EC4333 / EC4333HM: Financial Economics II

AY2024/2025 Semester 1

Homework Assignment 1

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Q1) Bond Prices and Yields

a)

The par yield, is a coupon rate such that the T-yearbond with the face value N is quoted at par (i.e.):

$$\sum_{i=1}^{T} \frac{c(T)}{[1+R(0,t)]^{t}} + \frac{1}{[1+R(0,t)]^{T}} = 1$$

$$\Longrightarrow c(T) = \frac{1-\frac{1}{[1+R(0,t)]^{T}}}{\sum_{i=1}^{T} \frac{1}{[1+R(0,t)]^{t}}}$$

Using the above, we have the following values for par-yield:

Maturity (years)	R(0, t)	(a) Par Yield
1	7.000%	7.000%
2	6.800%	6.807%
3	6.620%	6.636%
4	6.460%	6.487%
5	6.330%	6.367%
6	6.250%	6.293%
7	6.200%	6.246%
8	6.160%	6.209%
9	6.125%	6.177%
10	6.100%	6.154%

b)

A forward (zero-coupon) rate $F(0, t, s - t)$ is the rate at which you could sign a contract today to borrow or lend between periods t and s. That is,

$$F(0, t, s - t) = \left[\frac{[1+R(0,s)]^{s}}{[1+R(0,t)]^{t}}\right]^{1/(s-t)} - 1$$

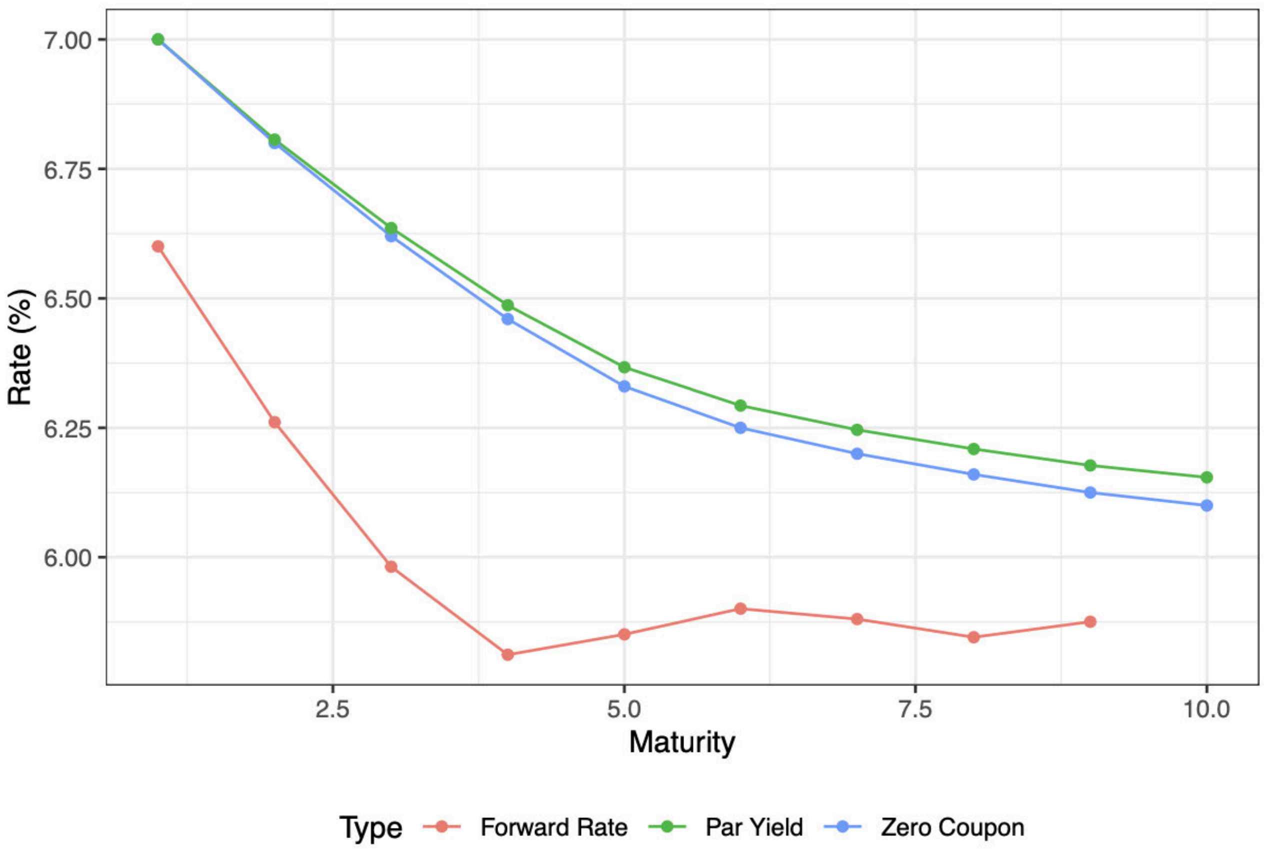
Since, we want to calculate the forward yield curve in one year, we have $s = t + 1$:

$$F(0, t, 1) == \frac{[1+R(0,t+1)]^{t+1}}{[1+R(0,t)]^{t}} - 1$$

Maturity (years)	R(0, t)	(b) Forward Rate
1	7.000%	6.600%
2	6.800%	6.261%
3	6.620%	5.981%
4	6.460%	5.812%
5	6.330%	5.851%
6	6.250%	5.900%
7	6.200%	5.880%
8	6.160%	5.845%
9	6.125%	5.875%
10	6.100%	NA

c)

In the lectures, we learned the following: when current zero-coupon rate curve is decreasing, the current par yield curve is above, and the forward zero-coupon rate curve is below the current zero-coupon rate curve. Here we verify the same:



The observed relationships between the zero-coupon rate curve, par yield curve, and forward yield curve provide insights into monetary policy and its impact on interest rates. In this scenario, the downward-sloping (decreasing) zero-coupon rate curve indicates expectations of lower future interest rates, which can signal that the central bank is pursuing or is expected to pursue a more accommodative monetary policy. The par yield curve, lying above the zero-coupon curve, reflects the yield on coupon-paying bonds, which typically demand higher yields to compensate for coupon payments made before maturity. On the other hand, the forward yield curve being below the current zero-coupon curve suggests that future interest rates are expected to be even lower.

This situation aligns with an easing monetary policy environment, where the central bank lowers interest rates to stimulate economic growth, often in response to slowing inflation or weak economic activity. The market's anticipation of this policy action is reflected in the lower forward rates. Hence, the downward-sloping yield curves can indicate an expectation of continued monetary easing and falling interest rates.

Note: All calculations done in codes.R and HW1 Excel.xlsm (sheet 1)

Q2) Bond Prices and Yields

a)

We fit a polynomial regression model (cubic polynomial) to minimise the sum of squared errors:

$$R^{-}(0, t) = a + bt + ct^{2} + dt^{3}$$

Th solution to the OLS is given by Stata:

```
. reg interest maturity maturity2 maturity3

      Source |       SS           df       MS      Number of obs   =        11
-------------+----------------------------------   F(3, 7)         =     53.88
       Model |  .000464703         3  .000154901   Prob > F        =    0.0000
    Residual |  .000020124         7  2.8749e-06   R-squared       =    0.9585
-------------+----------------------------------   Adj R-squared   =    0.9407
       Total |  .000484828        10  .000048483   Root MSE        =     .0017

------------------------------------------------------------------------------
    interest | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    maturity |   .0025493   .0018056     1.41   0.201    -.0017202    .0068188
   maturity2 |   .0001755   .0004079     0.43   0.680    -.0007891    .0011401
   maturity3 |  -.0000245   .0000256    -0.96   0.370     -.000085     .000036
       _cons |   .0707748    .001958    36.15   0.000     .0661449    .0754047
------------------------------------------------------------------------------

.
end of do-file
```

Note: Stata code used found in appendix Figure 2A

From the Stata output, we can see that the following are the best approximation that minimises the squared loss:

- a = 0.0707748
- b = 0.0025493
- c = 0.0001755
- d = -0.0000245

b)

