

Other Related Work

Related Work

Summarizing drawbacks of the related work:

- Time and space complexity for physics based approach increases with the size of environment and the resolution of its representation.
- Search based approach i.e. A* will provide a solution but at a cost of time and memory. Reducing the resolution of the environment might cause unsafe paths. Modified/Improvised A* still doesn't solve this problem.
- General RRT's although fast in open spaces, tend to considerably slow down in narrow spaces. Several improvements in sampling solve this issue, but doesn't take into account the robot model and motions which is vital for obtaining a collision free path in confined spaces.
- The automotive approaches solely try to solve specific scenarios. Scaling the approach to a large environment will yield inefficient results. Also, dynamic environment cannot be handled.
- Motion predictive control although handles dynamic obstacles, cannot solve confined space navigation without optimal reference path.

Related Work

Near Optimal Planning for Piano Mover's Problem

- **Requirements:** Map representing the environment, robot's dimensions, motion primitives (8-geometry maze router including rotation).

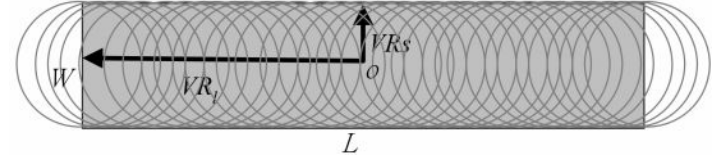


Figure 3: The rectangle model of a robot configuration.

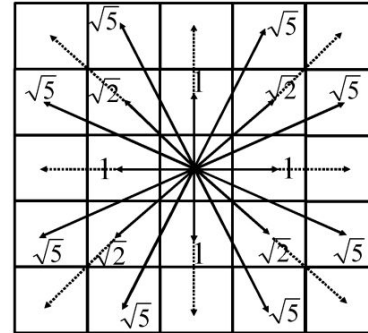


Figure 4: 8-geometry maze router.

1. Gene Eu Jan, Tong-Ying Juang, Jun-Da Huang, Chien-Min Su and Chih-Yung Cheng, "A fast path planning algorithm for piano mover's problem on raster," Proceedings, 2005 IEEE/ASME International Conference on Advanced Intelligent Mechatronics., 2005, pp. 522-527, doi: 10.1109/AIM.2005.1511035.

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- **Requirements:** Map representing the environment, robot's dimensions, motion primitives (8-geometry maze router including rotation).
- Environment is decomposed into cells to a particular resolution and each cell carries seven set of information.
- AT (Time of Arrival) uses the maze router to obtain cost from the start to current cell.
- Certain information are encoded before planning and the rest are assigned during planning.

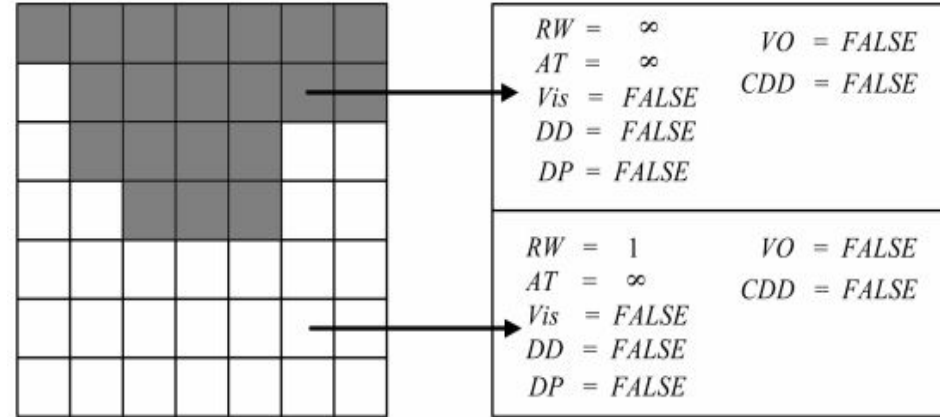


Figure 5: Cells data structures where RW (Regional Weight), AT (Time of Arrival), Vis (Visited), DD (Detection Diameter), DP (Detection Pie), VO (Virtual Obstacle), CDD (Collision-Detection Domain).

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- AT (Time of Arrival) uses the maze router to obtain cost from the start to current cell.
- Certain information are encoded before planning and the rest are assigned during planning.
- Path obtained by backtracking with the minimum value of time of arrival from the destination cell to the source cell taking into account the cost obtained by cell info.

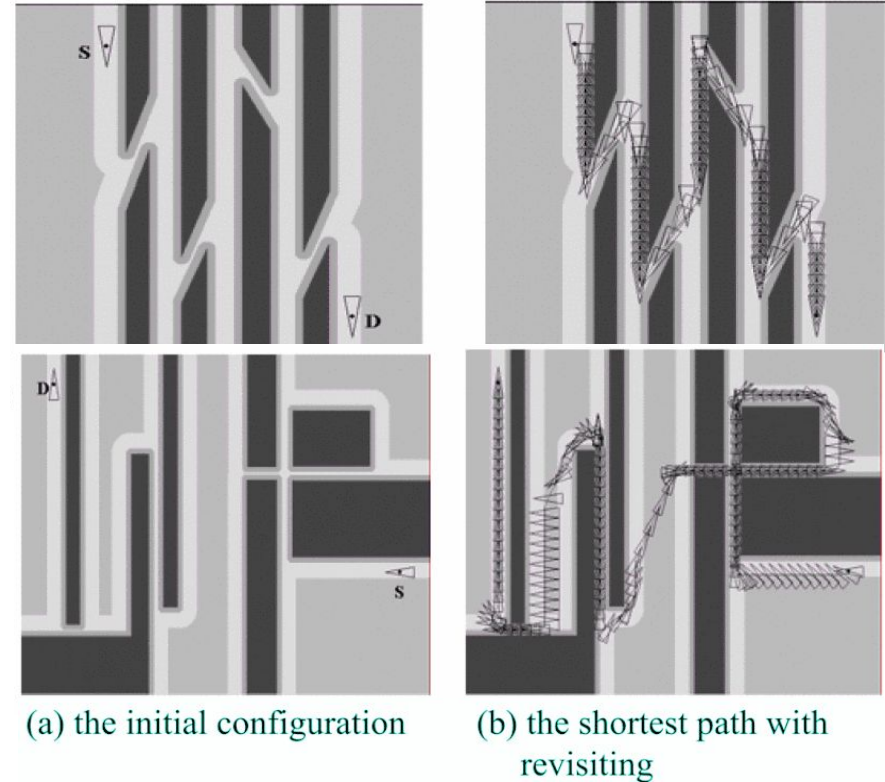


Figure 6: Illustration of the path planning of robot motion using rectangle model.

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- Certain information are encoded before planning and the rest are assigned during planning.
- Path obtained by backtracking with the minimum value of time of arrival from the destination cell to the source cell taking into account the cost obtained by cell info.
- **Drawbacks:** Complexity increases as the resolution or size of the environment or number of motion primitives increase. Not suitable for long range navigation.

