Advanced astrophysics notes

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October 3, 2019

30 september 2019

1 Introduction

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1.1 Topics

They are selected topics in stellar physics.

1. Stellar pulsations and Astroseismiology (dr. Michele Trabucchi);
2. stellar winds (dr. Paola Marigo);
3. final fates of massive & very massive stars (dr. Paola Marigo).

Basics in Stellar Physics: "astrophysics II" inside the bachelor's degree in astronomy (second semester). It can be taken as an optional course.

Material:

1. *Introduction to stellar winds* by Lamers, Cassinelli.
2. *Stellar Atmospheres: Theory and observations* (lecture notes from 1996).

and more on Paola Marigo's site.

1.1.1 Stellar Winds

Moving flows of materials ejected by stars. 20 to $2 \times 10^3\,\mathrm{km/s}$.

See *Bubble Nebula* in Cassiopea, there is a $45 M_\odot$ star ejecting stellar wind at $1700\,\mathrm{km/s}$.

Diagram: luminosity vs effective temperature. We see the *main sequence*. We can also plot the *mass loss rate*, $\dot{M} > 0$ in solar masses/year. An other important parameter is v_∞.

Diagram: mass loss (or gain) rate vs age of star.

Stellar winds affect stellar evolution, the dynamics of the interstellar medium, the chemical evolution of galaxies.

Momentum is approximately injected with $\dot{M} v$, kinetic energy with $\frac{[}{i}]12\dot{M}v^2$. Within $1 \times 10^8\,\mathrm{yr}$ around half of the infalling matter is reemitted.

1.1.2 Contents

We will start with the basic theory of stellar winds, and then: *coronal*, *line-driven* and *dust-driven* winds.

1.2 Final fates of massive & very massive stars

Masses over $10M_\odot$.

1.3 Stellar oscillations

...see slides.

Material: slides on moodle or Marigo's page.

1. *Pulsating stars* by Catelan & Smith (introductory);

2. *Theory of stellar pulsation* by Cox (harder).

Written exam, partial exam on stellar pulsation.

2 Variability in Astronomy

First observations of variable stars: ~ 1600, omicron-Ceti. It changes in magnitude by 6 orders of magnitude.

Others are found from the 1600 onwards, but since the XX century the reason is unknown. Is it *rotation*, *eclipses*?

For some it sure are eclipses, but the Cepheids are different. See δ-Cephei, asymmetric continuous curve. What if stars *pulsate*?

The *light curve* is the luminosity curve over time.

We can also look at the *phased* light curve. Of course we need the period: the phase is

$$\varphi = \frac{(t - t_0) \bmod \Pi}{\Pi} \tag{1}$$

where Π is the period. $E(t) = \lfloor (t - t_0)/\Pi \rfloor$ is the epoch.

We can then measure the period, but if the light curve is multiperiodic we can subtract the model from the curve to see if there are additional periods: this is *prewhitening*.

We can also look at the luminosity in Fourier space.

Of course there are issues with observational gaps (day-night, full moon): aliases; accuracy, duration of observations...

Also, the period can change in time.

Things have improved a great deal with large-scale surveys and space suveys.

2.1 Classification

By variability type: regular, semi-regular or irregular.

By intrinsic variability: extrinsic, external to the star: eclipses, transits, microlensing, rotation; intrinsic: rotation, eclipses (self-occultation), eruptive and explosive variables, oscillations, secular variations (?).

Whether rotation is to be considered intrinsic or extrinsic is a matter of taste.

Oscillations can be classified by several criteria.

The geometry can be radial (classical pulsators) or non-radial.

The restoring force can be the pressure gradient or the gravitational force (bouyancy, not gravitational waves).

The excitation mechanisms can be different.

The evolutionary phase and mass of the star can also be different.

3 Summary of stellar structure & evolution

Eulerian: properties of a gas are fields, the position is the position of an observer. To differentiate position with respect to time is meaningless: position is an independent variable. $f = f(r^i, t)$.

Lagrangian: we follow an element of fluid: $\mathrm{d}r^i \big/ \mathrm{d}t = v^i$. We can identify univocally these fluid elements.

When treating stellar structure & evolution, we look at mass layers $\mathrm{d}m$. $f = f(m, t)$. Do note that m is the mass of the whole full sphere under a certain layer, not the mass of the shell.