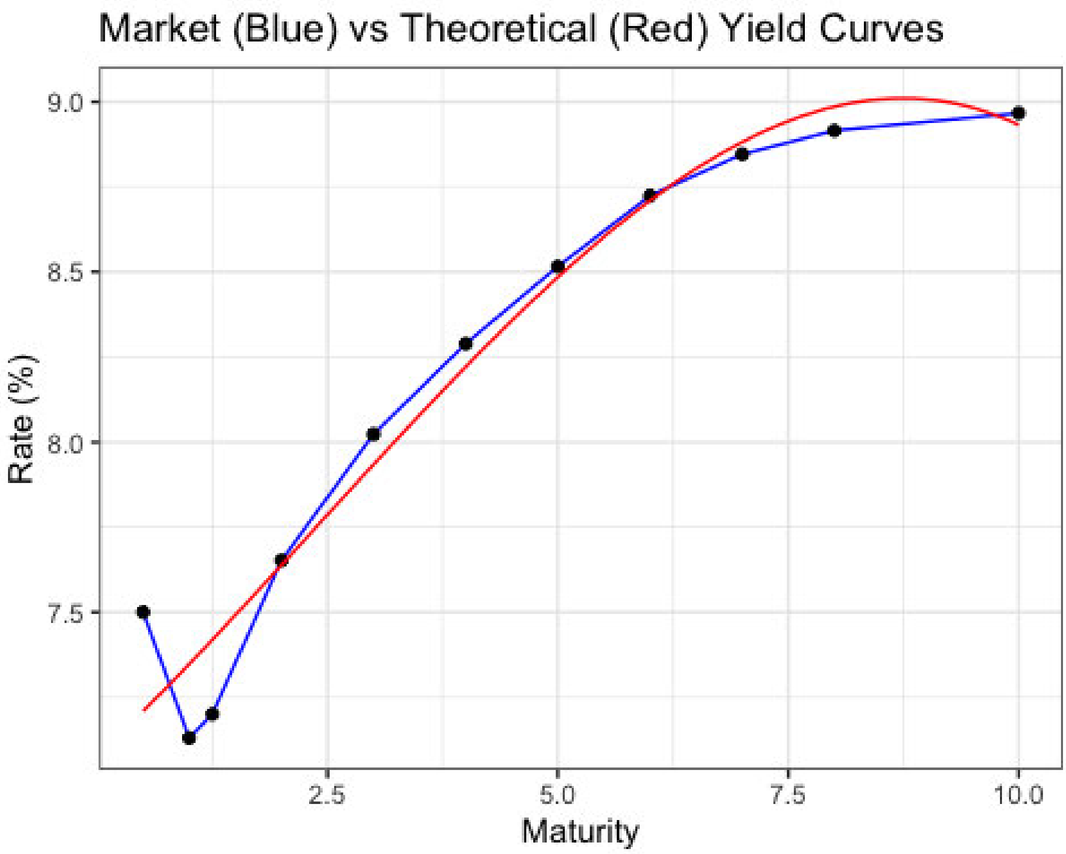
We compute the predictions with the values from part (a) as follows:

$$R(0, 0.8) = a + b * maturity + c * maturity^2 + d * maturity^3$$

Maturity	Calculations	Zero-coupon Rate R(0, maturity)
0.8	$R(0, 0.8) = a + 0.8b + 0.8^2c + 0.8^3d = 7.291\ \%$	7.2914%
1.5	$R(0, 1.5) = a + 1.5b + 1.5^2c + 1.5^3d = 7.491\ \%$	7.4911%
3.4	$R(0, 3.4) = a + 3.4b + 3.4^2c + 3.4^3d = 8.051\ \%$	8.0508%
5.25	$R(0, 5.25) = a + 5.25b + 5.25^2c + 5.25^3d = 8.545\ \%$	8.5451%
8.3	$R(0, 8.3) = a + 8.3b + 8.3^2c + 8.3^3d = 9.002\ \%$	9.0016%
9	$R(0, 9) = a + 9b + 9^2c + 9^3d = 9.007\ \%$	9.0074%

c)

Following are the curves:



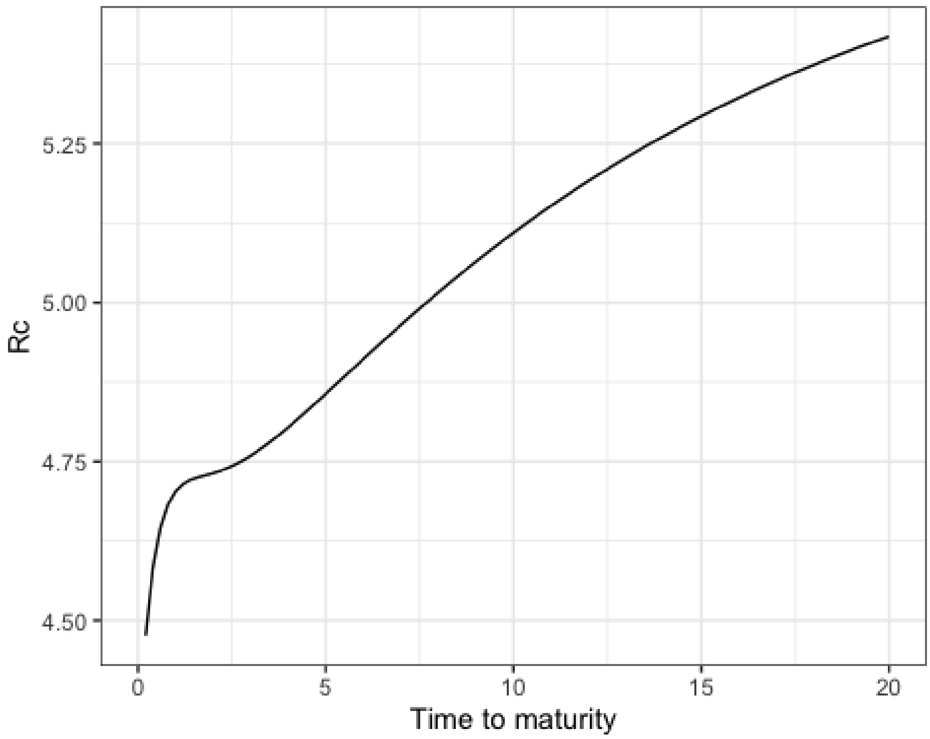
While there are some differences between the market and the theoretically fitted curve, the fit seems quite good overall.

Note: All calculations done in codes.R and HW1 Excel.xlsm (sheet 2).

Q3) Bond Prices and Yields

a)

We plot the curve using the given parameters with the extended Nelson Siegel model:



b)

We use the following equations to compute the price, level, slope and curvature $ duration respectively:

$$P_0 = \sum_{i=1}^{m} F_i e^{-\theta_i R(0,\theta_i)}$$

$$\frac{\partial P_0}{\partial \beta_0} = -\sum_{i=1}^{m} \theta_i F_i e^{-\theta_i R(0,\theta_i)}$$

$$\frac{\partial P_0}{\partial \beta_1} = -\sum_{i=1}^{m} \theta_i \left[\frac{1-exp\left(-\frac{\theta_i}{\tau_1}\right)}{\frac{\theta_i}{\tau_1}} \right] F_i e^{-\theta_i R(0,\theta_i)}$$

$$\frac{\partial P_0}{\partial \beta_2} = -\sum_{i=1}^{m} \theta_i \left[\frac{1-exp\left(-\frac{\theta_i}{\tau_1}\right)}{\frac{\theta_i}{\tau_1}} - exp\left(-\frac{\theta_i}{\tau_1}\right) \right] F_i e^{-\theta_i R(0,\theta_i)}$$

$$\frac{\partial P_0}{\partial \beta_3} = -\sum_{i=1}^{m} \theta_i \left[\frac{1-exp\left(-\frac{\theta_i}{\tau_2}\right)}{\frac{\theta_i}{\tau_2}} - exp\left(-\frac{\theta_i}{\tau_2}\right) \right] F_i e^{-\theta_i R(0,\theta_i)}$$

