EC4333 Financial Economics II

Department of Economics
National University of Singapore

Homework Assignment 1
Due: 11:59 pm, 19 September 2024

RULES You are encouraged to work on the homework in groups. If you choose to do so, only one copy needs to be submitted per group, and you must clearly list the names of all group members. Group members can come from either EC4333 or EC4333HM for this problem set. Submit a digital copy on Canvas before the deadline. Use Stata to complete the computational questions, and include a printout of the do-file at the end of your homework (DO NOT include logs or large outputs). Answers to computational questions should be presented in a report format, meaning they should be typed or written up and supplemented with relevant graphs, tables, and numbers as needed. Do not insert answers as comments within the do-file or provide screenshots of entire regression outputs when only a few specific numbers are required.

Bond Prices and Yields

1. We consider the following decreasing zero-coupon yield curve:

Maturity (years)	$R(0,t)$ **(%)**	**Maturity (years)**	$R(0,t)$ **(%)**
1	7.000	6	6.250
2	6.800	7	6.200
3	6.620	8	6.160
4	6.460	9	6.125
5	6.330	10	6.100

where $R(0,t)$ is the zero-coupon rate at date 0 with maturity t.

(a) Compute the par yield curve.
(b) Compute the forward yield curve in one year.
(c) Draw the three curves on the same graph. What can you say about their relative position?

What conclusions do you draw from this as regards the relationship existing between monetary policy and interest rates?

Deriving the Zero-Coupon Yield Curve

2. From the prices of zero-coupon bonds quoted in the market, we obtain the following zero-coupon curve:

Maturity (years)	$R(0,t)$ **(%)**
0.5	7.500
1	7.130
1.25	7.200
2	7.652
3	8.023
4	8.289
5	8.516
6	8.724
7	8.846
8	8.915
10	8.967

where $R(0,t)$ is the zero-coupon rate at date 0 with maturity t and $B(0,t)$ is the discount factor at date 0 with maturity t.

We need to know the value for $R(0,0.8)$, $R(0,1.5)$, $R(0,3.4)$, $R(0,5.25)$, $R(0,8.3)$ and $R(0,9)$ where $R(0,i)$ is the zero-coupon rate at date 0 with maturity i. We have to estimate them and test two different methods.

We postulate the following form for the zero-coupon rate function $\bar{R}(0,t)$:

$$\bar{R}(0,t) = a + bt + ct^2 + dt^3$$

(a) Estimate the coefficients a, b, c and d, which best approximate the given zero-coupon rates, using the following optimization program:

$$\min_{a,b,c,d} \sum_i \left(R(0,i) - \bar{R}(0,i) \right)^2$$

where $R(0,i)$ are the zero-coupon rates given by the market. Compare these rates $R(0,i)$ to the rates $\bar{R}(0,i)$ given by the model.

(b) Find the value for the six zero-coupon rates that we are looking for.

(c) Draw the two following curves on the same graph:

- the market curve by plotting the market points and
- the theoretical curve as derived from the prespecified functional form.

Hedging Interest-Rate Risk with Duration

3. On 24 August 2024, the values of the Nelson and Siegel Extended parameters are as follows

β_0	β_1	β_2	τ_1	β_3	τ_2
5.9%	-1.6%	-0.5%	5	1%	0.5

Recall from Lecture 3 that the continuously compounded zero-coupon rate $R^c(0,\theta)$ is given by the following formula:

$$
\begin{aligned}
R^c(0,\theta) = \beta_0 + \beta_1 & \left[\frac{1-\exp\left(-\frac{\theta}{\tau_1}\right)}{\frac{\theta}{\tau_1}}\right] \\
+ \beta_2 & \left[\frac{1-\exp\left(-\frac{\theta}{\tau_1}\right)}{\frac{\theta}{\tau_1}} - \exp\left(-\frac{\theta}{\tau_1}\right)\right] \\
+ \beta_3 & \left[\frac{1-\exp\left(-\frac{\theta}{\tau_2}\right)}{\frac{\theta}{\tau_2}} - \exp\left(-\frac{\theta}{\tau_2}\right)\right]
\end{aligned}
$$

(a) Draw the zero-coupon yield curve associated with this set of parameters.

(b) We consider three bonds with the following features. Coupon frequency is annual.

Bond	**Maturity (years)**	**Coupon (%)**
Bond 1	3	4
Bond 2	7	5
Bond 3	15	6

Compute the price and the level, slope, and curvature \$durations of each bond. Compute also the same \$durations for a portfolio with 100 units of bond 1, 200 units of bond 2, and 100 units of bond 3.

(c) The parameters of the Nelson and Siegel Extended model change instantaneously to become

β_0	β_1	β_2	τ_1	β_3	τ_2
5.5%	-1%	0.1%	5	2%	0.5

i. Draw the new zero-coupon yield curve.

ii. Compute the new price of the bond portfolio and compare it with the value given by the following equation:

New estimated price = former price$+\Delta\beta_0 \cdot D_{0,P} + \Delta\beta_1 \cdot D_{1,P} + \Delta\beta_2 \cdot D_{2,P} + \Delta\beta_3 \cdot D_{3,P}$

where $\Delta\beta_i$ is the change in value of parameter β_i, and $D_{i,P}$ is the \$duration of the bond portfolio associated with parameter β_i.

(d) Conclude.