Multimessenger astrophysics notes

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Tuesday 2020-3-10

Introduction

A questionnaire will be delivered in the middle of the course, for constructive criticism. It is optional, but we are asked to do it.

The professor is Elisa Bernardini, `elisa.bernardini@unipd.it`. Office at Paolotti, 3rd floor.

It is a very new field, there are no comprehensive textbooks.

Multimessenger Astrophysics is about using:

1. cosmic rays (p);
2. neutrinos (ν);
3. photons (γ) at all wavelengths;
4. gravitational waves (GW).

Each of these represents a certain fundamental force: strong, weak, electromagnetic and gravitational.

The first time neutrinos were detected for a certain astronomical event was with SN 1987 A. Another one was, recently, TXS 0506+056 and IC-170922A, also with neutrinos. Plus, one with GWs.

There will be *exercises*. They will not be asked in the exam, but small calculations might.

This course will assume some knowledge of fundamental particles and interactions, if we need support we should ask for it.

There are four suggested books, the professor prefers Spurio's and Perkins'. Longair will be used just for a specific topic: the interaction of cosmic rays.

The professor is a physicist, involved with antarctic experiments.

High Energy Physics is originally HE Astrophysics.

The flux (per unit area, time, energy, angle) of cosmic rays is roughly a powerlaw.

Open questions are: what are the sources of these particles? What is their acceleration mechanism?

A useful way to probe this is an astrophysical beam dump.

We can distinguish *astrophysics*, which uses electromagnetic radiation alone, and *astroparticle* physics which uses cosmic rays, neutrinos, high-energy gamma rays, searches for dark matter. A heuristic is that astrophysics measures energies in ergs, while astroparticle physics measures them in electronVolts.

Syllabus

1. Interactions of astroparticles;
2. acceleration, propagation, interaction of cosmic rays;
3. measurements and candidate sources for cosmic rays;
4. MultiMessenger approach: combining information from different types of particles and waves:
   (a) Gamma ray astrophysics;
   (b) multi-wavelength observations of astrophysical sources;
   (c) neutrino astrophysics;
   (d) GW.

Chapter 1

Interactions of astroparticles

1.1 Lorentz transformations

We start from the assumption that the transformations should be linear and that light propagates at the same velocity in each inertial frame. This directly implies that

$$ds^2 = -c^2\,dt^2 + dx^2 + dy^2 + dz^2\;, \tag{1.1}$$

the spacetime interval, is an invariant.

Then, if reference S' is moving with velocity v along the x axis with respect to S the transformation law is given by

$$t' = \gamma\Big(t - vx/c^2\Big) \tag{1.2}$$
$$x' = \gamma(x - vt) \tag{1.3}$$
$$y' = y \tag{1.4}$$
$$z' = z\,, \tag{1.5}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$. This implies the presence of time dilation and length contraction with respect to the rest frame. A moving observer will measure a *longer* duration than the stationary (comoving) one. A moving observer will measure a *shorter* duration than the stationary (comoving) one.

Exercise: muons A direct experimental confirmation of this is the observation of muons produced by cosmic rays in the earth's atmosphere.

Muons are produced around 10 km up by interactions of protons, in the Earth's atmosphere. After some time they decay into pions. The lifetime of muons is around $\tau_\mu \approx 2.2 \times 10^{-6}$ s. The lifetime of pions is $\tau_\pi \approx 2.6 \times 10^{-8}$ s.

We can say that, since the muons are relativistic, their velocity is $v \approx c$. So, the length they travel in their lifetime is $l_\mu = c\tau_\mu \approx 600$ m and $l_\pi = c\tau_\pi \approx 8$ m.

However, we did not account for time dilation! the typical energy of a μ particle is $E_\mu \approx 10\,\mathrm{GeV}$, so

$$\gamma = \frac{E}{m_\mu c^2} \approx 100\,, \tag{1.6}$$

so the time of decay in our frame is $t = \gamma\tau_\mu = 100 \times 2.2\,\mathrm{\mu s} \approx 60\,\mathrm{km}$, which is enough for them to reach the ground.

For the pions, on the other hand, their path will be something like $l_\pi = \gamma\tau_\pi c \approx 800\,\mathrm{m}$. Indeed, they are rare, and were detected using an emulsion field in high mountains.

In the muon's perspective, the length is contracted: so the muon sees $L = L'/\gamma$, and since $L' \approx 10\,\mathrm{km}$ we have $L \approx 100\,\mathrm{m}$.

