Interest Rate Modeling:

Short Rates to Market Models

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Why Interest Rate Modeling?

- Fundamental to valuation of bonds and other fixed income securities
- Combination of traditional econometric models with machine learning approaches
- Analysis of interest rate dynamics and their implications for risk management and investment strategies
- Master bond valuation techniques and contrast key interest rate models

Project Outline

Literature Review & Data Collection → **Model Design** → **Model Calibration** → **Data Analysis & Optimization** → **Bond Pricing**

Literature Review & Data Collection

Review short rate and market models

Gather STRIPS (Separate Trading of Registered Interest and Principal of Securities) and Treasury bonds data

Model Design

Implement Python code for Vasicek model, Cox-Ingersoll-Ross (CIR) model, Hull-White model, Brace-Gatarek-Musiela (BGM) model

Model Calibration

Calibrate to historical data and fine-tune and validate

Set up Monte Carlo framework and run simulations for models

Data Analysis & Optimization

Compare model outputs and optimize models with fitter functions

Bond Pricing

Use the best-performing model(s) to price various types of bonds

Characteristics of Data

Attribute	Data
Source	Bloomberg
Instrument	Principal STRIPS bonds
Maturity	Quarterly
Country/Region of Incorporation	United States
Coupon Type	Zero Coupon
Bond Type	US Treasuries (AA+) (option-free)

Short Rate Vs Market Models

Short Rate Models

- Short rate
  - Instantaneous interest rate at a given time for an infinitesimally short period
- Why short rate models?
  - Simple and tractable
  - Price ZCB, bond options, swaptions, and other financial instruments

One-Factor (1F) and Two-Factor (2F) Models

one-factor

$$dr(t) = [\theta(t) - ar(t)]\, dt + \sigma dW(t)$$

two-factor

$$dr_1(t) = [\theta_1(t) - a_1 r_1(t)]\, dt + \sigma_1 dW_1(t)$$
$$dr_2(t) = [\theta_2(t) - a_2 r_2(t)]\, dt + \sigma_2 dW_2(t)$$

$$r(t) = r_1(t) + r_2(t)$$

r(t) = short term interest rate

a = mean-reversion speed

θ = long-term mean interest rate level

σ = volatility

dW(t) = random Brownian motion

- 2F models provide more realistic curve behavior
- 2F are better at reproducing how real yield curves have behaved over time.

Vasicek Model

$$dr(t) = \kappa[\theta - r(t)]dt + \sigma dW(t), \quad r(0) = r_0$$

r(t) = short term interest rate

k = mean-reversion speed

θ = long-term mean interest rate level

σ = volatility

dW(t) = random Brownian motion

- Closed form solution
- Gaussian Tractability
- Possibility of negative interest rates

Cox-Ingersoll-Ross (CIR) Model

$$dr(t) = \kappa[\theta - r(t)]dt + \sigma\sqrt{r(t)}dW(t), \quad r(0) = r_0$$

r(t) = short term interest rate at time t

k = mean-reversion speed

θ = long-term mean interest rate level

σ = volatility

dW(t) = random Brownian motion

- Interest rates remain non-negative
- Non-central chi-squared distribution
- Tractability enables parameter calibration using historical data

Hull-White Model

$$dr(t) = [\theta(t) - a \cdot r(t)]dt + \sigma(t) \cdot dW(t), \quad r(0) = r_0$$

r(t) = interest rate at time t

a = mean reversion rate

θ(t) = interest rate term structure

t = time period

σ(t) = time dependent volatility

dW(t) = random market risk

- Time dependent parameters
- Allows for negative rates
- Normal distribution assumption may not reflect rate-behavior in extreme market conditions

Market Models

- Directly models market-observable forward rates, great for bond pricing
- Common use cases are for pricing caps, floors, or swaptions
- Brace-Gatarek-Musiela (BGM) model and Heath-Jarrow-Morton (HJM) model

