Calibration and Simulation of Interest Rate Models in MATLAB

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Outline

- Calibration to Market Data
- Calibration to Historical Data
- Counterparty Credit Risk
- Introduction to MATLAB Production Server
- User Story

Interest Rate Models

Short Rate Models
Vasicek
Cox-Ingersoll-Ross
Black-Derman-Toy
Black-Karasinski
Hull-White
G2++

Market Models
Heath-Jarrow-Morton
LIBOR Market Model

Interest Rate Models

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$$dr = (\theta(t) - ar)dt + \sigma dW(t)$$

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$$dr = (\theta(t) - ar)dt + \sigma dW(t)$$

G2++

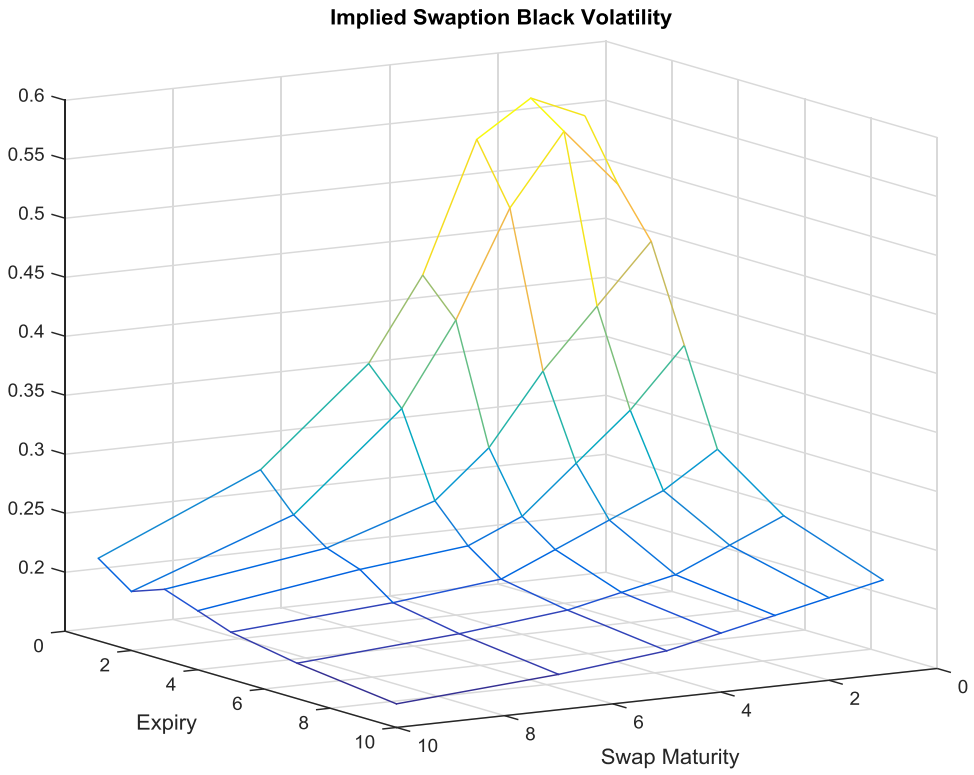
$$r(t) = x(t) + y(t) + \varphi(t)$$
$$dx(t) = -ax(t)dt + \sigma dW_1(t)$$
$$dy(t) = -bx(t)dt + \eta dW_2(t)$$
$$dW_1(t)dW_2(t) = \rho dt$$

Calibrate to Market Data

- Choose a set of liquid calibration instruments – typically caps, floors, swaptions.
- Find the set of model parameters that matches as closely as possible the observed prices.

$$\sum_{k=0}^{n} (P_i - \hat{P}_i(\theta))^2$$

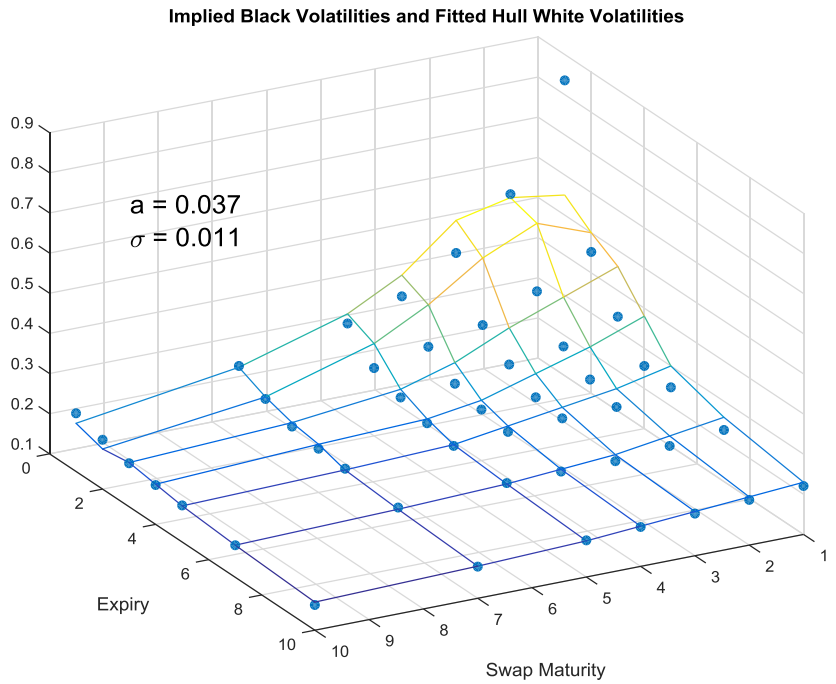
P_i: Market Price
$\hat{P}_i$: Model Price
θ: Model Parameters



Calibrate to Swaption Volatility Matrix

- Solve nonlinear data-fitting problems with `lsqnonlin`.
- Choose model and target.
- Flexibility with constraints, starting parameters, weights.

$$\sum_{k=0}^{n} (P_i - \hat{P}_i(\theta))^2$$




```
>> objfun = @(x) Price - swaptionbyhwcf(RateSpec,x(1),x(2),Strike,Exp,Mat));
>> x0 = [.1 .01];
>> lb = [0 0];
>> ub = [1 1];
>> HWParams = lsqnonlin(objfun,x0,lb,ub,options);
```

More Optimization Capabilities

- Brigo and Mercurio (2007) discuss using simulated annealing to calibrate G2++.
- Global Optimization Toolbox
  - Simulated Annealing
  - Pattern Search
  - Genetic Algorithm
  - Global Optimization
  - Multistart Framework

```
>> problem = createOptimProblem('lsqnonlin','objective',
       objfun,'x0',x0,'lb',lb,'ub',ub,'options',options);
>> ms = MultiStart;
>> [x,f] = run(ms,problem,20)
```

Brigo, D., & Mercurio, F. (2007). *Interest rate models-theory and practice: with smile, inflation and credit*. Springer.

