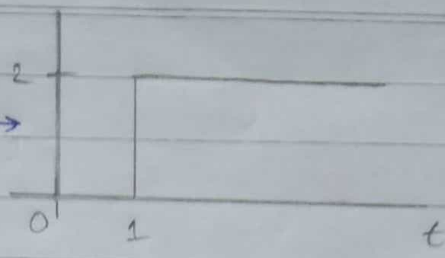
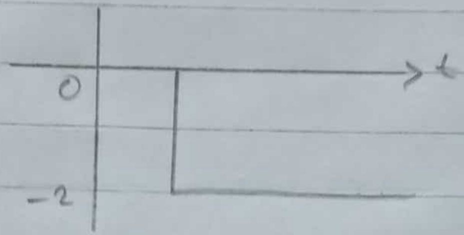
$2u(t-1) \rightarrow$



by using Amplitude Scaling

$-2u(t-1) \rightarrow$

desired signal

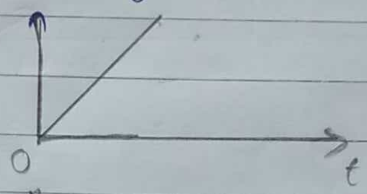


by using amplitude reversal

2) $3r(t-1)$

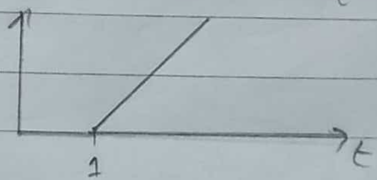
we know that the graph of

$r(t) \rightarrow$



slope is 1

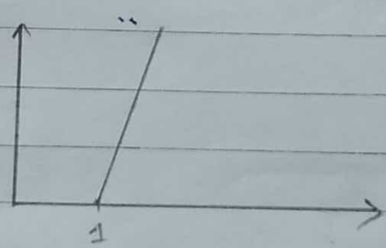
$r(t-1) \rightarrow$



by using right ~~Sigo~~ sided shifting

$3r(t-1)$

desired signal $\rightarrow$

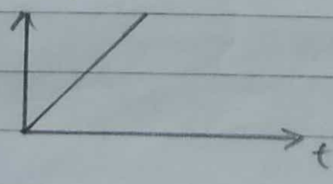


slope is 3

3) $r(-t+2)$

we know that the graph of

$r(t) \rightarrow$



slope = 1

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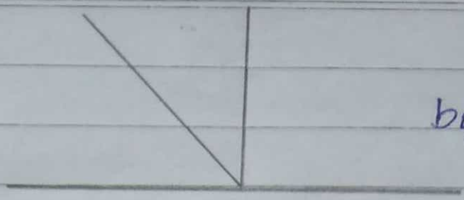
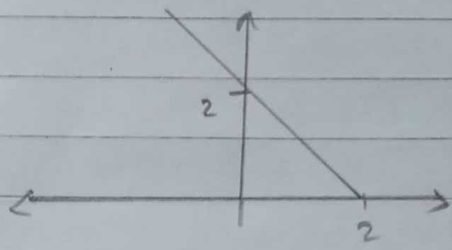
$r(-t) \longrightarrow$



