Radio Astronomy Techniques

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Contents

Chapter 1

Introduction

1.1 Some Fundamentals of Radiation

Radio waves are a form of electromagnetic (EM) radiation, just like visible light but at frequencies that are far too low for our eyes to detect. The laws of physics require that EM radiation contains waves of both electric and magnetic fields and that in a vacuum these fields must be perpendicular to each other and to the direction of propagation of the waves. These electric and magnetic fields both oscillate in their own planes. The two conjugate characteristics of radio waves are wavelength and frequency and for a single wave are related as:

$$\lambda\nu = c \tag{1.1}$$

Electromagnetic waves (radio or otherwise) are often created by a charged particle (usually an electron) when it undergoes an acceleration. The reason that we usually consider the electron to do the radiating, and not the proton, is that the electron will undergo a much greater acceleration than the proton with the same magnitude of force because of its much smaller mass. Light behaves both as a wave and a particle. The oscillating electric and magnetic fields that we described earlier is the wave description of light. We can also describe light by a particle we call the photon. The photon has no mass, and its energy, E, is given by

$$E = h\nu = \frac{hc}{\lambda} \tag{1.2}$$

1.1.1 The Radio Window

The radio window, as it can be called, ranges in frequency from 10 MHz $(1 \times 10^7 Hz)$ up to 300 GHz $(3 \times 10^{11} Hz)$, or in wavelengths from 30 m down to 1 mm. The energies of radio photons range from about 10^{-19} to 10^{-15} ergs (i.e., very tiny!!!). The boundaries of the radio window are due to atmospheric and ionospheric processes.
What happens at frequencies higher than the lower limit and higher

than the higher limit of the radio band?
At low frequencies (below about 10 MHz) free electrons in the ionosphere easily absorb and/or reflect radiation. Frequencies below this can be generated and transmitted on Earth, but they can't escape through the ionosphere. And likewise, waves with frequencies less than 10 MHz arriving from space cannot penetrate the ionosphere to reach ground-based radio telescopes. Frequencies below about 10 MHz must be observed from space-based radio telescopes.

At the high frequency boundary, absorption by H2O and O2 in our atmosphere blocks the radiation from space. An especially unfortunate aspect of this is that these and many other molecules are abundant in space and so it is of scientific interest to observe at these higher frequencies to probe the emission and absorption by these molecules in space (such as in star formation regions). Radio observatories that operate at mm wavelengths, therefore, are desirable and must be located at high altitude, dry locations (e.g. on mountain tops).

1.1.2 Spectroscopy

The radio-frequency spectrum appears to the user as a graph of intensity or flux density (i.e., the amount of radiation) vs. frequency or wavelength, as is often the case now for optical spectra taken with modern digital detectors.
There are three general types of spectra, as described by Kirchhoff's rules. These are given as:

- **Continuous Spectra:** When a radiation source emits at all frequencies over a range without breaks, the spectrum is called a continuous spectrum and the emitting object is called a continuum source. A classic example of a continuum source is an incandescent lamp. A plot of a continuous spectrum, for example, will show intensities over a broad range of frequencies, although the intensity of the emission can vary significantly.

- **Bright Line or Emission Line Spectra:** When a radiating object emits radiation only at some very specific frequencies, or wavelengths, the spectrum contains a set of discrete bright lines. These lines of light are called emission lines. The reason that a light source would emit only at some very specific frequencies is due to the quantum physics of atomic and molecular structure. An atom or molecule can be in an excited energy state and then spontaneously drop to a lower energy state. To conserve energy, it gives off a photon that carries the exact amount of energy that the atom or molecule loses. However, due to the laws of quantum mechanics, the internal energy levels of atoms and molecules are restricted to a set of discrete values. Thus, changes in energy (and hence the emitted photon energy or frequency) can only have certain specific values.

- **Dark Line or Absorption Line Spectra:** A more complicated and interesting case arises when the radiation from an intense continuum source passes through a cool, transparent gas of atoms or molecules. Some of the atoms or molecules in the gas can absorb photons from the continuum

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to raise them into a higher allowed energy level. The photons that they absorb must have exactly the same amount of energy that the atom or molecule gains, and so only the photons of certain, specific frequencies will be removed from the continuum. To an observer stationed beyond the cool gas, the spectrum will show the continuous spectrum of the background source with the radiation at these specific frequencies appearing darkened, thus the dark line nomenclature.

1.2 The Sky Coordinate System

1.2.1 Right Ascension and Declination

So, picture a globe of the sky, which is called the celestial sphere, with a globe of the Earth at its center. Figure 1.2.1 should help with this visualization. Extensions of the lines of longitude on the Earth produce similar lines on the sky, which we call lines of Right Ascension, often abbreviated either as RA or with the Greek letter α. Extensions of the lines of latitude make lines on the sky called declination, abbreviated either as Dec or δ. Since the apparent rotation of the sky is actually due to the rotation of the Earth, the poles of the celestial sphere are the points directly overhead the poles on the Earth. **The sky, then, appears to rotate like a sphere about an axis running through it from the north celestial pole to the south celestial pole.**

The details of the sky coordinate system, though, are a bit more complicated because of the rotation of the Earth. This rotation produces a constant relative motion between the lines of longitude and right ascension. Since this doesn't happen with the lines of latitude and declination, declination is easier to handle, and so we'll discuss it first.

It is natural to choose the 0°-line of declination to be the exact extension of the 0°-line of latitude. Just as we call the 0°-line of latitude on the Earth the equator, we denote the 0°-line of declination on the sky as the celestial equator. Then, the extension of the 10°-line of latitude to the sky points to the 10°-line of declination; likewise for the 20° line, the −10° line, and so forth. If you are standing at a latitude of +45°, for example, and you point at the spot straight overhead, you will be pointing at a spot on the sky of declination +45°.

Now let's consider the lines of RA; remember that they are complicated by the Earth's rotation. If at some moment in time we define the 0-line of RA to be the extension of the 0°-line of longitude, a minute later these lines will be shifted and, as the Earth rotates, they will continue to shift relative to each other. Of course, we want the lines of RA to be useful for defining the position of an object on the sky. So, we need the lines of RA to be fixed relative to the stars, not the Earth. What astronomers have chosen to do is to pick noon on the first day of Spring (approximately March 21) as the moment in time when the 0-line of RA aligns with the 0°-line of longitude (the latter passing through Greenwich, England). And then, through our knowledge of how the Earth rotates during the day, and how it moves around the Sun in its orbit throughout the year,
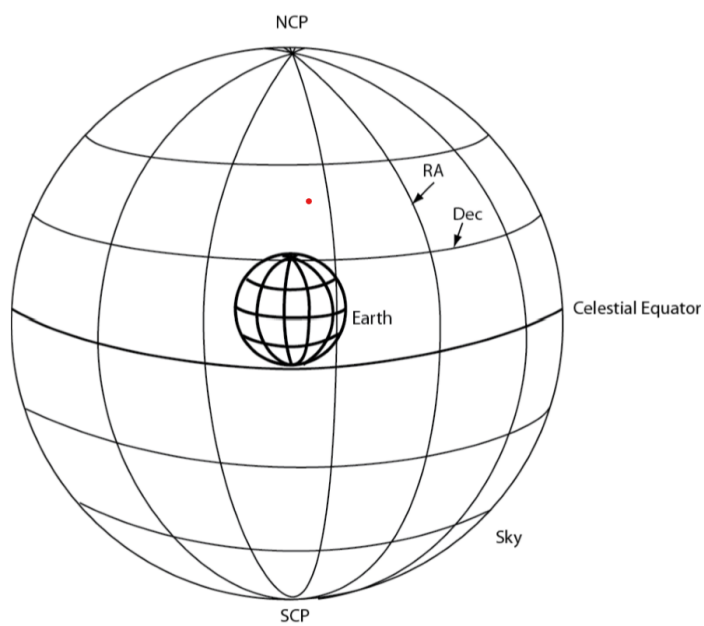
Figure 1.8: The celestial sphere, containing lines of RA and Dec, with the Earth at the center.

Figure 1.1: The celestial sphere, containing lines of RA and Dec, with the Earth at the center.

we can figure out by how much the lines of RA are shifted from the lines of longitude at any given moment of time during the year.

The units of right ascension are not what you would probably guess. Longitude and latitude are both given in degrees, and so it would be natural to expect the same with right ascension and declination; and declination is, in fact, given in units of degrees. Right ascension, though, has units of time; this needs some explanation. In short, using units of time for right ascension makes most calculations involving RA simpler, as will become apparent later. First, note that the Earth's longitudes don't behave like normal angles. In particular, the angular separation between adjacent longitude lines depends on the latitude where the separation is measured. Between 0° and 10° longitude there is an angular separation, as seen from the center of the Earth, of 10°, but only for points on the equator. Away from the equator, the angular separation between any two lines of longitude decreases with latitude and even becomes zero at the poles. The same, of course, occurs with the lines of right ascension. The angular separation between any two lines of right ascension is proportional to the cosine of the declination. Now, units of time turn out to be convenient to use for RA because the displacement between RA lines and longitude lines is related to the rotation of the Earth.

The units of RA are set up as follows. There are 24 hours of RA around the sky, and each hour is divided into 60 minutes of RA, and each minute contains 60 seconds of RA. Keep in mind, though, that 'one second of RA' does not equal 'one second of arc'. In fact, since there are 24 hours of RA around the sky and 360 degrees around a full circle, at the equator 1 hour of RA = 15° of arc, and so 1 minute of RA = 15 minutes of arc and 1 second of RA = 15 seconds of arc at the equator. Away from the equator, the general relation between the number seconds of RA and seconds of arc is given by

$$\text{seconds of arc} = 15\cos(\delta) \times \text{seconds of RA} \qquad (1.3)$$

where δ is the declination.

Another apparent oddity is that if you look at a map of the sky with North at the top then East is to the left side of the map. This is not due to some strange choice made by astronomers. This, in fact, is the same East as on maps of the ground. The reason for the apparent difference is that you, the observer, are between the ground and the sky and look in opposite directions to see each one. Try the following stretching exercise. Bend over and face the ground with the top of your head pointed to North. Stick your right arm out to point to the East. Now, without moving your arm or your body, twist your head around 18°o to look at the sky. (OK, you can't physically do that ... but you can imagine it.) If your right arm is still pointed to the East and the top of your head is to the North, with your head turned around 180° to look at the sky, what direction is East? Answer: to the left. As an additional oddity when you look at a map of the sky, RA increases to the left, which, at first, seems the reverse of what you would expect. We'll explain the reason for this in a little bit.
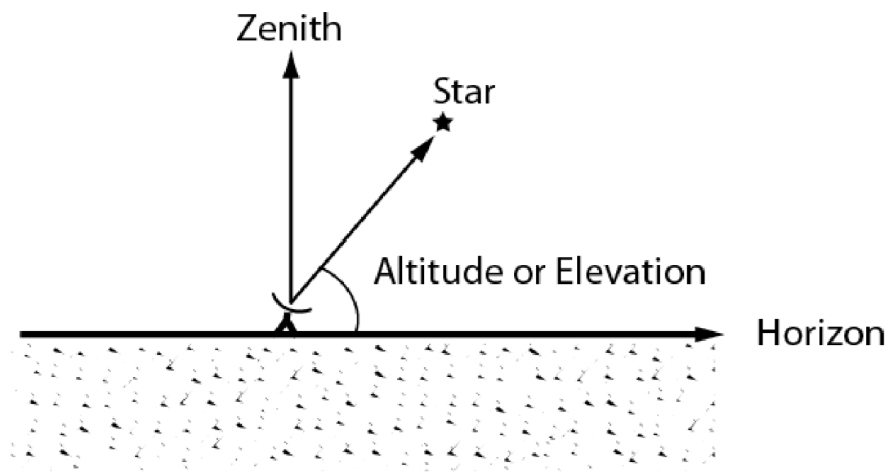
Figure 1.2: Schematic showing the relations between zenith, horizon, and altitude or elevation

1.2.2   Observer Centered Definitions

We also need a system that describes the way a particular observer (or telescope) sees the sky at any particular moment in time. So, we need definitions to describe an observer-centered coordinate system.

- **Horizon:** This defines the limit of what parts of the sky you can see at any particular moment. It is due to the ground and structures on the Earth blocking your view of the sky. If the Earth were transparent, you could also view the sky below your feet by looking through the Earth. Since the Earth is not transparent, you can't see this part of the sky, because it is below your horizon.

- **zenith:** This is the point in the sky directly overhead. Since the sky rotates continuously, this point on the celestial sphere continually changes, unless you are standing on either the North or South Pole.