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Today we introduce the concept of *invariant mass*: we consider a system of N particles, each with energy E_k and momentum $\vec{p}_k$. Their four-momenta are $p_k = \left(E_k/c, \vec{p}_k\right)$.

The total 4-momentum is

$$p_{\mathrm{tot}} = \sum_{k=1}^{N} p_k\,. \tag{1.7}$$

The scalar product between two of these is defined as

$$p_1 \cdot p_2 = \frac{E_1 E_2}{c^2} - \vec{p}_1 \cdot \vec{p}_2\,. \tag{1.8}$$

We define the quantity

$$\sqrt{s} = \sqrt{p_{\mathrm{tot}}^2} = \sqrt{\Big(\sum_k E_k\Big)^2 - \Big|\sum_k \vec{p}_k\Big| c^2}\,. \tag{1.9}$$

The variable s is called the center-of-mass energy, or invariant mass.

Let us consider a fixed target experiment: a particle 1 incident upon a screen, call it 2. Let us consider the frame in which the screen is at rest.

The momenta are $p_1 = \left(E_1/c, \vec{p}_1\right)$ and $p_2 = \left(m_2 c, \vec{0}\right)$. The total momentum is then

$$p_{\mathrm{tot}} = \left(\frac{E_1}{c} + m_2 c, \vec{p}_1\right), \tag{1.10}$$

so

$$\sqrt{s} = \left(\left(E_1 + m_2 c^2\right)^2 - p_1^2\right), \tag{1.11}$$

so

$$\sqrt{s} = \sqrt{m_1^2 c^4 + m_2^2 c^4 + 2E_1 m_2 c^2}\,. \tag{1.12}$$

If, instead, both particles are moving then

$$\sqrt{s} = \sqrt{m_1^2 c^4 + m_2^2 c^4 + 2\Big(E_1 E_2 - |p_1||p_2| c^2 \cos\theta\Big)}\,, \tag{1.13}$$

where θ is the angle between the particles' trajectories. For a head-on collision

$$\sqrt{s} = \sqrt{m_1^2 c^4 + m_2^2 c^4 + 2\Big(E_1 E_2 - |p_1||p_2| c^2\Big)}\,. \tag{1.14}$$

The flux of cosmic particles decreases approximately as a powerlaw with the energy [Pat, eq. 30.2]:

$$\text{flux} \approx 1.8 \times 10^4 \left(\frac{E}{1\,\text{GeV}}\right)^{-\alpha} \frac{\text{nucleons}}{\text{m}^2\text{srsGeV}}\,, \tag{1.15}$$

with $\alpha \approx 2.7$ generally, however there are high energy regions where this slope changes slightly.

We can detect cosmic rays at energies higher than the maximum ones of the LHC. The LHC COM energy is reported to be around $10^{17}\,\text{GeV}$: this seems strange! It is actually a consequence of the difference between fixed-target experiments (which describes the interaction between a cosmic ray and a nucleus on the Earth well) and experiments such as the LHC, in which two beams collide head on.

Exercise at the LHC the particles can be accelerated to energies of around

$$E \approx 7\,\text{TeV}\,, \tag{1.16}$$

so we want to compute the COM energy $\sqrt{s}$ for two protons colliding head-on, and we wish to compute the energy a cosmic ray would need to have in order to have $\sqrt{s}$ equal to that of the LHC.

For the LHC two protons with $E \approx 7\,\text{TeV}$ collide, so the total COM energy is $\sqrt{s} \approx 14\,\text{TeV}$.

Now, for the fixed target collision: the formula will be, taking proton 1 to be the cosmic ray and proton 2 to be the stationary proton, which has $\vec{p}_2 = 0$ and $\vec{E}_2 = m_p$ (setting $c = 1$):

$$\sqrt{s} = \sqrt{(E_1 + E_2)^2 + |\vec{p}_1 + \vec{p}_2|^2} \tag{1.17}$$

$$= \sqrt{E_1^2 + 2E_1 m_p + m_p^2 + m_p^2 - E_1^2} \tag{1.18}$$

$$= \sqrt{2m_p^2 + 2E_1 m_p}\,, \tag{1.19}$$

so by setting $\sqrt{s} = 14\,\mathrm{TeV}$ we find:

$$s = (14\,\mathrm{TeV})^2 = 2\left(E_1 m_p + m_p^2\right) \tag{1.20}$$

$$E_1 = \frac{s - 2m_p^2}{2m_p} \approx 10^8\,\mathrm{GeV}\,. \tag{1.21}$$

1.1.1 Interactions with matter

We wish to compute

1. the probability that a particle survives an interaction with matter;
2. and its *mean free path*: the average distance a particle travels before interacting.

