# Gravitational wave astrophysics notes

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# Contents



## Introduction

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Organization: the instructor for this first part is professor Michela Mapelli.
Structure of the course:

1. today we start with background on GW, theory and data;
2. tomorrow we will discuss BH formation from single stars;
3. on wednesday we will discuss BH formation form binary evolution;
4. next week we will discuss BH dynamics.

In the next part we will discuss neutron stars with professor Thomas Tauris.

There is no final grade for the exam, only a pass/fail. This year, there will be a google form with multiple choice questions.

### What is GWA?

It is a branch of astrophysics which studies the astrophysical characterization of GW sources. Mostly, it is about discussing binary black holes, binary neutron stars and NS-BH binaries, which emit in the frequency range we can observe right now.

The main open question is: what are the formation channels of the compact objects binaries observed by ground-base interferometers.

# Chapter 1

# Gravitational wave summary

From wikipedia: "GW are ripples in the curvature of spacetime which propagate as waves, travelling outward from the source, at the speed of light". The math describing these is that of GR: the main equations are the Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \,, \tag{1.1}$$

where $R_{\mu\nu}$ is the trace of the Riemann tensor, which is a measure of curvature; on the other hand $T_{\mu\nu}$ is a measure of the density of 4-momentum in spacetime, which acts as a source.

The metric $g_{\mu\nu}$ is used in order to measure distances.

The EFE are in general 10 second-order nonlinear PDEs. In the strong-curvature case, with rapidly moving sources we must use numerical solutions of the full nonlinear EFE.

In the weak field limit, with almost flat spacetime and slowly moving sources, we have analytic solutions.

We assume the perturbation looks like $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, then we can linearize the EFE. We can rewrite the linearized equations using the trace-reversed perturbation

$$\overline{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \,, \tag{1.2}$$

and then in a special coordinate system (in a special gauge) we get

$$\Box \overline{h}_{\mu\nu} = \frac{16\pi G}{c^4} T_{\mu\nu} + \mathcal{O}(h^2) \,, \tag{1.3}$$

which can be solved in general using the method of Green's functions, just like in electrodynamics, to find

$$\overline{h}^{\mu\nu} = \frac{4G}{c^4} \int \mathrm{d}^3 y \, \frac{T^{\mu\nu}(t, \vec{y})\big|_{\mathrm{ret}}}{\left|\vec{x} - \vec{y}\right|} \,, \tag{1.4}$$

where $t_{\mathrm{ret}} = t - \left|\vec{x} - \vec{y}\right| / c$.

This is general but not easy to visualize. In order to discuss it we first assume we are far from our source: so we can say that the support of the stress-energy tensor is all at a distance $r$ from us. So, we get

$$\overline{h}^{\mu\nu}(t, \vec{x}) = \frac{4}{r} \frac{G}{c^4} \int \mathrm{d}^3 y \, T^{\mu\nu}(t - r/c, \vec{y}) \,. \tag{1.5}$$

If the source is moving slowly, that is, with a nonrelativistic velocity, we get

$$\bar{h}^{ij}(t,\vec{x}) = \frac{2}{r}\frac{G}{c^4}\ddot{I}^{ij}(t-r/c)\,, \tag{1.6}$$

where

$$I^{ij}(t-r/c) = \int \mathrm{d}^3y\, x^i x^j \rho(t-r/c,\vec{y})\,. \tag{1.7}$$

For a full derivation see Hartle [Har03].

This means that not all accelerating masses produce GWs: only those with non-zero quadrupole moment do; there is a need for an asymmetry in mass distribution.

Let us consider the simplest case we can have: a binary system, with two stars of equal mass $M$ orbiting each other in a circular orbit of radius distance $a$ with angular velocity $\omega$. We can consider the center of mass and reduced mass coordinates.