Calibrate to Historical Data

- Use MLE to Calibrate to Short Rate Data
- Use Kalman Filter to Calibrate to Historical Yield Curve Data
- Import Data Using Datafeed Toolbox™ and the FRED Datafeed.

Aït-Sahalia, Y. (1999). Transition densities for interest rate and other nonlinear diffusions. *The Journal of Finance*, *54*(4), 1361-1395.

Park, F.C., “Implementing Interest Rate Models: A Practical Guide.” Capital Markets & Portfolio Research, Inc. white paper, 2004

Calibrate CIR Model using MLE

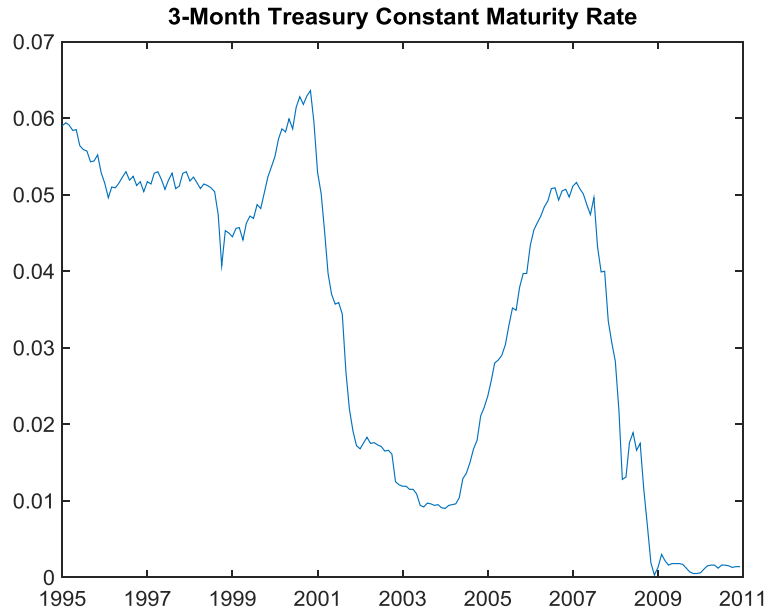
$$dr(t) = a(b - r)dt + \sigma\sqrt{r}dW(t)$$

a: mean reversion speed

σ: volatility of the short rate

b : level

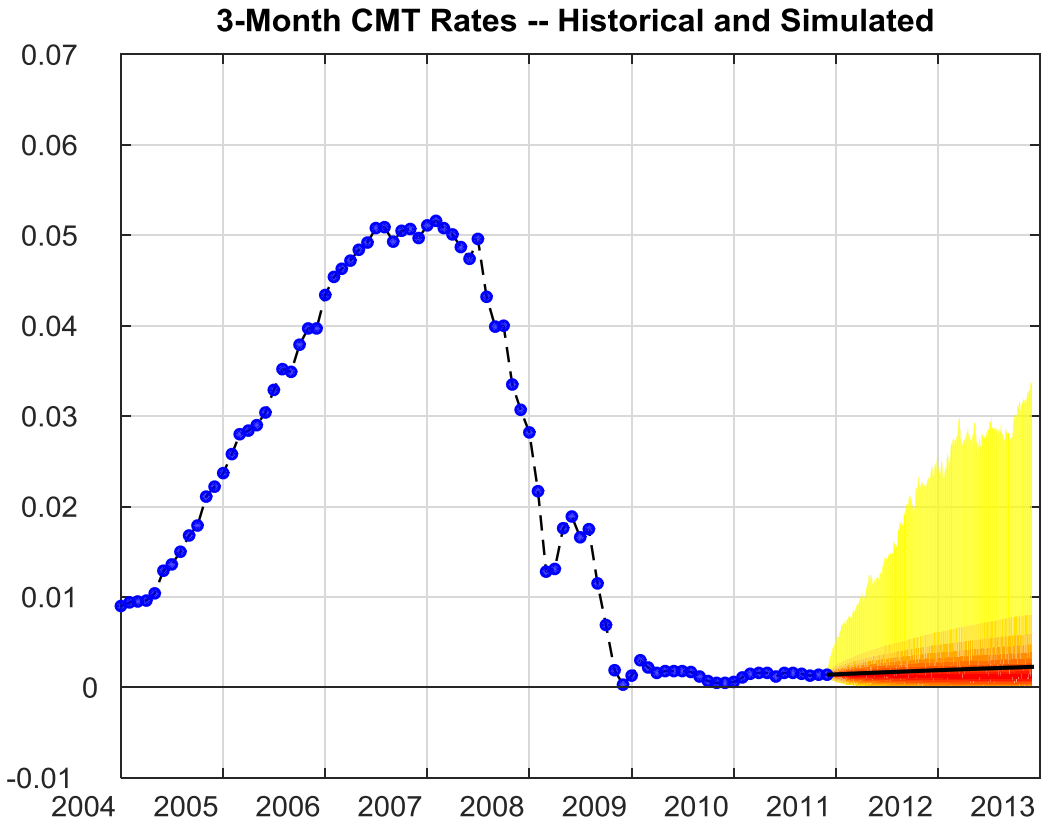
W: Brownian motion




```
>> ShortRateData = fetch(fred,'GS3M', '01-Jan-1995', '31-Dec-2010');
>> [CIR_Param,CIR_CI] = mle(ShortRates, 'pdf', {@cirpdf, dt},
        'start',x0,'lowerbound', [0 0 0], 'optimfun','fmincon')
CIR_Param =
    0.1285    0.0052    0.0523
CIR_CI =
   -0.0286   -0.0111    0.0471
    0.2856    0.0216    0.0576
```