These are both related to the *cross section*: it quantifies the probability of a certain interaction to occur between two particles — in which case we refer to a *partial* cross section — or of any interaction happening between the two particles, in which case we talk about a total cross section.

For clarity, let us focus on a specific example: a beam colliding on a target. We suppose that the beam is much broader than the target (so we can ignore border effects), and that the particles are uniformly distributed both in space and time.

Then, we define the incident flux F by:

$$F = \frac{\#\ \text{particles}}{\mathrm{d}t\,\mathrm{d}A}\,, \tag{1.22}$$

and is measured in $\mathrm{s}^{-1}\mathrm{m}^{-2}$. We want to consider the particles which are scattered in the solid angle $\mathrm{d}\Omega$. Scattering is intrinsically stochastic, so we will be looking at averages: specifically, we are interested in the average number of particles scattered per unit solid angle per unit time:

$$\left\langle \frac{\mathrm{d}N_s}{\mathrm{d}\Omega} \right\rangle\,; \tag{1.23}$$

for simplicity hereafter we will omit the signs of average, but they will always be implied. This quantity is measured in $\mathrm{sr}^{-1}\mathrm{s}^{-1}$.

There was no unit time mentioned by the professor, but we need it for the dimensions to work.

The differential cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$, which depends on the energy of the incoming particle and on the direction of scattering, is then defined as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E,\Omega) = \frac{1}{F}\frac{\mathrm{d}N_s}{\mathrm{d}\Omega}\,. \tag{1.24}$$

The total cross section is defined as

$$\sigma(E) = \int_{S^2} \frac{d\sigma}{d\Omega} \, d\Omega \,. \tag{1.25}$$

This is dimensionally an area: it quantifies the "size of the target" which must be hit in order to have an interaction.

Typically we deal with slabs of material with many scattering centers. So, we can define the number of scattering centers per unit beam area (dimensionally, an inverse area):

$$n\delta x \,, \tag{1.26}$$

where n is the number density of the beam, measured in m^{-3}, and δx is the thickness of the slab along the beam direction.

The number of particles per unit time which can interact is given by FA, where F is the flux while A is the cross-sectional area of the beam. The dimensions of FA are those of an inverse time.

Then, the average number of particles scattered per unit time and solid angle is given by

$$\frac{dN_s}{d\Omega}(\Omega) = FAN\delta x \frac{d\sigma}{d\Omega} \,, \tag{1.27}$$

while the total number of particles scattered per unit time is

$$N_s = FAN\delta x \sigma \,. \tag{1.28}$$

The probability of a particle in the beam to interact while going through a layer of thickness δx will then be given by

$$\mathbb{P} = \frac{\text{particles which do scatter per unit time}}{\text{available particles to scatter per unit time}} = \frac{N_s}{FA} = N\sigma\delta x \,. \tag{1.29}$$

We are usually interested in characterizing the probability of interaction in a thick layer. The probability of interacting in a large thickness x is given by

$$\mathbb{P}_{\text{interaction}} = 1 - \mathbb{P}_{\text{survival}} \,, \tag{1.30}$$

where $\mathbb{P}_{\text{survival}}$ quantifies the probability that the particle will not interact, i.e. "survive".

The probability of interaction in the region x to $x + dx$: this will be a multiple of dx, which we call $\mathbb{P} = w \, dx$.

Since the events are independent, the probability of surviving up to a depth $x + dx$ will then be given by

$$\mathbb{P}(x + dx) = \mathbb{P}(x)(1 - w \, dx) \tag{1.31}$$

$$= \mathbb{P}(x) + \frac{\mathrm{d}\mathbb{P}}{\mathrm{d}x}\,\mathrm{d}x\ , \tag{1.32}$$

which means

$$\mathrm{d}\mathbb{P} = -w\,\mathrm{d}x\ , \tag{1.33}$$

therefore, if the probability is normalized so that $\mathbb{P}(0) = 1$, we have

$$\mathbb{P}(x) = e^{-\int w \mathrm{d}x}\,. \tag{1.34}$$

Then, the probability of interaction will be

$$\mathbb{P}_{\text{int}} = 1 - e^{-\int w \mathrm{d}x}\,. \tag{1.35}$$

The mean free path is calculated as

$$\lambda = \langle x \rangle = \frac{\int x\mathbb{P}(x)\,\mathrm{d}x}{\int \mathbb{P}(x)\,\mathrm{d}x}\,, \tag{1.36}$$

and by expanding the exponential we can see that $w = 1/\lambda$.