- **altitude or elevation:** These are synonyms for the angular height of an object above your horizon at any given moment. When a star is on your horizon (so that it is either just rising or just setting) its altitude, or elevation, is 0° and when it is directly overhead – at the zenith – its altitude, or elevation, is 90°. Check out Fig. 1.2.2

- **azimuth:** This is the angular position perpendicular to the altitude, and defined as the angular position of an object along the horizon relative to due North. If the object is located north of the zenith, its azimuth is 0°. If it is south of the zenith, it has an azimuth of 180°. An azimuth
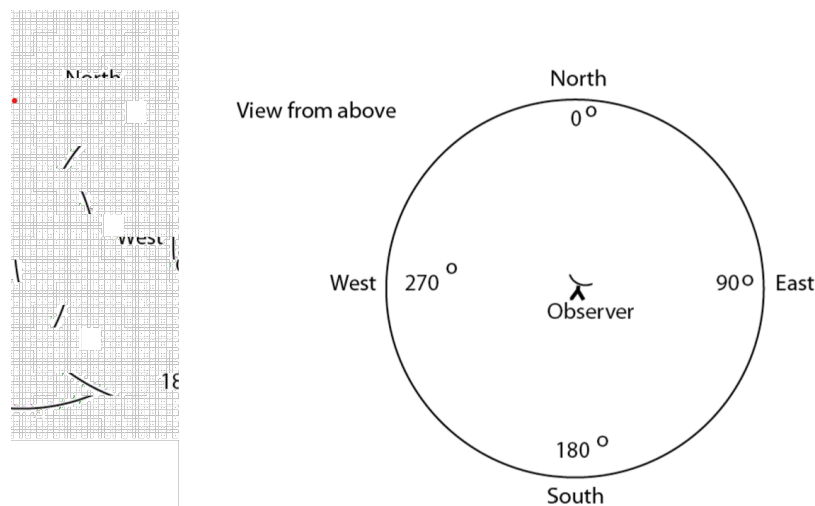
Figure 1.3: A view, from above, depicting azimuthal angles along with the cardinal directions around the horizon

of +90° is due East and 270° is due West. The angles of azimuth and their relation to the cardinal directions are shown in Figure 1.2.2. **Note that azimuth and altitude make a pair of angles that completely define an object's position in the sky relative to the observer. This last detail is important: all observers will see the same RA and Dec for an astronomical object, but the altitude and azimuth for the object will be different for observers at different locations, and indeed, even for the same observer, at different times of the day.**

- **meridian:** This is the line of RA that runs through your zenith. Equivalently, it is the line in the sky that runs through both your zenith and the celestial poles. Consider the following. As the Earth rotates, you will see the stars move across the sky from East to West. Consider, for a moment, just the stars that are at the same declination as your latitude; these are the stars that will pass directly overhead (i.e., through the zenith) on their trek westward. The moment that one of these stars is at the zenith is when it is highest in the sky (i.e., when it reaches its maximum elevation). Stars that are at different declinations will never pass directly overhead because the point directly overhead must have the same declination as your latitude (by definition of declination). All stars, regardless of their declination, will be highest in the sky at the moment that they cross your meridian. This is important because it is the best time to observe a particular object, and it also defines the mid-point of the time that an object is visible above the horizon (i.e., the moment in time halfway between rising and setting).

- **Transit (verb):** means to pass through the meridian. So, if your friend asks,"When does Mars transit?", he/she means at what time of day is Mars highest in the sky. This is a typical question whose real meaning is, "When is the best time to observe Mars?" Check your understanding: What is the azimuth of object when it transits? Assuming that the object does not pass through the zenith, then its azimuth will be either 0 degrees or 180 degrees at transit.

- **Hour Angle(HA):** An object's hour angle is the amount of time (in hours) since the object transited. For example, if the object is currently at the meridian, its hour angle at that moment is 0. If the object transited one hour ago, it has an HA of +1 h, and if it will transit in two and a half hours then its hour angle is -2.5 h.

- Local Sidereal Time(LST): This is defined as the RA of the meridian. While you are observing, the computer will keep track of your LST. This number is useful to help you keep track of what objects are transiting or when a particular object will transit. You must know an object's RA (as given in a catalog) in order to observe it. And, if you also know the LST, you can calculate the object's hour angle from:

$$HA = LST - RA \qquad (1.4)$$

The name LST suggests that this is a measure of time. In fact, it is, but it runs faster than the time we normally use. Consider the definition of the units of time that your watch or clock uses. One hour on your watch is defined as 1/24th of a day, which is defined as the average time period between transits of the Sun (i.e., from noon to noon). This method of measuring time is called solar time, because the Sun is used as the reference. Sidereal time uses the celestial sphere (or stars, other than the Sun) in the same way. One sidereal hour is 1/24th of the time period between transits of the same star. Time on the local sidereal clock is given by the RA of the meridian. For example, when the 12-hour line of RA is directly overhead, the local sidereal time is 12 hours. You don't want to use sidereal time as a real measure of time, though, because one hour of sidereal time is not equal to one hour of solar time. Each star rises 4 minutes earlier each day, meaning that a sidereal day is 4 minutes shorter than a solar day. This is due to the Earth's orbit because of the Earth's motion about the Sun, in the time it takes to rotate once it has moved 1/365th of the way around its orbit, and so the direction of the Sun has moved slightly. The Earth must rotate an additional four minutes for the Sun to reappear directly over the same place on the Earth.
An important point to remember is that a celestial object will transit when your LST is equal to the RA of the object. The Galactic Center, for example, has a RA of about 18 h, so it is highest in the sky (and best observed) at an LST near 18 h. Note, however, that because LST is slower' than solar time, the time of day when the Galactic Center transits
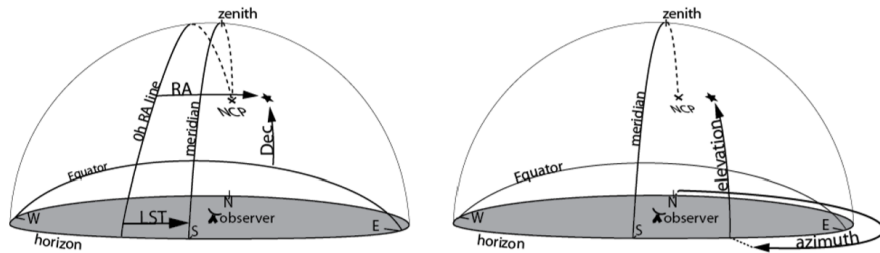
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Figure 1.11: – Depiction of sky coordinates as viewed by the observer (left) and observer-centered coordinates (right).

Figure 1.4: Depiction of sky coordinates as viewed by the observer (left) and observer centered coordinates (right)

will change throughout the year. In the northern hemisphere, 18 h LST is in the daytime during the winter months, while it occurs at night-time during the summer months, gradually shifting by 4 minutes every day

- **Universal Time(UT):** This is the solar time in Greenwich, England (i.e., the time on the clock where the longitude is 0o). This is useful for marking the exact time of an observation or an event. One doesn't need to know, then, from what time zone the observation was made. It defines a time that is the same for everyone on the Earth. Universal Time is particularly useful for situations in which the different time zones can cause confusion. Airlines, for example, actually schedule their flights using UT, but translate to local time for the customers.

  Now we can understand why hours, minutes, and seconds make a convenient choice of units for RA and why RA increases to the East. Suppose you wish to observe an object whose RA is 12 hours and 10 minutes. You look up at the sky and note that the object straight overhead has an RA of 4 hours and 10 minutes. Now you can ask, "How long will it be before my desired object transits?" The calculation you need to do is basically to treat the RA of the stars like the tick marks on a clock. Currently, the LST is 4:10 and the question is really asking when will the LST be 12:10. So the answer is simply 12:10 – 4:10 = 8 hours. (This does not account for the minor difference between the movement of your clock, which is based on the Sun, and the movement in LST, which is based on the stars.) Sources further to the East will transit at a later LST, hence their RA is larger; i.e., RA increases to the East.

  Check out Figure 1.2.2!

1.2.3   Apparent Size:

In addition to an object's position, how we see it also involves its apparent size. Our perception of size for distant objects is determined by the object's angular size, which is related to the object's linear size and distance. However, we do not generally know an astronomical object's distance, so we often cannot know its linear size; but we can perceive its angular size. In reality, objects in the sky appear to us as two-dimensional while angles, as we just discussed, are only one-dimensional. We need, therefore, a two-dimensional description of angular size.

This is called a solid angle, which we denote as Ω. To help gain an understanding of solid angle, consider a celestial object with a circular cross-sectional area small enough in angular diameter that $angular\ \ size(\theta) = \frac{diameter}{distance\ \ from\ \ observer}$ applies. The calculation of the solid angle of the object, then, is essentially the same as calculating the geometric area of a circle. (Hopefully, you remember that the area of a circle $= \pi R^2 = \pi(diameter/2)^2$.) Since the angular diameter of the circular object is θ(in radians), using our analogy between area and diameter of an object, its solid angle, Ω, is

$$\Omega = \pi \left(\frac{\theta}{2}\right)^2 = (\pi/4)\theta^2 \tag{1.5}$$

1.3   Basic Structure Of Traditional Radio Telescope:

1.3.1   Parabolic Reflector

Since radio waves are easily reflected by metallic surfaces but not easily refracted, no radio telescopes are refractors (in contrast to visible-wavelength telescopes where lenses are sometimes used). The radiation can then either be detected at the focal point of the dish or reflected back through the middle of the dish and focused and detected behind the dish. The former case is known as a prime focus telescope and the latter as a Cassegrain telescope.

Unlike visible light reflectors, radio dishes do not need highly polished surfaces. Radio waves, being much longer than visible waves, can be reflected by a much less precise surface. The general requirement for successful reflection of EM radiation is that irregularities in the reflecting surface be much smaller than the wavelength of the radiation. We explain in Chapter 3 that a successful reflection is obtained when

$$\delta l < \lambda/20 \tag{1.6}$$

where δl is the size of the surface irregularities of the reflector and λ is the wavelength of light that we wish to reflect Surface irregularities of the reflector and λ is the wavelength of light that we wish to reflect. At longer radio wavelengths,

in fact, the reflecting surface can even be a mesh, provided the holes in the mesh are sufficiently smaller than the wavelength of light to be reflected.

1.3.2 The Mount

There are two types of telescope mounts, which in reality are the same but with different tilts. All modern radio telescopes have altitude-azimuth or Alt-Az (also known as azimuth elevation or Az-El) mounts, meaning that the movement of the dish around one axis moves its pointing direction in altitude and movement around the other axis moves the pointing in azimuth.
The other type of mount, which is common for older and smaller visible-wavelength telescopes, is called an equatorial mount. With an equatorial mount, the axes correspond to the sky coordinates: movement around one axis moves the telescope in RA and around the other axis moves it in declination. An equatorial mount is really just an alt-az mount that is tilted so that the azimuth axis of the telescope points at the Celestial Pole rather than at the zenith.

1.4 Radio Maps

There are three common approaches to picturise the information obtained from the sky:

- **Contour maps:** The simplest way is with a contour map. An example of a contour map is shown in Figure 1.4. The contours are lines of constant intensity. An example of a contour map in which you may be familiar is a topographical map, in which the contours indicate the lines of constant elevation. Many closely spaced contours indicate a steep slope in a topographical map. In a similar way, a large number of closely spaced contours in an astronomical map indicates a region where the intensity is changing rapidly with the position.

- **Grey Scale Map:** A slightly more intuitive radio map is a grey scale map, in which no color is indicated, and the brightness variations in the source are directly indicated by the shades of grey, where black is the most intense and white is the least. A grey scale map of the same source as in Figure 1.4 is shown in Fig. 1.4. However, details in the structure are not revealed as well with a grey scale map as with a contour map, and so contour maps are often preferred for good analysis of the map. (One should be careful, though, not to present too many contours, especially for a source with complex structure. Overdoing contours with complex structure can sometimes make the image more confusing.) It is very common, and quite pleasing, to present maps with both the grey scale and the contour maps overlaid as in Fig. 1.4

- **False color maps:** Many radio astronomers are quite fond of presenting false color maps, as shown in Figure 1.4, in which the color does not

Figure 1.18: Contour map of the radio emission from the core region of the well-known radio source 3C279 at a wavelength of 6.0 cm. The contours displayed represent lines of intensities equal to 1, 2, 4, 8, 16, 32, and 64% of the maximum intensity in the map. (Map was made with data obtained from an observation with the Very Long Baseline Array in 2004.)
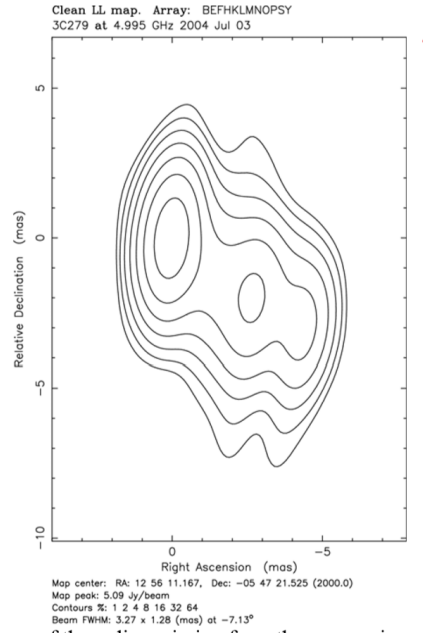
Figure 1.5: Contour map of the radio emission from the core region of the well-known radio source 3C279 at a wavelength of 6.0 cm. The contours displayed represent lines of intensities equal to 1, 2, 4, 8, 16, 32, and 64% of the maximum intensity in the map.