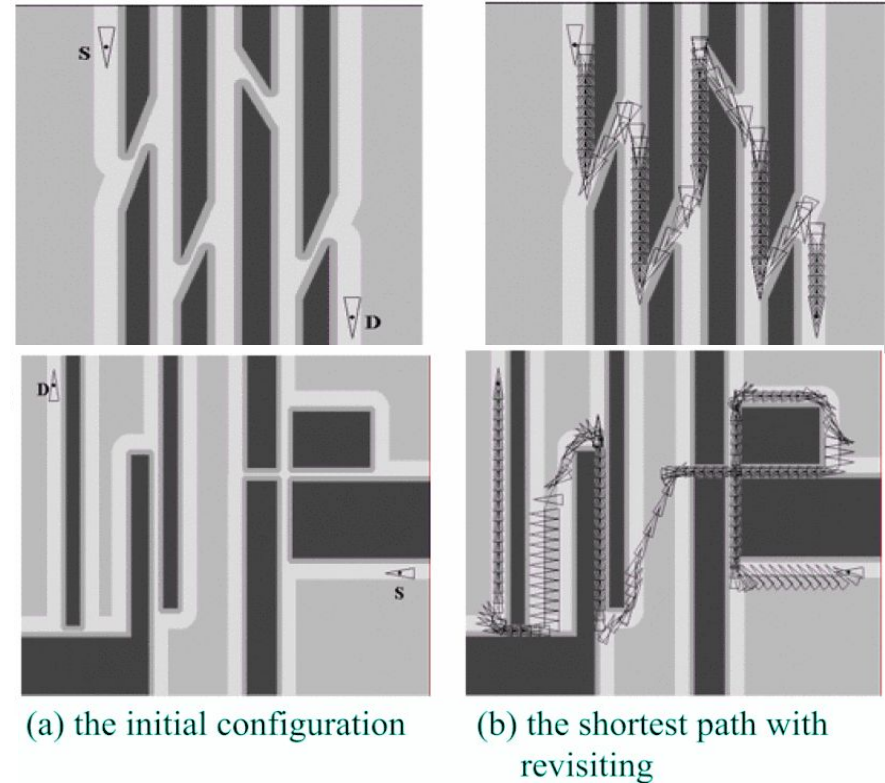


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Related Work

Cylindrical Algebraic Decompositions (CAD)

- **Requirements:** *Map representing the environment, robot's dimensions.*
- The problem considers a moving ladder ($[x, y]$, $[w, z]$) in configuration space.

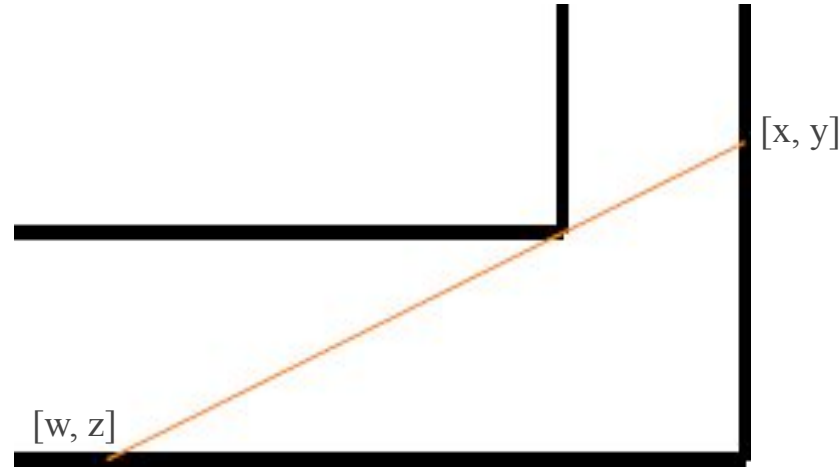


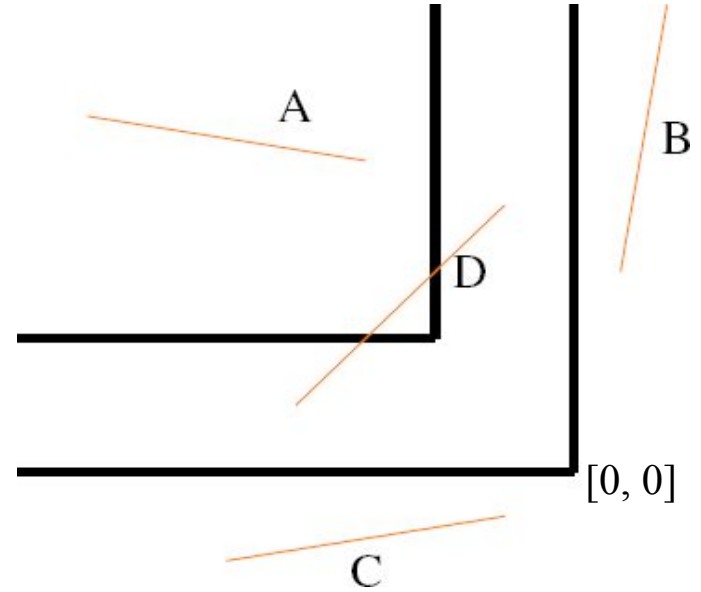
Figure 7: A configuration of a ladder in which the endpoints are in opposite branches of the corridor.

2. D. Wilson, J. H. Davenport, M. England and R. Bradford, "A "Piano Movers" Problem Reformulated," 2013 15th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing, 2013, pp. 53-60, doi: 10.1109/SYNASC.2013.14.

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- **Requirements:** Map representing the environment, robot's dimensions.
- The problem considers a moving ladder ($[x, y], [w, z]$) in configuration space (representing tight corners).
- First, express all possible invalid regions, then take its negation, 't' represents any point on the ladder.
- Quantifier Elimination by Partial CAD (QEPCAD) is used to construct cells and returning the equivalent quantifier-free expression.



$$\begin{aligned} & [x < -1 \wedge y > 1] \vee [w < -1 \wedge z > 1] \vee [x > 0] \\ & \vee [w > 0] \vee [y < 0] \vee [z < 0] \vee (\exists t)[0 < t \wedge t < 1 \\ & \wedge x + t(w - x) < -1 \wedge y + t(z - y) > 1]. \end{aligned}$$

Figure 8: Four canonical invalid positions of the ladder and their expression.

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- Each cell is described by a semi-algebraic set (a finite sequence of polynomial equations and inequalities).

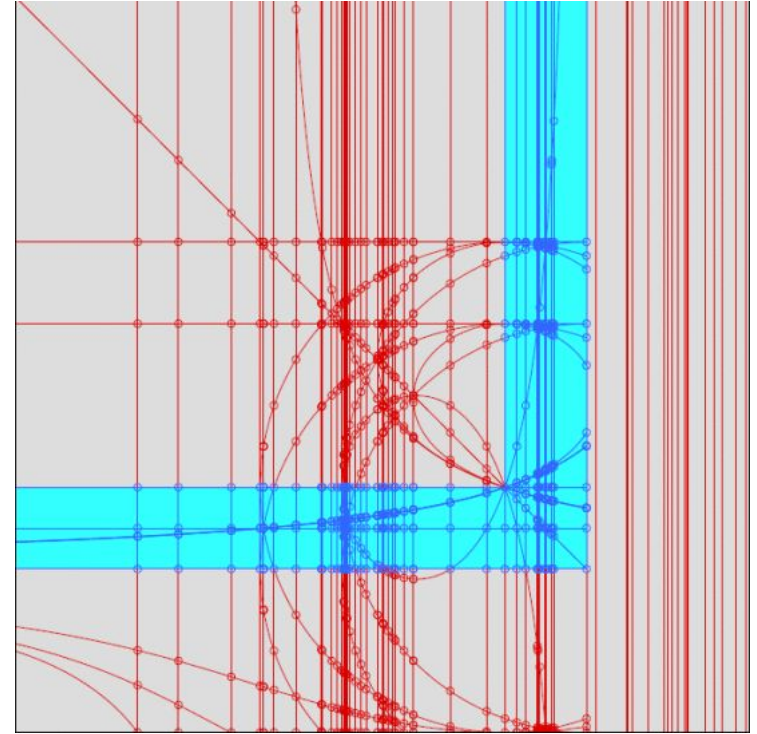


Figure 9: A two-dimensional CAD of just $[x,y]$ point in configuration space.