For bond 1,

$$m = 3,\ F_1 = F_2 = 4,\ F_3 = 104$$

$$P_1 \approx 97.618$$

$$\frac{\partial P_1}{\partial \beta_0} \approx -\ 281.583$$

$$\frac{\partial P_1}{\partial \beta_1} \approx -\ 212.86$$

$$\frac{\partial P_1}{\partial \beta_2} \approx -\ 56.409$$

$$\frac{\partial P_1}{\partial \beta_3} \approx -\ 47.086$$

For bond 2,

$$m = 7,\ F_1 = \cdots = F_6 = 5,\ F_7 = 105$$

$$P_2 \approx 99.601$$

$$\frac{\partial P_2}{\partial \beta_0} \approx -\ 604.559$$

$$\frac{\partial P_2}{\partial \beta_1} \approx -\ 337.789$$

$$\frac{\partial P_2}{\partial \beta_2} \approx -\ 171.249$$

$$\frac{\partial P_2}{\partial \beta_3} \approx -\ 48.579$$

For bond 3,

$$m = 15,\ F_1 = \cdots = F_{14} = 6,\ F_{15} = 106$$

$$P_3 \approx 106.622$$

$$\frac{\partial P_3}{\partial \beta_0} \approx -\ 1108.995$$

$$\frac{\partial P_3}{\partial \beta_1} \approx -\ 418.461$$

$$\frac{\partial P_3}{\partial \beta_2} \approx -\ 299.108$$

$$\frac{\partial P_3}{\partial \beta_3} \approx -\ 51.845$$

The table below summarises the results:

Bond	Price	Level $Duration	Slope $Duration	Curvature $Duraction (tau1)	Curvature $Duration (tau2)
Bond 1	97.62	-281.58322	-212.8598	-56.409283	-47.08554
Bond 2	99.60	-604.55916	-337.78905	-171.24851	-48.578524
Bond 3	106.62	-1108.9947	-418.46149	-299.10819	-51.845016

For a portfolio with 100 units of bond 1, 200 units of bond 2, and 100 units of bond 3,we calculate the price, level, slope and curvature $ duration by using linearity:

$$P_p = 100 \cdot P_1 + 200 \cdot P_2 + 100 \cdot P_3 = 40344.259$$

$$level_p = 100 \cdot level_1 + 200 \cdot level_2 + 100 \cdot level_3 = -259969.623$$

$$slope_p = 100 \cdot slope_1 + 200 \cdot slope_2 + 100 \cdot slope_3 = -130689.939$$

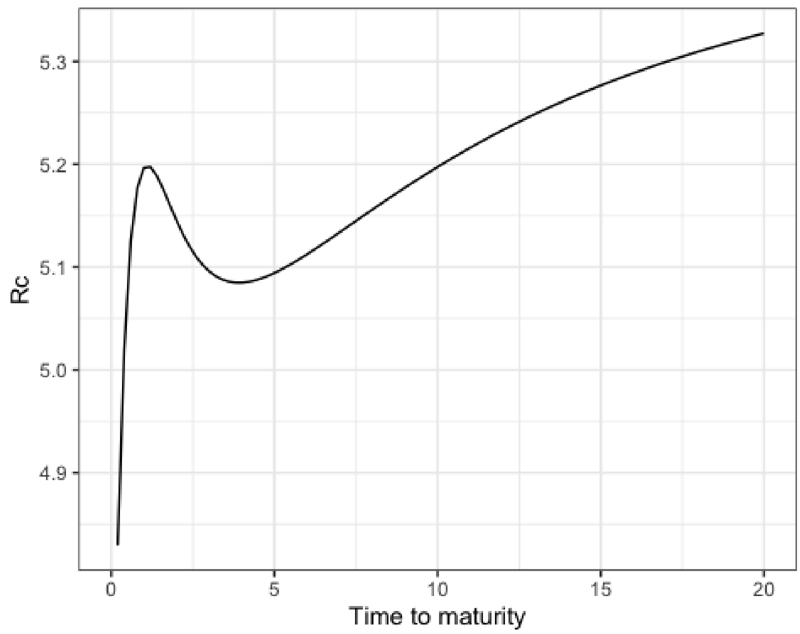
$$curvature_{p,\tau_1} = 100 \cdot curvature_{1,\tau_1} + 200 \cdot curvature_{2,\tau_1} + 100 \cdot curvature_{3,\tau_1} = -69801.449$$

$$curvature_{p,\tau_2} = 100 \cdot curvature_{1,\tau_2} + 200 \cdot curvature_{2,\tau_2} + 100 \cdot curvature_{3,\tau_2} = -19608.76$$

