Astroparticle physics notes

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Contents

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Introduction

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There are two courses, for Astrophysics and Cosmology and for Physics, which bear the same name. The other one is by Francesco D'Eramo: it assumes a knowledge of Quantum Field Theory.

This course, instead, only requires knowledge of Quantum Mechanics. The first part of this course is devoted to an introduction about the basics of Quantum Field Theory and gauge theories.

"By the end of the 20th century [. . .] we have a comprehensive, fundamental theory of all observed forces of nature which has been tested and might be valid from the Planck length scale of 10^{-33} cm to the edge of the universe 10^{28} cm". David Gross, 2007.

The task in APP is to be able to discuss such a fundamental theory.

First of all, we need to address the two standard models: the ΛCDM model for cosmology and the Standard Model of particle physics.

There are points of friction between the two Standard Models. There are also several questions: neutrinos' mass, what caused inflation. . .

These problems have a common denominator: the interplay between particle physics, cosmology and astrophysics. What we seek is new physics, beyond the two standard models.

Books: M.E. Peskin, "Concepts of Elementary Particle Physics". The book is addressed to students who are not experts in QFT and particle physics, rather, it provides the fundamental knowledge for these topics.

The exam is a colloquium, an oral exam, for which we can prepare a presentation on a specific topic. There is no issue if we do not precisely remember a specific formula, it is about going deep in the concepts.

An overview of the astroparticle physics landscape

Fundamental particles: the SM of particle physics Elementary particles make up ordinary matter. Fermions have spin 1/2, and are composed of quarks:

$$\begin{bmatrix} u & c & t \\ d & s & b \end{bmatrix} , \tag{1}$$

leptons:

$$\begin{bmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{bmatrix} . \tag{2}$$

The muon and tau particles are similar to electrons, but with higher mass.

These particles' interactions are mediated by 12 vector bosons, which have spin 1: these are "radiation" (the term is outdated).

1. gluons (g) mediate the strong nuclear interaction, there are 8 of them;
2. the $W^\pm$ and Z^0 bosons mediate the weak interaction;
3. the photon (γ) mediates the electromagnetic interaction.

There was a need for a mechanism to provide mass to the weak bosons and the fermions: this is accounted for by the Higgs boson, which is a scalar (that is, it has spin 0). This realizes the electroweak symmetry breaking.

The issue is that gravity is missing. In order to describe it in this scheme we would need a way to quantize it: all of these particles are actually excitations of quantum fields.

There are two marvelous 20th century theories, but they are not compatible.

Unification of interactions In 1687 Newton unified two domains of interactions: the terrestrial phenomena and the celestial phenomena, establishing the universality of gravitational interactions.

In 1865 Sir Maxwell unified electricity and magnetism into electromagnetism.

In 1967 Glashow, Weinberg and Salam propose the Standard Model of Particle Physics, unifying the Electromagnetic and Weak interactions. This is not a true unification: it is more appropriate to say that they are "mixed together" into the Electroweak interaction.

In the Standard Model, there is a kind of "frontier" around 100 GeV: below this energy, we see two interactions: the electromagnetic and the weak interaction. They are very much different: photons are massless, so the interaction has an infinite range, while the weak bosons are massive.

How can these be unified? We will see; above 100 GeV this apparent profound difference disappears in favour of the electroweak interaction. This is a phase transition.

Is the electroweak interaction above 100 GeV massless or not? Above this energy there is still a difference between the coupling constants of the two interactions. Above this energy, the W and Z bosons are no longer massive.

Above 100 GeV the strong interaction is separated from the electroweak one. Maybe there is an energy at which the electroweak interaction is unified with the strong one? We shall explore this topic: there are theories (Grand Unified Theories, GUT) in which there is such a unification.

As the energy increases, the coupling of the strongest interactions becomes weaker.

The energy scale, however, is very large: around 10^{16} GeV: this is a "science fiction" energy scale, it is extremely large. This is close to the Planck mass: $M_P \sim 10^{19}$ GeV, so we might not be able to describe this energy range with vanilla SM.

The Standard Model of Cosmology Now we can work backwards in our energy scale: as time progresses forward from the Big Bang, the energy of particles decreases.

The symmetry group of the Grand Unified Theory is broken, so we get subgroups; at each transition some symmetry is broken.

The EM + weak into electroweak transition is not speculative: we have observed it at the LHC. On the other hand, the electroweak + strong into GUT transition is speculative.