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- First, express all possible invalid regions, then take its negation, 't' represents any point on the ladder.
- Quantifier Elimination by Partial CAD (QEPCAD) is used to construct cells and returning the equivalent quantifier-free expression.
- Each cell is described by a semi-algebraic set (a finite sequence of polynomial equations and inequalities).
- **Drawbacks:** *Finding adjacency and connectedness of cells in the four-dimensional CAD is not currently possible with any existing technology. This is just a new way to formulate the problem and maybe could produce a solution in the future.*

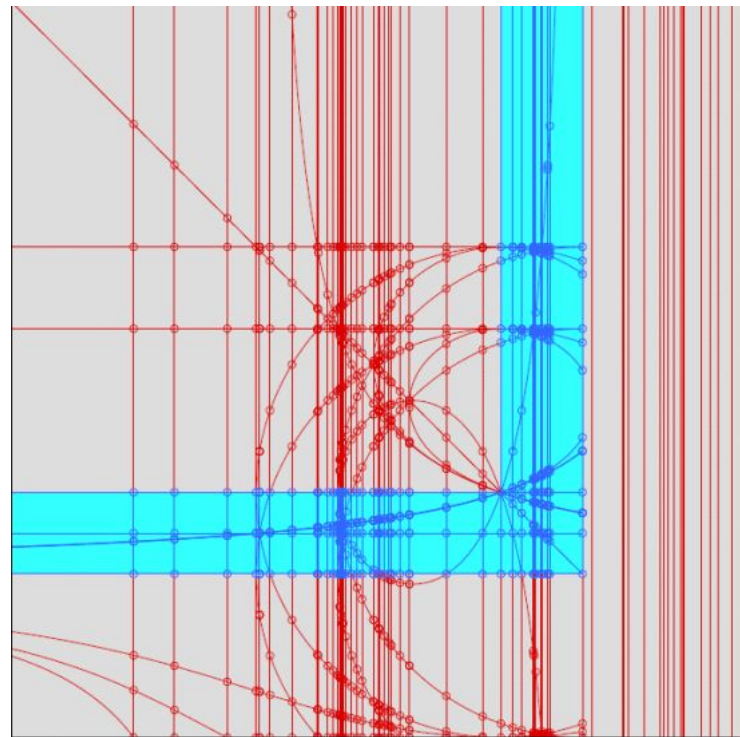


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Related Work

Locally Guided Multiple Bi-RRT*

- **Requirements:** *Map representing the environment, considers robot as a point.*
- Traditional RRTs consume high memory as well as time while finding a solution in cluttered environment.

3. X. Shu, F. Ni, Z. Zhou, Y. Liu, H. Liu and T. Zou, "Locally Guided Multiple Bi-RRT* for Fast Path Planning in Narrow Passages," 2019 IEEE International Conference on Robotics and Biomimetics (ROBIO), 2019, pp. 2085-2091, doi: 10.1109/ROBIO49542.2019.8961757.

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- Traditional RRTs consume high memory as well as time while finding a solution in cluttered environment.
- Sampling in narrow space is done using bridge test.

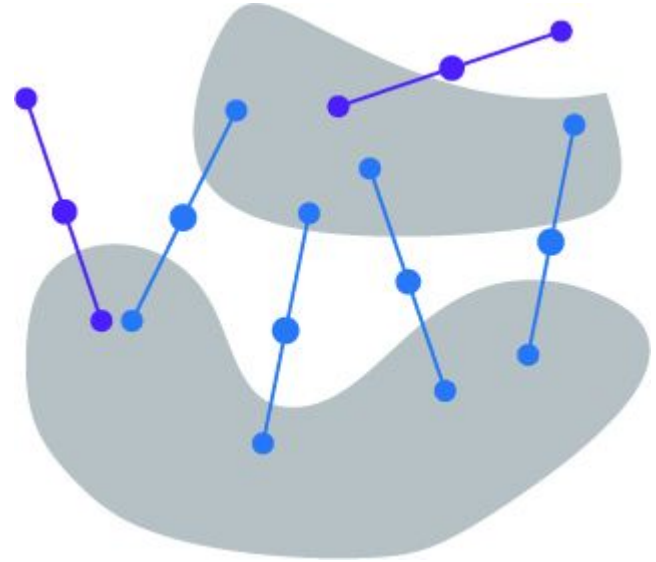


Figure 18: The principle of bridge test. The color in blue indicates passing the test, otherwise don't passing.

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- K-means++ clustering algorithm is employed to obtain the Identification Points (IP).

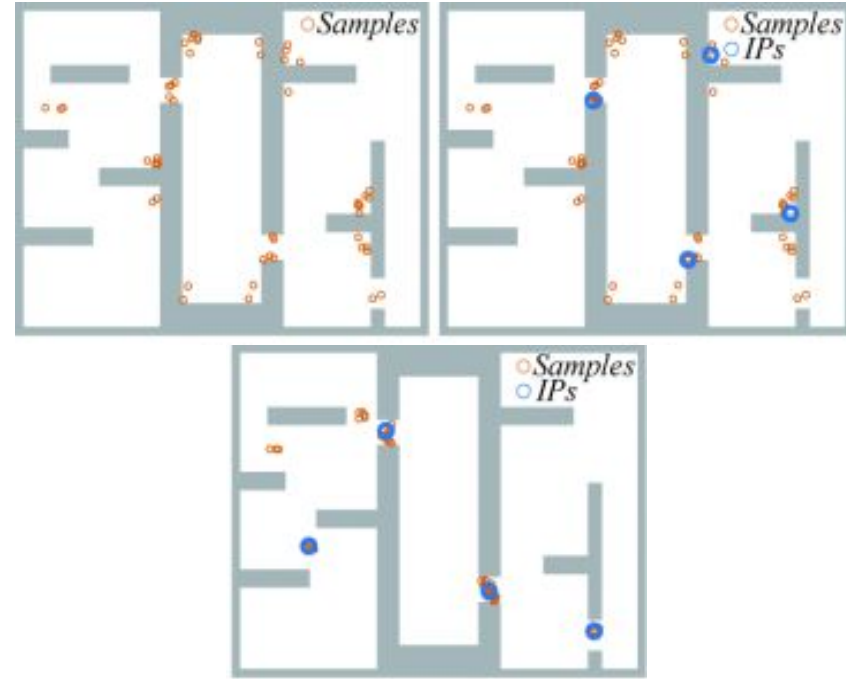


Figure 19: The improved bridge-test method with cluster analysis.

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- Sampling in narrow space is done using bridge test.
- K-means++ clustering algorithm is employed to obtain the Identification Points (IP).
- Generate local trees rooted at each IP and employ BRRT* to heuristically expand one by one.