Note: All calculations for 3 a) and 3 b) done in codes.R and HW1 Excel.xlsm (sheet 3)

c)

The new yield curve is shown below:



Portfolio in Part (C)

We use the NS model with the new parameters to first estimate the individual prices of the bond to compare with the new estimated price of the portfolio using the equation suggested by the question. The new price of the bonds using the NS model with updated parameters are:

$$P'_1 \approx 96.663$$

$$P'_2 \approx 98.486$$

$$P'_3 \approx 106.237$$

Thus, the new price of the bond portfolio is:

$$P'_p = 100 \cdot P'_1 + 200 \cdot P'_2 + 100 \cdot P'_3 = 39987.128$$

Now, we used the equation given in the question to estimate the new price of the portfolio and verify if it is consistent with the actual price change. The level, slope and curvature $duration of the portfolio was calculated as a weighted sum of the level, slope and curvature $duration of the individual bonds respectively. The weights are the number of units of bonds in the portfolio.

Bond 1	Bond 2	Bond 3	Old Portfolio Price	Level $Duration	Slope $Duration	Curvature $Duration (tau1)	Curvature $Duration (tau2)
100	200	100	40344.259	-259969.62	-130689.94	-69801.449	-19608.76

$$\Delta\beta_0 = \beta'_0 - \beta_0 = 0.0055 - 0.0059 =- 0.0040$$

$$\Delta\beta_1 = \beta'_1 - \beta_1 = - 0.010 - (- 0.0016) = 0.0060$$

$$\Delta\beta_2 = \beta'_2 - \beta_2 = 0.001 - (- 0.005) = 0.0060$$

$$\Delta\beta_3 = \beta'_3 - \beta_3 = 0.02 - 0.01 = 0.0100$$

$$\begin{aligned}\hat{P}' &= 40344.259 + (- 0.0040)(- 259969.62) + (0.0060)(- 130689.94) \\ &+ (0.0060)(- 69801.449) + (0.0100)(- 19608.76) \\ &\approx 39985.101\end{aligned}$$

We observe that the estimated price (39985.101) based on the level, slope and curvature durations is indeed consistent with the actual price (39987.128) using the NS model.

Note: All calculations for 3 c) done in codes.R and HW1 Excel.xlsm (sheet 4).

d)

The extended Nelson-Siegel model proves to be highly effective in the context of bond pricing, duration analysis, and hedging strategies. One of its key strengths, as discussed in lectures, lies in its ability to capture the overall shape of the yield curve—levels, slopes, and curvatures—through a relatively simple parameterization. This allows market participants to model how interest rates evolve and efficiently manage the risks associated with interest rate movements.

In this particular exercise, we observe that the new estimated price calculated using changes in the parameters and their associated $durations for the portfolio aligns very closely with the actual price change after the shift in the yield curve. This closeness demonstrates the accuracy of the Nelson-Siegel framework in predicting how changes in interest rate levels, slopes, and curvature will affect bond prices, making it a useful tool for hedging.

Appendix

Figure 2A

```
*************************************************
***   Deriving the Zero-Coupon Yield Curve      ***
*************************************************
clear

// Input data
matrix temp = (0.5, 0.07500 \ 1, 0.07130 \ 1.25, 0.07200 \ 2, 0.07652 \ 3, 0.08023 \ 4, 0.08289 \ 5, 0.08516 \ 6, 0.08724 \ 7, 0.08846 \ 8, 0.08915 \ 10, 0.08967)
matrix colnames temp = "maturity" "interest"

matrix list temp

svmat temp, names(col)

/* Q2a) */
gen t = _n

gen maturity2 = maturity^2
gen maturity3 = maturity^3

reg interest maturity maturity2 maturity3
```

Figure 2B

```
/* Q2b) */
// Predict
matrix temp = (0.8 \ 1.5 \ 3.4 \ 5.25 \ 8.3 \ 9)
matrix colnames temp = "new_maturity"
svmat temp, names(col)

rename maturity old_maturity
rename new_maturity maturity
replace maturity2 = maturity^2
replace maturity3 = maturity^3

predict pred, xb

rename maturity new_maturity
rename old_maturity maturity
replace maturity2 = maturity^2
replace maturity3 = maturity^3
```