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Clean LL map. Array: BEFHKLMNOPSY
3C279 at 4.995 GHz 2004 Jul 03

Map center: RA: 12 56 11.167, Dec: −05 47 21.525 (2000.0)
Map peak: 5.09 Jy/beam
Beam FWHM: 3.27 x 1.28 (mas) at −7.13°
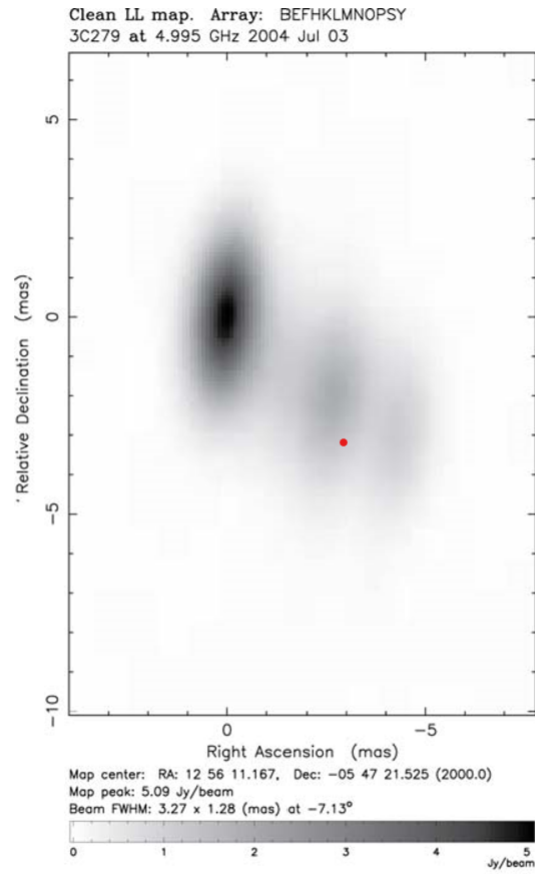
Figure 1.19: Grey scale map of the 6.0-cm emission from 3C279.

Figure 1.6: Grey scale map of the 6.0-cm emission from 3C279.
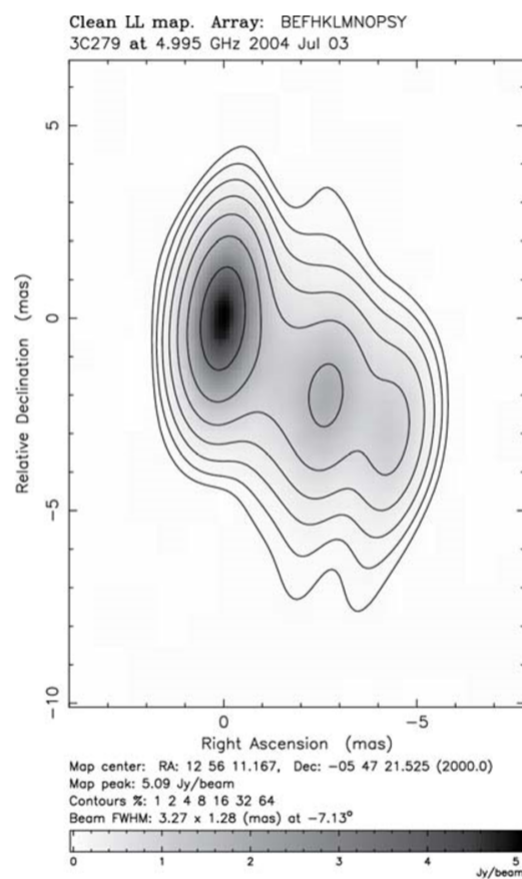
Figure 1.7: contour map of same source overlaid over greyscale

indicate different wavelengths but different levels of brightness. These images are usually accompanied with a color-wedge that indicates the relation between color and brightness.

Figure 1.8: False color map of the 6.0-cm emission from 3C279. The color wedge at the bottom indicates the transfer function relating color and brightness.

Chapter 2

Introduction To Radiation Physics

2.1 Measures of the Amount Of Radiation

2.1.1 Total Energy Emitted

One can describe a source's light output in terms of the total amount of energy emitted over a source's lifetime, at all frequencies, and in all directions. This, however, is clearly not the kind of measurement that we 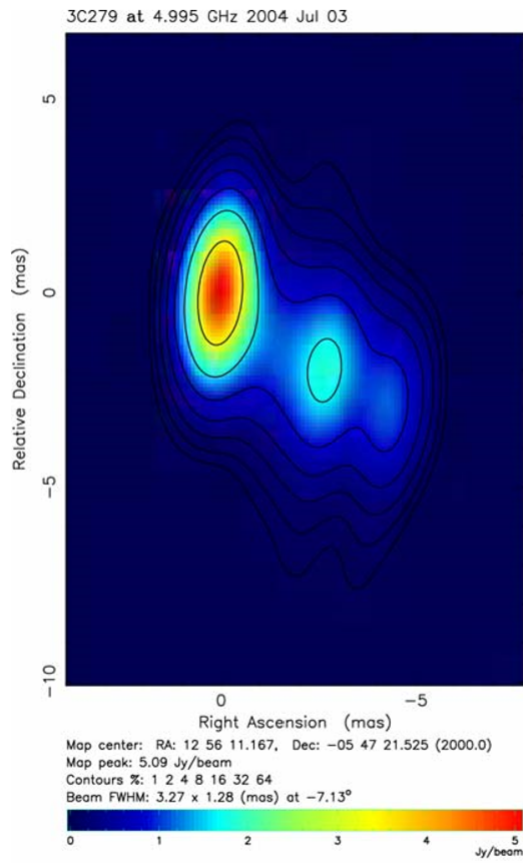can readily make. Since one can make measurements only over a finite time period, one can only describe the amount of energy detected in that time period. So that the measurement is independent of the time interval observed, a more useful quantity is luminosity or power, which is energy normalized by the time period.

2.1.2 luminosity(L)

Luminosity or power is the rate at which energy is emitted. Its S.I. units are J s^{-1} or watts (1 W = 1 J s^{-1}) and $erg s^{-1}$ in cgs. One calculates a source's luminosity or power by dividing the amount of energy emitted by the length of time over which the energy was emitted. This yields the rate of emission.
Luminosity, though, is not directly measurable because we don't detect all the radiation that was emitted by the source in any given second. The vast majority of the radiation is emitted in directions other than towards our telescope. This leads to our next quantity.

2.1.3 Flux(F)

The radiation power that we detect depends on the size of our telescope – the larger its cross sectional area, the more radiation will be detected – so we again wish to normalize our measurement, this time, by dividing by the area of the telescope. This gives the measure of flux, i.e. the amount of light energy per
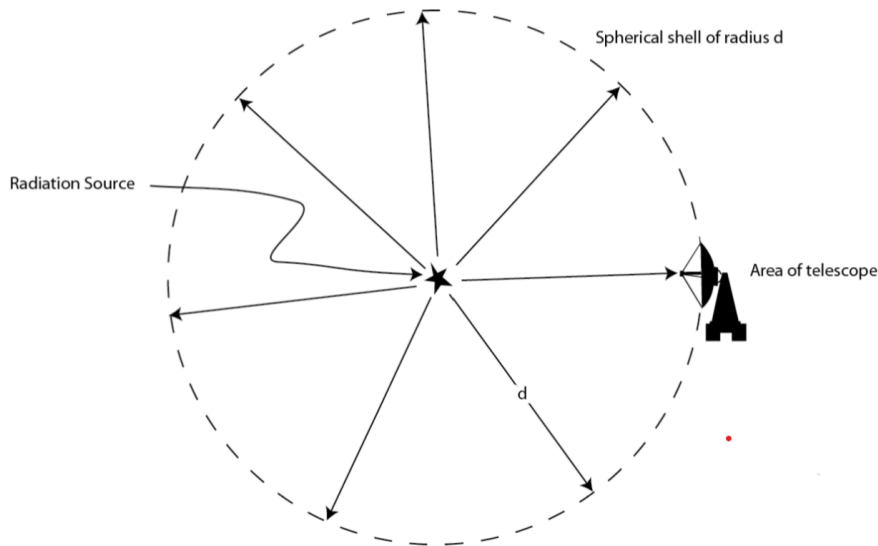
Figure 2.1: The radiation detected is the fraction of the total emitted that enters the small area of the telescope relative to the entire spherical shell of radius d.

unit time per unit area. The units of flux are J s^{-1} m^{-2} or W m^{-2} (SI) and ergs $s-1$ cm^{-2} (cgs).

We can determine the relation between the detected flux and the luminosity of the source by calculating the fraction of the total emitted radiation that enters our telescope. If the radiation from the astronomical source is emitted isotropically (i.e., equally in all directions) and the source is at a distance d, then the luminosity is spread out over a spherical shell of radius d (see Figure **??**). The fraction that we detect is given by the ratio of the effective area of our telescope (A_{eff}) to the area of the entire spherical shell. The surface area of the spherical shell is $4\pi d^2$, so the power detected, P, is

$$P = L\frac{A_{eff}}{4\pi d^2} \tag{2.1}$$

The radiation detected is the fraction of the total emitted that enters the small area of the telescope relative to the entire spherical shell of radius d.

$$F = \frac{L}{4\pi d^2} \tag{2.2}$$

Flux, also, is not a truly measurable quantity because we can't measure the radiation emitted at all frequencies over the entire EM spectrum. There is no detector in existence that can do that. We can only detect the radiation emitted over the tiny fraction of the electromagnetic spectrum to which our detector is sensitive.

2.1.4 Flux Density(F_λ or F_ν)

Flux density is the flux per unit frequency in the observed spectral range and it equals the detected flux divided by the width in frequency of the observation. So

$$F_\nu = \frac{F}{\Delta \nu} \tag{2.3}$$

where Δnu is the bandwidth or the range in frequency of the detected EM waves. **The symbol S_ν is often used by radio astronomers to represent flux density per unit bandwidth**. When working at visible wavelengths, astronomers tend to measure flux density in terms of wavelength rather than frequency, and so flux density is also often defined as flux per unit wavelength,

$$F_\lambda = \frac{F}{\Delta \lambda} \tag{2.4}$$

we'll only use F_ν , the flux per unit frequency, since that is the standard approach in radio astronomy. We call this flux density because it describes the density of flux in spectrum space. The mathematical relation between flux and flux density is

$$F = \int F_\nu d\nu \tag{2.5}$$

Flux density, at last, is a measurement that we can obtain directly and which all observers should agree upon, regardless of the telescope they use. The telescope measures the amount of power it receives, and this depends on the collecting area and the bandwidth. The flux density is the quantity about the source that we want to infer from these data. The amount of power a telescope collects from a source of given flux density is given by

$$P = F_\nu A_{eff} \Delta \nu \tag{2.6}$$

For astronomical sources at radio frequencies, the amount of energy we receive per unit time per unit area and per unit bandwidth is incredibly small, so both the SI and the cgs systems yield awkwardly small numbers for typical flux densities. To avoid carrying around excessively negative powers of ten in our calculations, radio astronomers have defined a unit of flux density, named after the father of radio astronomy, Karl Jansky. In units of the SI and cgs systems, a Jansky is defined as

$$1 \ Jy = 10^{-26} W \ m^{-2} Hz^{-1} = 10^{-23} \ ergs \ s^{-1} \ cm^{-2} \ Hz^{-1} \tag{2.7}$$

This is a very important unit to radio astronomy; it is a good number to commit to memory – along with its units.

2.1.5 Intensity(I_ν or I_λ)

The last important quantity regarding the amount of radiation is also the most fundamental. Intensity (or specific intensity), which is often referred to as surface brightness and sometimes just brightness, is the flux density per unit solid

angle.

Intensity has units of $W\ Hz^{-1}m^{-2}sr^{-1}$(for I_ν) or $W\ nm^{-1}\ Hz^{-1}m^{-2}sr^{-1}$(for I_λ)

. If you know the solid angular size of a source, you can calculate the average intensity of the source by dividing the measured flux density (F_ν) by the solid angle of the source (Ω). A general expression of intensity, then, is given by

$$I_\nu = \frac{F_\nu}{\Omega} \tag{2.8}$$

Intensity is, in fact, the correct description of the quantity that your eye measures and your brain interprets. When a light bulb appears especially bright (note the word that we use here: bright) your eye receives an especially large intensity of light. Consider how the appearance of the light is affected when you surround the bulb with a much larger soft-light globe? The total amount of light emitted is the same, but it comes from a larger surface, so the light seems less intense.