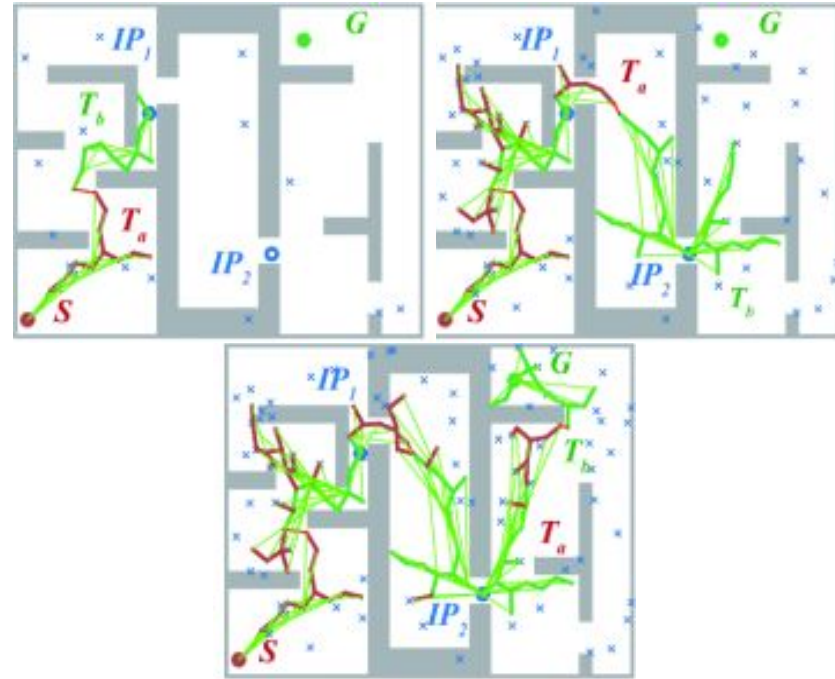


Figure 20: BRRT* applied between IPs and concatenated for a final path

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Related Work

Fast Bi-Directional Kinematic RRT

- **Requirements:** Map representing the environment, considers robot as a point.
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- Prunes nodes that have been failed to expand too many times.

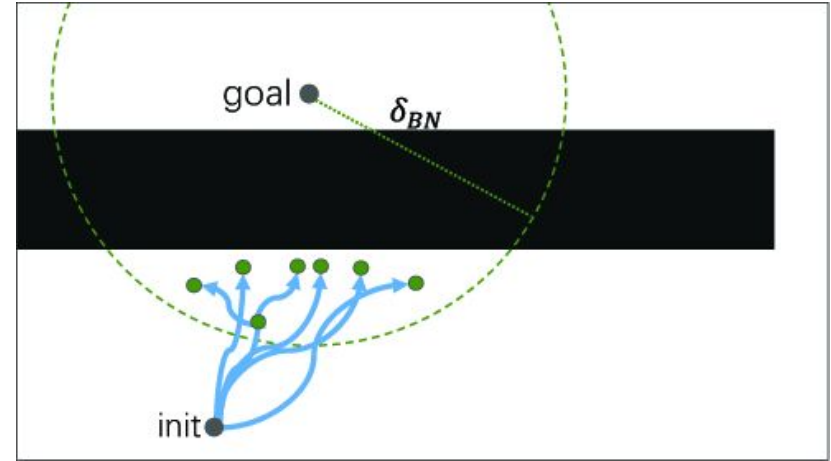


Figure 21: Nodes near obstacles failing to expand

4. J. Peng, Y. Chen, Y. Duan, Y. Zhang, J. Ji and Y. Zhang, "Towards an Online RRT-based Path Planning Algorithm for Ackermann-steering Vehicles," 2021 IEEE International Conference on Robotics and Automation (ICRA), 2021, pp. 7407-7413, doi: 10.1109/ICRA48506.2021.9561207.

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- Traditional RRTs consume high memory as well as time while finding a solution in cluttered environment.
- Prunes nodes that have been failed to expand too many times.
- Apply the idea of Rapid Random Vines to improve the performance of the algorithm for environments with narrow passages.

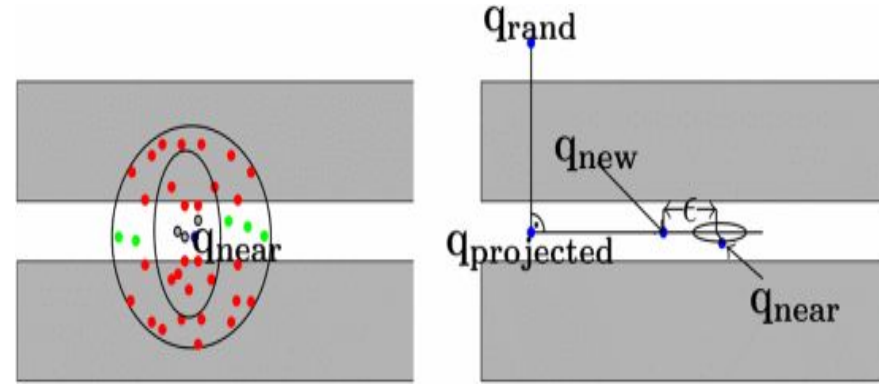


Figure 22: A vine node q_{near} is in a narrow passage and is not able to expand towards q_{rand} . After PCA has been conducted, the vine grows along the passage

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Related Work

Adaptive Rapidly-Exploring Random Tree Connect

- **Requirements:** Map representing the environment, considers robot as a point.
- Traditional RRTs consume high memory as well as time while finding a solution in cluttered environment.
- Nodes are sampled in unexplored area as much as possible at the beginning and the rate of random sampling is gradually decreased.

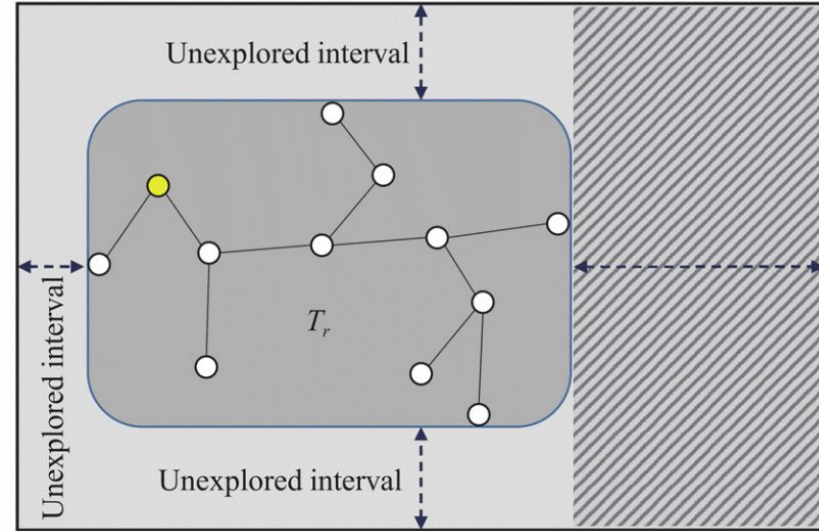


Figure 23: Rapidly expanding to unexplored regions

5. B. Li and B. Chen, "An Adaptive Rapidly-Exploring Random Tree," in IEEE/CAA Journal of Automatica Sinica, vol. 9, no. 2, pp. 283-294, February 2022, doi: 10.1109/JAS.2021.1004252.

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- Nodes are sampled in unexplored area as much as possible at the beginning and the rate of random sampling is gradually decreased.
- If the initial search fails (upto certain iteration), partially connected nodes from start and goal is expanded locally.