BGM Model

$$dL(t,T_i,T_{i+1}) = L(t,T_i,T_{i+1})\sigma_i(t)dW_i^{T_{i+1}}(t)$$

$dL(t,T_i,T_{i+1})$ = differential change in the forward LIBOR rate

$L(t,T_i,T_{i+1})$ = forward LIBOR rate

$\sigma_i(t)$ = time dependent volatility

$dW_i(t)$ = Brownian motion under the T_{i+1}-forward measure

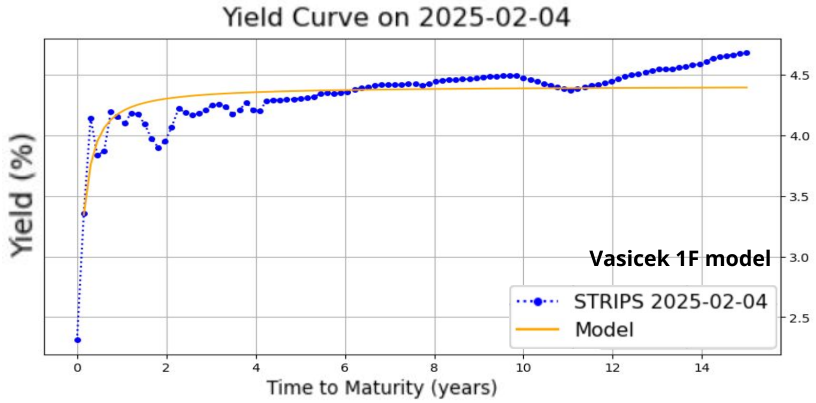
- Models forward LIBOR rates directly
- Captures correlation between different forward rates to realistically price multi-period interest rate derivatives

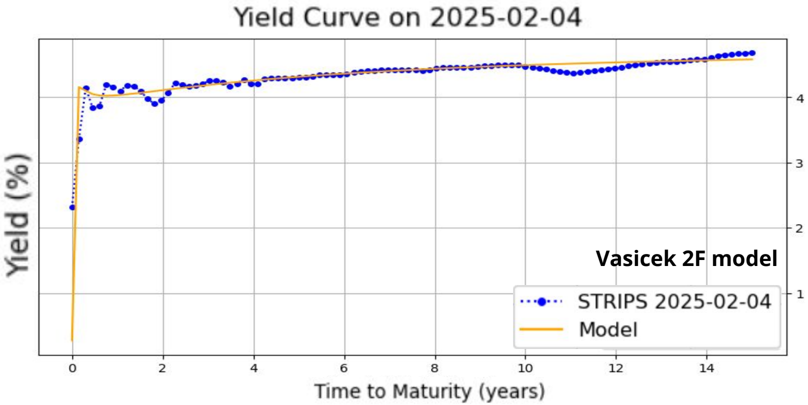
Model Performance

Model Performance Metrics

- **R-squared**: Measures the proportion of variance in the dependent variable explained by the model. Values closer to 1 indicate a better fit.
- **RMSE (Root Mean Squared Error)**: Represents the average prediction error in the same units as the dependent variable. Lower values indicate more accurate predictions.
- **AIC (Akaike Information Criterion)**: Balances model fit and complexity by penalizing the number of parameters. Lower AIC indicates a better model among competing alternatives.

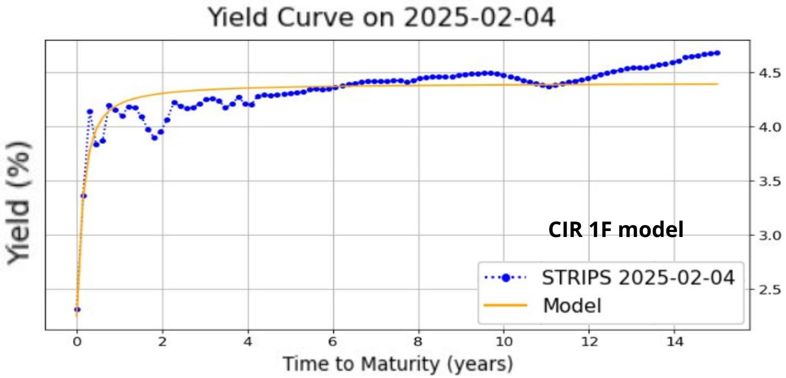
Vasicek 2F Model Captured Inverted Yield