Stochastic Differential Equation Models

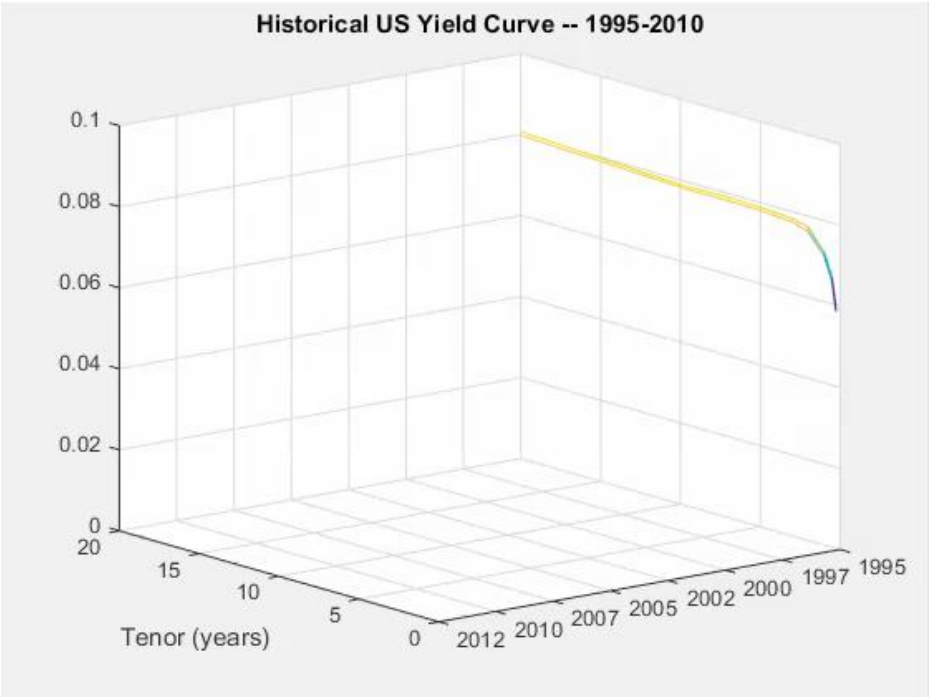
- Suite of models including : `bm, gbm, cir, hwv, heston, cev`
- Simulate methods
- Framework for creating custom models




```
>> CIR = cir(a, b, Sigma,'StartState',r0);
>> dt = 1/252;
>> nPeriods = 252*2;
>> nTrials = 10000;
>> Paths = simulate(CIR,nPeriods,'nTrials',nTrials,'DeltaTime',dt);
```

Calibrate using Kalman Filter

- Formulate models as state space systems.
- Use Kalman filter to estimate parameters.
- Estimate parameters from historical yield curves.



State Space formulation for G2++ Model

Transition Equation

$$x_t = Ax_{t-1} + B\mu$$

Measurement Equation

$$y_t = Cx_t + D\epsilon + E$$

$$A = \begin{bmatrix} e^{-a\Delta t} & 0 \\ 0 & e^{-b\Delta t} \end{bmatrix}$$

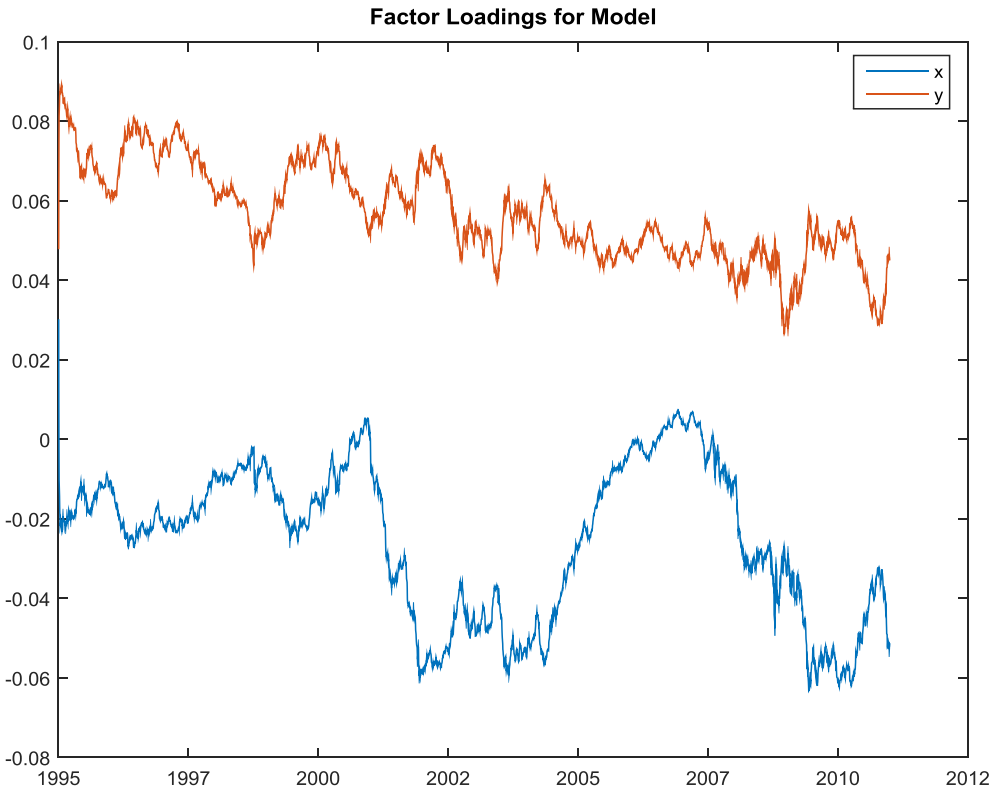
$$B = \begin{bmatrix} \sigma\sqrt{\frac{1-e^{-2a\Delta t}}{2a}} & 0 \\ 0 & \eta\sqrt{\frac{1-e^{-2b\Delta t}}{2b}} \end{bmatrix}$$

Park, F.C., “Implementing Interest Rate Models: A Practical Guide.” Capital Markets & Portfolio Research, Inc. white paper, 2004

State Space Model

New state space model, `ssm` in Econometrics Toolbox™.