In the lagrangian case, we use the convective derivative $\mathrm{d}/\mathrm{d}t = \partial_t + v^i \partial_i$ where v^i is the velocity defined before.

3.1 Equations of stellar structure

We write these in the spherically symmetric case.

The continuity equation is:

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \,. \tag{2}$$

Momentum conservation is given by:

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \,, \tag{3}$$

where

Energy conservation is given by:

$$\frac{\mathrm{d}L}{\mathrm{d}m} = \varepsilon - \varepsilon_\nu - \varepsilon_g \,, \tag{4}$$

where L is the luminosity, ε is the rate of nuclear energy generation per unit mass, while ε_ν is the rate of energy loss due to neutrino emission per unit mass, and ε_g is the work done by the gas per unit mass & time.

The energy transfer equation is:

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla \,, \tag{5}$$

where $\nabla = \partial \log T / \partial \log P$ is the temperature gradient, which has contributions from radiation, conduction, convection...

With the diffusion approximation, we can write the gradient as

$$\nabla = \nabla_{\mathrm{rad}} = \frac{3}{16\pi acG} \frac{\kappa_R L P}{m T^4} \,, \tag{6}$$

where a is a constant depending on the Stefan-Boltzmann constant and the speed of light.

where κ_R is the Rosseland mean opacity, given by

$$\frac{1}{\kappa_R} = \frac{\displaystyle\int_0^\infty \frac{\mathrm{d}B_\nu}{\mathrm{d}T} \frac{1}{\kappa_\nu} \,\mathrm{d}\nu}{\displaystyle\int_0^\infty \frac{\mathrm{d}B_\nu}{\mathrm{d}T} \,\mathrm{d}\nu} \tag{7}$$

Substituting in the result in (6) we get:

$$L = -\frac{64\pi^2 ac}{3} r^4 \frac{T^3}{\kappa} \frac{\partial T}{\partial m} \tag{8}$$

where κ is a generalized opacity, the harmonic mean of the Rosseland opacity κ_R and the convective opacity $\kappa_c = 4acT^3 / (3\rho\lambda_c)$.

Where λ_c is...?

If we need to deal with convection, this defies any simple modeling. There are instability criteria: where is it relevant? This is given by Ledoux's criterion,

$$\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} - \frac{\chi_\mu}{\chi_T} \nabla_\mu \,, \tag{9}$$

where:

$$\nabla_\mu = \frac{\mathrm{d}\log\mu}{\mathrm{d}\log P} \tag{10a}$$

$$\nabla_{\mathrm{ad}} = \left(\frac{\partial \log T}{\partial \log P}\right)_{\mathrm{ad}} \tag{10b}$$

$$\chi_\mu = \left(\frac{\partial \log P}{\partial \log \mu}\right)_{\rho,T} \tag{10c}$$

$$\chi_T = \left(\frac{\partial \log P}{\partial \log T}\right)_{\rho,\mu} \tag{10d}$$

which are thermodynamic parameters.

how are these called? What do they mean?

In the convective core, $\nabla \approx \nabla_{\mathrm{ad}}$, but outside of it we need something else.

Mixed-length theory model convection with "bubbles":

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Figure 22.8 in some PDF: run of adiabatic, radiation gradients vs $\log T$.

We compute ∇_{ad} and ∇_{rad} and see whether the region is convective or radiative.

We can move from the Eulerian and Lagrangian formalisms using the continuity equation. In the Eulerian formalism:

$$m(r) = \int_0^\infty 4\pi r^2 \rho(x)\, \mathrm{d}x \,. \tag{11}$$

We need several *constitutive equations* for the parameters ρ, c_P (heat capacity of stellar matter), the opacity κ, the nuclear transformation rate r_{ij} and the rate of generation of nuclear energy ε. These can all be considered as functions of P, T, μ.

We define:

$$\mu^{-1} = \sum_i (1+\nu_i) \frac{X_i}{A_i} \tag{12}$$

(CHECK)

The variables X, Y and Z represent the abundances of H, He and metals, and satisfy $X+Y+Z=1$.

We may need to know the metal mixture inside Z, but often we can approximate it as the Sun's distribution.

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho} \sum_j \left(r_{ji} - r_{ij}\right) \tag{13}$$

3.2 Classification of stars

Low mass stars have between 0.8 and 2 solar masses. Intermediate mass stars have masses between 2 and 8 $M_\odot$. Massive stars have masses of over $8 M_\odot$. (Add characteristics of these).