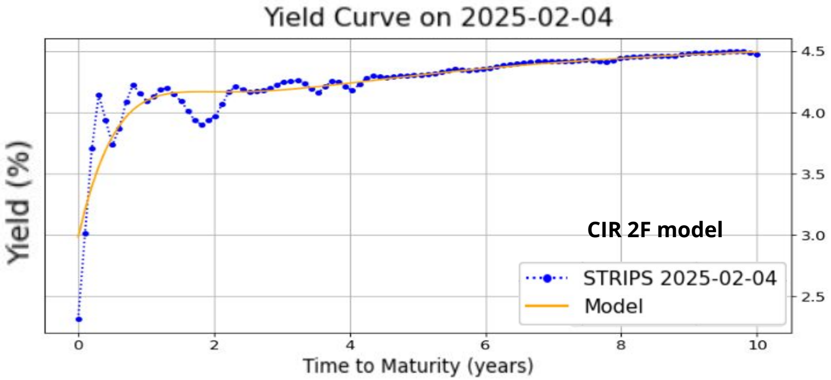




Vasicek	1F model	2F model
R-squared	0.76	0.93
RMSE	15.83%	8.61%
AIC	-161.57	-209.55

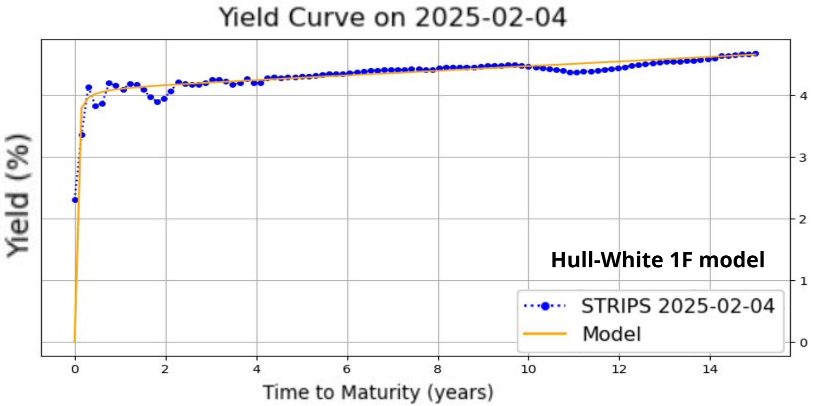
CIR Model Showed Moderate Fit

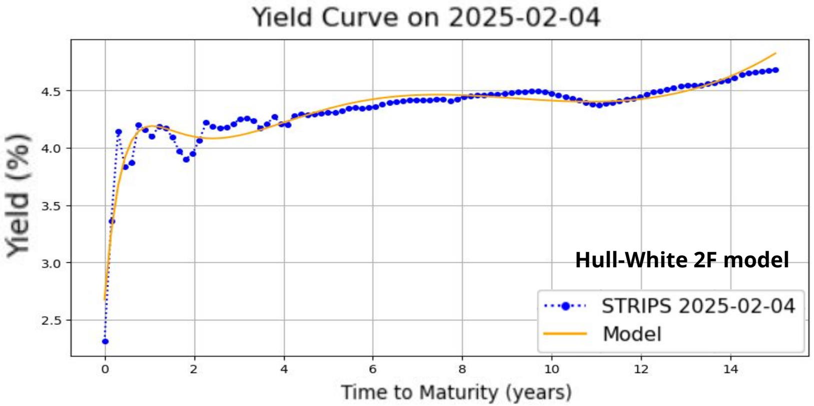




CIR	1F model	2F model
R-squared	0.76	0.79
RMSE	15.93%	15.01%
AIC	-160.99	-158.49

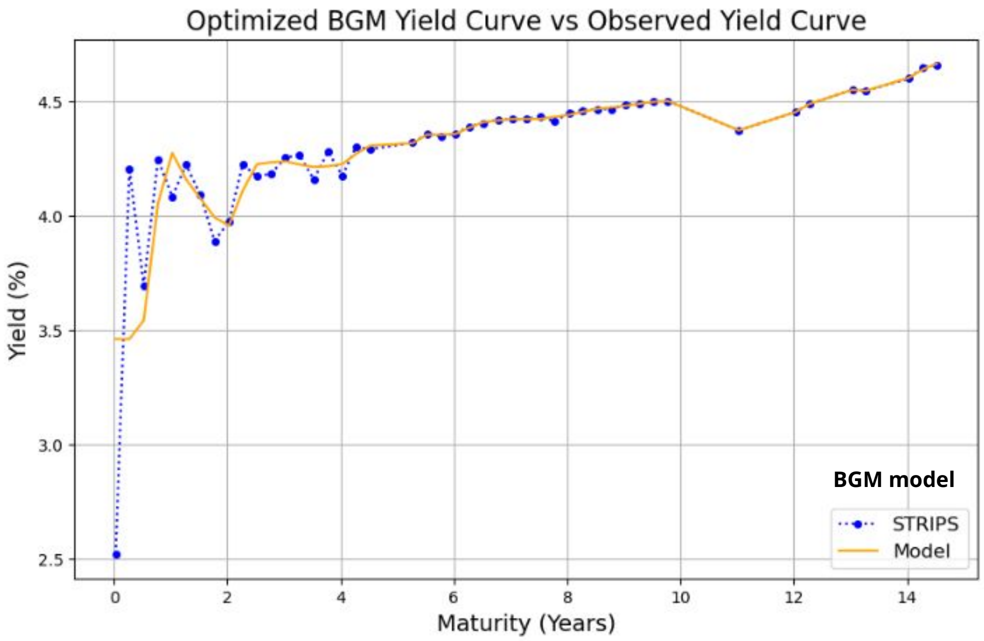
Hull-White 2F Model Mimicked the Yield Trend





Hull-White	1F model	2F model
R-squared	0.76	0.84
RMSE	15.98%	12.95%
AIC	-162.68	-172.09