When, in the expansion of the universe, we reach an energy per particle of ≈ 1 GeV we have a new transition: the quark-hadron transition, so free quarks become confined into hadrons such as protons and neutrons.

Around 1 MeV we have a new transition: nucleosynthesis.

Then, we reach recombination, which is when the radiation we see as the CMB is released.

There must be new physics somewhere: there is no room in the SM for dark matter, the matter-antimatter asymmetry, the mass of neutrinos.

Chapter 1

The particle physics Standard Model

For this lecture, a good reference is Peskin, chapter 2 [Pes19]. Wednesday 2020-3-11, compiled 2020-03-17

We wish to describe what we described yesterday as “matter” and “radiation”.

The problem is similar to the one we have in classical mechanics, an initial value problem: given the positions and velocities of the particles at a certain starting time t_0 we wish to compute their state at a later time t.

This classical description in which the particles are not wavelike fails at the microscopic level: we want to give a quantum description of such a system of particles. We will derive it from the classical description using the standard tools of quantization. We start with a refresher of the classical description.

1.0.1 The classical description of a system of particles

Our aim is to compute and solve the equations of motion. The usual approach is to use Hamilton's variational principle: it is the principle of least action, but it is not usually referred to as such: we are actually not *minimizing* the action but finding a *stationary point* for it. This could also be a maximum or a saddle point.

The action functional S depends on the coordinates $q_i(t)$ of the particles at at time t, on the derivatives of these positions $\dot{q}_i(t)$ which represent the velocities of the particles at a time t. We usually write $S\big[q_i(t), \dot{q}_i(t)\big]$.

If we fix $q(t_0)$ and $q(t_f)$, the positions at some initial and final time t_f, we can then trace out a path $q(t)$ and perturb it by $\delta q(t)$; we fix $\delta q(t_0) = \delta q(t_f) = 0$.

Under this perturbation of the path $q \to q + \delta q$, the action changes to $S \to S + \delta S$. We then ask that $\delta S = 0$.

S is an action: is dimensions are those of an energy times a time. In terms of the Lagrangian L, the action is defined as

$$S\big[q_i(t), \dot{q}_i(t)\big] = \int_{t_0}^{t_f} L(q_i(t), \dot{q}_i(t))\,\mathrm{d}t\ , \tag{1.1}$$

which means that the Lagrangian must have the dimensions of an energy. We will make use of a quantity called the Lagrangian density:

$$L = \int \mathscr{L}\Big(\phi(\vec{x}), \partial_\mu \phi(\vec{x})\Big)\, \mathrm{d}^3x \ . \tag{1.2}$$

From a finite number of particles we move to considering a field $\phi(\vec{x})$: this means that, in a certain sense, we are considering an infinite number of particles.

The dependence of the Lagrangian on the q_i and $\dot{q}_i$ shifted to a dependence on the spacetime coordinates x and their 4-derivatives $\partial_\mu x$. It could depend on many fields simultaneously, we omit this dependence for simplicity. Now, this Lagrangian density has the dimensions of an energy per unit volume.

Then, the action, computed in a region Ω of 4-dimensional spacetime, is

$$S = \int_\Omega \mathrm{d}^4x\, \mathscr{L}\Big(\phi(x), \partial_\mu(\phi(x))\Big)\,. \tag{1.3}$$

Now that we have established the notation, we can apply the action principle: we consider an infinitesimal variation of the field $\phi \to \phi + \delta\phi$. We require this variation to vanish not only at the initial and final time, but over all the boundary $\partial\Omega$:

$$\delta\phi\Big|_{\partial\Omega} = 0\,. \tag{1.4}$$

Then, imposing $\delta S = 0$ is equivalent to the Euler-Lagrange equations:

$$\frac{\partial \mathscr{L}}{\partial \phi(x)} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathscr{L}}{\partial\Big(\partial_\mu \phi(x)\Big)}\right) = 0\,. \tag{1.5}$$

If we have many fields ϕ_r, then we have a set of E-L equations for each of them. This is still classical: for example, classical (relativistic) electrodynamics is formulated in this way.