There are several aspects of intensity that are important to know:

- Flux density, F_ν , doesn't distinguish between the directions that the photons come from or travel to, whereas I_ν does. The directionality of the photons is important when studying the transfer of radiation through material, such as clouds of interstellar gas

- Intensity is independent of distance! This is hard to believe at first, but is easy to show. Intensity has two separate dependences on distance and they cancel with one another. Intensity depends on flux density and solid angle, as

$$I_\nu = \frac{F_\nu}{\Omega}$$
$$\text{where,}$$
$$F_\nu \sim \frac{L}{4\pi d^2 \Delta\nu}$$
$$\text{and,}$$
$$\Omega = \frac{\text{Cross-sectional area of source= A}}{d^2}$$
$$I_\nu \sim \frac{F_\nu}{\Omega} \sim \frac{Ld^2}{4\pi d^2 \Delta\nu A}$$

and, the d^2 cancel

$$I_\nu \sim \frac{L}{4\pi \Delta\nu A} \tag{2.9}$$

- Intensity is a direct measure of the object's surface brightness, i.e., the amount of energy radiated per second per unit area of the surface per unit solid angle right at the surface. Since intensity is independent of distance, the intensity one measures on Earth (or on Mars) will be the same as

that measured right at the surface. For example, consider two different objects–let's call them 'A' and 'B'–at the same distance with the same luminosity (L), flux (F), and flux density (F_ν), but which have different intensities. Let I_ν (A) ¿ I_ν (B). This can only occur when one of these sources is larger than the other so that Ω(B)¿Ω(A). Now consider what this means about the difference between the two sources. For B to have the same flux density even though it has a larger surface, it must be less intense. Intensity, we see, is directly related to the microscopic radiation processes of the object. If you measure a source's flux density and you can also measure the solid angle of the source (i.e., the source's size is large enough that you resolve its two-dimensional angular size) then you can get a direct handle on the microscopic radiation processes that produced the radiation. $1pc = 3.096 \times 10^{16} m$.

2.1.6 Relation Between Intensity and the Electric and Magnetic Field Waves

the energy that we measure is contained in the electric and magnetic fields. The energy flux of the radiation in terms of the fields is described by a quantity known as the Poynting vector, generally represented by S. And, since the strength of the magnetic field in the radiation is directly proportional to the strength of the electric field, the energy flux in the radiation can be expressed in terms of just the electric field strength.

$$S = \frac{1}{2} c \epsilon_0 E_0^2 (SI) \tag{2.10}$$

and

$$S = \frac{c}{8\pi} E_0^2 (cgs) \tag{2.11}$$

where the amplitude of the electric field waves is represented by E_0. (Note that a constant, ϵ_0, the permittivity of free space, appears in the SI expression but not in the cgs expression. This constant is not dimensionless; the definition of charge and electric field are quite different in the cgs system compared with the SI system, so changing systems is not as simple as just converting units.)

The important point to remember is that the intensity of the radiation is proportional to the square of the electric field in the waves, i.e.

$$I_\nu \propto E^2 \tag{2.12}$$

most radio telescope detectors produce a signal proportional to the square of the amplitude of the electric field so that the measured output is proportional to the power in the radiation.

2.2 Blackbody Radiation

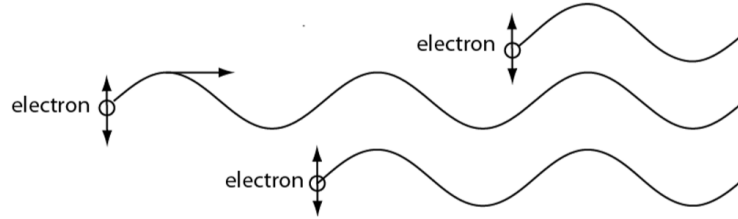I have skipped 2.3 and 2.4 for a future reading.

Figure 2.2: Schematic of the creation of coherent radiation. An oscillating electron emits a chain of sine waves which is joined by another identical chain of sine waves with exactly the same phase, and then another wave is added, in phase with the first two

2.3 Coherent Radiation

The radio-wavelength radiation emitted by an individual electron is undetectable by any radio telescope; the radiation we detect is the sum of many individual emission events by many individual electrons (or other charged particles). Now, there are two very different ways that electromagnetic waves emitted by separate electrons can combine; and the way they add together makes a huge difference in how we treat the radiation mathematically and also in its physical properties. let us imagine a single electron that is continuously oscillating at a fixed frequency, so that it continually emits electromagnetic waves at a single frequency or wavelength. A chain of sine waves, then, propagates away from the electron. Now imagine that after travelling some distance, this single chain of sine waves is joined by another chain of sine waves exactly in phase with it. And, a little further along, it is joined by another wave chain, and then another, as depicted in Figure 2.3

Eventually, the initial wave chain has gathered many other identical wave chains, all in phase with each other. Since all these wave chains are identical and with exactly the same phase, they add constructively, thereby amplifying the initial wave chain by a very large factor. This is, basically, what happens in a laser and the result is that at any specific position in space at any given instant in time, one can assign a frequency, wavelength, and phase. The resultant radiation is said to be coherent.

In contrast, imagine the light created by adding the radiation emitted by many unrelated electrons. Each electron emits its radiation at some random time, independent of the radiation emitted by the other electrons. This is what happens in an incandescent light bulb, for example, and in most astronomical sources. The composite radiation in this case will involve numerous sine wave chains, but the frequencies, directions of travel, and phases of all the separate wave chains at any given location in space and at any given moment in time will be completely unrelated. The resulting radiation, in this case, cannot be modeled by a single chain of sine waves. The composite radiation in this case is called incoherent.

In mathematical terms, coherence of radiation is often defined as when radiation at any location in space and time has a specific phase. However, the issue of coherence of radiation is actually more complex; its specific meaning depends on the context in which it is used. There are different forms of coherence and varying degrees of coherence. In the next section, we hope to convey a broader understanding of this phenomenon.

I am skimming through these sections for now and would come back later.

In conclusion, with incoherent light, the total intensity equals the sum of the component intensities, whereas with coherent light, the total intensity grows as the square of the sum of the component intensities.

Does this mean that we now have a problem with conservation of energy with coherent radiation? Shouldn't the intensity of N beams simply be N times the intensity of one beam, whether they are combined coherently or incoherently? Or do light beams added coherently magically have more energy than light beams added incoherently? The answer is no; the intensities of the coherent and incoherent beams differ, but their total energies are the same. Recall that intensity is flux of energy per unit solid angle per unit bandwidth. The total flux in a beam is the intensity integrated over solid angle and over the bandwidth, i.e.,

$$F_\nu = \int I_\nu d\Omega d\nu \qquad (2.13)$$

The high intensity of a coherent light beam is only present over a small range of wavelengths and the beam is very narrow in solid angle. Therefore, when the two intensities are integrated over frequency and solid angle they yield the same flux, or the same rate of energy flow.

2.4   Interference of Light

skipped, but will come back when appropriate.

2.5   Polarization

To describe this aspect, one talks of the polarization of the radiation. As an electromagnetic wave travels through vacuum, the direction of the wave propagation, of the electric field, and of the magnetic field must all be mutually perpendicular. Consider electromagnetic waves travelling along the +z axis. The oscillating electric field vector is perpendicular to this direction; we'll define these to be the $\pm y$ directions, and the magnetic field vector must be perpendicular to both of these, or the $\pm x$ directions. There are, however, an infinite number of directions that are perpendicular to the direction of travel and there is no law that determines in which of these directions the electric field must point - only that it be perpendicular to the magnetic field vector and to the direction of travel. So, for our +z-traveling waves, the electric field vector can also

oscillate in the $\pm x$ direction and the magnetic field vector then would oscillate in the $\pm y$ direction. In fact, the electric field can oscillate in any other direction in the x y plane, while the magnetic field oscillates along the perpendicular line in the x-y plane. Moreover, the electric field is not required to stay pointing in the same direction. For example, the electric field could rotate around the z axis as the wave propagates; (the magnetic field would then also rotate so as to stay perpendicular to the electric field).

But does this question of direction at all matter? To see why the direction of the electric field matters, consider the effect that the different electromagnetic waves discussed above have on an electron along the path of the radiation. The oscillating electric field will accelerate the electron, causing it to oscillate in the direction of the electric field. (The magnetic field only accelerates electrons that are already moving and the resultant acceleration of electrons with typical velocities is insignificant in comparison to that produced by the electric field). Now imagine that the electron is constrained to move only in the vertical direction (perhaps the electron is part of a very thin vertical wire antenna). Then, a chain of electromagnetic waves whose electric field points in the horizontal direction cannot accelerate this electron at all and so these waves will pass by the electron with no effect, whereas electromagnetic waves whose electric field oscillates in the vertical direction will cause the electron to oscillate. Even though these two sets of waves seem very similar in every way except for the direction in which the electric field vector points, they produce very different results. We see, therefore, that the direction of the electric field vector is indeed important.
we can treat any possible electric field as a resultant vector with two components lying in the x and y directions (see Figure 2.5). For any individual electromagnetic wave chain, then, we need only to measure the electric field in the x direction and the electric field in the y direction, and then we've measured the vector's components, which is sufficient for a complete description of the whole vector. These two base electric field measures we can call horizontal polarization and vertical polarization.

An alternative, and equally correct approach, is to view two different circular polarizations—one rotating clockwise and the other rotating counterclockwise—as the base components for describing any polarization. Just as a circularly polarized wave chain can be described as the sum of horizontally and vertically polarized wave chains, as we showed above, a linearly polarized wave chain can also be described by the sum of two oppositely rotating circularly polarized wave chains. And, the phase difference between the circular polarizations will determine the orientation of the resultant linearly polarized wave chain. Let's follow through with a particular example to see how this works. Consider Figure 2.12 as we discuss this example. Imagine two oppositely rotating circularly polarized wave chains in phase such that both their electric field vectors point upward (in the +y direction) at the same time. Then, at this time, the sum, or the total electric field, will point upward and will equal the linear sum of the two component electric fields. At this time, the resultant electric
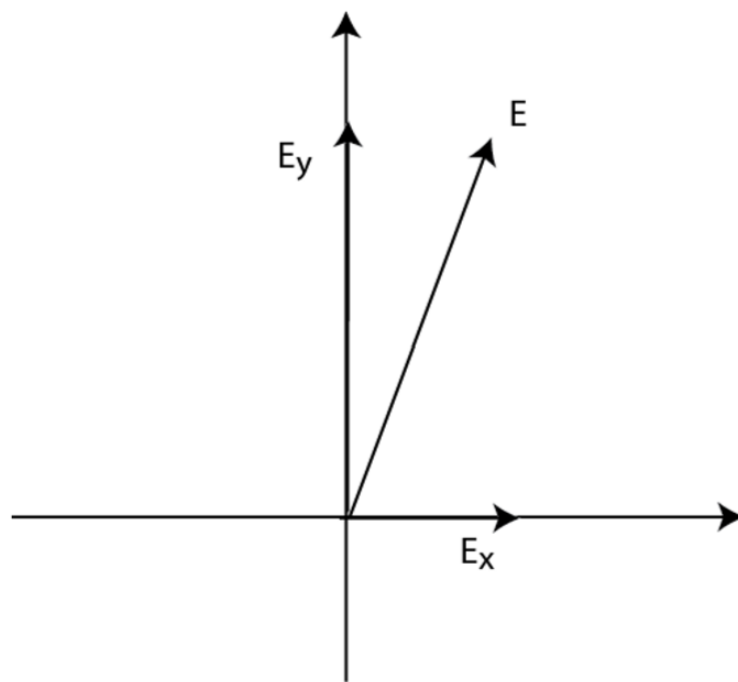
Figure 2.3: The total electric field vector of an electromagnetic wave can be fully described by its components along the x and y axes.

field has a maximum value. As the two circularly polarized wave chains rotate in opposite directions, they will gain opposite valued x-components which will cancel, while their y-components decrease. The resultant electric field, then, will still point in the +y direction, but will be smaller. As the two waves continue to rotate in opposite directions, the total E field vector will continue to decrease and stay in the +y direction. The total E field will continue to decrease until both of the rotating waves are horizontal, one pointing in the +x direction and one in the – x direction. At this moment the two component E fields completely cancel out, and so the resultant E field vector is zero. As the two waves continue to rotate they both gain -y components and the x components still cancel. The sum will reach a maximum again when the two vectors are lined up along the – y axis. We see, then, that the sum of these two opposite circularly polarized wave chains produces an oscillation only in the y direction, and so describes a linear polarization. In similar ways, a wave chain of any polarization can be described either by a sum of two perpendicular linear polarization functions of time and phase or by a sum of two oppositely rotating circular polarization functions of time and phase.