3.2.1 Low-mass star evolution

See slides for figures. What are Hayashi lines?

3.2.2 Intermediate-mass star evolution

Idem

4 Stellar oscillations

Today we will look at typical time-scales, the period-mean density relation, the energy equation and perturbation theory w/ linearization.

No derivations of the equations in the exam.

4.1 Time-scales

The *free fall* time scale is:

$$\tau_{\text{dyn}} \sim \left(\frac{R}{g}\right)^{1/2} = \left(\frac{R^3}{GM}\right)^{1/2}, \tag{14}$$

this is associated with pulsation. It is calculated using the travel time of a mass in free fall across the stellar radius accelerated by constant acceleration equal to surface acceleration.

We note that $\tau_{\text{dyn}} \propto \overline{\rho}^{-1/2}$. For the Sun it is about 25 min.

The *thermal* time scale is the relaxation time of deviations from thermal equilibrium:

$$\tau_{\text{th}} \sim E_{\text{th}}/L\,. \tag{15}$$

It is calculated as the time required for a star to irradiate all its energy.

Proof. Roughly, $3\int_V P\,\mathrm{d}V = -\Omega$ with $\Omega = -\int_M \dots$ □

Typically, $\tau_{\text{th}} \sim GM^2/(LR) \sim 10^7 \dots$ It is much larger than the dynamic time scale.

The *nuclear* time scale is even longer.

This allows us to say that oscillations will not be heavily affected by thermal conduction, and even less by nuclear processes: the pulsations will be almost *adiabatic*.

The best candidate for these oscillations are *sound waves*: is the adiabatic speed of sound roughly right?

The speed of sound is:

$$v_s^2 = \Gamma_1 \frac{P}{\rho} \tag{16}$$

If the gas follows the perfect equation of state

$$\frac{P}{\rho} = \frac{k_B}{m_H}\frac{T}{\mu}, \tag{17}$$

we get

$$v_s^2 = \frac{\Gamma_1 k_B T}{m_H \mu}\,. \tag{18}$$

Typical values are $\mu \sim (2X + 3Y/4 + Z/2)^{-2} \sim 0.6$, $\Gamma_1 = 5/3$, and $T_{\text{He}} \sim 4.5 \times 10^4\,\text{K}$.

So we get $v_s \sim 32.2\,\text{km/s}$.

The timescale is $\Pi \sim 2R/v_s \sim 22\,\text{d}$, while $\Pi_{\text{obs}} = 5.336\,\text{d}$, in terms of orders of magnitude it works.

We can use the equation for the sound speed in the virial theorem, and get:

$$\Omega = -3\frac{\int_M v_s^2/\Gamma_1 \, \mathrm{d}m}{\int_M \mathrm{d}m} M = -3\left\langle \frac{v_s^2}{\Gamma_1} \right\rangle M \tag{19}$$

If Γ_1 and v_s are independent, we can compute their averages separately.

This allows us to write Π wrt the moment of inertia, $I = \int_r r^2 \, \mathrm{d}m\,(r)$:

$$\Pi \sim \left(\frac{I_{\mathrm{osc}}}{-\Omega}\right)^{1/2} \tag{20}$$

This is further evidence that we are dealing with a dynamical phenomenon.

Of course the speed of sound changes throughout the interior of the star. We compute the period as the travel time of sound waves throughout the diameter:

$$\Pi = 2\int_0^R \mathrm{d}t(r) = 2\int_0^R \frac{\mathrm{d}r}{\sqrt{\Gamma_1(r)P(r)/\rho(r)}}\,, \tag{21}$$

since $\mathrm{d}t = \mathrm{d}r\,/v_s$.

We also rewrite the differential equation for P substituting $m = \rho r$. Doing this we get

$$\Pi = \overline{\rho}^{-1/2}\sqrt{\frac{3\pi}{2\Gamma_1 G}} \tag{22}$$

Which confirms Ritter's relation $\Pi \propto \overline{\rho}^{1/2}$.

This works for acoustic modes, but if we consider non-radial g-modes it stops working, such as variables of type ZZ Ceti.

4.2 The energy equations

Just an overview for now: we will consider the star as a thermodynamic engine.