Gravitational wave astrophysics notes

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Contents

Introduction

Monday 2020-3-23, compiled 2020-04-01

Organization: the instructor for this first part is professor Michela Mapelli.

Structure of the course:

1. today we start with background on GW, theory and data;
2. tomorrow we will discuss BH formation from single stars;
3. on wednesday we will discuss BH formation form binary evolution;
4. next week we will discuss BH dynamics.

In the next part we will discuss neutron stars with professor Thomas Tauris.

There is no final grade for the exam, only a pass/fail. This year, there will be a google form with multiple choice questions.

What is GWA?

It is a branch of astrophysics which studies the astrophysical characterization of GW sources. Mostly, it is about discussing binary black holes, binary neutron stars and NS-BH binaries, which emit in the frequency range we can observe right now.

The main open question is: what are the formation channels of the compact objects binaries observed by ground-base interferometers.

Chapter 1

Gravitational wave summary

From wikipedia: "GW are ripples in the curvature of spacetime which propagate as waves, travelling outward from the source, at the speed of light". The math describing these is that of GR: the main equations are the Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \,, \tag{1.1}$$

where $R_{\mu\nu}$ is the trace of the Riemann tensor, which is a measure of curvature; on the other hand $T_{\mu\nu}$ is a measure of the density of 4-momentum in spacetime, which acts as a source.

The metric $g_{\mu\nu}$ is used in order to measure distances.

The EFE are in general 10 second-order nonlinear PDEs. In the strong-curvature case, with rapidly moving sources we must use numerical solutions of the full nonlinear EFE.

In the weak field limit, with almost flat spacetime and slowly moving sources, we have analytic solutions.

We assume the perturbation looks like $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, then we can linearize the EFE. We can rewrite the linearized equations using the trace-reversed perturbation

$$\overline{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \,, \tag{1.2}$$

and then in a special coordinate system (in a special gauge) we get

$$\Box \overline{h}_{\mu\nu} = \frac{16\pi G}{c^4} T_{\mu\nu} + \mathcal{O}(h^2) \,, \tag{1.3}$$

which can be solved in general using the method of Green's functions, just like in electrodynamics, to find

$$\overline{h}^{\mu\nu} = \frac{4G}{c^4} \int \mathrm{d}^3 y \, \frac{T^{\mu\nu}(t, \vec{y})\big|_{\text{ret}}}{\left|\vec{x} - \vec{y}\right|} \,, \tag{1.4}$$

where $t_{\text{ret}} = t - \left|\vec{x} - \vec{y}\right| / c$.

This is general but not easy to visualize. In order to discuss it we first assume we are far from our source: so we can say that the support of the stress-energy tensor is all at a distance r from us. So, we get

$$\overline{h}^{\mu\nu}(t, \vec{x}) = \frac{4}{r} \frac{G}{c^4} \int \mathrm{d}^3 y \, T^{\mu\nu}(t - r/c, \vec{y}) \,. \tag{1.5}$$

If the source is moving slowly, that is, with a nonrelativistic velocity, we get

$$\overline{h}^{ij}(t,\vec{x}) = \frac{2}{r}\frac{G}{c^4}\ddot{I}^{ij}(t-r/c)\,, \tag{1.6}$$

where

$$I^{ij}(t-r/c) = \int \mathrm{d}^3y\, x^i x^j \rho(t-r/c,\vec{y})\,. \tag{1.7}$$

For a full derivation see Hartle [**hartleGravityIntroductionEinstein2003**].

This means that not all accelerating masses produce GWs: only those with non-zero quadrupole moment do; there is a need for an asymmetry in mass distribution.

Let us consider the simplest case we can have: a binary system, with two stars of equal mass M orbiting each other in a circular orbit of radius distance a with angular velocity ω. We can consider the center of mass and reduced mass coordinates.

The components of the second mass moment are found from $x(t) = a\cos(\omega t)$, $y = a\sin(\omega t)$, $z = 0$:

$$I_{xx} = Ma^2\big[1+\cos(2\omega t)\big] \tag{1.8a}$$
$$I_{yy} = Ma^2\sin(2\omega t) \tag{1.8b}$$
$$I_{zz} = Ma^2\big[1-\cos(2\omega t)\big]\,, \tag{1.8c}$$

so we get

$$\overline{h}^{ij} \sim \frac{2}{r}\frac{G}{c^4}(2\omega)^2 Ma^2 \begin{bmatrix} \cos\big((2\omega(t-r/c))\big) & \sin\big((2\omega(t-r/c))\big) & 0 \\ \sin\big((2\omega(t-r/c))\big) & -\cos\big((2\omega(t-r/c))\big) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.9a}$$

which we can simplify using

$$\omega = \sqrt{\frac{2GM}{a^2}}\,, \tag{1.10}$$

so we can see that the frequency of the GW is *twice* ω: $\omega_{GM} = 2\omega$.

We also get the two polarizations, h_+ and $h_\times$.

