

Astroparticle physics notes

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0.1 Introduction

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There are two courses, for Astrophysics and Cosmology and for Physics, which bear the same name. The other one is by Francesco D'Eramo: it assumes a knowledge of Quantum Field Theory.

This course, instead, only requires knowledge of Quantum Mechanics. The first part of this course is devoted to an introduction about the basics of Quantum Field Theory and gauge theories.

“By the end of the 20th century [...] we have a comprehensive, fundamental theory of all observed forces of nature which has been tested and might be valid from the Planck length scale of 10^{-33} cm to the edge of the universe 10^{28} cm”. David Gross, 2007.

The task in APP is to be able to discuss such a fundamental theory.

First of all, we need to address the two standard models: the Λ CDM model for cosmology and the Standard Model of particle physics.

There are points of friction between the two Standard Models. There are also several questions: neutrinos' mass, what caused inflation. . .

These problems have a common denominator: the interplay between particle physics, cosmology and astrophysics. What we seek is new physics, beyond the two standard models.

Books: M.E. Peskin, “Concepts of Elementary Particle Physics”. The book is addressed to students who are not experts in QFT and particle physics, rather, it provides the fundamental knowledge for these topics.

The exam is a colloquium, an oral exam, for which we can prepare a presentation on a specific topic. There is no issue if we do not precisely remember a specific formula, it is about going deep in the concepts.

0.1.1 An overview of the astroparticle physics landscape

Fundamental particles: the SM of particle physics Elementary particles make up ordinary matter. Fermions have spin 1/2, and are composed of quarks:

$$\begin{bmatrix} u & c & t \\ d & s & b \end{bmatrix}, \quad (1)$$

leptons:

$$\begin{bmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{bmatrix}. \quad (2)$$

The muon and tau particles are similar to electrons, but with higher mass.

These particles' interactions are mediated by 12 vector bosons, which have spin 1: these are "radiation" (the term is outdated).

1. gluons (g) mediate the strong nuclear interaction, there are 8 of them;
2. the W^\pm and Z^0 bosons mediate the weak interaction;
3. the photon (γ) mediates the electromagnetic interaction.

There was a need for a mechanism to provide mass to the weak bosons and the fermions: this is accounted for by the Higgs boson, which is a scalar (that is, it has spin 0). This realizes the electroweak symmetry breaking.

The issue is that gravity is missing. In order to describe it in this scheme we would need a way to quantize it: all of these particles are actually excitations of quantum fields.

There are two marvelous 20th century theories, but they are not compatible.

Unification of interactions In 1687 Newton unified two domains of interactions: the terrestrial phenomena and the celestial phenomena, establishing the universality of gravitational interactions.

In 1865 Sir Maxwell unified electricity and magnetism into electromagnetism.

In 1967 Glashow, Weinberg and Salam propose the Standard Model of Particle Physics, unifying the Electromagnetic and Weak interactions. This is not a true unification: it is more appropriate to say that they are "mixed together" into the Electroweak interaction.

In the Standard Model, there is a kind of “frontier” around 100 GeV: below this energy, we see two interactions: the electromagnetic and the weak interaction. They are very much different: photons are massless, so the interaction has an infinite range, while the weak bosons are massive.

How can these be unified? We will see; above 100 GeV this apparent profound difference disappears in favour of the electroweak interaction. This is a phase transition.

Is the electroweak interaction above 100 GeV massless or not? Above this energy there is still a difference between the coupling constants of the two interactions. Above this energy, the W and Z bosons are no longer massive.

Above 100 GeV the strong interaction is separated from the electroweak one. Maybe there is an energy at which the electroweak interaction is unified with the strong one? We shall explore this topic: there are theories (Grand Unified Theories, GUT) in which there is such a unification.

As the energy increases, the coupling of the strongest interactions becomes weaker.

The energy scale, however, is very large: around 10^{16} GeV: this is a “science fiction” energy scale, it is extremely large. This is close to the Planck mass: $M_P \sim 10^{19}$ GeV, so we might not be able to describe this energy range with vanilla SM.