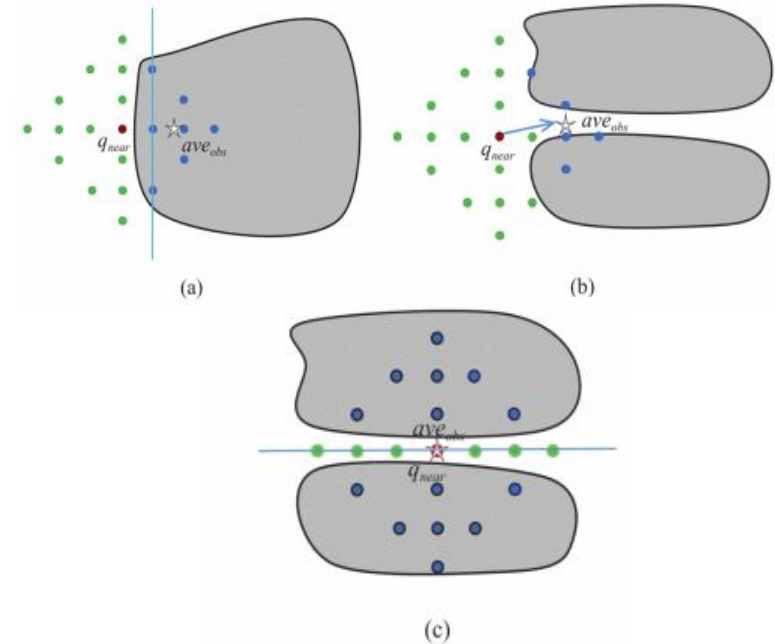


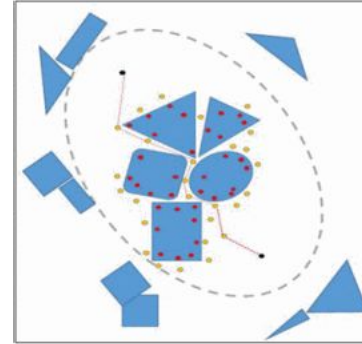
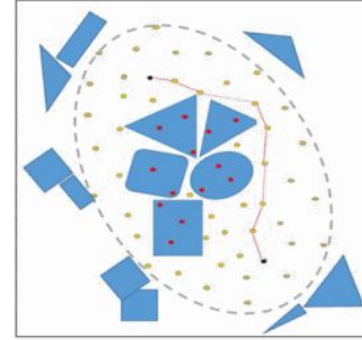
Figure 24: Behavior at narrow spaces