The amplitudes are of the order

$$h = \frac{1}{2}\left(h_+^2 + h_\times^2\right)^{1/2} \sim \frac{8G^2}{c^4}\frac{M^2}{ra}\,, \tag{1.11}$$

so with a solar mass binary, orbiting at a distance of a solar radius a kiloparsec away we get

$$h \sim 10^{-21}\left(\frac{M}{M_\odot}\right)^2\left(\frac{1\,\mathrm{kpc}}{r}\right)\left(\frac{R_\odot}{r}\right). \tag{1.12}$$

The amplitude is called the *strain*: the bigger it is, the easier the detection.

The easiest events to detect involve large masses, are close, and involve very close inspirals.

The crucial thing is that GW emission implies energy loss: this can be express iin the Keplerian limit as

$$E_{\mathrm{orb}} = -G\frac{m_1 m_2}{2a}\,. \tag{1.13}$$

The variable thing here is the radius a: it decreases, until coalescence. As the semimajor axis decreases, the frequency increases.

In order to quantify this, we can use the expression for the power emitted:

$$P_{GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{1}{a^5}m_1^2 m_2^2 (m_1 + m_2)\,, \tag{1.14}$$

so we can write

$$P_{GW} = \frac{\mathrm{d}E_{\mathrm{orb}}}{\mathrm{d}t} = \frac{G m_1 m_2}{2a^2}\frac{\mathrm{d}a}{\mathrm{d}t}\,, \tag{1.15}$$

which means

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{64}{5}\frac{G^3}{c^5}\frac{1}{a^3}m_1 m_2 (m_1 + m_2)\,, \tag{1.16}$$

which we can integrate in order to find the necessary time for coalescence:

$$t_{GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{a^4}{m_1 m_2 (m_1 + m_2)}\,. \tag{1.17}$$

Here we neglected the eccentricity, if we account for it we need to multiply by a term $(1-e^2)^{7/2}$.

This is an extremely long timescale. If $a = 1\,\mathrm{Au}$ and $m_1 = m_2 = M_\odot$ we get timescales like $t_{GW} \sim 10^{17}\,\mathrm{yr}$!

The aforementioned equations hold for the “inspiral phase”. The phases we usually distinguish are

1. inspiral, when the Keplerian approximation still holds;
2. merger, when the distance between the objects is closer than the ISCO;
3. ringdown, when the objects have coalesced.

We can use post-Newtonian techniques for the inspiral, then full numerical relativity for the merger, and “black hole perturbation methods” for the ringdown. The ISCO has a radius is

$$r_{ISCO} = 6\frac{G(m_1 + m_2)}{c^2}\,, \tag{1.18}$$

and we can get a good estimate of the frequency using the Keplerian formula:

$$\omega_{GW,ISCO} = 2\sqrt{\frac{G(m_1 + m_2)}{r_{ISCO}^3}}\,, \tag{1.19}$$

which works surprisingly well, better than a factor of 2.

See Abbott et al [**ligoscientificcollaborationandvirgocollaborationObservationGravitationalWaves2016**] for graphs.

With the sensitivity of current ground-based detectors we can detect small BH and NS mergers.

SMBH mergers are too low frequency, but we could maybe see extremely high mass ratio mergers.

We could see GWs from neutrons stars with crustal asymmetries, with $10\,\text{Hz}$ to $10^{-2}\,\text{Hz}$.

Asymmetric supernova explosions would also be in this frequency range, but with too low amplitude.

In this course we will focus on binary compact objects.

1.1 Observations of GW

In the first two observations by LIGO-VIRGO we have seen 10 BBHs and 1 binary neutron star.

The third observing run is ongoing, it will finish about now.

From these observations we know that

1. BBHs exist;

2. BBHs can merge in a Hubble time;

3. Massive BHs (with mass $> 20M_\odot$) exist.

As the observation number increases, the range of the observations increases. For O3, we will be able to see up to around $110\,\text{Mpc}$: this number is derived discussing a test well-oriented $1.3M_\odot$ neutron star, which would give a SNR of around 12. This is roughly proportional to $m^{5/3}$: the more massive the object, the larger the horizon: we will be able to see further for more massive objects.

What are the parameters which are directly observables (neglecting eccentricity):

1. 2 masses;

2. 6 spin components;

3. polarization;

4. inclination of binary with respect to interferometers;

5. 2 angles (RA, DEC);

6. redshift;

7. reference time;

8. phase at a reference time.

1.1.1 Masses and spins

The "no hair theorem" states that BHs are unique up to mass and spin, since charge is negligible. Conventionally, we have $m_1 > m_2$. LIGO-VIRGO measure two masses:

$$m_{\text{chirp}} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \tag{1.20a}$$

$$M = m_1 + m_2 \,, \tag{1.20b}$$

where m_{chirp} defines the change of the frequency in the inspiral phase. The total mass is measured by measuring the frequency at merger.

Generally, we can measure well one of these but not the other. For high-mass mergers, we have the merger at the central frequency in the LIGO-VIRGO range, at around 400 Hz. For low-mass mergers, the merger is at a higher frequency: this means that we can see more of the inspiral, since it stays in the high-sensitivity range longer.