The components of the second mass moment are found from $x(t) = a\cos(\omega t)$, $y = a\sin(\omega t)$, $z = 0$:

$$I_{xx} = Ma^2\big[1+\cos(2\omega t)\big] \tag{1.8}$$

$$I_{yy} = Ma^2\sin(2\omega t) \tag{1.9}$$

$$I_{zz} = Ma^2\big[1-\cos(2\omega t)\big]\,, \tag{1.10}$$

so we get

$$\bar{h}^{ij} \sim \frac{2}{r}\frac{G}{c^4}(2\omega)^2 Ma^2 \begin{bmatrix} \cos\big((2\omega(t-r/c))\big) & \sin\big((2\omega(t-r/c))\big) & 0 \\ \sin\big((2\omega(t-r/c))\big) & -\cos\big((2\omega(t-r/c))\big) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.11}$$

which we can simplify using

$$\omega = \sqrt{\frac{2GM}{a^2}}\,, \tag{1.12}$$

so we can see that the frequency of the GW is *twice* $\omega$: $\omega_{GM} = 2\omega$.

We also get the two polarizations, $h_+$ and $h_\times$.

The amplitudes are of the order

$$h = \frac{1}{2}\left(h_+^2 + h_\times^2\right)^{1/2} \sim \frac{8G^2}{c^4}\frac{M^2}{ra}\,, \tag{1.13}$$

so with a solar mass binary, orbiting at a distance of a solar radius a kiloparsec away we get

$$h \sim 10^{-21}\left(\frac{M}{M_\odot}\right)^2\left(\frac{1\,\mathrm{kpc}}{r}\right)\left(\frac{R_\odot}{r}\right). \tag{1.14}$$

The amplitude is called the *strain*: the bigger it is, the easier the detection.

The easiest events to detect involve large masses, are close, and involve very close inspirals.

The crucial thing is that GW emission implies energy loss: this can be express iin the Keplerian limit as

$$E_{\mathrm{orb}} = -G\frac{m_1 m_2}{2a}\,. \tag{1.15}$$

The variable thing here is the radius $a$: it decreases, until coalescence. As the semimajor axis decreases, the frequency increases.

In order to quantify this, we can use the expression for the power emitted:

$$P_{GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{1}{a^5}m_1^2 m_2^2(m_1+m_2)\,, \tag{1.16}$$

so we can write

$$P_{GW} = \frac{\mathrm{d}E_{\mathrm{orb}}}{\mathrm{d}t} = \frac{Gm_1 m_2}{2a^2}\frac{\mathrm{d}a}{\mathrm{d}t}\,, \tag{1.17}$$

which means

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{64}{5}\frac{G^3}{c^5}\frac{1}{a^3}m_1 m_2(m_1+m_2)\,, \tag{1.18}$$

which we can integrate in order to find the necessary time for coalescence:

$$t_{GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{a^4}{m_1 m_2(m_1+m_2)}\,. \tag{1.19}$$

Here we neglected the eccentricity, if we account for it we need to multiply by a term $(1-e^2)^{7/2}$.

This is an extremely long timescale. If $a = 1\,\mathrm{Au}$ and $m_1 = m_2 = M_\odot$ we get timescales like $t_{GW} \sim 10^{17}\,\mathrm{yr}$!

The aforementioned equations hold for the "inspiral phase". The phases we usually distinguish are

1. inspiral, when the Keplerian approximation still holds;
2. merger, when the distance between the objects is closer than the ISCO;
3. ringdown, when the objects have coalesced.

We can use post-Newtonian techniques for the inspiral, then full numerical relativity for the merger, and "black hole perturbation methods" for the ringdown. The ISCO has a radius is

$$r_{ISCO} = 6\frac{G(m_1+m_2)}{c^2}\,, \tag{1.20}$$

and we can get a good estimate of the frequency using the Keplerian formula:

$$\omega_{GW,ISCO} = 2\sqrt{\frac{G(m_1+m_2)}{r_{ISCO}^3}}\,, \tag{1.21}$$

which works surprisingly well, better than a factor of 2.

See Abbott et al [LIG+16b] for graphs.

