EC4333 Financial Economics II

Department of Economics
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Homework Assignment 2
Due: 11:59 pm, 29 October 2024

RULES You are encouraged to work on the homework in groups. If you choose to do so, only one copy needs to be submitted per group, and you must clearly list the names of all group members. Group members can come from either EC4333 or EC4333HM for this problem set. Submit a digital copy on Canvas before the deadline.

Decision Under Uncertainty

1. Consider the following utility function where w denotes wealth:

$$u(w) = a - be^{\gamma w}, \gamma > 0.$$

   (a) Discuss whether the investor is non-satiable and/or risk-adverse.

   (b) Calculate the absolute and relative risk aversion coefficients of the utility function. Does the form of risk aversion intuitively make sense?

2. Consider a game of chance in which a coin is tossed repeatedly until a head appears. The probability of getting a head is p, and the trials are independent. Let the random variable N denote the number of trials on which the first head occurs.

   (a) Show that the probability of obtaining the first head on the n-th trial is given by:

$$\mathbb{P}(N = n) = p(1-p)^{n-1}$$

   (b) If a reward $w = p^{-n}$ is paid and the utility of this reward is described by the utility function $u(w) = \ln(w)$, find the expected utility of the reward for this game, i.e., $\mathbb{E}(u(x))$.

3. Desmond is a promising basketball talent currently playing in Singapore. If he avoids injury until the end of the season, he will be offered an NBA contract worth 1 million Singapore dollars. However, if he gets injured, he can only work as a coach in Singapore, receiving a contract worth 10,000 Singapore dollars. There is a 10% probability that he gets injured and a 90% probability that he stays healthy until the end of the season.

   (a) What is his expected utility if his utility function is $u(w) = \ln(w)$?

   (b) Suppose Desmond can buy insurance at a price of G that will pay him 990,000 Singapore dollars in case of injury. What is the maximum price G that Desmond would be willing to pay for this insurance?

Stochastic Dominance

4. In the proof of Theorem 2 from Lecture 5 (see Slide 40), we claim that when $\mathbb{E}_F(x) = \mathbb{E}_G(x)$, i.e., two risky assets have the same mean, the following holds:

$$\int_{\underline{w}}^{t} [F(s) - G(s)]\, ds \Big|_{t=\underline{w}}^{t=\bar{w}} = 0.$$

   Provide a rigorous proof for this claim.

   Hint: Show that when x with the CDF F has non-negative support, $\mathbb{E}_F(x) = \int_{\underline{w}}^{\bar{w}} [1 - F(s)]\, ds$.

5. You are given the following information on investment options A and B.

   Investment A: Outcomes (W) in $[0, 1]$ follow a uniform distribution; and

   Investment B: Outcomes (W) in $[0, 1]$ are as follows:

$$\text{Prob}(W \leq w) = \begin{cases} \sqrt{w}, & \text{for } 0 \leq w \leq 0.16 \\ 0.5, & \text{for } 0.16 < w \leq 0.6 \\ 0.75, & \text{for } 0.6 < w \leq 0.8 \\ 0.9, & \text{for } 0.8 < w \leq 1.0 \\ 1.0, & \text{for } w \geq 1.0 \end{cases}$$

   (a) Based on the FOSD rule, discuss which option is preferred.

   (b) Based on the SOSD rule, discuss which option is preferred.

Mean-Variance Portfolio Choice

6. The aim of this question is to show that when an individual's wealth is normally distributed with mean μ and variance σ^2, they only care about the mean and variance when maximizing their expected utility. In other words, this question serves as a motivation for Markowitz's mean-variance portfolio choice theory. For the following questions, assume the utility function is $u(w)$.

   (a) Find the Taylor expansion of the utility function around a certain wealth level w_0 up to the third order.

   (b) Find the expected utility $\mathbb{E}(u(w))$ using the approximation from the previous question and then replace w_0 with 0, as it is an arbitrary value.

   (c) Show that the expected utility $\mathbb{E}(u(w))$ is a function of μ and σ^2.

7. You are given the following information on the expected returns $E(R)$ and variance/covariance matrix Σ of returns in a four-asset world (say, A, B, C and D):

$$E(R)^\top = (0.20 \quad 0.15 \quad 0.15 \quad 0.20)$$

$$\Sigma = \begin{pmatrix} 0.25 & -0.15 & 0.15 & 0.05 \\ -0.15 & 0.21 & -0.15 & -0.15 \\ 0.15 & -0.15 & 0.20 & 0.05 \\ 0.05 & -0.15 & 0.05 & 0.25 \end{pmatrix}$$

(a) Explain clearly how you could construct the global minimum variance portfolio. Obtain its expected return and variance.

(b) Construct the minimum variance portfolio which has an expected (target) return of 22%. Compute the corresponding variance as well.