The detected BH mergers span the range from around 10 to $60 M_{\odot}$.

The masses of the NS detected through GW are consistent with those measured through X-ray binaries; on the other hand the BHs measured with GWs are heavier than those measured by EM measurements.

The spins of the BHs are defined by the vector

$$\vec{S} = \frac{\vec{J} c}{G m^2} \,, \tag{1.21}$$

which goes from $S = 0$ for Schwarzschild to $S = 0.998$ for tan extremal Kerr.

LIGO and VIRGO are not sensitive to individual spins. They can measure two combinations: if $\hat{L}$ is the BBH orbital angular momentum, we define the effective spin as

$$\chi_{\text{eff}} \frac{\left(m_1 \vec{S}_1 + m_2 \vec{S}_2 \right)}{m_1 + m_2} \cdot \hat{L} \,, \tag{1.22}$$

which ranges from -1 to 1. This is measures since it affects the phase of the GWs, while the orthogonal component of the spin affects precession:

$$\chi_p = \frac{1}{B_1 m_1^2} \max \left(B_1 S_{1,\perp}, B_2 S_{2,\perp} \right) \,, \tag{1.23}$$

where...

These are really uncertain. There is heavy degeneracy between the value $q = m_2 / m_1$ and the effective spin.

What does a 2D pdf plot showing degeneracy look like in general?

Tuesday 2020-3-24, compiled 2020-04-01

We start with a brief summary of GW observations. The current bandwidth is between 10 Hz to 1×10^4 Hz, but at the edges there is heavy noise. The range which is actually good is 100 Hz to 1000 Hz.

Figure from [**ligoscientificcollaborationandvirgocollaborationBinaryBlackHole2016**]: frequency evolution of mergers, compared to detector sensitivity.

We see a correlation between the mass of the observed BHs and the distance: this is due to an observational bias.

With only the two LIGO detectors there is huge degeneracy in RA-DEC, if we add VIRGO the situation improves a lot.

Rate estimations need assumptions about the population.

The BNS rate is 10 to 100 times larger than what was expected. For BHNS mergers, we only have an upper limit.

For BBHs, we have 10 to $140\,\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$.

We get an upper limit for the rate of intermediate mass black holes, IMBHs, which have masses between 100 to $1\,000\,000 M_\odot$. The upper limit is very stringent for a merger of two $100 M_\odot$ BHs with aligned spins. For larger BHs, the strain is larger but the frequency gets out of the range of the detector.

Events with a False Alarm Rate of $\lesssim 10^{-8}\,\mathrm{Hz}$ are directly sent out as public alerts by LIGO-VIRGO. On the site GRACEDB we can get informations for alerts. The alerts are flagged with labels describing the masses of the two objects, as calculated by the rough low-latency analysis. This is to help the EM observers.

The MassGap is when one of the objects is between 3 and $5 M_\odot$; this is because it is believed that there is a mass gap between the heavier neutron stars and the lightest BHs in that range.

Chapter 2

The formation of compact objects from single star evolution

The idea of this lecture is to understand a figure, showing the mass of the remnant as a function of the M_{ZAMS} of the star, and the metallicity.

The observations of $> 20 M_\odot$ BHs was a surprise: before 2016, models did not usually predict these (however, a Polish team and the Padua team predicted it).

Massive hot stars lose mass through line driven winds. These stellar wind models underwent major upgrades in the last 10 years — see Vink+ 2016 for details.

These winds are based on the coupling of radiation to ions through spectral absorption lines. This then depends on the metallicity of the star: $\dot{M} \propto Z^\alpha$, with $\alpha \sim 0.5 \div 0.9$.

Iron lines are the most important. Thompson scattering becomes dominant when the star is close to the electron scattering Eddington limit.

Knowing this, we can evolve a star with varying metallicity.

Stellar winds depend both on Thompson electron scattering and on resonant metal lines. The α in $\dot{M} \propto Z^\alpha$ changes like:

$$\alpha \approx 0.85 \qquad \Gamma < 2/3 \tag{2.1a}$$

$$\alpha \approx 2.45 - 2.4\Gamma \qquad \Gamma > 2/3\,. \tag{2.1b}$$

When there is less metallicity dependence, even the low-metallicity stars lose lots of mass, like the high-metallicity ones.

The question is: does the star explode as a CCSN or not? If yes, we get a NS or a low-mass BH. If no, we have a direct collapse BH: this can be massive!

When the iron core forms, electron degeneracy pressure is the only thing holding up the core. When it reaches the Chandrasekhar mass, it collapses, and is then stopped by neutron degeneracy pressure.

This creates a bounce shock. There is a boundary: above it neutrinos fly away, below it the shock is stalled.

The question is: how is this shock revived? it must be, since we observe CCSNe.

Maybe there is convection. This should be simulated with 3D simulations, which are very computationally expensive.