- Supports time-invariant and time-varying, linear state-space models.
- Perform univariate and multivariate time-series data analysis.
- Functionality to: `estimate, filter, smooth, simulate, forecast`

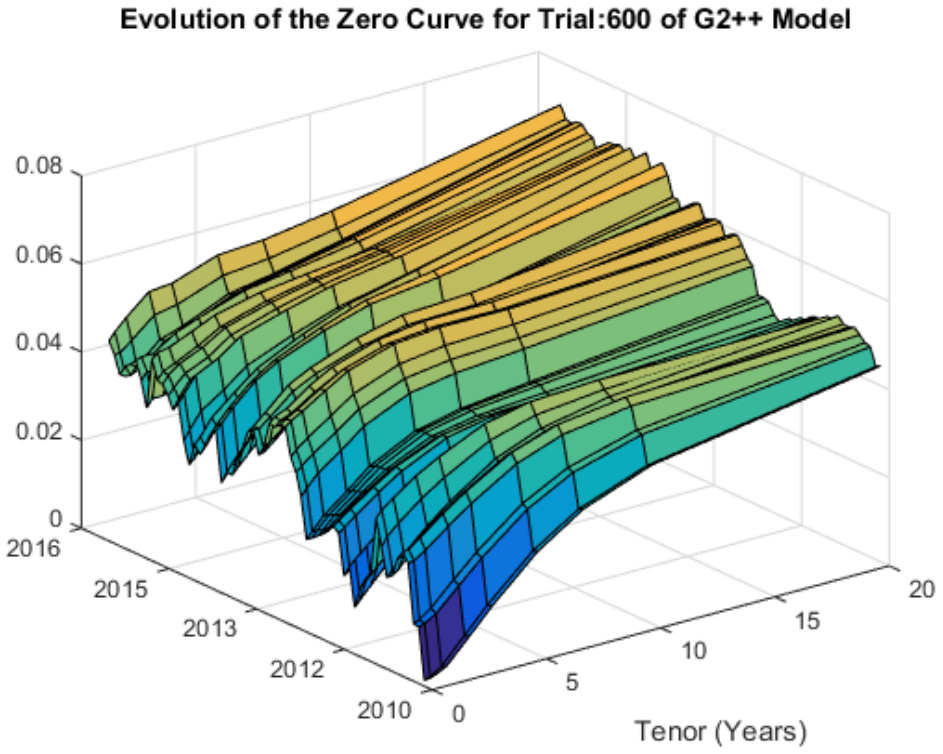



```
G2PPSSM = ssm(@(Params)g2ppssm(Params,dT,tenor));
[G2PP,Param] = estimate(G2PPSSM,Data,x0,'lb',lb,'ub',ub);
```

Interest Rate Model Simulation

Specify models and simulate entire term structure

- Support for Hull White, G2++ and LiborMarketModel.
- `simTermStructs` simulates entire term structure.




```
>> G2PP = LinearGaussian2F(RateSpec,a,b,sigma,eta,rho);
>> nDates = 24;
>> DT = 1/12;
>> nTrials = 10000;
>> Paths = simTermStructs(G2PP,nDates,'NTRIALS',nTrials,'DeltaTime',DT);
```

Swap Portfolio

- Store data in a MATLAB Table.
- Easy to read in data.
- Tabular display.

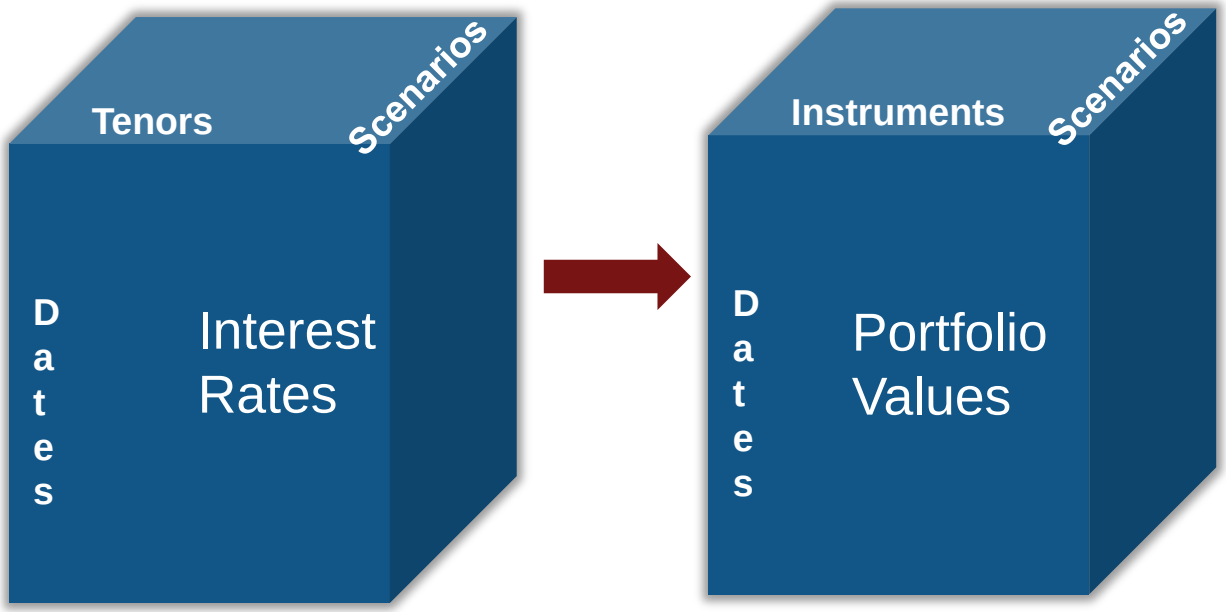
```
>> SwapPort = readtable('SwapPortfolio.xlsx')

SwapPort =
```

Notional	Maturity	RecType	PayType	RecRate	PayRate	RecReset	PayReset
1e+07	'1/15/2018'	1	0	0.031	10	12	12
5e+06	'2/15/2018'	0	1	20	0.032	12	12
1e+06	'3/15/2019'	1	0	0.033	30	12	12
2e+06	'4/15/2019'	0	1	40	0.034	12	12
1e+07	'5/15/2020'	1	0	0.036	50	12	12
7e+06	'6/15/2020'	0	1	65	0.036	12	12
7.5e+06	'7/15/2021'	1	0	0.0385	70	12	12
8e+06	'8/15/2021'	0	1	75	0.04	12	12
3e+06	'9/15/2022'	1	0	0.039	85	12	12
3.5e+06	'10/15/2022'	0	1	95	0.04	12	12

```
>>
```

Valuing the Portfolio

- Value portfolio using `swapbyzero`
- Use `parfor` to loop over simulation dates.