Slope 1
by using time reversal

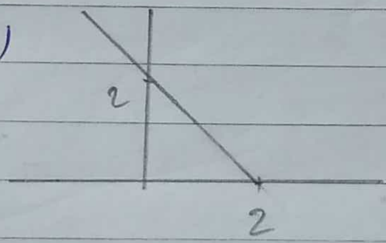
$r(-t+2) = (r(-(t-2))) \longrightarrow$



slope = 1
by using right sided shifting

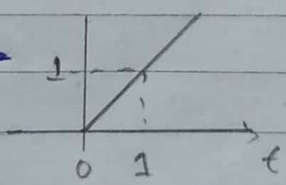
required signal is

$r(-t+2)$



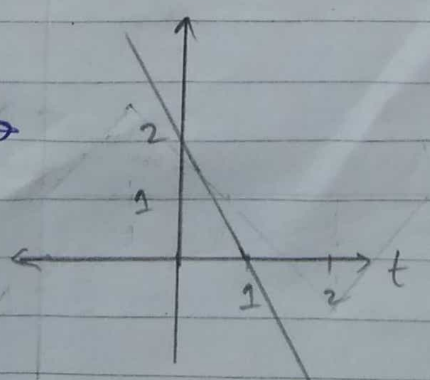
4) We know that the graph of

$r(t) \Longrightarrow$



$r(t) - 2r(t-1) + 2r(t-3) - r(t+4)$

$-2r(t-1)$
$\longrightarrow$



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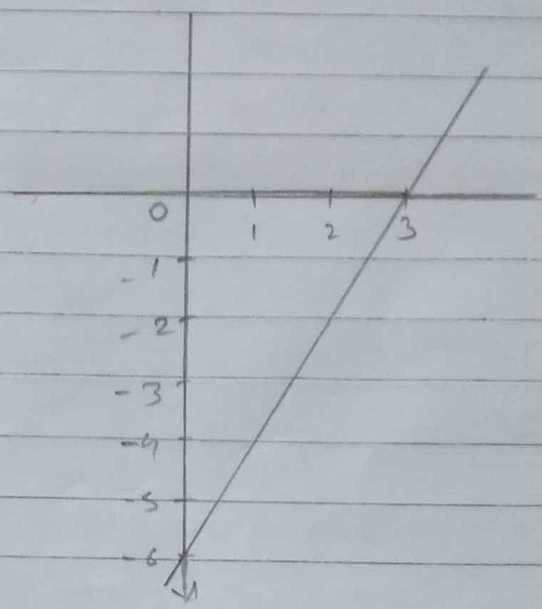
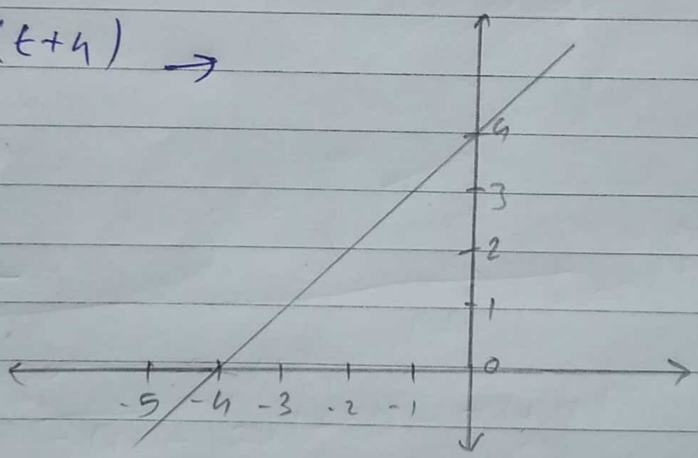
$2r(t-3) \longrightarrow$



$-r(t+4) \rightarrow$



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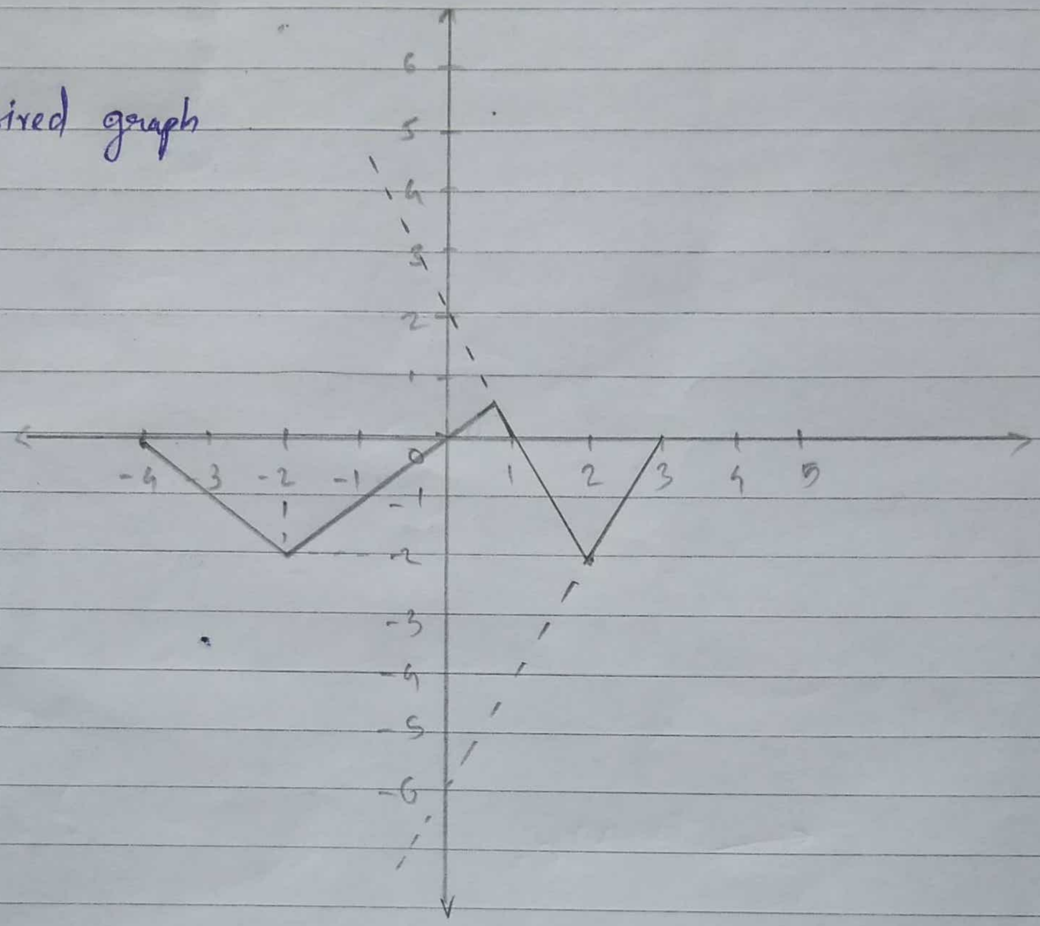
$r(t) - 2r(t-1) + 2r(t-3) - r(t+4)$