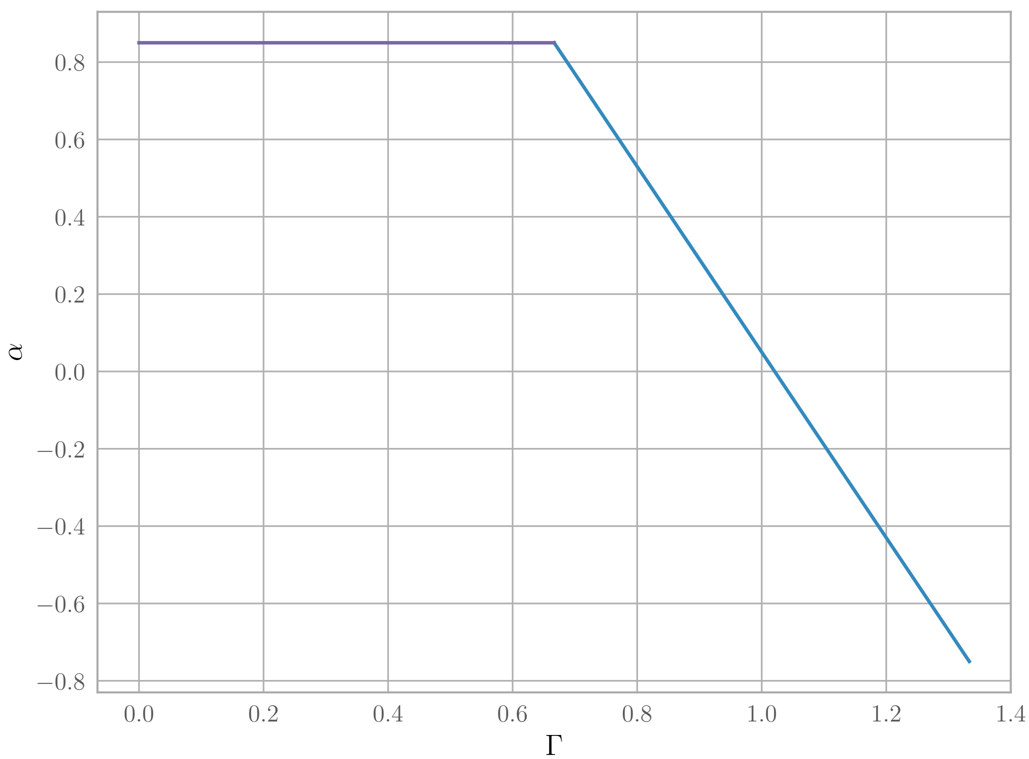

Figure 2.1:

We can get a lower bound on the mass of the envelope: if it is very massive, the star directly collapses. The estimate gives $M > 50 M_\odot$.

The simplest models use the C-O core mass as a parameter. If it is larger than $8 \div 12 M_\odot$ the SN explosion fails.

Another criterion is the compactness parameter:

$$\xi_M = \frac{M/M_\odot}{R(M)/1 \times 10^6\,\mathrm{m}}, \tag{2.2}$$

which is a multiple of the ratio $R_{\text{Schwarzschild}}/R$.

Simulations show that the time for the collapse has an inverse dependence on the compactness parameter.

There is a correlation between ξ and M_{CO}. However there are researchers who are not convinced that compactness is a good enough estimator.

The mass of the remnant also depends on the fallback, which is not well understood.

How fast is the explosion? More importantly: by how much is the revival delayed with respect to the collapse?

Also, there are also PISNe. These leave no remnant. If there is pulsational instability, we have less final mass since more of it is shed.

Also, we have electron-capture SNe, which form NSs and will be discussed with professor Tauris.

To wrap up: the main idea is that if there is low metallicity, lower than $0.5Z_\odot$ then stellar winds are quenched, there is larger pre-SN mass, and therefore direct collapse is more likely, so we will have more massive BHs.

This was first discussed by Heger+ 2003. We have plots showing the final mass versus Zero-Age Main Sequence Mass. Black line: no mass loss. Blue line: mass of the star at collapse. Red line: mass of the remnant. At high metallicity, the mass of the star as collapse is very low. A zero metallicity, we have two regimes: above a threshold of about $40M_\odot$ there is direct collapse; below this we have a SN with BH formation.

What happens for intermediate metallicities? It is difficult: we need to account for metals, and we do not have calibration stars in the near universe.

This depends on the model of stellar winds, but the literature is quite convergent on that. The only big uncertainty is due to rotation and magnetic fields.

The model also depends on the model of CCSNe, but the main uncertainty is in the low-mass end. This is crucial for our understanding of NS and low-mass BH formation.

In this area, we cannot be really predictive.

Are the masses of the detected BHs evidence for population III stars? See Hartwig [**hartwigGravitationalWavesRemnants2016**]: it is not very likely.

Wednesday 2020-3-25, compiled 2020-04-01

To summarize yesterday's lesson: we discussed the main physical processes affecting the mass of BHs and NSs.

We know that the mass of a star is not constant over time: it decreases because of mass loss, and the final mass which is still bound at collapse is the budget we have to form the compact object.