Since we also have the relation $\mathrm{d}\mathbb{P} = -w\,\mathrm{d}x = -N\sigma\,\mathrm{d}x$ (minus sign since we are considering survival, not scattering), we can identify

$$w = \frac{1}{\lambda} = N\sigma\,, \tag{1.37}$$

therefore

$$\lambda = \frac{1}{N\sigma}\,. \tag{1.38}$$

The number density of scattering centers can be derived as

$$N = N_A \frac{\rho}{\omega_A}\,, \tag{1.39}$$

where $N_A \approx 6.022 \times 10^{23}\,\mathrm{mol}^{-1}$ is Avogadro's number, while ω_A is the mean molecular weight.

If a beam of particles crosses matter, its intensity will be reduced. If N is the number of particles in the beam, it will decrease in a length $\mathrm{d}x$ by

$$\frac{\mathrm{d}N}{N} = \rho \frac{N_A}{\omega_A} \sigma\,\mathrm{d}x\ . \tag{1.40}$$

Often we define a different quantity, similar to the mean free path:

$$\lambda' = \rho\lambda = \frac{\rho\omega_A}{\sigma\rho N_A} = \frac{\omega_A}{\sigma N_A}\,, \tag{1.41}$$

which is measured in $\mathrm{kg/m^2}$. This is useful since often we need to deal with mediums of variable density, so it is easier if we do not need to account for it.

1.2 Interactions of astroparticles

change section name?

We must be able to describe the whole lifetime of the astroparticle: the processes affecting it in the astrophysical environment, close to the source ①; the processes happening in the atmosphere ②.

In the Moodle there is a figure for the opacity of the atmosphere for different photon energies.

Also, we must describe the processes happening when we are observing the particles. This is described by the physics of particle detectors ③.

We start with ③.

1.2.1 Astroparticles and detectors

We must consider nuclei (cosmic rays), photons, neutrinos and gravitational waves. We will neglect gravitational waves.

For nuclei, we need to account for the strong, weak and electromagnetic interactions; the strong interaction is the dominant process.

For photons, we just need to account for the electromagnetic interaction.

For neutrinos, we just need to account for the weak interaction.

For a proton-proton interaction, the strong-interaction cross section is typically $\sigma_{pp} \approx 45\,\text{mb}$, where $\text{b} = 10^{-28}\,\text{m}^2$ is a barn, the approximate area of a uranium nucleus.

We can read a research article, in which they measure the proton-air cross section [Col12]. In this work, figure 2 shows the cross section of proton-air interaction. It is on the order of the hundreds of millibarn, increasing with energy.

We can calculate the quantity λ' we defined before:

$$\lambda' = \frac{\omega_A}{N_A \sigma} \approx 93\,\text{g/cm}^2\,. \tag{1.42}$$

This is much lower than $\rho_{\text{atm}}/L_{\text{atm}}$: therefore a cosmic ray will typically not survive to the ground.

Let us present a simple model to derive the proton-proton cross section if we have the proton-air one. We approximate the proton-air cross section to be constant in energy. We can say that, to first approximation, $\sigma = \pi R^2$ with $R = R_T + R_p$. The radius of an atomic nucleus can be approximated by

$$r_T^{(A)} = r_0 A^{1/3}\,, \tag{1.43}$$

where $r_0 \approx 1.2\,\text{fm}$. Using this, we can get an estimate of the proton-proton cross section, to be compared with collider experiments.

Moving on to ②: the typical interaction length for pair production is given by

$$\lambda_\gamma = \frac{9}{7} x_0 \,, \tag{1.44}$$

since electrons interact in matter by giving off brehmssrahlung radiation, a process which is related to pair production. The quantity $x_0 \approx 36\,\mathrm{g/cm^2}$: the radiation length for electrons in matter. So, $\lambda_\gamma \approx 47\,\mathrm{g/cm^2}$ for $E > 10\,\mathrm{GeV}$.

In order to get information about these quantities, we can use data from the Particle Data Group, at http://pdg.lbl.gov.

Bibliography

[Col12] Pierre Auger Collaboration. "Measurement of the Proton-Air Cross-Section at $\sqrt{s}=57$ TeV with the Pierre Auger Observatory". Version 2. In: *Physical Review Letters* 109.6 (Aug. 10, 2012), p. 062002. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.109.062002. arXiv: 1208.1520. URL: http://arxiv.org/abs/1208.1520 (visited on 03/11/2020).

[Pat] C. Patrignani. *Cosmic Rays*. URL: http://pdg.lbl.gov/2017/reviews/rpp2017-rev-cosmic-rays.pdf (visited on 03/16/2020).