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Related Work

Obstacle-Guided Sampling for RRTs

- **Requirements:** Map representing the environment, considers robot as a point.
- Traditional RRTs consume high memory as well as time while finding a solution in cluttered environment.
- Gaussian obstacle-based sampling strategy is extended resulting in denser sample distribution near obstacles while retaining an underlying uniform spread over the free space.



```
1  $x_{found} \leftarrow \emptyset$ 
2 if IsValid( $x_{mean}$ ) then
3    $x_{found} \leftarrow x_{mean}$ 
4 else
5   for count = 1 to count = limit do
6      $x_{temp} \leftarrow \text{Gaussian}(x_{mean}, stdev)$ 
7     if IsValid( $x_{temp}$ ) then
8        $x_{found} \leftarrow x_{temp}$ 
9       if count  $\geq \frac{limit}{2}$  then
10          $\mathcal{N} \leftarrow \{x_{found}\}$ 
11         break
12 return  $x_{found}$ 
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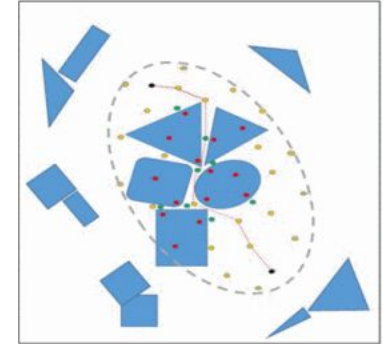


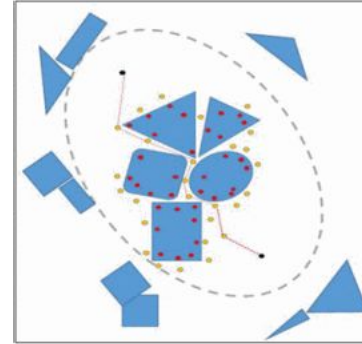
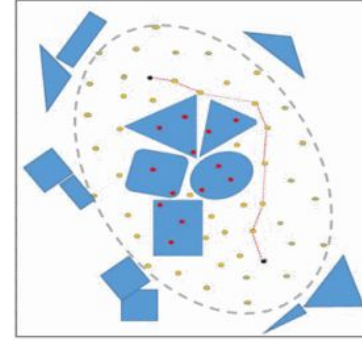
Figure 25: Gaussian obstacle-based sampling strategy applied over obstacles

6. Z. Meng, H. Qin, H. Sun, X. Shen and M. H. Ang, "Obstacle-guided informed planning towards robot navigation in cluttered environments," 2017 IEEE International Conference on Robotics and Biomimetics (ROBIO), 2017, pp. 332-337, doi: 10.1109/ROBIO.2017.8324439.

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- Gaussian obstacle-based sampling strategy is extended resulting in denser sample distribution near obstacles while retaining an underlying uniform spread over the free space.
- **Drawbacks:** Although these improvised RRTs are fast, these approaches do not account for the orientation or dimensions of the robot.



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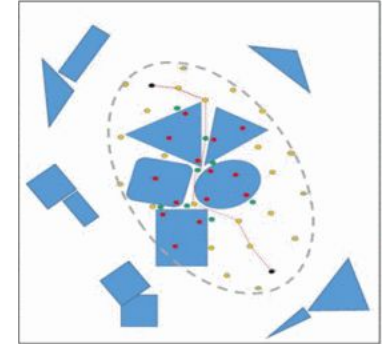


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Spline-Based Planning

- **Requirements:** *Map representing the environment, dynamic obstacles information.*
- Approach defines an optimal control problem to find the trajectory in confined spaces and dynamic environment.

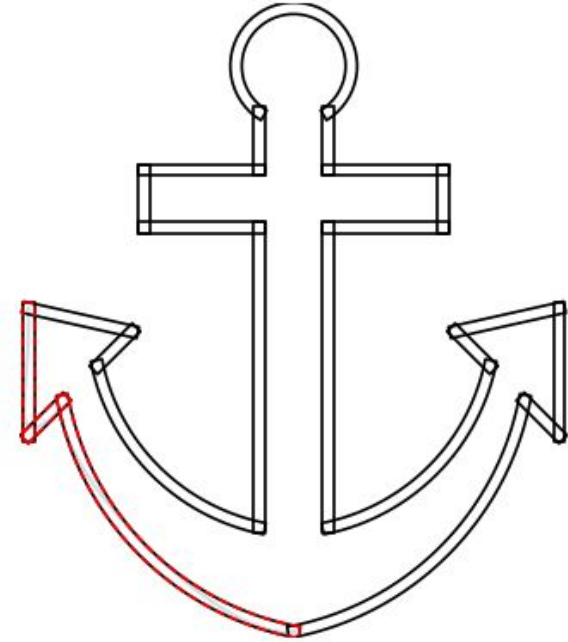


Figure 35: Spline planning in narrow spaces.

7. T. Mercy, R. Van Parys and G. Pipeleers, "Spline-Based Motion Planning for Autonomous Guided Vehicles in a Dynamic Environment," in IEEE Transactions on Control Systems Technology, vol. 26, no. 6, pp. 2182-2189, Nov. 2018, doi: 10.1109/TCST.2017.2739706.

Related Work

Spline-Based Planning

- **Requirements:** *Map representing the environment, dynamic obstacles information.*
- Approach defines an optical control problem to find the trajectory in confined spaces and dynamic environment.
- Given start and end points and obstacles position, the algorithm creates a potential field about the obstacles.

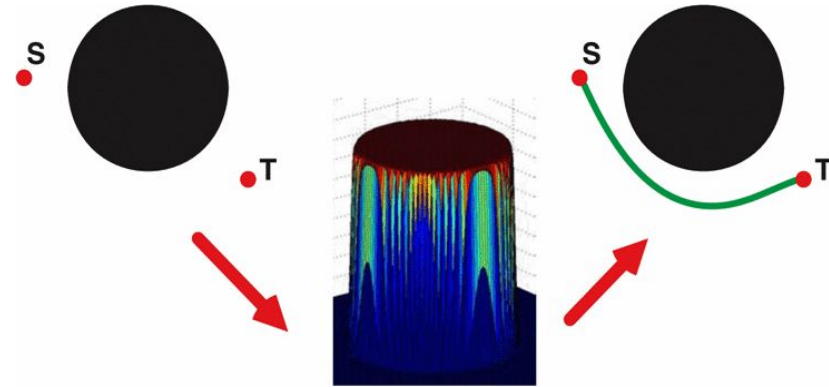


Figure 36: Generating spline by creating potential field.

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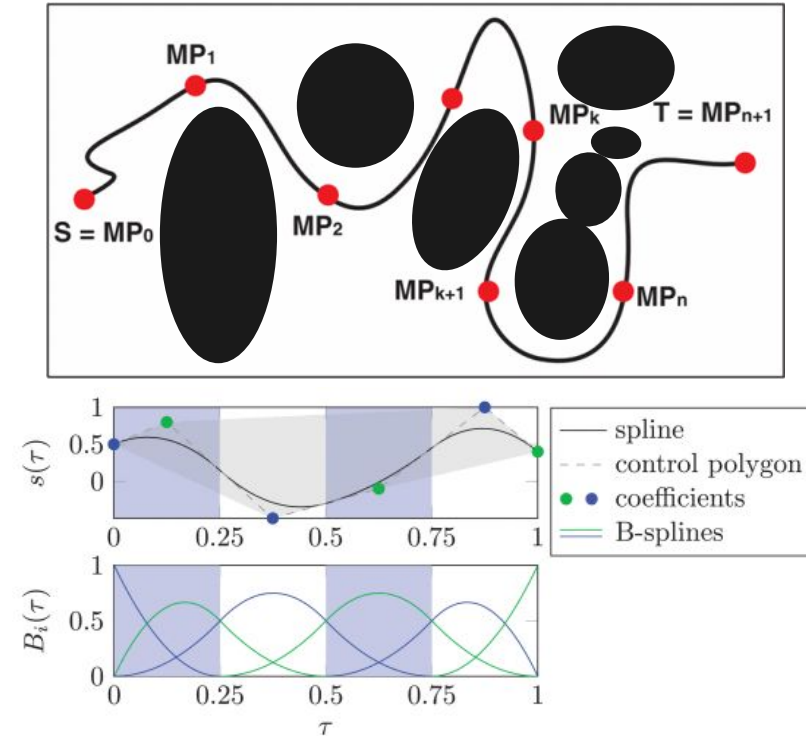


Figure 37: Optimizing spline globally.

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- The initial straight line path from 'S' to 'T' is expressed with 'via points' equally spaces and each point is pushed to a free space.
- To account for obstacle movement, the constraints include a linear prediction model for every obstacle and the algorithm updates the spline continuously.