CCSNe are crucial: for every star, if is explodes as a supernova it can form a NS or a light BH, while if the explosion fails we can have formation of more massive BHs.

In the picture from yesterday, we did not discuss PISNe. Now, we also include very massive stars, with masses $M > 150M_\odot$.

Pair Instability leaves no remnant, Pulsational Pair Instability just leads to more mass loss. If we also consider these, we get a mass gap between 60 to $120M_\odot$ in the remnants.

Only metal-poor stars can develop He-cores which are heavy enough to start pair creation. PISNe create a V-shaped signature for metal-poor stars, around 60 to $100M_\odot$ ZAMS mass.

Around $230M_\odot$ ZAMS mass, we have a jump to huge remnants for extremely low-metallicity stars. Here, pair creation starts, and the switch-on of the nuclear burning is not enough to counteract the collapse: all of the star just falls in on itself.

If this is indeed the case, it could be a formation channel for intermediate-mass BHs, which could merge to form SMBHs.

So, we expect to see no BHs between 60 to $120M_\odot$: currently, LIGO-VIRGO observations are consistent with this.

What about spin? It is not well-understood. We expect that the spin of the Compact

Object is related to the spin of the core, at collapse. However, part of this spin will be lost during the SN explosion. If we have accretion disks or jets, some angular momentum is lost.

If we have direct collapse, then the spin will be conserved.

However, if we take the final angular momentum of the core and assume it collapses directly to a BH, this gives almost only maximally rotating BHs.

This is definitely not what we observe: although we have uncertainty, most of the spins are consistent with being small [**theligoscientificcollaborationGWTC1GravitationalWaveTransient2019**].

This is derived with GR SN collapse models, but these are usually not fine enough to really account for the remnant.

Spin direction can be changed by the SN kick — when the SN explodes, the remnant rebounds, if there is no SN kick, the spin remains in the same direction: this can be tested, if we observe the SN head-on then we expect the spin to be head-on as well.

2.1 Formation of BH and NS binaries

The initial separation for a binary to start losing significant energy from GW radiation must be $\lesssim 50 R_\odot$. How do the compact objects get this close?

A way is for the stars to start off gravitationally bound. The second scenario is one in which two BHs which come from unbound stars, in a dense environment.

2.1.1 Primordial binaries

We know that massive stars tend to form in pairs: 70 to 90 % of massive stars are in a binary.

The issue is that evolutionary processes can affect the binary.

We can have mass transfer.

Mass transfer from wind

A massive star loses a lot of mass from wind, more or less isotropically, and if it is orbited by a companion then the companion will intercept some of this. How small is the cross section? If we do some analytical calculations, we see that if 1 is the star emitting wind, while 2 is the accretor, we have

$$\left| \frac{\dot{M}_{2A}}{\dot{M}_{1W}} \right| \propto \left(\frac{v_{\text{orb}}}{v_W} \right)^4 . \tag{2.3}$$

Typically the ratio of velocities is of the order $1/10$, so this is very inefficient. For Wolf-Rayet stars we have high orbital velocities.

Roche lobe

In the corotating frame, a stationary corotating test particle experiences an acceleration given by the potential

$$\phi_R = \frac{Gm_1}{\left|\vec{r} - \vec{d}_1\right|} + \frac{Gm_2}{\left|\vec{r} - \vec{d}_2\right|} + \frac{1}{2}|\vec{\omega} \times \vec{r}|^2\,, \tag{2.4}$$

and it can be seen that the dependence on the masses is actually a dependence on the mass ratio $q = m_1/m_2$, where 1 is conventionally the donor while 2 is the accretor. We can draw the equipotential surface, and we have an equipotential surface which is shaped like an 8.

Then, a good approximation for the critical radius for the donor is

$$\frac{r_1}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \log\left(1 + q^{1/3}\right)}\,, \tag{2.5}$$

where a is the semimajor axis of the binary orbit.

If a star fills its Roche lobe, then mass can flow from one star to the other.

This matter flow is not radial, from Coriolis force it can form an accretion disk.

The big question is whether the Roche lobe overflow is stable. Recall, there are three important timescale in stellar evolution: the dynamical timescale, the Kelvin-Helmholtz thermal timescale, and the nuclear timescale.

Is the transfer stable across these timescales (that is, across which of these)?

Let us assume that the mass transfer is conservative, then it will be able to change the mass ratio, but non the total mass nor the angular momentum. So, we have the conserved quantity

$$(m_1 m_2)^2 a = \text{const}\,. \tag{2.6}$$

Therefore, at $m_1 = m_2$ there is a minimum for the semimajor axis a.

So, let us consider a RLO system with

parameters

So, we have instability if the star 1 cannot shrink fast enough to keep it into hydrodynamical equilibrium. Then, it is unstable over a dynamical timescale. This causes a common envelope to form, or directly a merger.

If, after having shrunk, the donor is not in thermal equilibrium anymore, then we have instability on a thermal timescale. This causes the donor to expand.