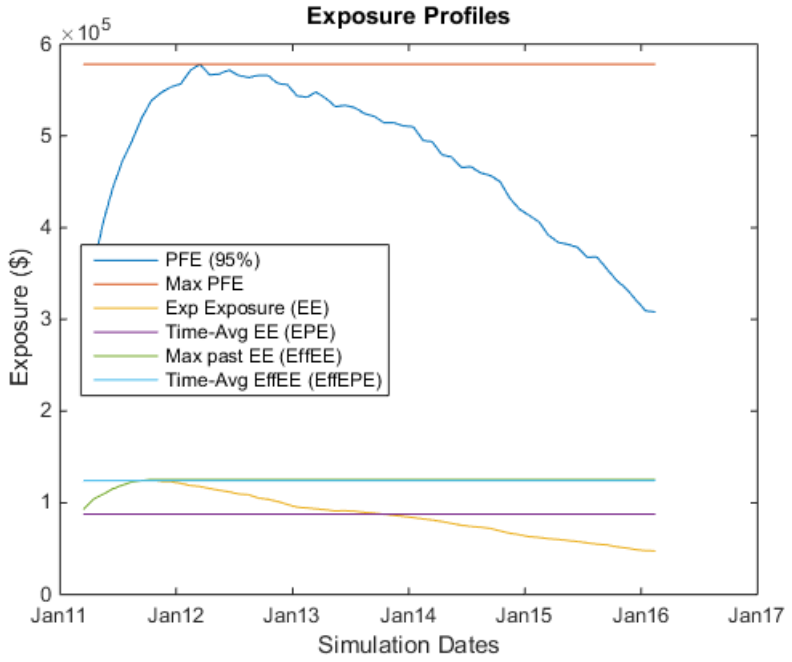



```
>> Values = zeros(nDates,nSwaps,nTrials);
>> parfor dateidx=1:nDates
       Values(dateidx,:,:) = swapbyzero(...)
   end
```

Counterparty Credit Risk Functions

Compute exposures and CCR profiles

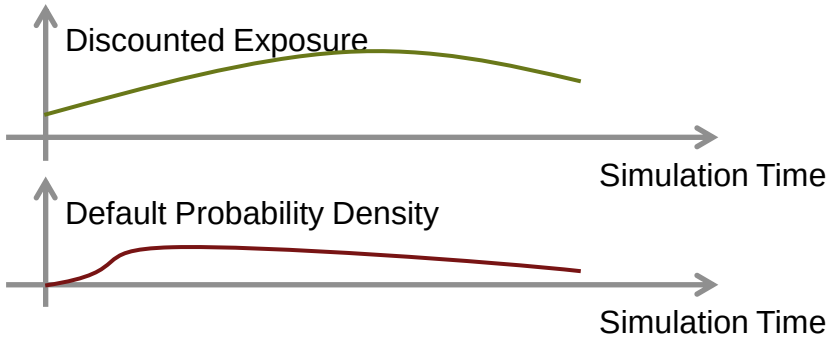
- Support for computing credit exposures.
- Support for computing various credit exposure profiles, including potential future exposure and expected exposure.




```
>> Exposures = creditexposures(Values);
>> Profiles = exposureprofiles(SimDates,Exposures);
```

Computing Credit Valuation Adjustment

- Compute exposure from `exposureprofiles`
- Compute default probabilities from `cdsbootstrap`



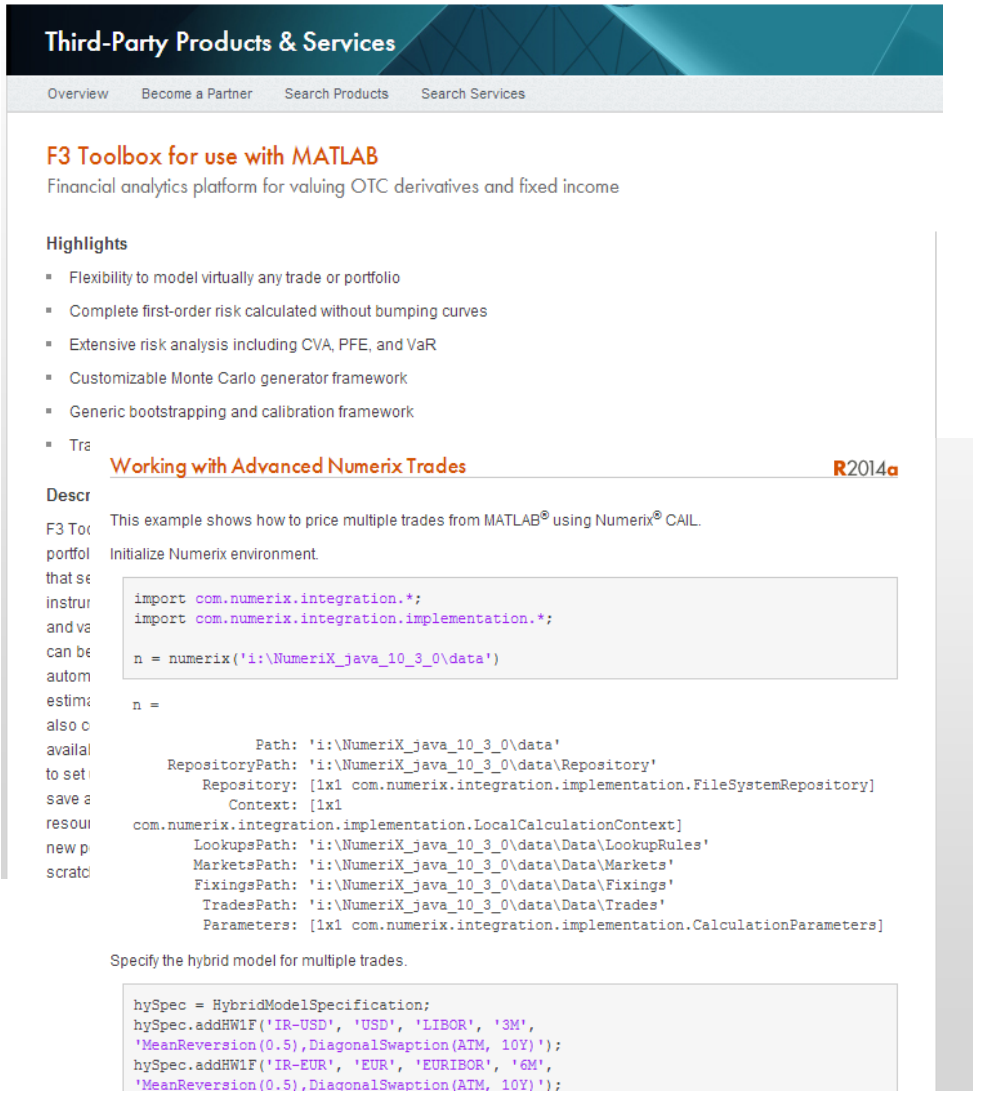
$$CVA = (1 - R)\int_0^T DiscExp(t)dPD(t)$$

```
>> ProbData = cdsbootstrap(ZeroCurve,CDSData,Settle);
>> Recovery = 0.4;
>> CVA = (1-Recovery)*sum(discEE(2:end,:) .* diff(ProbData(:,2)));
CVA =

      12244.32
```