desired graph



5)

~~$r(t) - 2u(t-1) - r(t-2)$~~

~~$r(t)$~~

~~$-2u(t-1)$~~

Q. 7

(a)

1) Casual and non casual systems

<u>Casual systems</u> : A system is said to be casual if it does not respond before the input

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is applied. In other words in a casual systems, the output at any time depends only on the values of the input signal up to and including that time and doesn't depend on future values of the input

Ex $y(t) = x(t-1) + x(t)$

Here if $y \rightarrow t = 2sec$

$y(2) = x(1) + x(2)$

past values ↑ ↑ present values

Hence this signal is casual and the system is Casual System

Non casual system : A System whose present response depends on future values of the inputs is called as a non. casual system. Thus cannot be implemented in real. time

Ex: $y(n) = \sin(n)\,x(n+1)$

there for any value of n output depends on future inputs Hence System is non Casual

2) Memory and Dynamic systems

Memory systems :- A system is called memory system if the output from the system is dependent

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on the past inputs (or future inputs) to the system

Ex: $y(n) = x(n-1) + x[n]$

for values of n the System depends on past Values. Hence the system is memory System

Dynamic systems :- If the output of System depends only on present values of input then the system is known as memory less system or dynamic system

ex: $y(t) = x(t)$

Here the ~~o/p~~ output depends only on present values Hence the System is dynamic

3) Stable and Unstable

Stable system :- A system is said to be stable if it follows BIBO stablity, i.e., bounded output stablity is a form of stablity for linear signals and systems that take inputs. If a system is BIBO stable, then the output will be bounded for every input to the system that is bounded

Ex $h[n] = \delta[n - n_0]$
$h(t) = \delta(t - t_0)$

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} |\delta[k-n_0]| = 1 < \infty$$

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$$\int_{-\infty}^{\infty} |h(\tau)| d\tau = \int_{-\infty}^{\infty} |\delta(\tau - t_0)| d\tau = 1 < \infty$$

Here are two signals which are DT and CT and the output is bounded for input hence system is stable system

Unstable system :- for a bounded input, if the output is unbounded in the system then it is said to be unstable system

Ex : for LTID system
$y = |y| e^{i\beta}$ and $y^n = |y|^n e^{i\beta n}$

Since the magnitude of $e^{i\beta n}$ is 1 it is not necessary to be considered, Therefore, in case of $|y|^n$ if $|y| > 1$, $y^n \to \infty$ as $n \to \infty$
Hence the system is unstable system

Q 7
(b)

To find even and odd components of the following signals

1) $x(t) = \cos t + \sin t + \cos t \sin t$

we know that

even component : $x_e(t) = \left[\frac{x(t) + x(-t)}{2} \right]$

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odd component : $x_o(t) = \left[\frac{x(t) - x(-t)}{2}\right]$

taking eq^n (1) we have

$$x_e(t) = \left[\frac{x(t) + x(-t)}{2}\right]$$

As $x(t) = \cos t + \sin(t) + \cos t \sin t$

$x(-t) \Rightarrow \cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$

$\Rightarrow \cos(t) - \sin(t) - \cos t \sin t$

$\left\{ \begin{array}{l} \cos(-t) = \cos(t) \\ \sin(-t) = -\sin(t) \end{array} \right\}$