With the sensitivity of current ground-based detectors we can detect small BH and NS mergers.

SMBH mergers are too low frequency, but we could maybe see extremely high mass ratio mergers.

We could see GWs from neutrons stars with crustal asymmetries, with $10\,\mathrm{Hz}$ to $10^{-2}\,\mathrm{Hz}$.

Asymmetric supernova explosions would also be in this frequency range, but with too low amplitude.

In this course we will focus on binary compact objects.

## 1.1 Observations of GW

In the first two observations by LIGO-VIRGO we have seen 10 BBHs and 1 binary neutron star.

The third observing run is ongoing, it will finish about now.

From these observations we know that

1. BBHs exist;
2. BBHs can merge in a Hubble time;
3. Massive BHs (with mass $> 20M_\odot$) exist.

As the observation number increases, the range of the observations increases. For O3, we will be able to see up to around $110\,\mathrm{Mpc}$: this number is derived discussing a test well-oriented $1.3M_\odot$ neutron star, which would give a SNR of around 12. This is roughly proportional to $m^{5/3}$: the more massive the object, the larger the horizon: we will be able to see further for more massive objects.

What are the parameters which are directly observables (neglecting eccentricity):

1. 2 masses;
2. 6 spin components;
3. polarization;
4. inclination of binary with respect to interferometers;
5. 2 angles (RA, DEC);
6. redshift;
7. reference time;
8. phase at a reference time.

### 1.1.1 Masses and spins

The "no hair theorem" states that BHs are unique up to mass and spin, since charge is negligible. Conventionally, we have $m_1 > m_2$. LIGO-VIRGO measure two masses:

$$m_{\text{chirp}} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \tag{1.22}$$

$$M = m_1 + m_2 \,, \tag{1.23}$$

where $m_{\text{chirp}}$ defines the change of the frequency in the inspiral phase. The total mass is measured by measuring the frequency at merger.

Generally, we can measure well one of these but not the other. For high-mass mergers, we have the merger at the central frequency in the LIGO-VIRGO range, at around 400 Hz. For low-mass mergers, the merger is at a higher frequency: this means that we can see more of the inspiral, since it stays in the high-sensitivity range longer.

The detected BH mergers span the range from around 10 to $60 M_{\odot}$.

The masses of the NS detected through GW are consistent with those measured through X-ray binaries; on the other hand the BHs measured with GWs are heavier than those measured by EM measurements.

The spins of the BHs are defined by the vector

$$\vec{S} = \frac{\vec{J} c}{G m^2} \,, \tag{1.24}$$

which goes from $S = 0$ for Schwarzschild to $S = 0.998$ for tan extremal Kerr.

LIGO and VIRGO are not sensitive to individual spins. They can measure two combinations: if $\hat{L}$ is the BBH orbital angular momentum, we define the effective spin as

$$\chi_{\text{eff}} \frac{\left( m_1 \vec{S}_1 + m_2 \vec{S}_2 \right)}{m_1 + m_2} \cdot \hat{L} \,, \tag{1.25}$$

which ranges from $-1$ to 1. This is measures since it affects the phase of the GWs, while the orthogonal component of the spin affects precession:

$$\chi_p = \frac{1}{B_1 m_1^2} \max \left( B_1 S_{1,\perp}, B_2 S_{2,\perp} \right) \,, \tag{1.26}$$

where…

These are really uncertain. There is heavy degeneracy between the value $q = m_2 / m_1$ and the effective spin.

What does a 2D pdf plot showing degeneracy look like in general?

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We start with a brief summary of GW observations. The current bandwidth is between 10 Hz to $1 \times 10^4$ Hz, but at the edges there is heavy noise. The range which is actually good is 100 Hz to 1000 Hz.

Figure from [LIG+16a]: frequency evolution of mergers, compared to detector sensitivity.

We see a correlation between the mass of the observed BHs and the distance: this is due to an observational bias.