For binary compact objects, the most important thing is the dynamical instability. If at least one of the two stars already has a CO core, then as they form a common envelope then the drag of the envelope increases, which means than the cores tend to spiral in.

Then, either they form a single star, or the envelope is removed and the two cores are left spiralling each other.

A naked core is the same thing as a WR star, so it can collapse to a BH. Then, we can get a binary BH system. The envelope is crucial in robbing the system of energy, so that it can merge within a Hubble time.

This seemed unlikely, which was the reason why people in astrophysics were doubtful of the possibility to see BBHs.

Now this is kind of better understood, but it is not possible to study it on thermal timescales.

Is the envelope actually ejected? LIGO and VIRGO exclude a scenario in which there still is an envelope, at least for the events which we did not see electromagnetically.

There is a scenario by O'Connor: the BHs form from the split of the core of a single progenitor, and merge immediately: this is definitely rejected. Another scenario is about the formation of the secondary BH when there is still a common envelope. This, as mentioned, is excluded when we do not see the EM counterpart, since there would be no way to quench it.

See Thorne-Zytkow stars.

Friday 2020-3-27, compiled 2020-04-01

Common envelope: Webbink formalism. It does not really capture all the physics.

If the parameter α is small, then we need to shrink the binary a lot in order to eject the envelope.

How can we constrain this α? We do not really get good constraints from GW and EM observations, however in simulations the product $\alpha\lambda$ is very important.

This formalism is definitely not enough. How can we do without it? An alternative to common envelope theory is given by chemically homogenous evolution. A strong chemical gradient usually is associated with a strong density gradient; if we have chemically homogeneous stars their radius is smaller, so they do not surpass their Roche limit.

How can stars be chemically homogeneous? They can rotate very fast.

The stars can overfill the Roche lobe without entering common envelope, because of their rotation: the behavior is changed, conventian

This model predicts heavy BHs, with aligned spins (unless the SN resets them). A natal kick can be due to: asymmetry in mass ejection during core collapse, asymmetry in neutrino emission during core collapse, Blaauw mechanism. Tha last one can be due to symmetric SN: the decrease in mass can perturb the binary evolution.

Is the supernova explosion usually strong enough to push the other star outward? No, this is not usually a relevant effect.

If we fit the 3D velocity distribution of SNe we get a Maxwellian with $\sigma \sim 265\,\mathrm{km/s}$. Usually stellar rotation is neglected, Whoosley's Kepler code accounts for this. On the other hand, the proper motion of the center of mass before the explosion is accounted for by removing the velocity of nearby stars.

This is debated, but we can say that usually the velocities are of order $\gtrsim 100\,\mathrm{km/s}$. BHs should experience less kick than Neutron Stars, from conservation of linear momentum by a factor 10.

A good approximation is

$$v_{\text{kick, BH}} = (1 - f_{\text{fb}}) v_{\text{kick, NS}} \,. \tag{2.7}$$

Also, we can say that

$$v_{\text{kick, CO}} \propto \frac{m_{\text{ej}}}{m_{\text{CO}}} \,. \tag{2.8}$$

Also, of course, we must account for GW decay: we get that GW emission produces circularization (e decreases) and orbital decay (a decreases).

[summary of isolated binary evolution]

2.2 Dynamics of stars and Black Holes in Dense Stellar Systems

Why dynamics? Well, massive stars form in star clusters, which are dynamically active places. Actually, they are the most dynamic place in the universe.

People usually see a dichotomy between the sparse galactic field vs globular clusters, which are dynamical, old but somewhat rare.

We also have young star clusters and open clusters: these are dynamical and short lived; this is the place in which we see most of star formation. Many stars which are currently in the field were formed there

We also have nuclear star clusters: they are also dynamical, long lived and typically host SMBHs.

If we put density vs mass in a log-log plot, we see a linear (so, powerlaw) correlation: from the low mass, low density we have the solar neighborhood, young clusters, globular clusters and galactic nuclei.

When we have close encounters, they are much more often between an object and a binary: a binary has a much larger cross section.

Binaries are energy reservoirs:

$$E_{\text{int}} = \frac{1}{2}\mu v^2 - \frac{Gm_1 m_2}{r}\,, \tag{2.9}$$

where r and v are the relative separation and velocity. We also have $E_{\text{int}} = -E_{\text{binding}}$. This energy can be exchanged with the "intruder". This can result in binary shrinking: the binding energy increases, which means that the semimajor axis decreases, since

$$E_{\text{bind}} = \frac{Gm_1 m_2}{2a}\,. \tag{2.10}$$

This can also result in a dynamical exchange of a partner.

The probability of the exchange is very strongly a function of the mass of the intruder: it looks like $\mathbb{P} \sim [m_{\text{intruder}} > m_1]$.

An intruder can also "ionize" the binary if it comes in with a critical minimal velocity.