The Standard Model of Cosmology Now we can work backwards in our energy scale: as time progresses forward from the Big Bang, the energy of particles decreases.

The symmetry group of the Grand Unified Theory is broken, so we get subgroups; at each transition some symmetry is broken.

The EM + weak into electroweak transition is not speculative: we have observed it at the LHC. On the other hand, the electroweak + strong into GUT transition is speculative.

When, in the expansion of the universe, we reach an energy per particle of ≈ 1 GeV we have a new transition: the quark-hadron transition, so free quarks become confined into hadrons such as protons and neutrons.

Around 1 MeV we have a new transition: nucleosynthesis.

Then, we reach recombination, which is when the radiation we see as the CMB is released.

There must be new physics somewhere: there is no room in the SM for dark matter, the matter-antimatter asymmetry, the mass of neutrinos.

Chapter 1

The particle physics Standard Model

For this lecture, a good reference is Peskin, chapter 2 [Pes19].

We wish to describe what we described yesterday as “matter” and “radiation”.

The problem is similar to the one we have in classical mechanics, an initial value problem: given the positions and velocities of the particles at a certain starting time t_0 we wish to compute their state at a later time t .

This classical description in which the particles are not wavelike fails at the microscopic level: we want to give a quantum description of such a system of particles. We will derive it from the classical description using the standard tools of quantization. We start with a refresher of the classical description.

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1.0.1 The classical description of a system of particles

Our aim is to compute and solve the equations of motion. The usual approach is to use Hamilton’s variational principle: it is the principle of least action, but it is not usually referred to as such: we are actually not *minimizing* the action but finding a *stationary point* for it. This could also be a maximum or a saddle point.

The action functional S depends on the coordinates $q_i(t)$ of the particles at time t , on the derivatives of these positions $\dot{q}_i(t)$ which represent the velocities of the particles at a time t . We usually write $S[q_i(t), \dot{q}_i(t)]$.

If we fix $q(t_0)$ and $q(t_f)$, the positions at some initial and final time t_f , we can then trace out a path $q(t)$ and perturb it by $\delta q(t)$; we fix $\delta q(t_0) = \delta q(t_f) = 0$.

Under this perturbation of the path $q \rightarrow q + \delta q$, the action changes to $S \rightarrow S + \delta S$. We then ask that $\delta S = 0$.

S is an action: its dimensions are those of an energy times a time. In terms of the Lagrangian L , the action is defined as

$$S[q_i(t), \dot{q}_i(t)] = \int_{t_0}^{t_f} L(q_i(t), \dot{q}_i(t)) dt, \quad (1.1)$$

which means that the Lagrangian must have the dimensions of an energy. We will make use of a quantity called the Lagrangian density:

$$L = \int \mathcal{L}(\phi(\vec{x}), \partial_\mu \phi(\vec{x})) d^3x. \quad (1.2)$$

From a finite number of particles we move to considering a field $\phi(\vec{x})$: this means that, in a certain sense, we are considering an infinite number of particles.

The dependence of the Lagrangian on the q_i and \dot{q}_i shifted to a dependence on the spacetime coordinates x and their 4-derivatives $\partial_\mu x$. It could depend on many fields simultaneously, we omit this dependence for simplicity. Now, this Lagrangian density has the dimensions of an energy per unit volume.

Then, the action, computed in a region Ω of 4-dimensional spacetime, is

$$S = \int_\Omega d^4x \mathcal{L}(\phi(x), \partial_\mu(\phi(x))). \quad (1.3)$$

Now that we have established the notation, we can apply the action principle: we consider an infinitesimal variation of the field $\phi \rightarrow \phi + \delta\phi$. We require this variation to vanish not only at the initial and final time, but over all the boundary $\partial\Omega$:

$$\delta\phi \Big|_{\partial\Omega} = 0. \quad (1.4)$$

Then, imposing $\delta S = 0$ is equivalent to the Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \phi(x)} - \frac{\partial}{\partial x^\mu} \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi(x))} \right) = 0. \quad (1.5)$$

If we have many fields ϕ_r , then we have a set of E-L equations for each of them. This is still classical: for example, classical (relativistic) electrodynamics is formulated in this way.