Third Party Interfaces

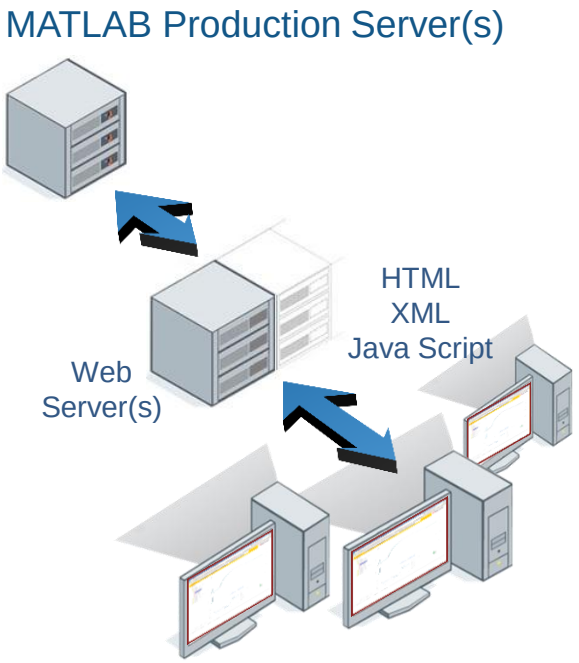
Access third party analytics

- Support for accessing Numerix® instruments and risk models.
- Support for accessing FinCAD through the F3 Toolbox for MATLAB.



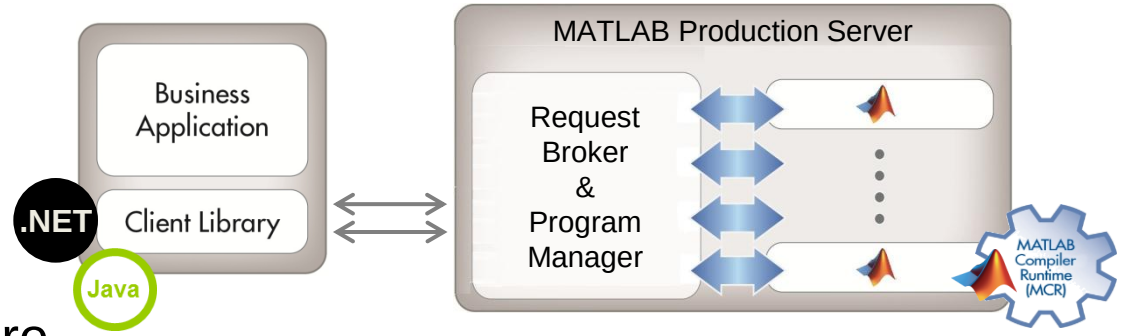
MATLAB Production Server™

- Directly deploy MATLAB programs into production
  - Centrally manage multiple MATLAB programs and runtime versions
  - Automatically deploy updates without server restarts
- Scalable and reliable
  - Service large numbers of concurrent requests
  - Add capacity or redundancy with additional servers
- Use with web, database and application servers
  - Lightweight client library isolates MATLAB processing
  - Access MATLAB programs using native data types



What is MATLAB Production Server?

- Enterprise framework for running packaged MATLAB programs



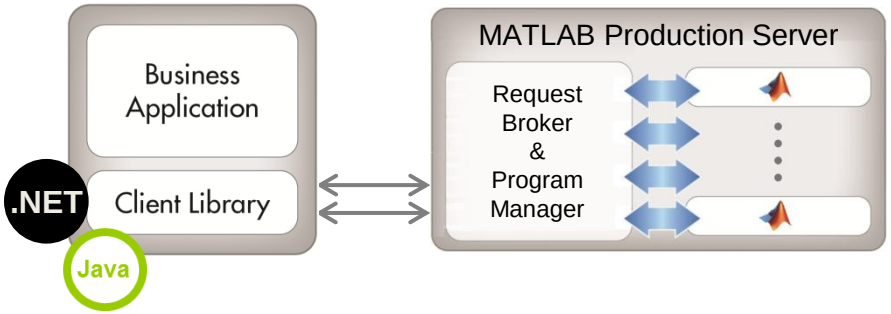
- Server software
  - Manages packaged MATLAB programs & worker pool
- Runtime libraries
  - MATLAB Compiler Runtime (MCR)
- Lightweight client library (.NET & Java)
  - Request MATLAB programs (functions)

Use Case: Risk Manager

- Web Interface
- Monitor exposure profiles and CVA for each counterparty.
- Different simulation models, calibration methods
- CVA sensitivity analysis
- What-if scenarios, stress scenarios

Client Library

- Request MATLAB programs running on MATLAB Production Server



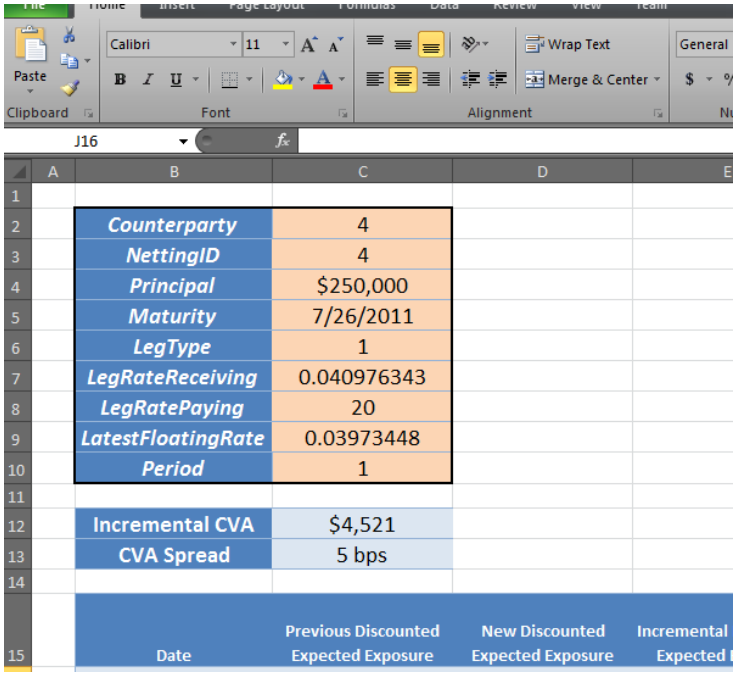
- Two libraries available

.NET	MathWorks.MATLAB.ProductionServer.Client.dll
Java	mps_client.jar

  - HTTP or HTTPS based communication protocol
  - Automatic data marshalling between .NET or Java types and MATLAB types
  - Dynamic Invocation API