Will the binary usually acquire or lose energy? We can answer statistically: this is given by Heggie's law: hard binaries, which have E_b higher than the average kinetic energy of a star in the star cluster:

$$\frac{Gm_1 m_2}{2a} > \frac{1}{2}\langle m \rangle \sigma^2\,, \tag{2.11}$$

where σ is the velocity dispersion, will tend to become harder, and similarly soft binaries will tend to become softer.

Now, we move to the rate of three-body encounters. What is the cross section of three-body encounters?

We can define $b_{\max}$ as the maximum impact parameter so that there is a nonzero energy exchange between star and binary.

A thing we can do is to ask that the maximum pericenter distance be of the order of the semimajor axis. This gives us an expression for the cross section $b_{\max}$, and also for the area $\Sigma = \pi b_{\max}^2$.

Then we can estimate the typical rate of interaction. Typically, as long as the binary is hard and sufficiently massive (so that exchanges are unlikely) we will have

$$\frac{\Delta E_b}{E_b} \propto \frac{m_3}{m_1 + m_2}\,, \tag{2.12}$$

so we write the constant as

$$\xi = \frac{m_1 + m_2}{m_3} \frac{\langle \Delta E_b \rangle}{E_b} \sim 0.1\,. \tag{2.13}$$

Now we can express the average variation of the binding energy as

$$\frac{\mathrm{d}E_b}{\mathrm{d}t} = \xi \frac{\langle m \rangle}{m_1 + m_2} E_b \frac{2\pi G(m_1 + m_2) n a}{\sigma} = \pi \xi G^2 \frac{\rho}{\sigma} m_1 m_2\,, \tag{2.14}$$

so we get that a hard binary hardens at a constant rate.

The hardening rate is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{a}\right) = \frac{2}{G m_1 m_2} \frac{\mathrm{d}E_b}{\mathrm{d}t} = 2\pi G \xi \frac{\rho}{\sigma} \tag{2.15a}$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -2\pi G \xi \frac{\rho}{\sigma} a^2\,, \tag{2.15b}$$

which notably does not depend on the masses anymore; also, we can use it to derive a timescale:

$$t_h = \left| \frac{a}{\dot{a}} \right| = \frac{1}{2\pi G \xi} \frac{\sigma}{\rho} \frac{1}{a}\,, \tag{2.16}$$

which only depends on the local velocity dispersion σ, mass density ρ, semimajor axis a and the parameter of efficiency ξ.

So we have hardening: this allows for the binary to be shrunk enough so that it start to efficiently emit GWs.

We can equate this timescale t_h with t_{GW}: we can equate the two in order to find the time at which the two effects are equal. We can combine the two to get

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -2\pi \xi \frac{G\rho}{\sigma} a^2 - \frac{64}{5} \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 (1 - e^2)^{7/2}} a^{-3}\,. \tag{2.17}$$

This formalism is very accurate, especially if the masses of the binary objects are very large compared to the other objects, and if the variation of σ and ρ is on longer timescales than the evolution of the binary.

An other important prediction is that BHs are favoured in exchanges since they are very massive. More than 90 % of BBHs are formed in young star clusters via exchange.

It is very difficult for isolated binaries to form events like the heaviest ones we know.

The eccentricity is usually very small, since GW emission causes circularization.

The spins tend to be aligned, more often than not, especially for heavy binaries.

Kozai-Lidov resonance: if we have a tight binary orbited by a third object, this can oscillate. This does not affect the semimajor axis of the binary, however is changes the eccentricity.

If we introduce gravitational corrections, these systems can suddenly collapse: then we predict to see very eccentric BBHs.

Intermediate mass BHs: in a young star cluster there is process called dynamical friction, by which the most massive stars tend to sink to the center. This may lead to the formation of a very massive star, which could possibly collapse to an intermediate mass black hole.

IMBHs could also be formed by repeated mergers in a cluster. We will, however need to account for relativistic kicks.

In lecture 5 there would be some exercises in population synthesis codes. In lecture 8 we have a code which generates realistic initial conditions for realistic three-body encounters.

Chapter 3

Thomas Tauris' part

Monday 2020-3-30, compiled 2020-04-01

The topics will be:

1. X-ray Binaries and Recycling Millisecond Pulsars;
2. Spin and B-field evolution of Neutron Stars;
3. Formation of BNSs;
4. BNS and GW at low and high frequencies.

3.1 X-ray Binaries and Recycling Millisecond Pulsars

For a review: van den Heuvel & Tauris 2006, and also a textbook by the same authors in 2020-2021.

Based on the observations in the sixties, we can deduce that most of the X-ray sources are accreting neutron stars. Their luminosities were huge, on the orders of $L_X \sim 10^{37}$ erg/s.

If they were normal stars or white dwarfs, they would need to accrete huge amounts of material, and that much material would obscure the radiation emitted. So, they can only be neutron stars or black holes.

In '67, then, the first radio pulsar was then discovered.

The accretion luminosity can come from either the release of gravitational binding energy and nuclear burning at the surface of the object.

High-mass X-ray binaries: something like a $16 M_\odot$ star accreting onto a $1.3 M_\odot$ compact object. The dynamics of these is driven by wind accretion, and Roche-lobe overflow.

