Programming Task - Heat Diffusion

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1 Governing Equation

The governing equation for two-dimensional unsteady-state heat conduction problem is given by :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

$$T = 20°C \qquad (\in \Gamma) \tag{2}$$

$$T = \begin{cases} 40°C, & \text{if } (x-0.5)^2 + (y-0.5)^2 < 0.2, t = 0 \\ 20°C, & \text{otherwise, } t = 0 \end{cases} \tag{3}$$

where $T(x, y, t)$ is the transient temperature, $\alpha = \frac{k}{\rho C}$ is the thermal diffusivity of the material and Γ is the boundary of the domain.

2 Discretization

A finite difference (FD) scheme has been used. Assuming uniform discretization, the step length of x-axis is $\Delta x = x_{i+1} - x_i$ and y-axis is $\Delta y = y_{j+1} - y_j$. The step length is controlled by the input of number of divisions,n, such that i,j = 0,1,2...,n. The temporal term on the right hand side can be represented by a first order forward difference approximation to give:

$$\left(\frac{\partial T}{\partial t}\right)^n_{i,j} = \frac{T^{n+1}_{i,j} - T^n_{i,j}}{\Delta t}$$

where $n = 0, 1, 2..$ denotes the time instant and $\Delta t = 5s$ is the time step.Using a second order central difference scheme, the spatial terms on the left hand side are approximated as

$$\frac{\partial^2 T}{\partial x^2}^n_{i,j} = \frac{T^n_{i+1,j} - 2T^n_{i,j} + T^n_{i-1,j}}{(\Delta x)^2}$$

$$\frac{\partial^2 T}{\partial y^2}^n_{i,j} = \frac{T^n_{i,j+1} - 2T^n_{i,j} + T^n_{i,j-1}}{(\Delta y)^2}$$

Assuming $\Delta X = \Delta y$, ultimately, the temperature at a point at any time instant can be given by :

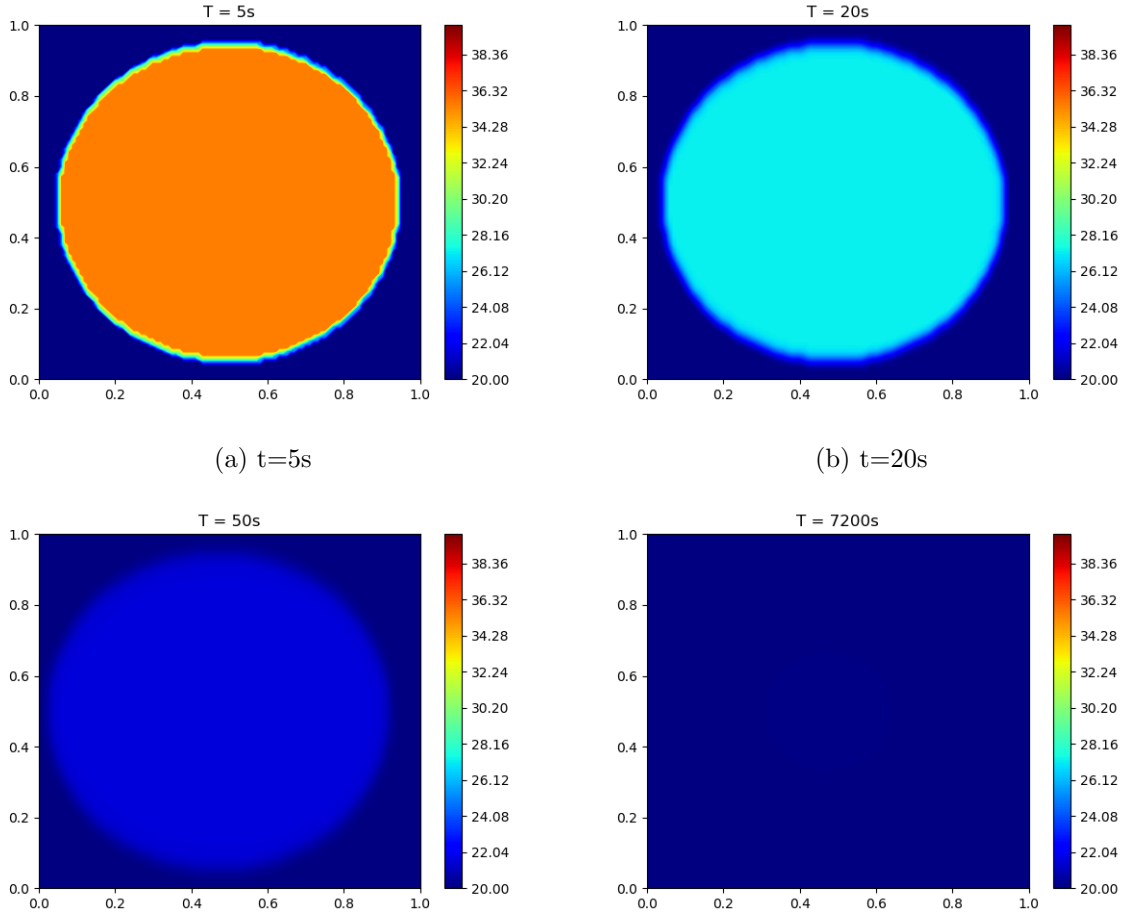
$$T^{n+1}_{i,j} = \tau(T^n_{i+1}j + T^n_{i-1}j + T^n_{i,j+1} + T^n_{i,j-1}) + (1 - 4\tau)T^n_{i,j}$$

where $\tau = \frac{\alpha \Delta t}{(\Delta x)^2}$

3 Assumptions

1) The material is assumed to be stainless steel with $\alpha = 4.5 \times 10^{-6} m^2/s$. This gives a stable time step of $\Delta t = 5s$ for $\Delta x = 0.01$

4 Results



(a) t=5s

(b) t=20s

(c) t=50s

(d) t=7200s

Figure 1: Transient Temperature Profile