The two possible directions for circular polarizations are generally called left circular polarization and right circular polarization. However, there is, unfortunately, an inconsistency in these definitions between different disciplines, which often causes a fair bit of confusion. The reason for the inconsistency involves the different images one gets from different perspectives. One who imagines the view of an approaching circularly polarized wave chain will define the rotation of the electric field in the opposite sense from one who imagines the view of a receding wave chain as seen from the source. Now, since the initial development of radio astronomy was accomplished by electrical engineers, especially those who are practiced in designing radio transmitters, the definitions and conventions used by radio astronomers to describe radio waves were established from the perspective of the source, viewing the radio waves traveling away. The radio astronomer's definitions of right and left circular polarization (in the conventions established by the IAU (International Astronomy Union)), therefore, are the same as that given in the conventions of the IEEE (Institute of Electrical and Electronics Engineers). In this convention, right circular polarization describes an electromagnetic wave whose electric field rotates clockwise as the wave comes out of the transmitter when viewing the wave from the transmitter and in the direction of propagation. However, this means that the observer, who views the wave from the opposite direction, will see the electric field rotate counterclockwise. So, in terms of the observation of a circularly polarized radio signal, right circular polarization means that the electric field vector, as it enters the antenna, is observed to turn counterclockwise with time. Another, easy way to remember this is that a right circularly polarized wave obeys the right-hand rule in that when the thumb points in the direction of propagation, the curl of the fingers indicates the direction that the electric field turns.

There is confusion in these definitions because in most other fields of physics right and left circular polarization are defined in terms of the perspective of an approaching wave and so are given the opposite definitions from those used by

radio astronomers. Since radio astronomy is now considered a branch of physics there is a natural tendency to hold onto the definitions found in physics texts. And, since radio astronomers naturally think of themselves as observing the radio waves as they enter the antenna, not creating them and sending them off to space, the physicist's definition, conceptually, makes more sense. But, this is not the standard definition used in the field. The lesson for us here is to remember the origin of the field that we're working in, which in this case is electrical engineering, and that the standard definitions in the field are influenced by the field's history.

In an actual observation of radiation from space, all the wave chains that enter your telescope are not going to have exactly the same polarization. What you detect may have no net polarization, in which case it is called unpolarized radiation, or it may have a greater intensity in one polarization (horizontal vs. vertical, or left circular vs. right circular) in which case a percent polarization of the radiation is a useful measure.

Analysis and discussion of polarized radiation is easier with the use of another set of polarization terms which we will now discuss in the next subsection. This is the set of Stokes Parameters.

2.5.1   Stokes Parameters:

The first parameter, labeled 'I', is equal to the total intensity of all the radiation. So, I equals the sum of the intensities of all orthogonal polarizations. With the +z axis assigned to the direction of propagation of the wave, the electric field vector must be in the x-y plane, and so is described using only two orthogonal bases. We have introduced two commonly used sets of independent bases (horizontal and vertical linear polarizations or left and right circular polarizations), so we can write I as

$$I = I_x + I_y \tag{2.14}$$
$$or \ \ I = I_l + I_r \tag{2.15}$$
$$\tag{2.16}$$

where I_x and I_y are the intensities of any two orthogonal linear polarizations and I_l and I_r are the intensities of the left circular and right circular polarizations. The second parameter, labeled 'Q', is equal to the difference in the intensities of the two linear polarizations, so

$$Q = I_x - I_y \tag{2.17}$$

The third parameter, 'U', is very similar to the second, but involves a rotation of the x and y axes by 45°. If we call these shifted axes 'a' and 'b', shown in Figure ??, so that I_a is the intensity of light that is linearly polarized along an axis halfway between the x and y axes and I_b in the intensity of light polarized 90° relative to a, then
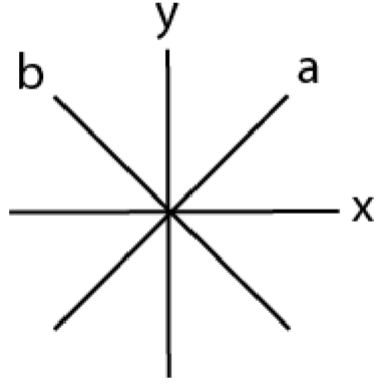
$$U = I_a - I_b \tag{2.18}$$

Figure 2.4: Two sets of orthogonal axes for measuring polarization of radiation. The x-y axes are used to calculate the Stokes Q and the a-b axes are used for the Stokes U.

The fourth parameter, 'V', is equal to the difference in the intensities of the left and right circular polarizations, so

$$V = I_R - I_L \tag{2.19}$$

Stokes V, alone, is a measure of the amount of net circular polarization.
Q and U determine the amount of linear polarization. To be more precise, the intensity of the net linear polarization, which we'll call L, is given by summing Q and U in quadrature, i.e.,

$$L = \sqrt{Q^2 + U^2} \tag{2.20}$$

One convenience of the Stokes Parameters should already be apparent; they provide a means of quantifying the linear and circular polarization intensities at the same time, And, if the detected radiation has no net polarization, linear or circular, then

$$Q = U = V = 0 \tag{2.21}$$

The Case of Fractional Polarization:
If there is no net circular polarization but there is non-zero linear polarization, then V = 0 and the fractional amount of linear polarization is given by L / I. If there is no net linear polarization but a non-zero circular polarization, then Q = U = 0 and the fractional amount of circular polarization is $|V|$/ I. If Q and/or U is non-zero and V is non-zero then the net polarization is elliptical. The total fractional polarization can, in general, be expressed as

$$\frac{\sqrt{Q^2 + U^2 + V^2}}{I} \tag{2.22}$$

With elliptical polarization, the orientation angle of the major axis of the ellipse, relative to the x and y axes (as defined above), is called the polarization angle, and is often denoted by θ, where θ is the angle starting from the +x axis and moving toward the +y axis. The polarization angle is related to Q and U by

$$\theta = \frac{1}{2}\tan^{-1}(U/Q) \tag{2.23}$$

If there is a net linear polarization, the direction of the polarization in the x-y plane is the same as this angle (since a line is just a flattened ellipse). Circular polarization, by its nature, has no major axis and so the polarization angle is is nonsensical.

You will also want to quantify the degree of ellipticity, which is commonly described by an angle denoted as β in which $\tan\beta$ is equal to the ratio of the amplitude of the electric field along the minor axis to that along the major axis of the ellipse, i.e., $\tan\beta$ = Eminor/Emajor. For pure circular polarization $\beta = \pm\pi/4$ radians and for pure linear polarization $\beta = 0$. The parameter β is determined from Stokes Parameters by

$$\beta = \frac{1}{2}\tan^{-1}(V/Q) \tag{2.24}$$

Stokes Parameters are advantageous to use in describing polarization largely because they are all measures of intensity. As such, there is no phase of the waves to worry about and they can easily be added, averaged, and manipulated algebraically. In contrast, consider the difficulty in calculating the average position angle of linearly polarized radiation from a number of measurements, keeping in mind that polarization position angles that differ by 180° represent the same radiation. Imagine you had two measurements yielding polarization position angles of 1° and 179°, which in terms of the physical orientation of the electric fields differ by only 2°. When you algebraically average them you infer a position angle of 90°, which is orthogonal to the correct answer. The proper approach is to keep track of the Stokes parameters, average their values, and then calculate the polarization angle

Chapter 3

Radio Telescopes

A typical radio telescope consists of a primary reflector (or dish), feed, transmission line, and receiver. We note that radio telescopes operating at very long wavelengths (typically 1 meter or longer) can take a very different form. Most radio telescopes are fully steerable, mounted on Alt-Az, (also called Az-El) mounts, and can point to any direction in the sky. A computer controls the motion of the telescope and continuously translates the sky coordinates of an astronomical object into current altitude and azimuth positions. We will not discuss further the mechanics of how a radio telescope is mounted or moves.
Some sections are skipped!
The primary reflector serves two important functions. First, it collects and focuses the radiation from astronomical sources, making faint sources more detectable. The amount of radiation collected depends on the telescope's effective area (A_{eff}), which is closely related to the physical area of the primary reflector. the power, P, of radiation collected from an astronomical source of flux density F_ν is given by

$$P = F_\nu A_{eff} \Delta \nu \qquad (3.1)$$

where the bandwidth, $\Delta \nu$, is the range of frequencies being detected. The bandwidth, as we will discuss later, is determined by the receiver. The larger the physical area of the reflector, the more power is collected from an astronomical source and the more readily faint sources can be detected. As we will see, a number of factors impact the amount of radiation that enters the receiver and so the effective area of a radio telescope is always smaller than its geometrical area.

The second function of the primary reflector is to provide directivity, which is a telescope's ability to differentiate the emission from objects that are close to each other, in angle on the sky. When using a single radio telescope to make a map (as discussed in Chapter 4), the directivity determines the resolution in the map. The directivity of a telescope depends, largely, on the diameter of the primary reflector and is governed by the principle of diffraction. Diffraction, in fact, limits the directivity of all telescopes regardless of wavelength or

frequency. The directivity of a radio telescope is commonly described as the telescope's beam pattern, which is the topic of our next section.

3.0.1 Beam Pattern

The beam pattern is a measure of the sensitivity of the telescope to incoming radio signals as a function of angle on the sky. This is similar to what optical astronomers often call the pointspread function. So the telescope is not only just sensitive to a point in space, its a non zero distribution of sensitivity over a a set of directions. The term "beam pattern" derives from the idea of a beam of radio waves leaving a transmitting antenna. Because the sensitivity pattern is the same, whether the antenna receives or transmits – a principle known as the reciprocity theorem – we are free to describe the pattern either way.

Ideally, we would like each feed in our telescope to collect radio signals from only one direction in the sky, so that when we point the telescope in a specific direction, the power detected through each feed corresponds to the radiation coming from only that spot on the sky. Unfortunately, this is not possible due to the diffraction of light. Diffraction is a phenomenon of all types of waves, whether light waves, sound waves or water waves, that occurs when the waves pass through an aperture (or opening in which only a fraction of the wave front is not blocked).

The derivation in the appendix shows that with the case of a uniformly illuminated rectangular aperture, total destructive interference occurs for the sum of all the rays when the source is located λ/D radians from the central axis, where D is the distance across the aperture. **skipped some part** For a uniformly-illuminated circular aperture, like that of a typical optical telescope, the total collected power is zero when the source is $1.22(\lambda/D)$ radians from the central axis.

A sample plot of the sensitivity of a parabolic reflector as a function of angle is shown in Figure 3.0.1

To understand these, return to the situation shown in where the phase difference is of order of $2\pi\frac{L\theta}{\lambda}$ where L be the distance between two interfering waves and θ is the angle when moved by sources causes the path difference resulting in interference of waves, and note that when $\theta = \lambda/L$ the phase difference is 2π radians (one complete cycle) and so the waves from these two points are in phase again and add constructively. Considering the entire aperture, though, one finds that there is only partial constructive interference. The sensitivity of the telescope, therefore, has a small increase after it first reaches zero, as shown in Figure 3.0.1. A telescope, then, can detect power from a source a good ways off-axis, although with much less sensitivity. As the off-axis angle increases further, the response of the telescope goes through a series of peaks and valleys in which there is partial constructive and destructive interference. These off-axis responses are called sidelobes and are undesirable as they can add confusion to observations. For instance, we could detect as much power from
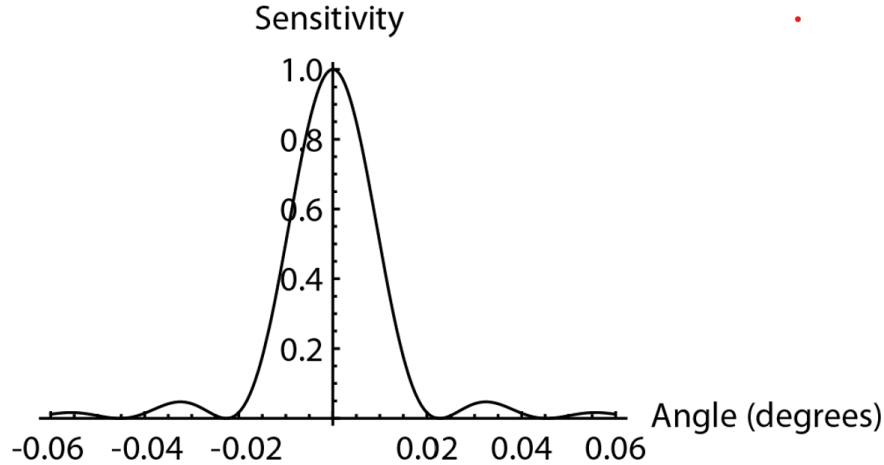
Figure 3.1: A typical profile of the relative sensitivity, or response pattern, of a radio telescope as a function of angle, θ, relative to the telescope's central axis. The beam shown is for a uniformly-illuminated reflector with a diameter of 40 m observing at 1.4-cm wavelength.

a bright source located in one of these sidelobes as we detect for a faint source on-axis. The central peak of the sensitivity pattern, in contrast, is often called the main beam. The telescope's sensitivity pattern, like that shown in Figure 3.0.1, is called the Airy pattern.