The momenta are

$$\pi(x) = \frac{\partial \mathscr{L}}{\partial \dot{\phi}(x)}\,, \tag{1.6}$$

where a dot denotes a time derivative and using this we define the Hamiltonian density by

$$\mathscr{H}(x) = \pi(x)\dot{\phi}(x) - \mathscr{L}(\phi, \partial_\mu \phi)\,, \tag{1.7}$$

and similarly to the Lagrangian we have the full Hamiltonian H

$$H = \int \mathrm{d}^3x\, \mathscr{H}\,. \tag{1.8}$$

Now, these fields (ϕ and π) are classical fields: we will apply classical quantization to them. They will need to satisfy the classical commutation relations:

$$\left[\phi(\vec{x},t), \pi(\vec{x}',t)\right] = i\hbar\delta(\vec{x}-\vec{x}') \tag{1.9}$$
$$\left[\phi(\vec{x},t), \phi(\vec{x}',t)\right] = \left[\pi(\vec{x},t), \pi(\vec{x}',t)\right] = 0\,. \tag{1.10}$$

1.0.2 A relativistic reminder

the energy-momentum four-vector is

$$p^\mu = \begin{bmatrix} E \\ \vec{p}c \end{bmatrix}\,, \tag{1.11}$$

where the greek index μ can take values from 0 to 3.

The metric signature used here is the the mostly minus one. So, $p^\mu q_\mu = E_p E_q - \vec{p}\cdot\vec{q}$, since we raise and lower indices using the metric $\eta_{\mu\nu}$.

The square norm of the 4-momentum is $p\cdot p = p^2 = E^2 - \left|\vec{p}\right|^2 c^2$. It is Lorentz invariant.

In the rest frame of the observer, $p^\mu = \left[E_0, \vec{0}\right]$, and this E_0 is just (c^2 times) the mass of the particle: this is the *definition* of mass.

When the relation is satisfied we have

$$p^2 = E^2 - \left|p\right|^2 c^2 = \left(mc^2\right)^2 \tag{1.12}$$
$$E = c\sqrt{\left|p\right|^2 + (mc)^2}\,. \tag{1.13}$$

When this relation is satisfied we say we are *on shell*: for virtual particles, instead, this might not be satisfied.

We will use natural units: $\hbar = c = 1$.

This means that we equate energies (eV) and angular velocities (Hz); also we equate times (s) and lengths (m).

The rest energy of the electron is $m_e \approx 511\,\text{keV}$. Let us consider an electron with a momentum p equal to its mass m_e: then, its uncertainty in position is of the order

$$\frac{\hbar}{pc} = \frac{1}{m_e} \approx 4\times 10^{-11}\,\text{cm}\,. \tag{1.14}$$

The dimensions of the lagrangian density, in natural units, are those of an energy to the fourth power, or a length to the -4, or a mass to the fourth.

Another useful exercise is to calculate the coupling of the electromagnetic field:

$$V(r) = \frac{e^2}{4\pi\epsilon_0 r} = \frac{e^2}{4\pi}\frac{1}{r}\,, \tag{1.15}$$

since we set $\epsilon_0 = \mu_0 = 1$. We can introduce the electromagnetic α: this is

$$\alpha = \frac{e^2}{4\pi} \times \frac{1}{\hbar c}\,. \tag{1.16}$$

This then becomes adimensional: $\alpha \approx 1/137$. It represents the strength of the electromagnetic interaction: the strength of the coupling of the photon to the electron. The fact that it is $\sim 10^{-2}$ is important: it allows us to work in a perturbative way, in powers of α.

What is the coupling of the strong and weak interactions? This will be discussed.

Next time, we will discuss symmetries and symmetry breaking.

1.1 Symmetries and conservation laws

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Our aim is to describe the fundamental constituents of matter with a Quantum Field Theory. The method used to derive the equations of motion is a variational principle: we will find a Lagrangian density for various particles, and then apply the variational principle to find their equations of motion.

A guiding principle on the description of these fundamental particles is based on using their symmetries. We have Nöether's theorem in Quantum Field Theory: from these symmetries we are able to find conserved quantities.

These symmetries are described with groups, since we can compose their application; the theory describing groups is very rich. For this lecture we will base ourselves on Peskin's chapter 2 [Pes19].

A group G is a set of elements endowed with an operation. The set of elements can be either discrete or continuous. Most of the time, in theoretical physics, we use continuous groups.

We distinguish:

1. **spacetime** symmetries: groups which transform our coordinate system for spacetime, such as Lorentz and Poincaré transformations;

2. **internal** symmetries: groups which transform a certain field, or a certain property of our quantum system.