With only the two LIGO detectors there is huge degeneracy in RA-DEC, if we add VIRGO the situation improves a lot.

Rate estimations need assumptions about the population.

The BNS rate is 10 to 100 times larger than what was expected. For BHNS mergers, we only have an upper limit.

For BBHs, we have 10 to $140\,\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$.

We get an upper limit for the rate of intermediate mass black holes, IMBHs, which have masses between 100 to $1\,000\,000 M_\odot$. The upper limit is very stringent for a merger of two $100 M_\odot$ BHs with aligned spins. For larger BHs, the strain is larger but the frequency gets out of the range of the detector.

Events with a False Alarm Rate of $\lesssim 10^{-8}\,\mathrm{Hz}$ are directly sent out as public alerts by LIGO-VIRGO. On the site GRACEDB we can get informations for alerts. The alerts are flagged with labels describing the masses of the two objects, as calculated by the rough low-latency analysis. This is to help the EM observers.

The MassGap is when one of the objects is between 3 and $5 M_\odot$; this is because it is believed that there is a mass gap between the heavier neutron stars and the lightest BHs in that range.

# Chapter 2

# The formation of compact objects from single star evolution

The idea of this lecture is to understand a figure, showing the mass of the remnant as a function of the $M_{\text{ZAMS}}$ of the star, and the metallicity.

The observations of $> 20 M_{\odot}$ BHs was a surprise: before 2016, models did not usually predict these (however, a Polish team and the Padua team predicted it).

Massive hot stars lose mass through line driven winds. These stellar wind models underwent major upgrades in the last 10 years — see Vink+ 2016 for details.

These winds are based on the coupling of radiation to ions through spectral absorption lines. This then depends on the metallicity of the star: $\dot{M} \propto Z^{\alpha}$, with $\alpha \sim 0.5 \div 0.9$.

Iron lines are the most important. Thompson scattering becomes dominant when the star is close to the electron scattering Eddington limit.

Knowing this, we can evolve a star with varying metallicity.

Stellar winds depend both on Thompson electron scattering and on resonant metal lines. The $\alpha$ in $\dot{M} \propto Z^{\alpha}$ changes like:

$$\alpha \approx 0.85 \qquad \Gamma < 2/3 \tag{2.1}$$

$$\alpha \approx 2.45 - 2.4\Gamma \qquad \Gamma > 2/3\,. \tag{2.2}$$

When there is less metallicity dependence, even the low-metallicity stars lose lots of mass, like the high-metallicity ones.

The question is: does the star explode as a CCSN or not? If yes, we get a NS or a low-mass BH. If no, we have a direct collapse BH: this can be massive!

When the iron core forms, electron degeneracy pressure is the only thing holding up the core. When it reaches the Chandrasekhar mass, it collapses, and is then stopped by neutron degeneracy pressure.

This creates a bounce shock. There is a boundary: above it neutrinos fly away, below it the shock is stalled.

The question is: how is this shock revived? it must be, since we observe CCSNe.

Maybe there is convection. This should be simulated with 3D simulations, which are very computationally expensive.