The width of the central peak of the Airy pattern is used to define the angular resolution of a singledish telescope. By custom, we measure the angular width of this peak between the two points where the received power falls to one-half the on-axis value. This angle we call the full width at half maximum (FWHM) of the main beam of the telescope.
The solid angle described by the main beam is considered to be the range over which the telescope can detect radio emission; that is, we can detect off-axis objects anywhere within this solid angle, albeit with reduced sensitivity, if they are located close to or inside the half power points. Stronger sources may be detectable even further from the on-axis direction, but customarily we use the half power points to define the angular dimensions of the beam. **This same angle defines the angular resolution of the telescope, because we cannot discern detailed structure of objects at smaller angular sizes. This same angle defines the angular resolution of the telescope, because we cannot discern detailed structure of objects at smaller angular sizes.** Our definition of the resolution angle, the FWHM of the main lobe of the antenna pattern, which is fairly standard in the radio astronomy community, is

not the same quantity used in visible wavelength astronomy. The canonical resolution angle of a visible-wavelength telescope is $\Delta\theta_{res}$=1.22 λ/D, which is the expression we gave above as the angle where the sensitivity pattern (of a circular, uniformly-illuminated aperture) first equals zero. This is also defined as the radius of the Airy disk. (Note that the plot in Figure 3.0.1 is one-dimensional profile, while there are two angles on the sky, and so the central peak can be thought of as a disk, and the angle to the first null is its radius.) The radius of the Airy disk, in fact, is larger than the FWHM of its central peak, which equals 1.02 λ/D. (The visible-wavelength astronomy convention follows the Rayleigh Criterion, which defines the minimum angle of separation between two sources to be resolved as when the center of the Airy disk of one source is in the first null of the other.)

Usually a small angular resolution is desirable, as it means that astronomical sources close together in angle on the sky can be distinguished or that fine angular detail can be discerned within a source. Since the FWHM of the main lobe is inversely proportional to the diameter of the reflector, we have that

large diameter telescopes not only collect more power from an astronomical source, but also provide better angular resolution.

3.1 Noise, Noise Temperature and Antenna Temperature

How do we describe noise in observation?
All the components in the receiver, especially the amplifiers, generate their own electrical signals that propagate through the receiver and are unrelated to the signal from the astronomical source. The power measured coming out of the detector, then, includes these extra signals, which interferes with our ability to detect and measure the power of the radiation from the astronomical source. These extra signals are undesirable, but cannot be avoided. We call this unwanted signal noise. We need to be aware of the noise, and know how to account for it. It is also desirable for the receiver to be designed so as to minimize the noise as much as possible.
Concept of Equivalent Temperature!
Characterizing noise signals generated in electrical circuits has, of course, always been of great interest in electronics. Nyquist in 1928, for example, found that a resistor in the circuit will add electrical noise with a power per Hz that depends solely on the resistor's temperature. For this reason, the electronic power in a circuit, in general, can be described in terms of an equivalent temperature, Tequiv, which equals the temperature of a resistor that would produce the same amount of power as the resistor. Following this convention, radio astronomers also describe the power travelling in the transmission lines and receiver in terms

of an equivalent temperature given by

$$T_{equiv} = \frac{P}{k\Delta\nu} \tag{3.2}$$

where k is Boltzmann's constant and $\Delta\nu$ is the bandwidth of the radiation with power P. The above equation can also be viewed as using the Boltzmann constant (which has units of energy per Kelvin) as a conversion factor, to express energy (or power per Hz) in units of temperature.

What is Antenna and Noise Temperature?

Some of the detected power is due to the astronomical source, which was converted by the antenna to electronic power in the transmission line. We call the equivalent temperature of the power that the antenna delivers to the transmission line, the antenna temperature, T_A. The far majority of the detected power, though, is due to noise from the receiver components. We describe the total noise power by the noise temperature, T_N, and each component in the receiver is characterized by its own noise temperature.

We need, now, to discuss how the final power measured in the output of the receiver relates to the antenna temperature and the noise temperature of each of the components. Keep in mind that both the source signals and the noise are affected by the amplification and losses that occur along the path through the receiver. The equivalent temperature of the final power output, then, is not simply the sum of the equivalent temperatures of all the sources in the path.

The power measured by the receiver:

Let's first focus on the signal from the astronomical source and see how its power is affected by the processes in the receiver. At each stage, the source signal is either amplified (when passing through an amplifier), or reduced by a loss (such as in a transmission line or in the mixer). We can use the gain, G, defined by Equation P_{out}/P_{in}, for each step; when passing through an amplifier, G ¿ 1, and when there is a loss, G ¡ 1. For example, if we assign a gain of G1 to the first element, which is the RF amplifier, then the power in the source signal after this stage is

$$P = G_1 k \Delta\nu T_A$$

By the time the signal exits the detector, the power due to the radiation entering the antenna has been amplified by a net gain, G, and so

$$P = Gk\nu T_A \tag{3.3}$$

Note that the antenna temperature describes the power in the input radiation before any amplification. Even though the amount of power increases when the signal is passed through an amplifier, the radiation is still described by the same equivalent temperature. Therefore, regardless of the amount of amplification in the system, the input radiation power will still be described by the same antenna temperature. Now, since we need to compare the noise power with the power from the astronomical source, we must describe both powers in the same way. In particular, an amplifier's noise temperature is defined by the equivalent

temperature of the noise power as if it were introduced at the input to the amplifier, and hence it is amplified along with the astronomical signal. Imagine, for example, that the receiver contained only an amplifier of gain G1 and noise temperature TN1. Then, the power output would be

$$P = G_1 k \Delta\nu T_A + G_! k \Delta\nu T_{N1} = G_! k \Delta\nu (T_A + T_{N1})$$

Now, if we have two amplifiers in succession, the first characterized by G_1 and T_{N1}, and the second by G_2 and T_{N2}, then the noise power due to the first is amplified by a factor of G2 along with the noise produced by the second amplifier. So the total noise power coming out of the second amplifier

$$P_N = G_2 G_1 k \Delta\nu T_{N1} + G_2 k \Delta\nu T_{N2} \tag{3.4}$$

the total gain in a succession of devices is just the product of the individual gains, so the total gain here is $G = G_1 G_2$. Hence, we can define the total noise temperature (T_N) in this sequence of devices by

$$P_N = G k \Delta\nu T_N \tag{3.5}$$

and so the total noise temperature relates to the individual noise temperatures as

$$T_N = T_{N1} + \frac{T_{N2}}{G_1} \tag{3.6}$$

Note that the noise contribution of the second amplifier is reduced by the gain of the first amplifier. The total noise power, including all elements in the signal path, then, is obtained by considering the G and T_N of all the elements in succession and applying the idea behind Equation (3.6). In general, for many elements in succession, we have:

$$G = G_1 G_2 G_3 \cdots \tag{3.7}$$

and

$$T_N = T_{N1} + \frac{T_{N2}}{G_1} + \frac{T_{N3}}{G_1 G_2} + \cdots \tag{3.8}$$

The total noise produced by all the components in the receiver is called the receiver noise temperature, T_{rec}.
Note that a lossy element, such as a mixer or transmission line, has G ¡ 1, which, in effect, increases the contribution to the total noise temperature from all the components that come after it.
How could a loss cause an increase in the noise temperature?
Remember that the noise temperature describes the power before amplification (or loss), which makes it easier to compare the noise temperature and the antenna temperature. The effect of a loss is to decrease the power in the source signal, and therefore any noise generated later in the signal path will appear larger, relative to the source signal, than if there had not been that loss, and so the value T_N of these later elements must now be larger.

The RF amplifier, the first device the radiation enters immediately after the feed, therefore, is the most critical in determining the total noise temperature. Since RF amplifiers usually have gains of at least a factor of 100 (or 20 dB), the contributions to the noise temperature of all other elements in the receiver and detector are reduced by at least a factor of 100. A noisy mixer, for example, does not add that much to the total noise temperature, provided that its loss is not too great. It is, therefore, extremely important that the RF amplifier has as much gain and as little noise as possible.

What is the total power entering the detector?
Recall that the antenna temperature is defined before all the amplification. Therefore, the total power entering the detector is given by

$$P = Gk\Delta\nu(T_A + T_N) \tag{3.9}$$

Considering the responsivity of the detector, we have the voltage out of the detector is

$$V = \alpha Gk\Delta\nu(T_A + T_N) \tag{3.10}$$

For the majority of astronomical sources, and with most receivers, it is the case that $T_A \ll T_N$. Therefore, even if we observe blank sky, the power out of the receiver is not zero because of the noise power. How do we solve for truncate the noise component when taking observations?
For these reasons, we cannot readily make total power measurements, but instead, we must make switched power measurements in which we measure the difference in voltage between when the telescope is aimed at the astronomical source, (called an on-source observation), and when it is aimed at blank sky (called an off-source observation). The latter observation is made by pointing the telescope more than a beam width away from the source. Switched observations are discussed in more detail in the next chapter.

It is tempting to believe that the switched observations completely subtract off the noise power, and hence that noise is not really a concern. However, this is not the case. The switched observations remove the offset in the measured power caused by the noise, but the fluctuations in the noise power still affect our measurement, and dominate our uncertainty in the antenna temperature. It is these fluctuations that limit the sensitivity of a radio telescope to detect a faint astronomical source.

The Statistics of Noise Signals

the variance is the mean square deviation from the average. Following common convention, we will use σ to indicate the standard deviation, and thus σ^2 as the variance. *The higher standard deviation, the more spread out the function is.*. And, therefore, for single-measurement experiments, the uncertainty equals the square root of the variance of that type of measurement.

What comprises fluctuations while measurement of power?

Now, when measuring the amount of power in electromagnetic radiation, the variance in the measure of that power depends on two effects. Since the amount of power in the radiation is proportional to the number of photons arriving per second, the variance in power must relate to fluctuations in the arrival rate of the photons. The power in the radiation is also proportional to E2 in the waves, and so the variance also depends on the fluctuations in the waves. The former effect is a consequence of the particle aspect of light while the latter is often called the wave noise and is a purely classical effect. When both effects are included in the statistics, the variance is found to have two terms; the variance due to fluctuations in the photon arrival rate depends on the number of photons and the wave noise depends on the number squared.

Or,

$$\sigma^2 \propto n + n^2 \tag{3.11}$$

where,

$$n = \frac{1}{\exp\left(h\nu/kT\right) - 1}$$

which (you hopefully recognized is the last factor in the Planck function) is proportional to the number of photons. Note that the magnitude of n is less than one when $h\nu$¿kT and greater than one when $h\nu$¡kT. Therefore, at higher frequencies, the first term in Equation 3.11 dominates and the uncertainty is proportional to the square root of the number of photons, i.e.

$$\sigma \propto \sqrt{n} \tag{3.12}$$

while at lower frequencies, the second term dominates and the uncertainty is proportional to the number of photons,

$$\sigma \propto n \tag{3.13}$$

If you are familiar with observations at visible frequencies, you may recall that one generally counts the number of photons – such as when measuring the brightness in the individual CCD pixels – and that the uncertainty in that count is directly proportional to the square root of the count. This is described by Poisson statistics, which describe the data when counting discrete objects. At radio frequencies, though, when the photon energies are so low that the statistics falls into the wave regime, the uncertainty in the number of photons N is directly proportional to N. Similarly, with the measured power we have

$$\sigma_P \propto P_N \tag{3.14}$$

with any measurement involving random errors, the uncertainty in the measure decreases by averaging more values, by a factor equal to the square root of the number of measurements. Similarly, by making the measurement for more seconds or by increasing the bandwidth, we make many independent measurements of the power. In general,

the number of independent measurements made over a time period of Δt and bandwidth $\Delta \nu$ is given by $\Delta t \Delta \nu$. Therefore, in an observation with a bandwidth $\Delta \nu$ and integration time of Δt, the uncertainty in the power measured, σ_P, is given by

$$\sigma_P = \frac{P_N}{\sqrt{\Delta t \Delta \nu}} \tag{3.15}$$

The power measured is that of both the signal from the astronomical source and the total receiver noise power, however there may also be some additional unwanted noise contributions. Since the receiver noise power is usually much greater than the power from the astronomical source, it is the fluctuations of the noise power that limits the ability to detect a weak astronomical source. For this reason, receiver noise, measured by the noise temperature, is a very important parameter to radio astronomers. The power measured is that of both the signal from the astronomical source and the total receiver noise power, however there may also be some additional unwanted noise contributions