For our set to be a group, we need to be able to define an operation — we will usually call it multiplication — between the elements of the group, such that if $a, b \in G$ then $ab \in G$. Also, we must have

1. associativity: $(ab)c = a(bc)$;

2. existence of the identity $\mathbb{1}$, such that $\mathbb{1}a = a\mathbb{1} = a$;

3. existence of inverses: there exists a^{-1} such that $aa^{-1} = a^{-1}a = \mathbb{1}$.

What is of interest to us is the association of the group with a transformation which is a symmetry: this is called a *representation*.

We call a transformation a symmetry if, after performing the transformation, the dynamics of the system do not change.

For a quantum mechanical system, we are interested in the observables: these are described by operators, whose eigenvalues are the observations we make, and which in the Heisenberg picture evolve like

$$\frac{\mathrm{d}}{\mathrm{d}t} O(t) = [H, O(t)] \,; \tag{1.17}$$

if an operator commutes with the Hamiltonian, $[H, O] = 0$, then the operator is constant.

If we perform a transformation in the form

$$|\psi\rangle \to |\psi'\rangle = U |\psi\rangle \,, \tag{1.18}$$

then the operators will change by

$$O \to O' = U^\dagger O U \,. \tag{1.19}$$

Note that whether we have $U^\dagger O U$ or $U O U^\dagger$ does not matter, since we ask observables O to be Hermitian, so $O = O^\dagger$.

We know that these transformations must always be unitary, because the conservation of probability implies that we must have $\langle \psi | \psi \rangle = \text{const}$: so,

$$U^\dagger U = \mathbb{1} \,. \tag{1.20}$$

This can be also stated as $U^\dagger = U^{-1}$.

So, the function associating a unitary operator U to an element g of the group is called its *unitary representation*.

A transformation G is a symmetry if $\forall a \in G$ we have

$$[U(a), H] = 0 \,, \tag{1.21}$$

that is, the unitary representation of the group element always commutes with the Hamiltonian.

If we have a state $|\psi\rangle$ with energy $H|\psi\rangle = E|\psi\rangle$, then the transformation commuting with the Hamiltonian means that $|\psi'\rangle = U|\psi\rangle$ has the same energy.

Now, we can move to an example, taken from Peskin [Pes19]. Consider the discrete group $\mathbb{Z}_2$, which only has the elements 1 and -1, with the same multiplication rules as those we would have if these elements were integers. So, the group is closed with respect to multiplication. It can be easily checked that this is indeed a group based on our definition.

In order for this to be of interest to us, we can consider a quantum mechanical system and find a unitary representation acting on its Hilbert space.

Let us suppose we have a QM system with a basis made of two states $\left|\pi^+\right\rangle$ and $\left|\pi^-\right\rangle$. Let us define the *charge conjugation* operator C, by:

$$C\left|\pi^+\right\rangle = \left|\pi^-\right\rangle \qquad \text{and} \qquad C\left|\pi^-\right\rangle = \left|\pi^+\right\rangle . \tag{1.22}$$

So, we can find a unitary representation of $\mathbb{Z}_2$ in this system: we need to define $U(1)$ and $U(-1)$. We define

$$U(1) = \mathbb{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad U(-1) = \sigma_x - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{1.23}$$

where the matrices are to be interpreted as acting on vectors expressed to the basis $\{|\pi_+\rangle, |\pi_-\rangle\}$.

So, we can say that our unitary representation looks like

$$\mathbb{Z}_2 \to \{\mathbb{1}, C\} . \tag{1.24}$$

Now, if $[C, H] = 0$ (and $\mathbb{1}$ commutes with H, which is always the case) then we say that "H has the symmetry $\mathbb{Z}_2$".

The interesting question to determine will be whether this is actually the case for our given group.

Groups can be subdivided into abelian and non-abelian ones. A group is abelian if for every a, b in G we have $ab = ba$, or equivalently, $[a, b] = 0$. It is not if this is not the case, that is, there exist a, b such that $ab \neq ba$.