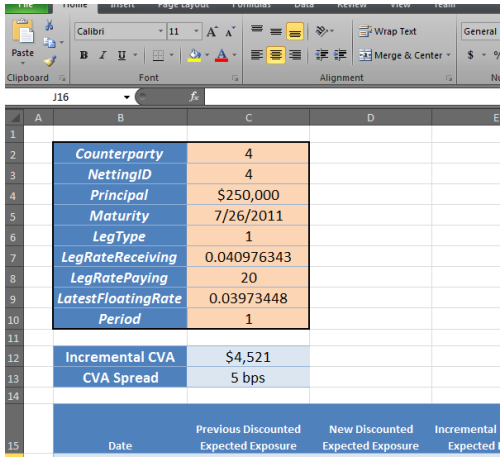
Use Case: Trader

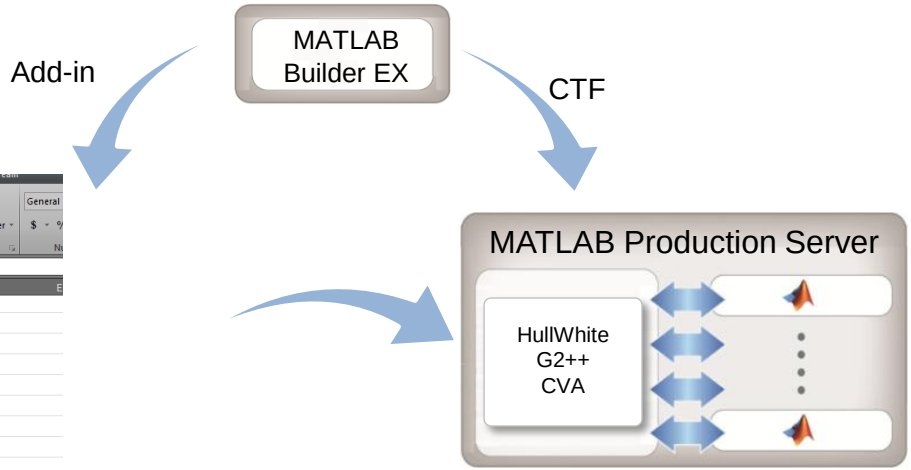
- Spreadsheet front end
- Analyze a new trade
  - Use different models and calibration methods
  - Compute CVA
  - Compute new exposures



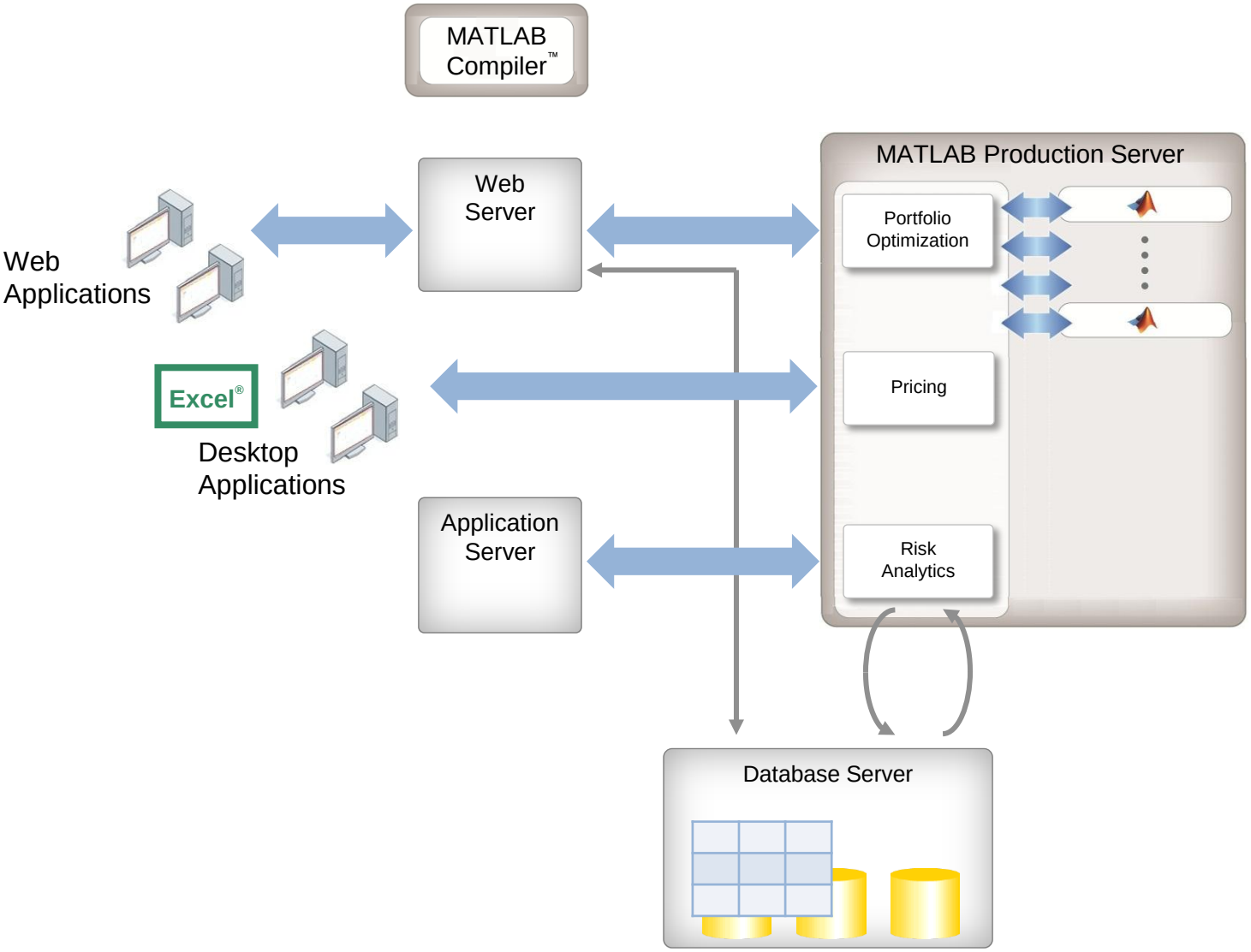
Desktop Application

- Request analytics on MATLAB Production Server from Microsoft® Excel
- MATLAB Builder™ EX
  - Generate CTF and corresponding add-in
  - Support 32 and 64bit Excel environments with same CTF/add-in pair
  - Deploy without admin rights