BGM Model Showed Less Error in Yield Prediction

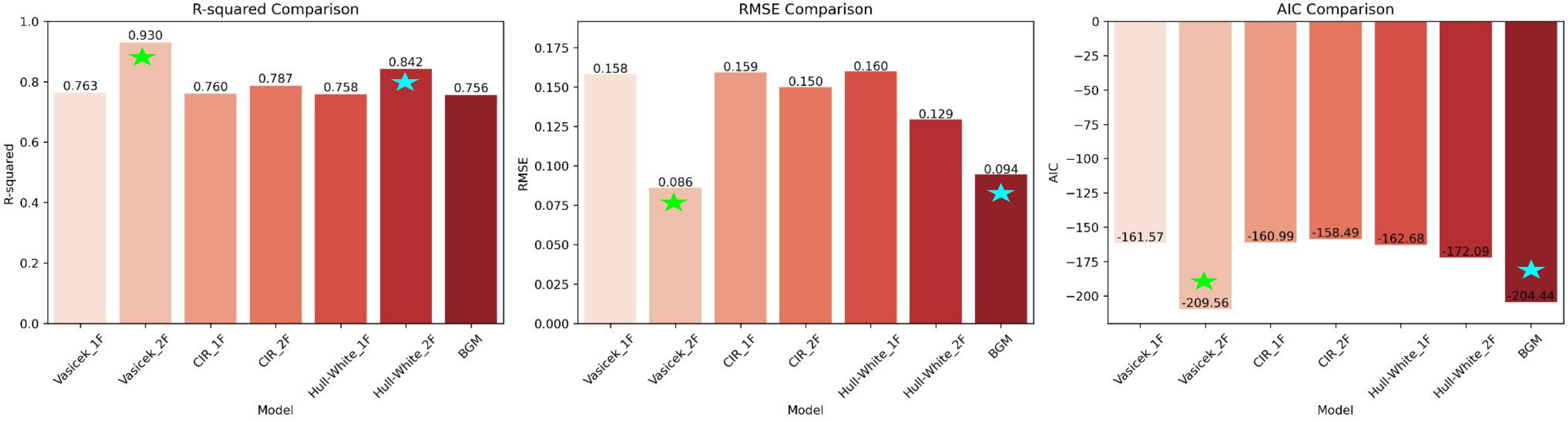


BGM	
R-squared	0.76
RMSE	9.44%
AIC	-204.44

Model Performance

Models	R-squared	RMSE	AIC
Vasicek 1F model	0.76	15.83%	-161.57
Vasicek 2F model	**0.93**	**8.61%**	**-209.56**
CIR 1F model	0.76	15.93%	-160.99
CIR 2F model	0.79	15.01%	-158.49
Hull-White 1F model	0.76	15.98%	-162.68
Hull-White 2F model	**0.84**	12.95%	-172.09
BGM model	0.76	**9.44%**	**-204.44**

Model Performance



★ - Best performing model
★ - Second best performing model

Bond Pricing

Bond Pricing

$$\text{Price of ZCB: } P = \frac{F}{(1+r)^n}$$

P = price of zero coupon bond

F = face value of bond

r = yield to maturity of bond

n = number of time periods

$$\text{Price of CB: } P = \sum_{t=1}^{N} \frac{C}{(1+r)^t} + \frac{F}{(1+r)^N}$$