The condition on the elements directly translates to a condition on the matrices of the unitary representation. If we have commuting matrices, we can simultaneously diagonalize them: for example, in the case of $\mathbb{Z}_2$ we can go to a basis in which

$$C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{1.25}$$

specifically the states need to be

$$|\pi_1\rangle = \frac{|\pi^+\rangle + |\pi^-\rangle}{\sqrt{2}} \qquad \text{and} \qquad |\pi_2\rangle = \frac{|\pi^+\rangle - |\pi^-\rangle}{\sqrt{2}}, \tag{1.26}$$

since then $C|\pi_1\rangle = |\pi_1\rangle$ (we write $C = +1$) and $C|\pi_2\rangle = -|\pi_2\rangle$ (we write $C = -1$). We will often use this notation, confusing operator and eigenvalue.

In the case of nonabelian groups it is not in general possible to diagonalize all the matrices; we can however write the matrices as a block matrix:

$$U_R = \begin{bmatrix} U_1 & 0 & 0 \\ 0 & U_2 & 0 \\ 0 & 0 & \dots \end{bmatrix} . \tag{1.27}$$

Do note that some elements of a nonabelian group can commute: for example, in the rotation group we have

$$[J^i, J^j] = \epsilon^{ijk} J_k \,, \tag{1.28}$$

so if we take $i = j$, that is, we consider rotations along the same axis, they will commute since then the Kronecker symbol is equal to zero.

1.1.1 Space translations

An element of the group can be written as

$$U(a) = e^{iaP} \,, \tag{1.29}$$

where the operator P, whose eigenvalue is the momentum, is called the generator of the transformation.

If we take a plane wave, for example,

$$\langle x | p \rangle = e^{ipx} \,, \tag{1.30}$$

then we have

$$\langle x | \, U(a) \, | p \rangle = e^{ip(x-a)} \,. \tag{1.31}$$

If our system is invariant under translations, then Nöether's theorem tells us that the momentum is conserved.

In order to be a physical observable P needs to be Hermitian: $P = P^\dagger$.

So, the adjoint of the unitary transformation is

$$U^\dagger(a) = \sum_n \left(\frac{(-iaP)^n}{n!} \right)^\dagger = \sum_n \frac{(iaP^\dagger)^n}{n!} = e^{iaP^\dagger} = e^{iaP} = U^{-1}(a) \,, \tag{1.32}$$

which confirms the fact that the transformation is unitary.

Let us say that the momentum operator P commutes with the Hamiltonian: $[P, H] = 0$. Then,

$$[U(a), H] = 0 \,. \tag{1.33}$$

All this is to say that a constant of motion O corresponds to an operator O which commutes with the Hamiltonian.

An example: the group G of 3D rotations. They depend on a continuous parameter $\vec{\alpha}$, just like translations depended on the parameter a.

The rotation is written as

$$U(\vec{\alpha}) = e^{-i\vec{\alpha} \cdot \vec{J}} \,, \tag{1.34}$$

where the components of the angular momentum have the following commutation relations:

$$[J^i, J^j] = \epsilon^{ijk} J^k \,. \tag{1.35}$$

We can multiply rotation matrices:

$$U(\vec{\beta})U(\vec{\alpha}) = U(\vec{\gamma}) \,. \tag{1.36}$$

This space of 3D rotations is called SO(3), since every rotation corresponds to a 3x3 matrix which is a rotation matrix — so it is orthogonal and has determinant 1.

We can look for 1D representations of the generators J^i: we get $J^i = 0$, which means that we are not actually performing a rotation. Which states transform this way? These are scalar states, spin 0.

For 2D representations, we have

$$J^i = \frac{1}{2}\sigma^i \,, \tag{1.37}$$

where the σ^i are the Pauli matrices.

We can also find 3D spin-1/2 representations.

A rotation in 2D, represented by an element of SO(2), corresponds to a phase shift, so we can say that it is equivalent to an element of U(1). This then allows us to see that SO(2) is abelian.

In general, we write for a unitary $n \times n$ unitary representation

$$U(n) \to e^{-i\alpha^n t^a} \,, \tag{1.38}$$

where the generators t^a are Hermitian matrices corresponding to Hermitian operators. In particular, one of these is the identity: $t^0 = \mathbb{1}$.

So, we omit it and say that we have $n^2 - 1$ generators for the SU(n) group. We shall see that each of these generators corresponds to a particle, and for the weak interaction we will have $2^2 - 1 = 3$ particles, while for the strong one we will have $3^2 - 1 = 8$.

Bibliography

[Pes19] M. E. Peskin. *Concepts of Elementary Particle Physics*. Oxford Master Series in Particle Physics, Astrophysics and Cosmology. Oxford University Press, 2019.