3.2 Very Low Frequency Radio Astronomy

Since its inception, radio astronomy has tended toward higher observing frequencies. Early on, this was an obvious way to achieve higher angular resolution, since $\theta_{resolution} \propto \lambda/D$. As the field evolved, other reasons for observing at high frequencies also became apparent; particularly important was the fact that many molecules and interstellar dust emit radiation at shorter radio wavelengths. These and other factors pushed radio astronomers to develop telescopes and receivers working at ever higher frequencies, and the trend is likely to continue. As technology improves, the distinction between radio and the far infrared regimes is increasingly blurred, as radio techniques (in particular, coherent receivers) are being applied to even shorter wavelengths

This renaissance is mostly driven by scientific motivations, but as we shall see, there are important technological developments involved as well, with advances in computer technology being even more important than those in radio-frequency electronics. For example, the first major radio telescope initiative of this century – the Square Kilometer Array – is based on the design concept known as "Small-D, Large-N", in which very many antennas (large N), each one of relatively small size (small D) work in concert, their individual signals being combined and processed by computers, which (currently) enjoy an exponential growth in capacity. **skipped sections and subsections!**

3.2.1 The Low Frequency Window

The lower frequency limit for terrestrial radio astronomy is about 10 MHz. This limit is determined by the plasma frequency of the ionosphere (see Vol. II), which is usually in the range of 3 to 5 MHz. Below this cut-off frequency, radio waves arriving at the Earth are reflected outward by the ionosphere, thus never

reaching radio telescopes on the surface. The same effect occurs to low frequency radio waves generated on the Earth, reflecting these waves from the ionosphere back to the surface. This is the principle behind long distance short wave radio communication. Occasionally it is possible to observe as low as 3 – 5 MHz (when the free electron density in the ionosphere is unusually low) but reliable observing frequencies are somewhat higher, typically about 10 MHz. The upper frequency limit for "very low frequency" is somewhat arbitrary. The range from 300 MHz to about 600 MHz is where the transition from low frequency to high frequency methods and technology occurs. We adopt 300 MHz ($\lambda = 1$ m) as a reasonable value.

3.2.2   Antennas

skipped sections and subsections!

Chapter 4

Single-Dish Radio Telescope Observations

There are two principal – but greatly different - methods for making radio astronomy observations: single-dish observations and observations made with multiple telescopes that are combined together using interferometric techniques. In this chapter we give a thorough discussion of the basic considerations of radio observations made with a single telescope, generally referred to as single-dish observations.

The realtion that relates the power collected by the telescope to the flux density, F_ν, of the source radiation.

$$P = F_\nu A_{eff} \Delta\nu \tag{4.1}$$

where F_ν is the flux density of the source, A_{eff} is the effective area of the telescope and $\Delta\nu$ is the bandwidth over which the detection occurs

4.1 Basic Measurements with single dish telescope

4.1.1 Switched Observations

Imagine that you point a single-dish radio telescope at a known astronomical radio source and read the amount of power measured by the detector. Is all of this measured power due to the astronomical radio source? No. As explained in Chapter 3, even when the telescope is pointed at a bright radio source, the power measured by the detector is dominated by noise – primarily due to the electrical components in the receiver, but also, depending on the wavelength and the particulars of the observation, to background radiation from the sky and/or thermal radiation from the ground and the dish. Therefore, the total detected power is actually a very poor measure of the power emitted by the

radio source. **In fact, the measure of a non-zero power, alone, is not proof that you have even detected the astronomical source.**

To determine the amount of detected power due solely to the astronomical source, one must subtract off the power due to receiver noise and other sources of unwanted radiation. This is a fundamental principle of single-dish radio observations – all single-dish observations require the subtraction of the noise signal.

How do we solve this issue? This is accomplished by a process known as switching, in which the power is measured also when observing a nearby patch of sky that does not contain the astronomical source. The power measurement when the telescope is pointed at the source is generally called the on observation, and the measured voltage is denoted as V_{on}, and the measurement when pointed away from the source is the off observation, with voltage equal to V_{off}. Another form of switching, called frequency switching, which can be used for spectral line observations is discussed in Section 4.4 .

Let us see, how we remove this offset.

radio astronomers usually express the power measured from an astronomical source by its equivalent temperature called antenna temperature (T_A) while the noise power produced by the telescope receiver components is referred to as the noise temperature (T_N). We lump all of this unwanted power together, and call it the system temperature (T_{sys}). The voltage measured by the detector when the telescope is pointed at the source, or V_{on},

$$V_{on} = \alpha G k \Delta\nu (T_A + T_{sys}) \tag{4.2}$$

where α is the responsivity of the detector (and has units of V/W), G is the dimensionless total receiver gain, k is Boltzmann's constant, and $\Delta\nu$ is the observing bandwidth (set by the bandpass filter). Similarly, the voltage when pointed off-source, V_{off},

$$V_{off} = \alpha G k \Delta\nu T_{sys} \tag{4.3}$$

Therefore, we can remove the system offset term by:

$$V_{on} - V_{off} = \alpha G k \Delta\nu T_A \tag{4.4}$$

which is independent of the noise. So to find T_A, one then only needs to determine the conversion between temperature and voltage. **Instead of substituting in all the factors involved ($\alpha G k \Delta\nu$) this is accomplished by a calibration of the system temperature, which we explain in the next subsection.**

Variation of gain of Radio Receiver and corresponding correction required?

There is one other important issue about switching that we must address first. The G in Equations presented above is the total gain of the radio receiver, which varies over time and can cause significant errors if not accounted for. A variation in G of a few percent causes a deviation in the conversion from detected

power, in volts, to antenna temperature, in Kelvins, in Equation V_{on} by a few percent. However, since the detected power from the source, $V_{on} - V_{off}$, is a tiny fraction of the total detected power V_{on}, a small variation in G, causing a small change in V_{on}, translates into a tremendous error in $Von - Voff$, and could even result in a negative value for the calculated antenna temperature. **To prevent gain variations from producing such enormous errors, the on and off observations must both be made regularly throughout the observation**

1.2.1, 1.2.3, 1.3.1, 1.3.2, 1.3.3, 1.3.7, 1.3.10, 1.3.12, 1.3.14 and 1.4.
Michelson Interferometer How does angular width of star determines the type of image obtained?
If the angular width [1] of the star is small compared with the spacing in between adjacent maxima, the image of the star is crossed by alternate dark and light bands, known as interference fringes. If, however, the width of the star[2] is comparable to the spacing between maxima, one can visualize the resulting image as being formed by the superposition of images from a series of points across the star. The maxima and minima of the fringes from different points do not coincide, and the fringe amplitude is attenuated. Fringe visibility is defined as

$$\mathbf{V}_M = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \leq 1 \tag{4.5}$$

Let $I(l, m)$ be the two-dimensional intensity of the star, or of a source in the case of a radio interferometer. (l, m) are coordinates on the sky, with l measured parallel to the aperture spacing vector and m normal to it. The fringes provide resolution in a direction parallel to the aperture spacing only. In the orthogonal direction, the response is simply proportional to the intensity integrated over solid angle. Thus, the interferometer measures the intensity projected onto the l direction, that is, the one dimensional profile $I_1(l)$ given by

$$I_1(l, m) = \int I(l, m) dm \tag{4.6}$$

Michelson and Peas used mainly the circular disk model to interpret their observations and determined the stellar diameter by varying the aperture spacing of the interferometer to locate the first minimum in the visibility function.

4.2 two element radio interferometer

baseline: the spacing between the antennas

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[1] Angular width of an object depends on how far we are from that object, the smaller angular width would mean, we are very far

[2] example:this is true for maybe the sun

What are the Consequences of this offset?
The resulting drifts of the offsets in the output level degrade the detectability
of weak sources and the accuracy of measurement of the fringes.
Do we have any solution to this stupidity?
Phase switching by Ryle, helped to remove the unwanted components of the
receiver output, leaving only the required fringe oscillations

Phase Switching Interferometer

What is the methodology behind Phase switching interferometer?
If $V1$ and $V2$ represent the signal voltages from the two antennas, the out-
put from the simple adding interferometer is proportional to $(V_1 + V_2)^2$. In
the phase-switching system, **the phase of one of the signals is periodi-
cally reversed**, so the output of the detector alternates between $(V_1 + V_2)^2$
and $(V_1 - V_2)^2$. The frequency of the switching is a few tens of hertz, and a
synchronous detector takes **the difference between the two output terms**,
which is **proportional to** $V_1 V_2$. Thus, the output of a phase-switching inter-
ferometer is **the time average of the product of the signal voltages**; that
is, it is proportional to the cross-correlation of the two signals.
What is a correlator?
In Modern interferometry the circuitry responsible for multiplication and aver-
aging of the two signals in called **the Correlator**.
How does the phase switching affect the power term?
In the traditional receiver the power was proportional to the squared sum of the
two signals

$$P \propto [V \sin(2\pi\nu_0 t) + V \sin(2\pi\nu_0 (t - \tau))]^2$$
$$\implies P = P_0 \left[1 + \cos(2\pi\nu_0\tau)\right]$$

this power is power received by detector

where, P_0 is power generated by antenna alone (in absence of source)[3]

In case of phase-switching the cross term stays the constant term in power
equation disappears, and the output only contains the fringes.

What is th significance of removal of constant term?
The removal of the constant term greatly reduces the sensitivity to instrumental gain variation, and it becomes practicable to install amplifiers at the antennas to overcome attenuation in the transmission lines. This advance resulted in the use of longer antenna spacings and larger arrays.

Optical Identifications and Calibration Sources

So you know, these discreet sources, how do you study them in optical?
For the study of sources in optical, we would need high precision location of these sources. So, the principal method then in use for position measurement with interferometers was to determine the time of transit of the central fringe using an east west baseline, and also the frequency of the fringe oscillations, which is proportional to the cosine of the declination

The need for absolute calibration of the antennas and receiving system rapidly disappeared after a number of compact radio sources were identified with optical objects. Optical positions accurate to $\sim 1''$ could then be used, and observations of such sources enabled calibration of interferometer baseline parameters and fringe phases.

4.3   Van Cittert-Zernike Theorem

In this section we would learn about Van Cittert-Zernike Theorem. An optical term **Mutual Coherence** is used.

First, we would clear off the air around the Mutual Coherence function, For a field $E(t)$ measured at points 1 and 2, the **mutual coherence function** is given as

$$\Gamma_{12}(u, v, \tau) = \lim_{T \to \infty} \int_T^T E_1(t)E^\star(t - \tau)dt \qquad (4.7)$$

Here, u and v are expressed in wavelength and represent coordinates of spacing between two measurement point. **For zero time offset, $\Gamma_{12}(u, v, 0) \equiv \mathbf{V}(u, v)$; the complex visibility function for radio case**

Let us consider a system, which consists of an extended, quasi-monochromatic , incoherent source and let the mutual coherence of the radiation is measured at the point P_1 and P_2 in a plane normal to the direction of source.
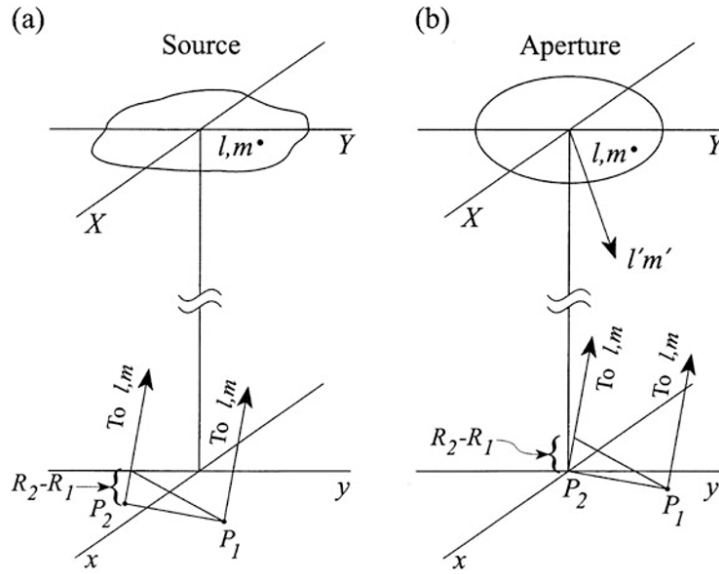
The diagram (b), represents the same system , but with the source replaced **by an aperture of identical shape and size and illuminated from behind by a spatially coherent wavefront.**

Note point:

- The distribution of the electric field amplitude over the aperture is proportional to the intensity distribution over the source.

- Fraunhoffer diffraction pattern is observed in plane containing P_1 **and** P_2

- P_2 lies on the maximum of diffraction pattern, even though relative position for P_1 and P_2 is basically the same.

VCZ theory is based on the the fact that the behavior of both the mutual coherence and the Fraunhofer diffraction can be represented by similar Fourier transform relationships.

4.3.1 Mutual Coherence of an Incoherent Source

Now, consider the same system we mentioned above in Fig. 4.3.1

- Consider source to be located in a distant plane given by (X,Y).