P = price of coupon bond

C = coupon payment

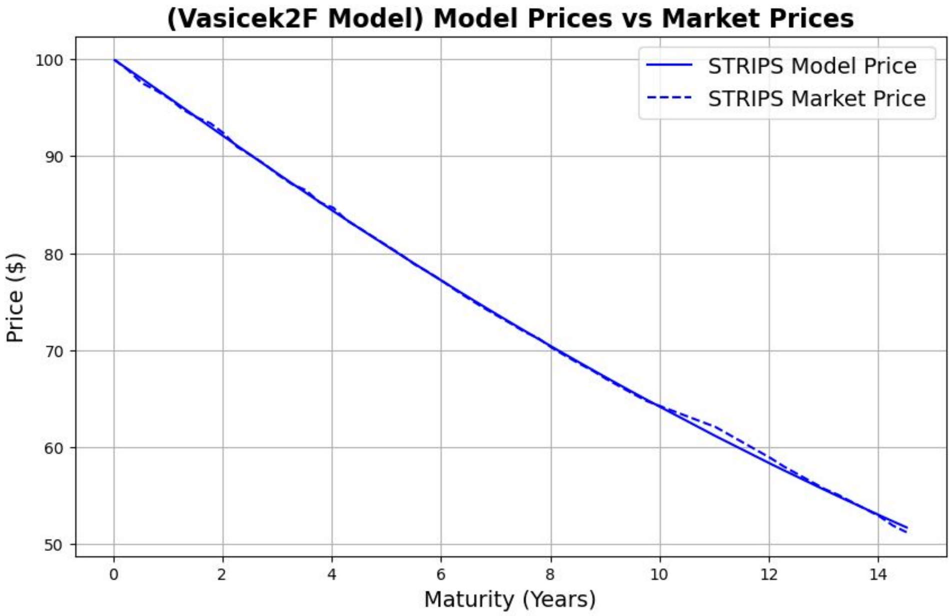
F = face value of bond

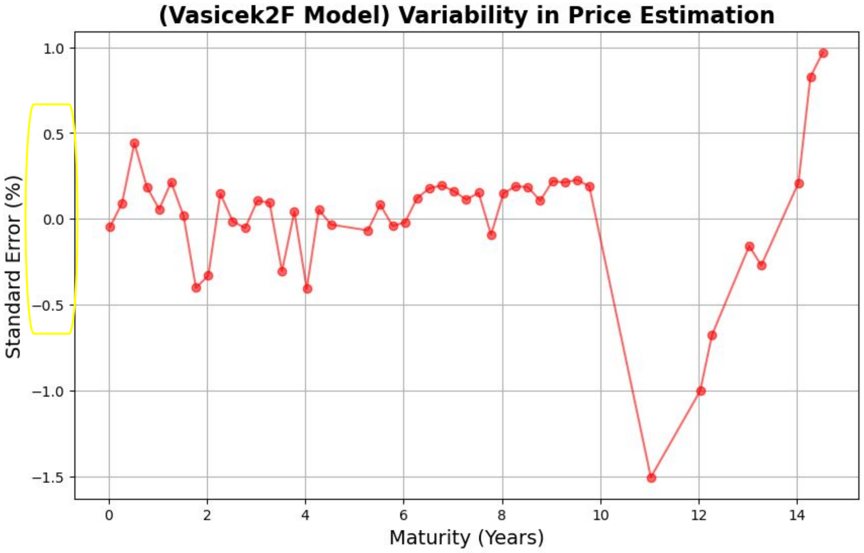
r = yield to maturity of bond