Be-star X-ray binaries: a star undergoing de-cretion, orbited by a NS which enters in the decretion disk: regular X-ray emission.

Low mass X-ray binaries: they are very long-lived, the donor star has a mass of less than a solar mass. Their periods are very short. The material accretes on top of the NS, and can undergo runaway nuclear fusion. These bursts typically last a minute or so.

We can have Roche-lobe overflow at different stages in stellar evolution: if it does while it is hydrogen burning, helium burning or even after we have cases A, B and C.

It is really hard to account for all the processes in compact binary evolution.

We define the exponents

$$\zeta = \frac{\partial \log R}{\partial \log M}, \tag{3.1}$$

which we can use to define some initial stability criteria.

If there is mass loss ($\beta > 0$) then we can have the orbit shrinking even if the period decreases.

3.1.1 Recycling and MilliSecond Pulsars

How do they form? We have the $P - \dot{P}$ diagram. P means period. MSPs are in the low P, low $\dot{P}$ region.

Tuesday 2020-3-31, compiled 2020-04-01

3.2 Spin and B-field evolution of NSs

What is the structure of neutron stars? Their radius is of the order of 15 km, but this is very much a field of investigation. We have a carbon atmosphere of the height of a few centimeters, the core has nuclear density, of the order of $10^{15}\,\mathrm{g/cm^3}$.

The spin frequencies are of the order of 700 Hz to 0.05 Hz. Their magnetic fields are of the order of $10^{13}\,\mathrm{G}$, they lose rotational energy at something like $\dot{E} \sim 10^5 L_\odot$.

Their masses are of the order $1.4 M_\odot$. They are like a giant atomic nucleus, with $A \sim 10^{57}$ and densities of the order of 2 to 10 times the density of nuclei.

They are very precise clocks. Although they are slowly slowing down, they do so predictably.

The radio emission is of the order 0.48 Jy, and if we were to interpret it as blackbody radiation we get absurdly high temperatures. There must be a different mechanism.

The total amount of radiation detected from radio pulsars is very little.

See radio telescope in Sardinia!

We have detected radio pulsars mostly on our side of the galaxy, since the radio waves have a hard time going through the galactic center.

3.2.1 Pulse profiles

Sometimes we have a single pulse, sometimes we have two, and they can have different shapes.

The emission in different wavelengths can be misaligned, depending on the geometry. Radio, optical and X-ray photons can

See [**shapiroBlackHolesWhite1983**].

We can use the scattering and the dispersion relation in the ISM to gauge the distance of the pulsars. The distance is one over the slope of the curve of phase to frequency.

This needs a measure of the density of free electrons on the ISM, so it is quite uncertain, it has an uncertainty of around 20 %.

We can constrain the stellar wind mass-loss rate of a B-star using pulsars.

What is the angular resolution with which we can determine the positions of these?

Spin evolution of pulsars: we have a rotating dipole, so there is emission of radiation.

We can give a characteristic age by $\tau = P/2\dot{P}$.

We know that the strain of the GWs is given by

$$h_{\mu\nu} = \frac{2G}{c^4 d} \ddot{Q}_{\mu\nu} \,, \tag{3.2}$$

so we need an asymmetric mass distribution rotating.

We know that

$$L_{GW} \sim \frac{G}{5c^5} \left\langle \dddot{Q}_{\mu\nu} \dddot{Q}_{\mu\nu} \right\rangle \,, \tag{3.3}$$

and typically

$$\dddot{Q} \sim \frac{MR^2}{T^3} \sim \frac{Mv^3}{R} \,, \tag{3.4}$$

where M, v and R are characteristic masses, velocities and radii of the system.

A neutron star with a mountain with height 1 mm has a L_{GW} of 60 orders of magnitude larger than that of a 20 m steel cylinder spinning at the speed of sound.

We can measure the braking index:

$$n = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2} \,, \tag{3.5}$$

which determines the evolution of the pulsar in the $P - \dot{P}$ diagram. $n = 3$ is what we would have for a pure dipole, while if we had only decay by gravitational radiation we would see $n = 5$.

The true age of a pulsar is given by

$$t = \frac{P}{(n-1)\dot{P}} \left[1 - \left(\frac{P_0}{p} \right)^{n-1} \right] . \tag{3.6}$$

Only a tiny part of the lost energy goes to radio waves.

The slope in the $P, \dot{P}$ diagram is given by $2 - n$.

Magnetars: a type in neutron star with a very high magnetic-field decay: they cannot be radio pulsars since their X-ray luminosity is $\gg \dot{E}_{\text{rot}}$.

Largest magnetar γ-ray burst: the atmosphere was measured to be fluctuating at its same period!

We can measure *proton* synchrotron emission lines.

There is evidence that over time the pulsars tend to align the spin axis and the N-S magnetic axis. This is very difficult to measure.

What would be the frequency range of the GW emission

Hulse-Taylor pulsars. We have many ways to measure masses of binary NS systems.

Wednesday 2020-4-1, compiled 2020-04-01