- Now, the radiated field is measured at points P_1 and P_2 in the (x,y) plane that is parallel to the source in the plane. **When we talk in case of radio, basically P_1 and P_2 refer to interferometer antenna locations.**

- For, convinence to specify a point in (X,Y) plane, with respect to (x,y) we use direction cosines (l,m).
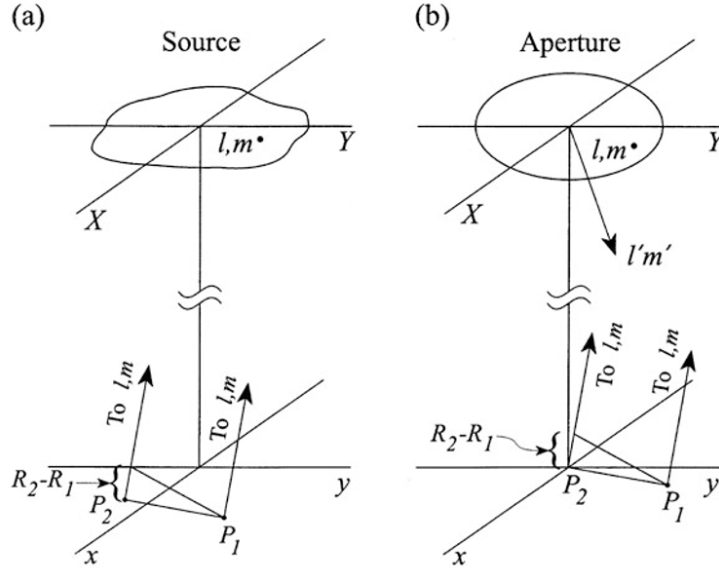
Now, the fields at P_1 and P_2 resulting from a single element of source is

$$E_1(l, m, t) = \varepsilon(l, m, t - \frac{R_1}{c}) \frac{\exp[-j2\pi\nu(t - R_1/c)]}{R_1} \tag{4.8}$$

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and

$$E_2(l, m, t) = \varepsilon(l, m, t - \frac{R_2}{c})\frac{\exp[-j2\pi\nu(t - R_2/c)]}{R_2} \tag{4.9}$$

Here

- $\varepsilon(l, m, t)$ is the complex amplitude of the electric field at the source at element (l,m).

- R_1 and R_2 are distances from the elememt from P_1 and P_2

- Exponential term will represent the phase change in light traversing the paths from source to P_1 and P_2.

Now, the complex cross-correlation of the field voltages at P_1 and P_2 due to radiation from the element at (l,m) for the specific case of zero time offset is given as

$$\langle E_1(l, m, t)E_2^\star(l, m, t)\rangle = \left\langle \varepsilon(l, m, t - \frac{R_1}{c})\varepsilon^\star(l, m, t - \frac{R_2}{c})\right\rangle$$
$$\times \frac{\exp\left[-j2\pi\nu(t - R_1/c)\right]\exp\left[-j2\pi\nu(t - R_2/c)\right]}{R_1 R_2}$$

$$= \left\langle \varepsilon(l, m, t)\varepsilon^\star(l, m, t - \frac{R_2 - R_1}{c})\right\rangle \frac{\exp\left[j2\pi\nu((R_1 - R_2)/c)\right]}{R_1 R_2} \tag{4.10}$$

> If the source is spatially incoherent , this means terms of form $\left\langle E_p(l_p, m_p, t)E_q^\star(l_q, m_q, t)\right\rangle$, where p and q refer to different elements of the source, is 0.

Simplification: if the quantity $\frac{R_2 - R_1}{c}$ is small compared with the reciprocal receiver bandwidth $\implies$ we can ignore, this term in amplitude terms for ε. Therefore the above equation for cross-correlation is reduced to:

$$\langle \varepsilon(l, m, t)\varepsilon^\star(l, m, t)\rangle \frac{\exp\left[j2\pi\nu((R_1 - R_2)/c)\right]}{R_1 R_2} \tag{4.11}$$

- $\langle \varepsilon(l, m, t)\varepsilon^\star(l, m, t)\rangle$ is a measure of **time-averaged intensity**, $I(l, m)$ of the source.

So, now we found cross-correlation for one element and for finding mutual coherence function of fields at points P_1 and P_2, we integrate over the source, using ds to represent an element of area within the (X, Y) plane:

$$\Gamma_{12}(u, v, 0) = \int_{source} I(l, m)\frac{\exp\left[j2\pi\nu((R_1 - R_2)/c)\right]}{R_1 R_2}ds \tag{4.12}$$

Note: Here, u and v are measured in wavelengths and are *x and y components of spacing between the points P_1 and P_2* .

- R_1 & R_2 is the differential distance in the path lengths from (l,m) in the source to P_1 and P_2.

- Coordinates for P_1 and P_2 are (x_1, y_1) and (x_2, y_2) respectively.

- Hence, $u = (x_1 - x_2)v/c$ and $u = (x_1 - x_2)v/c$.

- Now. $\implies (R_1 - R_2) = (ul + vm)c/v$

- now, as source is far away from P_1 and P_2, $R_1 \simeq R_2 \simeq R$.

- Here,R is the distance between (X,Y) and (x,y) origins.

Now if $ds = R^2 dl dm$ then (4.9) is given by:

$$\Gamma_{12}(u, v, 0) = \int \int_{source} I(l, m)\exp{-j2\pi(ul + vm)} dl dm \qquad (4.13)$$

Note: as the integral outside boundary is zero, therefore we can extend to infinity.

> And hence, the mutual coherence which was equivalent to complex visibilty is the fourier transform of the intensity distribution function of the source. **This is Van Cittert Zernike** theorem.

Chapter 5

Analysis of the Interferometer Response

The abstract to this chapter would be prepared once I ma done with the chapter. **This chapter is being considered from 2017 book for radio**

5.1 Fourier Transform Relationship between Intensity and Visibility

5.1.1 General Case

What we would like to do is to derive the relationship between intensity and visibility in a coordinate-free form and then show that we can achieve a similar form to a Fourier transform when we choose a certain coordinate system.units Suppose that antennas track a source under observation, From the Fig. 5.1.1 , we indicate the phase reference position(**Phase tracking center**) by unit vector s_0.

Now, for one polarization, an element of source of solid angle $d\Omega$ at position $s = s_0 + \sigma$ has power given by
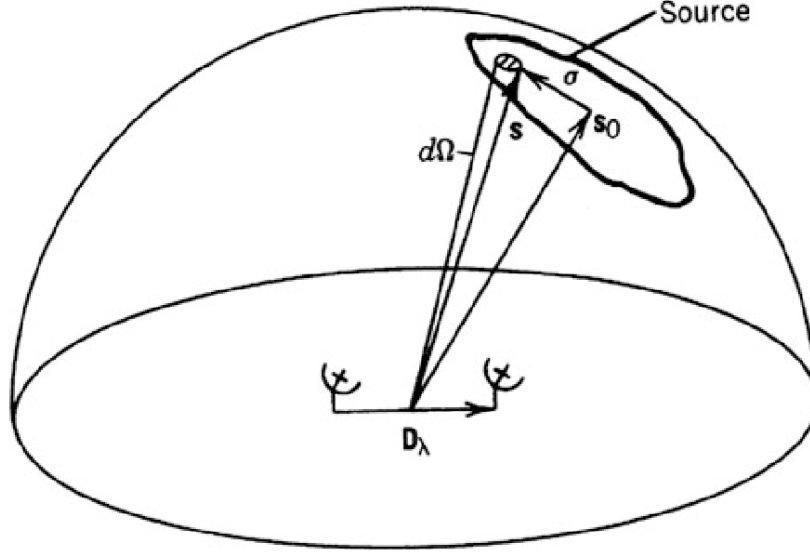
$$P = \frac{1}{2}A(\sigma)\Delta\nu d\Omega \tag{5.1}$$

Here.

- $A(\sigma)$ is effective collecting area of each Antenna.

- $I(\sigma)$ is intensity distribution as observed from the distance. It has $W \ \ m^{-2} \ \ Hz^{-1} \ \ sr^{-1}$

- $\Delta\nu$ is the bandwidth of the receiving systems

Assumptions made here,

- The author has made assumptions in eq. (2.1) and (2.2 **Look into it.**)

- The source is assumed to be at far field.

Now the correlator output is $\propto$ received power $\times$ fringe term $\cos(2\pi\nu\tau_g)$.
The baseline measurements us specified by $\mathbf{D}_\lambda$ is measured in wavelenghs and
$\nu\tau_g = \mathbf{D}_\lambda.\mathbf{s} = \mathbf{D}_\lambda.(\mathbf{s_0} + \sigma)$.
Finally, we can write correlator output as

$$r(D_\lambda.s_0) = \Delta\nu \int_{4\pi} A(\sigma)I(\sigma)\cos[2\pi\mathbf{D}_\lambda.(\mathbf{s_0} + \sigma)]d\Omega \qquad (5.2)$$

$$= \Delta\nu \cos[2\pi\mathbf{D}_\lambda.(\mathbf{s_0})] \int_{4\pi} A(\sigma)I(\sigma)\cos[2\pi\mathbf{D}_\lambda.(\sigma)]d\Omega$$

$$-\Delta\nu \sin[2\pi\mathbf{D}_\lambda.(\mathbf{s_0})] \int_{4\pi} A(\sigma)I(\sigma)\sin[2\pi\mathbf{D}_\lambda.(\sigma)]d\Omega \qquad (5.3)$$

> **Note:** the integration of the response to the element $d\Omega$ over the source
> in (5.3) requires the assumption that the source is spatially incoherent,
> that is, that the radiated waveforms from different elements $d\Omega$ are un-
> correlated. **This assumption is justified for essentially all cosmic
> radio sources.**

Now, let us assume, that A_0 be the antenna collecting area in direction s_0
in which the beam is pointed.
What we no is now introduce $A_N(\sigma) = A(\sigma)/A_0$ and consider the modified

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intensity distribution $A_N(\sigma)I(\sigma)$. We define complex visibility as

$$\mathbf{V} = |\mathbf{V}| \tag{5.4}$$

5.2 Calibration

Why do we need to do Calibration? The purpose of calibration is to remove,in so far as possible,the effects of instrumental and atmospheric factors in the measurements.Such factors depend largely on the individual antennas or antenna pairs and their associated electronics, so correction must be applied to the visibility data before they are combined into an image.

5.2.1 Complex Gains

A properly designed synthesis array will preserve a linear relationship between the measured visibilities $V'(u,v)$ and the actual visibilities $V(u,v)$. It is expressed as:

$$V_{ij}(t) = G_{ij}(t)V_{ij}(t) + \epsilon_{ij}(t) + \eta_{ij}(t) \tag{5.5}$$

So we have two unrequired components:

- Offset(ϵ_{ij})

- Stochastic noise term(η_{ij})

G_{ij} is the complex gain for the *ij-th* baseline. The equations are complex and represents the two output of each correlator the *sine and cosine* componen
In terms of individual antennas the complex gains can be expressed as gains of individual antennas:

$$G_{ij}(t) = g_i(t)g_j^s tar(t) = a_i(t)a_j(t)\exp\left[i\left(\phi_i(t) - \phi_j(t)\right)\right] \tag{5.6}$$

Here $a(i)$ is the **multiplicative gain of i-*th* antenna**. When the array is optimized by measuring **a point source** at the phase center.

   So what happens when we have a large strong source as calibrator?
For a point source, amplitude is constant and expected phase difference is zero , any differences in the a_i and any non zero phase difference must be due to antennas itself. **For N antennas: we have to measure G_{ij} fro N(N-1)/2 baselines**. But first we need to optimize the array and this requires some initial calibration.

- **Antenna Pointing and Gain**: our equation for inverting the visibilities assumes that the normalized antenna primary beam distribution $A(l,m)$ is independent of time and identical for each antenna.
  Ideally the antenna tracking center must follow the same intended position for all antennas (n general it can refer to the phase tracking center.) . The error in this tracking can cause reduced sensitivity and distortions

in extended objects.

For point sources: the tracking accuracy should be $0.1\theta_{FWHM}$ where, θ_{FWHM}= angular size of FWHM of the primary beam.

Extended objects like Sun, covers a large part of the primary beam , the pointing accuracy should be better θ_{FWHM} .

This antenna pointing error is the difference between the actual pointing position and the desired one. The pointing error is a directional dependence due to

- misalignment of antenna rotation axes
- gravitational deformation
- non perpendicular axes.
- atmospheric refraction
- wind loading
- differential heating of the structure.

- **Delay Calibration**:an error in delay of only a few nanoseconds will cause the different parts of the observing band to partially interfere (different frequencies in the band arrive with different phase), leading to a drop in the amplitude. Obviously, we need to have the delays optimized.

- **Time and Place:** The time and location of the antennas must be known to relatively high accuracy.