Integrating with IT systems



Developing a New Interest Rate Model

Intuitive Analytics

Challenge

Develop a new interest rate model based on work by Deguillaume, Rebonato and Pogudin (2013).

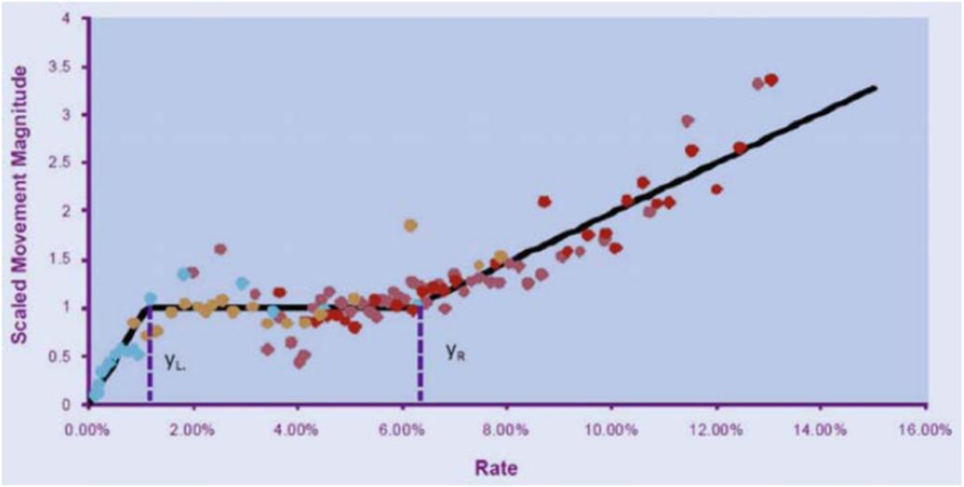


Figure 11. Blue: Japanese yen; red: sterling; maroon: US$; orange: Swiss franc.

Deguillaume, N., Rebonato, R., & Pogudin, A. (2013). The nature of the dependence of the magnitude of rate moves on the rates levels: a universal relationship. *Quantitative Finance*, *13*(3), 351-367.

Developing a New Interest Rate Model

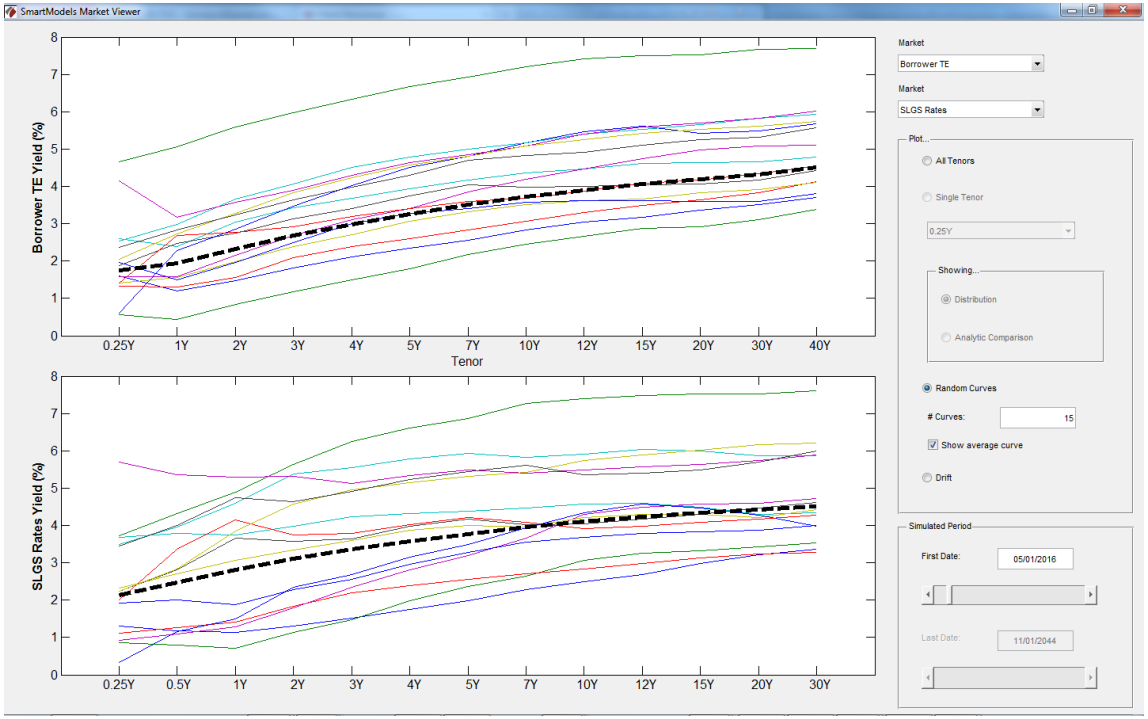
Intuitive Analytics

Solution

The model was developed in MATLAB and can be calibrated to multiple currencies and markets.

Deployment

The model was then deployed to customers via the SmartModels Excel Add-In interface.



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Summary

- Calibration Approaches
  - Market Data: `lsqnonlin`
  - Historical Data: `mle, ssm`
- Monte Carlo Simulation in MATLAB
  - `bm, gbm, cir, hwv, heston, cev`
  - `HullWhite1F, LinearGaussian2F, LiborMarketModel`
- Counterparty Credit Risk
  - `creditexposures, exposureprofiles`
  - `cdsbootstrap`
- Enterprise deployment with MATLAB Production Server


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