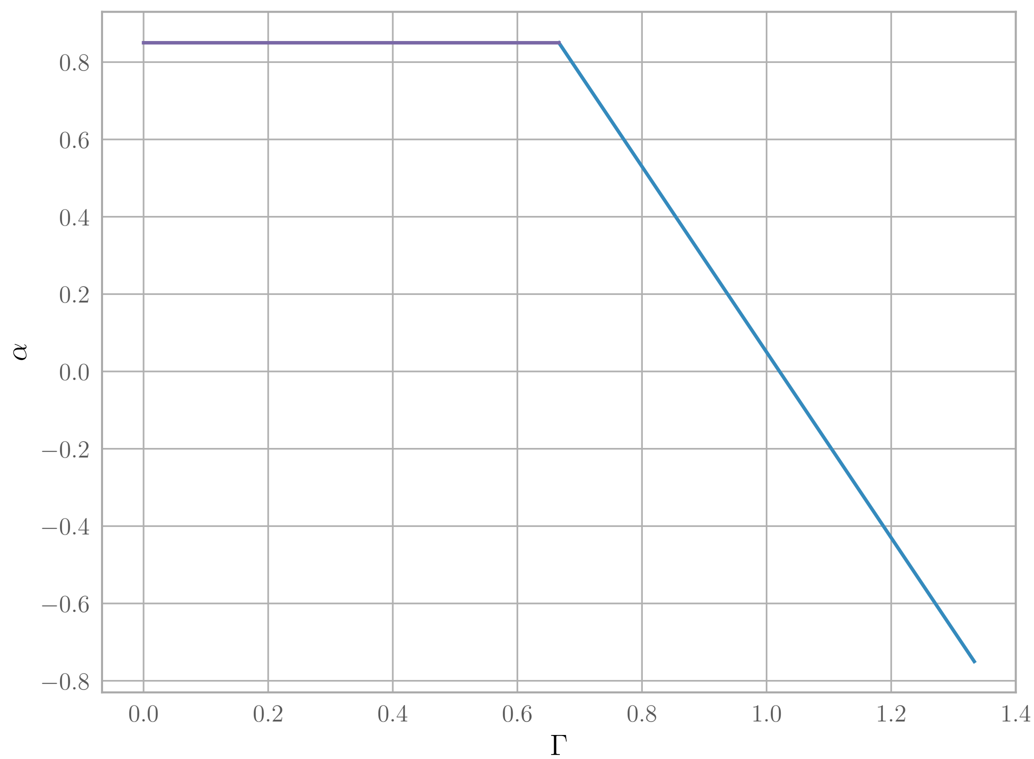


Figure 2.1:

We can get a lower bound on the mass of the envelope: if it is very massive, the star directly collapses. The estimate gives $M > 50M_{\odot}$.

The simplest models use the C-O core mass as a parameter. If it is larger than $8 \div 12M_{\odot}$ the SN explosion fails.

Another criterion is the compactness parameter:

$$\xi_M = \frac{M/M_{\odot}}{R(M)/1 \times 10^6\,\mathrm{m}}, \tag{2.3}$$

which is a multiple of the ratio $R_{\text{Schwarzschild}}/R$.

Simulations show that the time for the collapse has an inverse dependence on the compactness parameter.

There is a correlation between $\xi$ and $M_{CO}$. However there are researchers who are not convinced that compactness is a good enough estimator.

The mass of the remnant also depends on the fallback, which is not well understood.

How fast is the explosion? More importantly: by how much is the revival delayed with respect to the collapse?

Also, there are also PISNe. These leave no remnant. If there is pulsational instability, we have less final mass since more of it is shed.

Also, we have electron-capture SNe, which form NSs and will be discussed with professor Tauris.

To wrap up: the main idea is that if there is low metallicity, lower than $0.5Z_{\odot}$ then stellar winds are quenched, there is larger pre-SN mass, and therefore direct collapse is more likely, so we will have more massive BHs.

This was first discussed by Heger+ 2003. We have plots showing the final mass versus Zero-Age Main Sequence Mass. Black line: no mass loss. Blue line: mass of the star at collapse. Red line: mass of the remnant. At high metallicity, the mass of the star as collapse is very low. A zero metallicity, we have two regimes: above a threshold of about $40M_{\odot}$ there is direct collapse; below this we have a SN with BH formation.

What happens for intermediate metallicities? It is difficult: we need to account for metals, and we do not have calibration stars in the near universe.

This depends on the model of stellar winds, but the literature is quite convergent on that. The only big uncertainty is due to rotation and magnetic fields.

The model also depends on the model of CCSNe, but the main uncertainty is in the low-mass end. This is crucial for our understanding of NS and low-mass BH formation.

In this area, we cannot be really predictive.

Are the masses of the detected BHs evidence for population III stars? See Hartwig [Har+16]: it is not very likely.

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