- **Drawbacks:** *Dynamic obstacles velocity and positions are previously known, which in reality has to be detected, which would pose a problem while using laser scans. Vehicle dimension isn't taken into account. Here, splines solve forwards motion planning.*

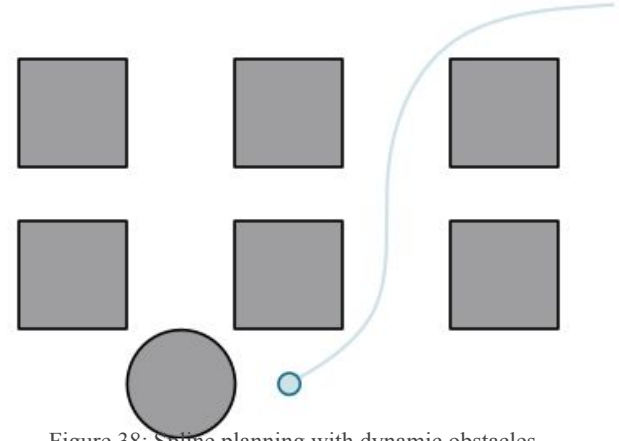


Figure 38: Spline planning with dynamic obstacles.

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Related Work

Model Predictive Path Planning Based on Projected C-Space

- **Requirements:** Map representing the environment, vehicle dimensions and kinematics.
- The vehicle model, and the other physical limitations such as the input bounds and safety constraints are considered in the optimization problem i.e. state equations of the system.
- Collision avoidance constraints are described in the projected configuration space transforming a rectangular collision area into a circle.

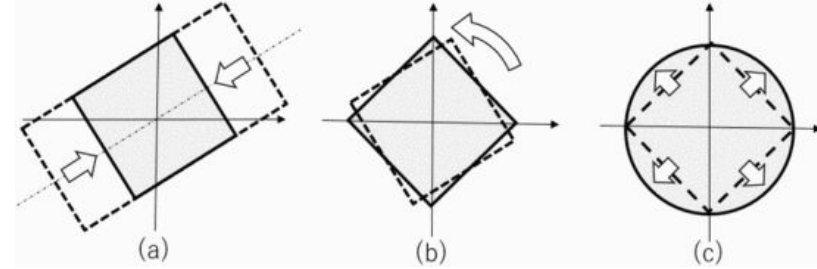


Figure 26: Coordinate transformation to map rectangular collision avoidance area to circular shape.

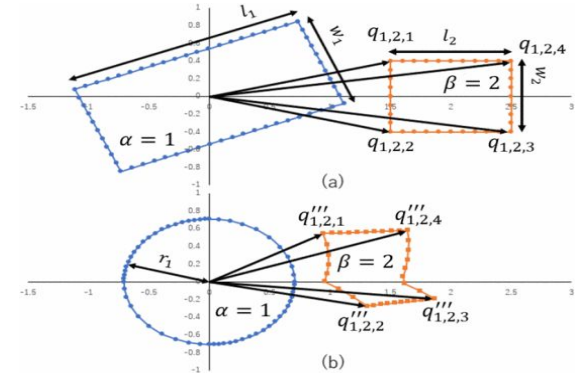


Figure 27: Transformation from rectangle to circle original rectangle collision area (a) is transformed to circle form (b).

8. T. Yamaguchi, T. Ishiguro, H. Okuda and T. Suzuki, "Model Predictive Path Planning for Autonomous Parking Based on Projected C-Space," 2021 IEEE International Intelligent Transportation Systems Conference (ITSC), 2021, pp. 929-935, doi: 10.1109/ITSC48978.2021.9564599.

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- **Requirements:** *Map representing the environment, vehicle dimensions and kinematics.*
- The vehicle model, and the other physical limitations such as the input bounds and safety constraints are considered in the optimization problem i.e. state equations of the plants.
- Collision avoidance constraints are described in the projected configuration space transforming a rectangular collision area into a circle.
- Finally, the optimization problem for the path planner based on the Model Predictive Control scheme is solved for reverse parking scenario.

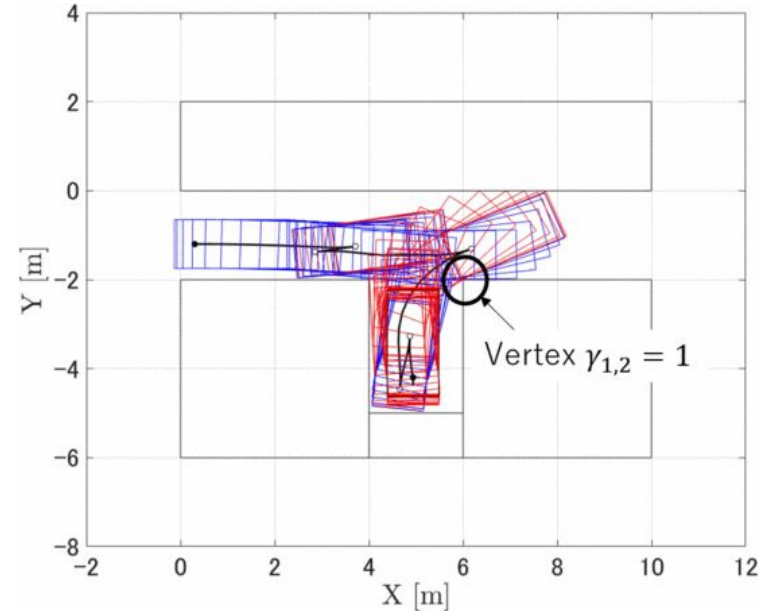


Figure 28: Vehicle position and posture in reverse parking.

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Related Work

Model Predictive Path Planning Based on Projected C-Space

- **Requirements:** *Map representing the environment, vehicle dimensions and kinematics.*
- The vehicle model, and the other physical limitations such as the input bounds and safety constraints are considered in the optimization problem i.e. state equations of the plants.
- Collision avoidance constraints are described in the projected configuration space transforming a rectangular collision area into a circle.
- Finally, the optimization problem for the path planner based on the Model Predictive Control scheme is solved for reverse parking scenario.
- **Drawbacks:** *Obstacles are to be know and in polygon form to apply transformation. Works well in short range planning, as the number of obstacles increases so does the complexity.*

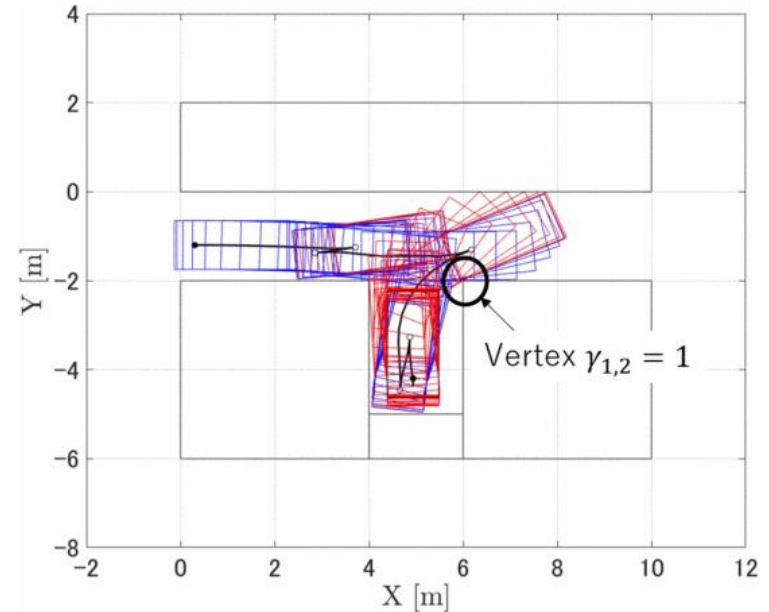


Figure 28: Vehicle position and posture in reverse parking.

8. T. Yamaguchi, T. Ishiguro, H. Okuda and T. Suzuki, "Model Predictive Path Planning for Autonomous Parking Based on Projected C-Space," 2021 IEEE International Intelligent Transportation Systems Conference (ITSC), 2021, pp. 929-935, doi: 10.1109/ITSC48978.2021.9564599.