t = time of coupon payment

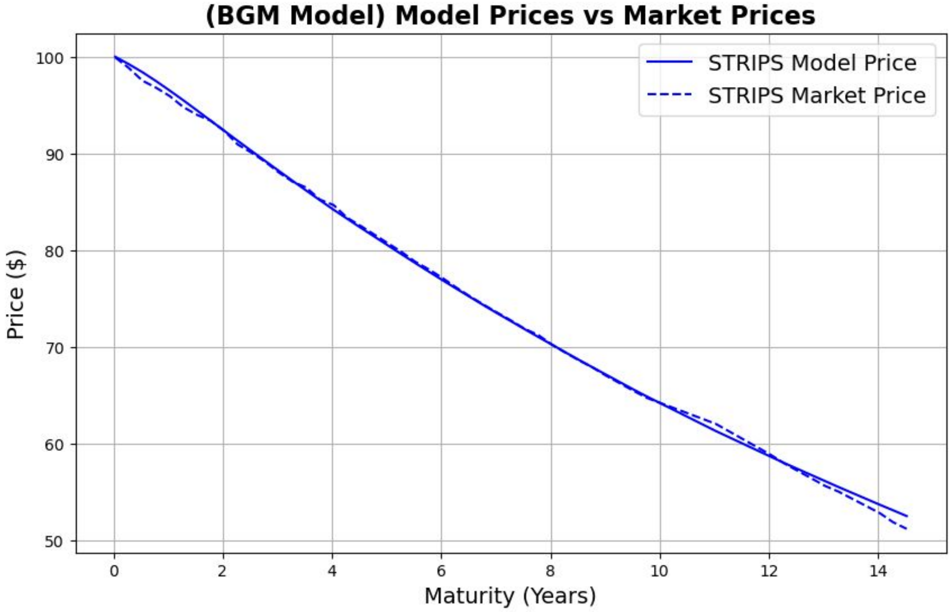
N = number of time periods

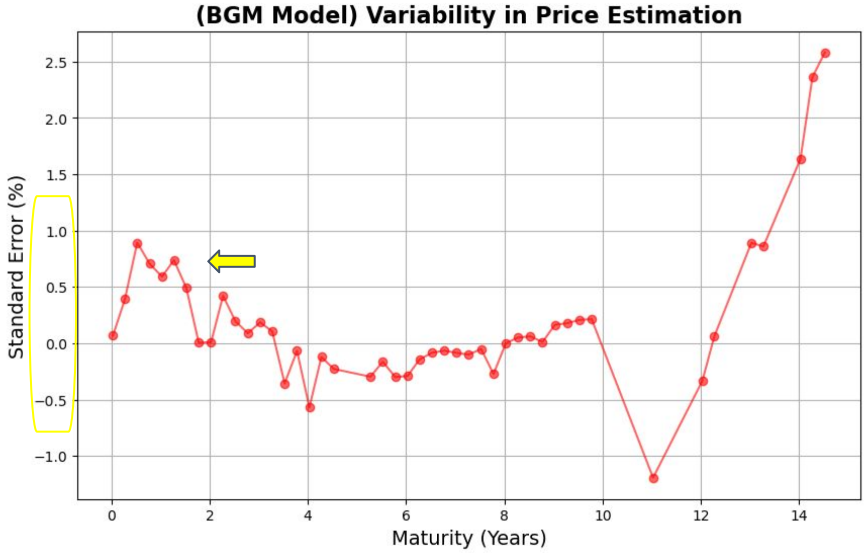
Vasicek Model: Estimated Prices vs Market Prices