$\Rightarrow \cos t - \sin t - \cos t \sin t$

Hence, $x_e(t) = \left[\frac{x(t) + x(-t)}{2}\right]$

$$= \left[\frac{\cos t + \cancel{\sin t} + \cancel{\cos t \sin t} + \cos t - \cancel{\sin t} - \cancel{\cos t \sin t}}{2}\right]$$

$$= \frac{\cancel{2}\cos t}{\cancel{2}}$$

$$= \cos t$$

Therefore $\boxed{x_e(t) = \cos t}$

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x_o = odd component

$$x_o(t) = \left[\frac{x(t) - x(-t)}{2}\right]$$

So, $$x_o(t) = \left[\frac{\cancel{\cos t} + \sin t + \cos t \sin t - (\cancel{\cos t} - \sin t - \sin t \cos t)}{2}\right]$$

$$= \left[\frac{2\sin t + 2\sin t \cos t}{2}\right]$$

$$\Rightarrow \sin t + \sin t \cos t$$

Hence $\boxed{x_o(t) = \sin t + \sin t \cos t}$

Q. 7 (b)

2) $x[n] = \{-2, 1, 2, -1, 3\}$

$x[n]$ can be written as

$$x[n] = -2\delta(n+2) + \delta(n+1) + 2\delta(n) - \delta(n-1) + 3\delta(n-2) \quad \text{—} \; 1$$

Now

$$x[n] = -2\delta(-n+2) + \delta(-n+1) + 2\delta[-n] - \delta(-n-1) + 3\delta(-n-2)$$

$$x[-n] = -2\delta(n-2) + \delta(n-1) + 2\delta(n) - \delta(n+1) + 3(n+2)$$

even component

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_e[n] = \frac{-2\delta(n+2) + \delta(n+1) + 2\delta(n) - \delta(n-1) + 3\delta(n-2) - 2\delta(n-2) + \delta(n-1) + 2\delta(n) - \delta(n+1) + 3\delta(n+2)}{2}$$

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$$= \delta[n+2] + 4\delta[n] + \delta(n-2)$$

$$x_e[n] = \left\{ \frac{1}{2}, 0, 2, 0, \frac{1}{2} \right\}$$

odd component

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

$$x_o[n] = \frac{-2\delta(n+2) + \delta(n+1) + 2\delta(n) - \delta(n-1) + 3(\delta(n-2)) + 2\delta(n-2) - \delta(n-1) + 2\delta(n) + \delta(n+1) - 3\delta(n+2)}{2}.$$

$$= \frac{-5(\delta(n+2)) + 2\delta(n+1) + 0 - 2\delta(n-1) + 5\delta(n-2)}{2}$$

$$x_o[n] = \left\{ -\frac{5}{2}, 1, 0, -1, \frac{5}{2} \right\}$$

Q.5 To calculate energy

$$x(t) = 2\frac{\sin(\pi t)}{t}$$

Given that $x(t) = \frac{2\sin(\pi t)}{T}$

We know that

$$A \operatorname{Rect}\left(\frac{t}{\tau}\right) \xrightarrow{FT} \frac{A\tau \sin(t\pi/2)}{(t\pi/2)}$$

Therefore by duality theorem

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$$AT \sin(t\,T/2) \xrightarrow{FT} 2\pi A\, Rect\left(\frac{\omega}{T}\right) \rightarrow 2\pi A\, Rect\left(\frac{\omega}{T}\right)$$

Let $T = 2\pi$

$$\frac{A\, 2\pi \sin(\pi t)}{\pi(t)} \longrightarrow 2\pi A\, Rect\left(\frac{\omega}{2\pi}\right)$$

$$\Rightarrow \frac{\sin \pi t}{t} \longrightarrow \pi\, Rect\left(\frac{\omega}{2\pi}\right)$$

Now by parsevals energy theorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |x(j\omega)|^2\, d\omega$$

$$\Rightarrow E = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[2\pi\, Rect\left(\frac{\omega}{2\pi}\right)\right]^2 d\omega$$

$$\Rightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} 4\pi^2\, d\omega$$

$$\Rightarrow \frac{4\pi^2}{2\pi}[\omega]_{-\pi}^{\pi} = 4\pi^2$$

Hence energy is $4\pi^2$ units

Q.3 (b) Energy signal is the signal which has finite energy and averge power $x(t)$ is energy signal if

$$0 < E < \infty \quad \text{and} \quad P = 0$$

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$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Power signal is the signal which has finite ~~∞~~ average power and infinite energy $x(t)$ is power signal if

$$0 < P < \infty \text{ and } E = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} |x(t)|^2 \, dt$$

$$P = \lim_{N \to \infty} \frac{1}{N} \sum_{n=\frac{N}{2}}^{N/2} |x[n]|^2$$

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