The momenta are

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)}, \quad (1.6)$$

where a dot denotes a time derivative and using this we define the Hamiltonian density by

$$\mathcal{H}(x) = \pi(x)\dot{\phi}(x) - \mathcal{L}(\phi, \partial_\mu \phi), \quad (1.7)$$

and similarly to the Lagrangian we have the full Hamiltonian H

$$H = \int d^3x \mathcal{H}. \quad (1.8)$$

Now, these fields (ϕ and π) are classical fields: we will apply classical quantization to them. They will need to satisfy the classical commutation relations:

$$[\phi(\vec{x}, t), \pi(\vec{x}', t)] = i\hbar\delta(\vec{x} - \vec{x}') \quad (1.9)$$

$$[\phi(\vec{x}, t), \phi(\vec{x}', t)] = [\pi(\vec{x}, t), \pi(\vec{x}', t)] = 0. \quad (1.10)$$

1.0.2 A relativistic reminder

the energy-momentum four-vector is

$$p^\mu = \begin{bmatrix} E \\ \vec{p}c \end{bmatrix}, \quad (1.11)$$

where the greek index μ can take values from 0 to 3.

The metric signature used here is the mostly minus one. So, $p^\mu q_\mu = E_p E_q - \vec{p} \cdot \vec{q}$, since we raise and lower indices using the metric $\eta_{\mu\nu}$.

The square norm of the 4-momentum is $p \cdot p = p^2 = E^2 - |\vec{p}|^2 c^2$. It is Lorentz invariant.

In the rest frame of the observer, $p^\mu = [E_0, \vec{0}]$, and this E_0 is just (c^2 times) the mass of the particle: this is the *definition* of mass.

When the relation is satisfied we have

$$p^2 = E^2 - |\vec{p}|^2 c^2 = (mc^2)^2 \quad (1.12)$$

$$E = c\sqrt{|\vec{p}|^2 + (mc)^2}. \quad (1.13)$$

When this relation is satisfied we say we are *on shell*: for virtual particles, instead, this might not be satisfied.

We will use natural units: $\hbar = c = 1$.

This means that we equate energies (eV) and angular velocities (Hz); also we equate times (s) and lengths (m).

The rest energy of the electron is $m_e \approx 511$ keV. Let us consider an electron with a momentum p equal to its mass m_e : then, its uncertainty in position is of the order

$$\frac{\hbar}{pc} = \frac{1}{m_e} \approx 4 \times 10^{-11} \text{ cm}. \quad (1.14)$$

The dimensions of the lagrangian density, in natural units, are those of an energy to the fourth power, or a length to the -4 , or a mass to the fourth.

Another useful exercise is to calculate the coupling of the electromagnetic field:

$$V(r) = \frac{e^2}{4\pi\epsilon_0 r} = \frac{e^2}{4\pi} \frac{1}{r}, \quad (1.15)$$

since we set $\epsilon_0 = \mu_0 = 1$. We can introduce the electromagnetic α : this is

$$\alpha = \frac{e^2}{4\pi} \times \frac{1}{\hbar c}. \quad (1.16)$$

This then becomes adimensional: $\alpha \approx 1/137$. It represents the strength of the electromagnetic interaction: the strength of the coupling of the photon to the electron. The fact that it is $\sim 10^{-2}$ is important: it allows us to work in a perturbative way, in powers of α .

What is the coupling of the strong and weak interactions? This will be discussed. Next time, we will discuss symmetries and symmetry breaking.

Bibliography

- [Pes19] M. E. Peskin. *Concepts of Elementary Particle Physics*. Oxford Master Series in Particle Physics, Astrophysics and Cosmology. Oxford University Press, 2019.