BGM Model: Estimated prices vs Market prices



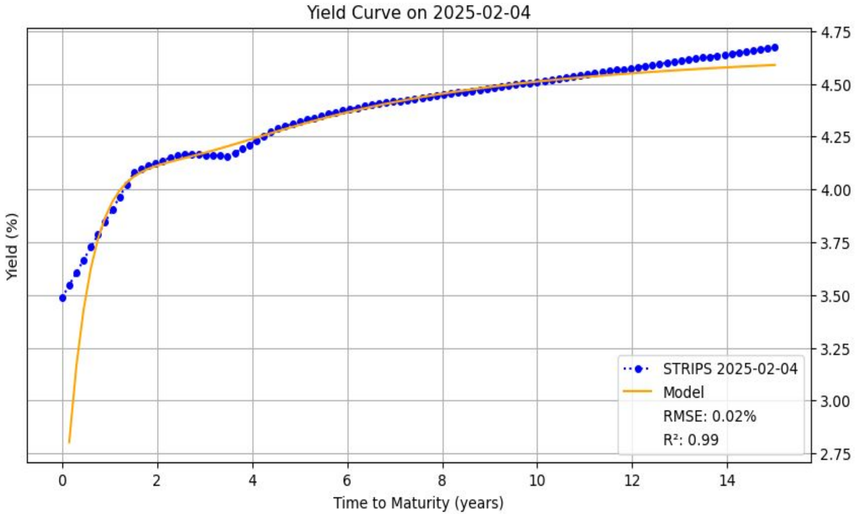


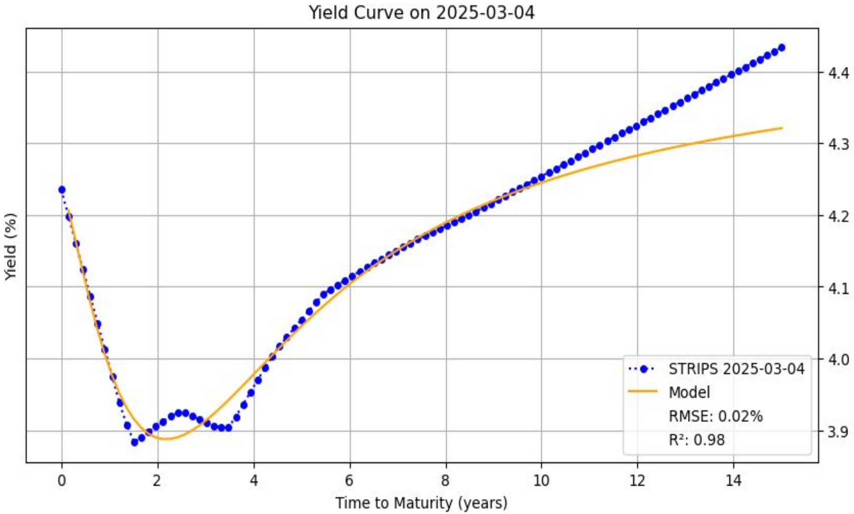
Future Work

Bond Pricing with BGM Model

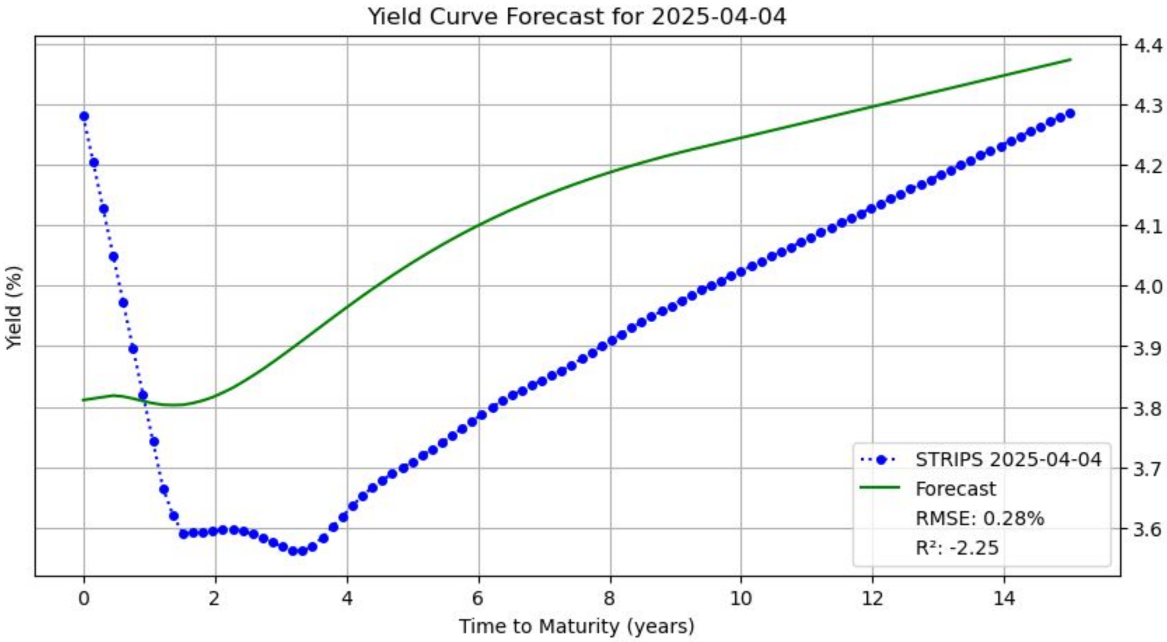
Security	Maturity	Coupon Frequency	Market Price	Model Price (Estimated)
T-bill	3 months	zero-coupon	95.81	98.42
Treasury Bond	10 year	semi-annual	104.75	101.28
Treasury Bond	5 year	semi-annual	101.17	98.14
Treasury Bond	2 year	semi-annual	92.32	91.75
TIPS	2 year	semi-annual	104.84	92.82
Corporate Bond (A)	7 year	semi-annual	103.01	101.68
Corporate Bond (BBB)	10 year	semi-annual	100.43	106.75

Interest Rate Forecasting with Vasicek 2F Model





Vasicek Model Forecast Looks Inflationary



Future Work

- Understand the limitations of each model
- Extend to more market models like HJM (Heath-Jarrow-Morton) model
- Enhance the analysis to handle bonds with coupon payments
- Interest rate forecasting for investment decisions

Q & A

Thank you!

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