Programming Task - Heat Diffusion

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August 8, 2020

1 Governing Equation

The governing equation for two-dimensional unsteady-state heat conduction problem is given by :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

$$T = 20°C \qquad (\in \Gamma) \tag{2}$$

$$T = \begin{cases} 40°C, & \text{if } (x-0.5)^2 + (y-0.5)^2 < 0.2, t = 0 \\ 20°C, & \text{otherwise, } t = 0 \end{cases} \tag{3}$$

where $T(x, y, t)$ is the transient temperature, $\alpha = \frac{k}{\rho C}$ is the thermal diffusivity of the material and Γ is the boundary of the domain.

2 Discretization

An explicit finite difference (FD) formulation has been used. Assuming uniform discretization, the step length of x-axis is $\Delta x = x_{i+1} - x_i$ and y-axis is $\Delta y = y_{j+1} - y_j$. The step length is controlled by the input of number of divisions,n, such that i,j = 0,1,2...,n. The temporal term on the right hand side can be represented by a first order forward difference approximation to give:

$$\left(\frac{\partial T}{\partial t}\right)_{i,j}^{n} = \frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t}$$

where $n = 0, 1, 2..$ denotes the time instant and $\Delta t = 5s$ is the time step.Using a second order central difference scheme, the spatial terms on the left hand side are approximated as

$$\frac{\partial^2 T^n}{\partial x^2}_{i,j} = \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{(\Delta x)^2}$$
$$\frac{\partial^2 T^n}{\partial y^2}_{i,j} = \frac{T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n}{(\Delta y)^2}$$

Assuming $\Delta X = \Delta y$, ultimately, the temperature at a point at any time instant can be given by :

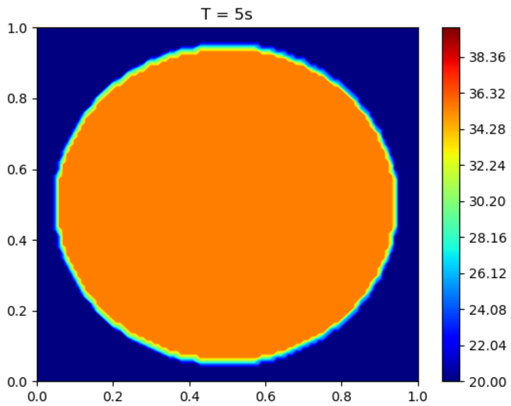
$$T_{i,j}^{n+1} = \tau(T_{i+1}^n j + T_{i-1}^n j + T_{i,j+1}^n + T_{i,j-1}^n) + (1 - 4\tau)T_{i,j}^n$$

where $\tau = \frac{\alpha \Delta t}{(\Delta x)^2}$

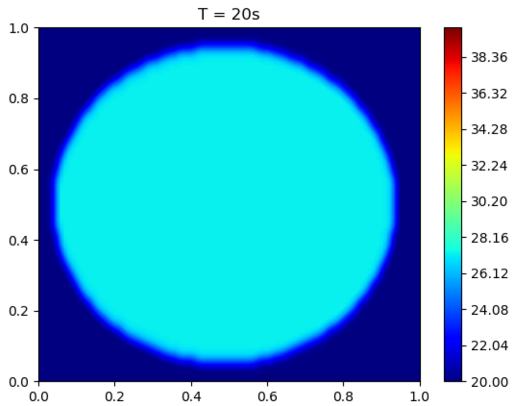
3 Assumptions

1) The material is assumed to be stainless steel with $\alpha = 4.5 \times 10^{-6} m^2/s$. This gives a stable time step of $\Delta t = 5s$ for $\Delta x = 0.01$

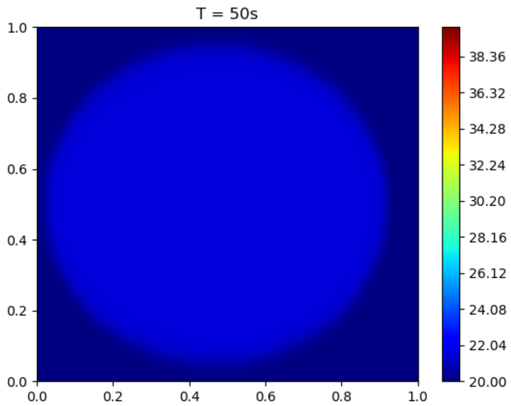
4 Results



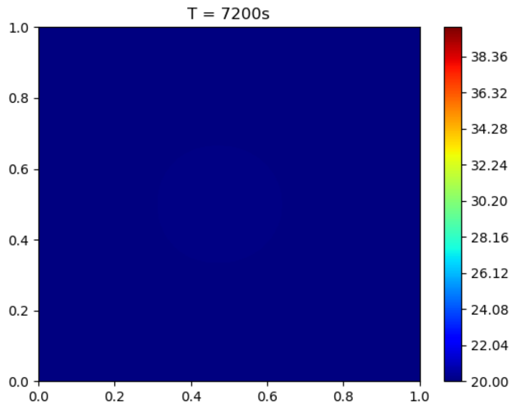
(a) t=5s



(b) t=20s



(c) t=50s



(d) t=7200s

Figure 1: Transient Temperature Profile