Numerical methods notes

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The course is held by Michela Mapelli: a team leader in GW astronomy.

The first lessons are about basic Linux and Python, I will not take notes now, I will start later on.

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1 Sorting

Useful for astrophysics since we often deal with large files.

Bubble sort We loop through the file, and swap each pair if it is not ordered.

It is $O(n^2)$ in general.

Selection sort We look for the minimum and put it at the beginning, then scan the remaining array.

It is $O(n^2)$ in general.

Quicksort

1. We pick an element, the *pivot*;
2. we reorder the array so that all elements less than the pivot come before it;
3. we do this recursively to the subarrays to the left and right of the pivot.

It is $O(n^2)$ in the worst case, $O(n \log n)$ usually.

Merge sort We divide the array into small subarrays, and merge them to produce larger subarrays.

It is $O(n^2)$ in the worst case, $O(n \log n)$ usually.

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One can time a bash command using

```
time [Command]
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2 Linear systems

We want to solve a linear system, in the form $A\vec{x} = \vec{b}$, with unknown $\vec{x}$. How do we solve this numerically? We can transform our system to an equivalent one, by

1. exchanging two rows;
2. multiplying an equation by a nonzero constant;
3. adding an equation to another.

These allow us to do Gaussian elimination, and LU decomposition. These are *direct methods*.

Another class is that of *indirect methods*: we start with an *ansatz* and refine it. These are easier to implement, more generally applicable, more efficient if the matrix is sparse. They, however, do not always converge.

An example is the Gauss-Seidel method.

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2.1 Gauss-Seidel

We can rewrite $\sum_j A_{ij} x_j = b_i$ as

$$x_i = \frac{1}{A_{ii}} \left(b_i - \sum_{j \neq i} A_{ij} x_j \right) , \tag{1}$$

and the algorithm works by starting with and *ansatz*, updating it with this formula, and iterating. The update can be written more generally as

$$x_i^{n+1} = \frac{\omega}{A_{ii}} \left(b_i - \sum_{j \neq i} A_{ij} x_j^n \right) + (1 - \omega) x_i^n , \tag{2}$$

with the *relaxation parameter* ω. Do note that n is not an exponent but an iteration number.

A good choice for ω after the 5th iteration:

$$\omega_{\text{opt}} = \frac{2}{1 + \sqrt{1 - (\Delta x^{k+p} / \Delta x^{k})^{1/p}}}, \tag{3}$$

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Implementation of Gauss-Seidel.