Related Work

Multistage Hybrid A* and Numerical Optimal Control

- **Requirements:** Map representing the environment, robot's dimensions.
- Given the start and end points along with the obstacles, the map is discretized to an occupancy grid.

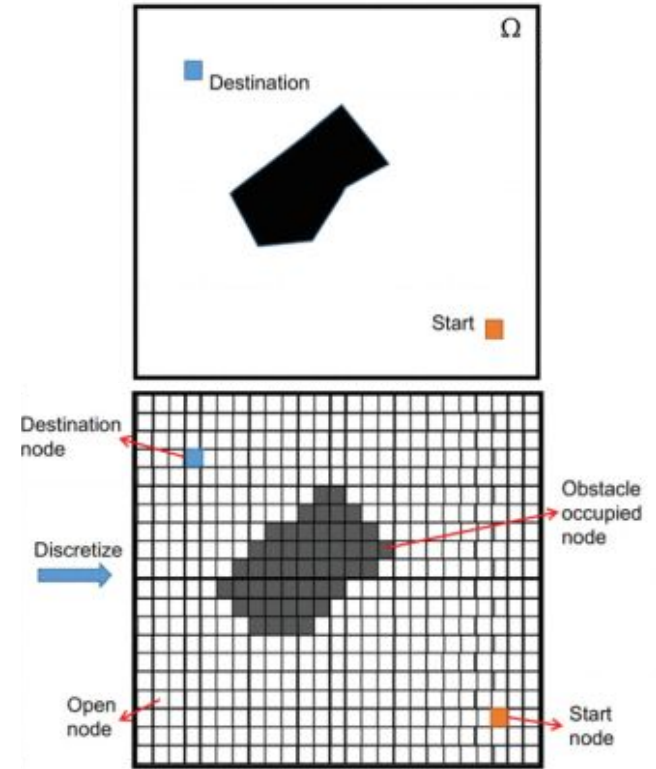


Figure 10: Discretized map.

9. W. Sheng, B. Li and X. Zhong, "Autonomous Parking Trajectory Planning With Tiny Passages: A Combination of Multistage Hybrid A-Star Algorithm and Numerical Optimal Control," in IEEE Access, vol. 9, pp. 102801-102810, 2021, doi: 10.1109/ACCESS.2021.3098676.

Related Work

Multistage Hybrid A* and Numerical Optimal Control

- **Requirements:** Map representing the environment, robot's dimensions.
- Given the start and end points along with the obstacles, the map is discretized to an occupancy grid.
- Hybrid A* initially find a path using regular A*, then finds narrow passages and divides the path into segments, kinematically feasible subpaths (Reeds–Shepp curves) are found and finally are combined together.
- Improved Safe Travel Corridor (STC) - Based Trajectory Optimization is used to solve the optimal control problem. First-order explicit Runge-Kutta method is applied to discretize and Interior Point Method (IPM) is chosen as the NLP solver.

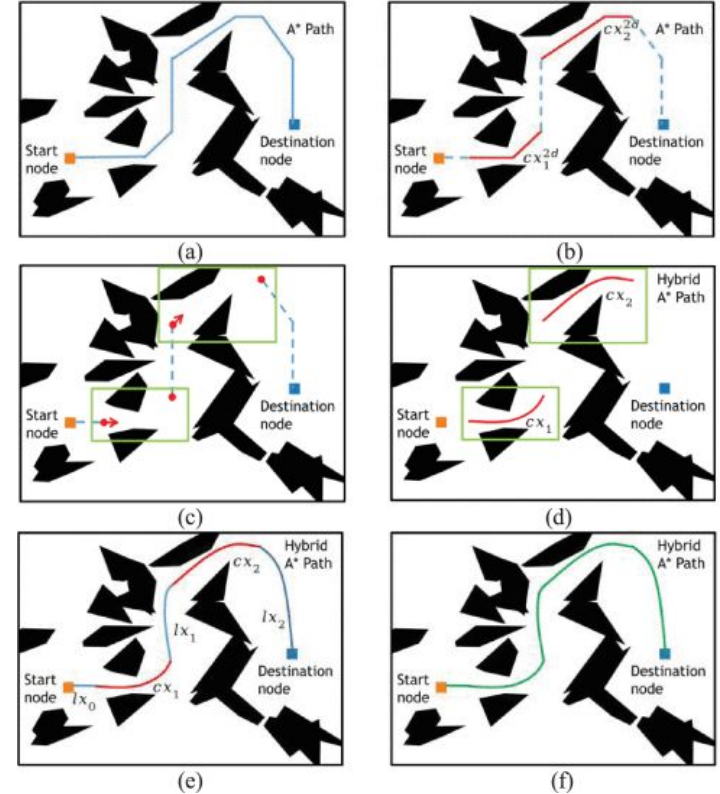


Figure 11: Schematics on the steps in the multistage hybrid A* algorithm.

9. W. Sheng, B. Li and X. Zhong, "Autonomous Parking Trajectory Planning With Tiny Passages: A Combination of Multistage Hybrid A-Star Algorithm and Numerical Optimal Control," in IEEE Access, vol. 9, pp. 102801-102810, 2021, doi: 10.1109/ACCESS.2021.3098676.

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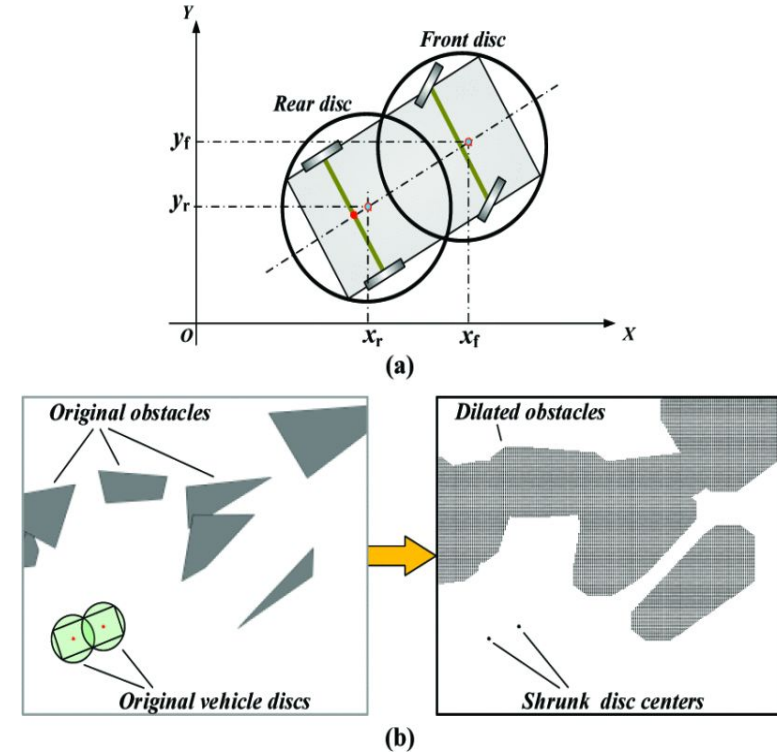


Figure 12: Visualizing the altered environment and robot.

9. W. Sheng, B. Li and X. Zhong, "Autonomous Parking Trajectory Planning With Tiny Passages: A Combination of Multistage Hybrid A-Star Algorithm and Numerical Optimal Control," in IEEE Access, vol. 9, pp. 102801-102810, 2021, doi: 10.1109/ACCESS.2021.3098676.

Related Work

Multistage Hybrid A* and Numerical Optimal Control

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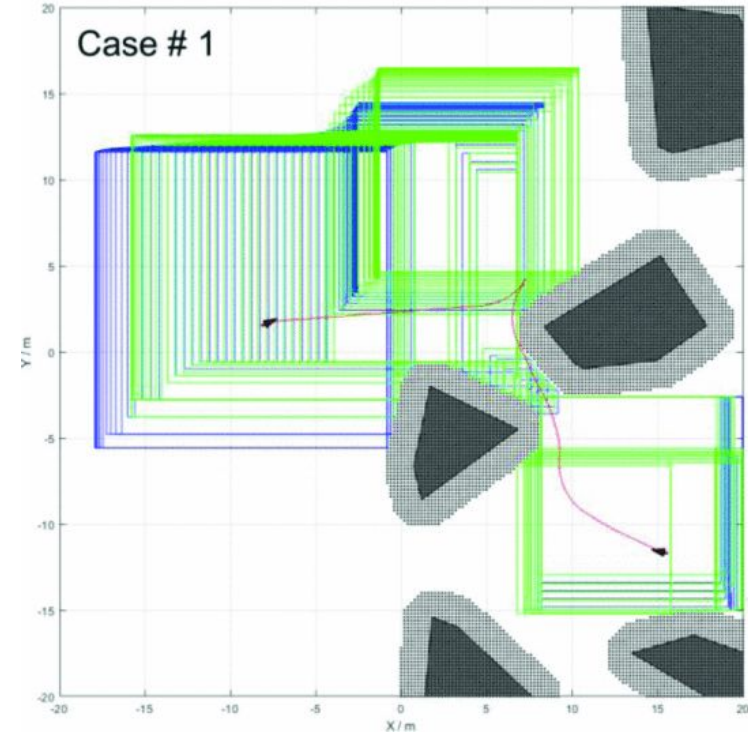


Figure 13: Applied STC to both ends of the vehicle.

9. W. Sheng, B. Li and X. Zhong, "Autonomous Parking Trajectory Planning With Tiny Passages: A Combination of Multistage Hybrid A-Star Algorithm and Numerical Optimal Control," in IEEE Access, vol. 9, pp. 102801-102810, 2021, doi: 10.1109/ACCESS.2021.3098676.

Related Work

Multistage Hybrid A* and Numerical Optimal Control

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- Hybrid A* initially find a path using regular A*, then finds narrow passages and divides the path into segments, kinematically feasible subpaths (Reeds–Shepp curves) are found and finally are combined together.
- Improved Safe Travel Corridor (STC) - Based Trajectory Optimization is used to solve the optimal control problem. First-order explicit Runge-Kutta method is applied to discretize and Interior Point Method (IPM) is chosen as the NLP solver.
- **Drawbacks:** *Initial coarse path could lead to local minimas. To obtain safer path the number of STCs should be increased which would increase complexity.*

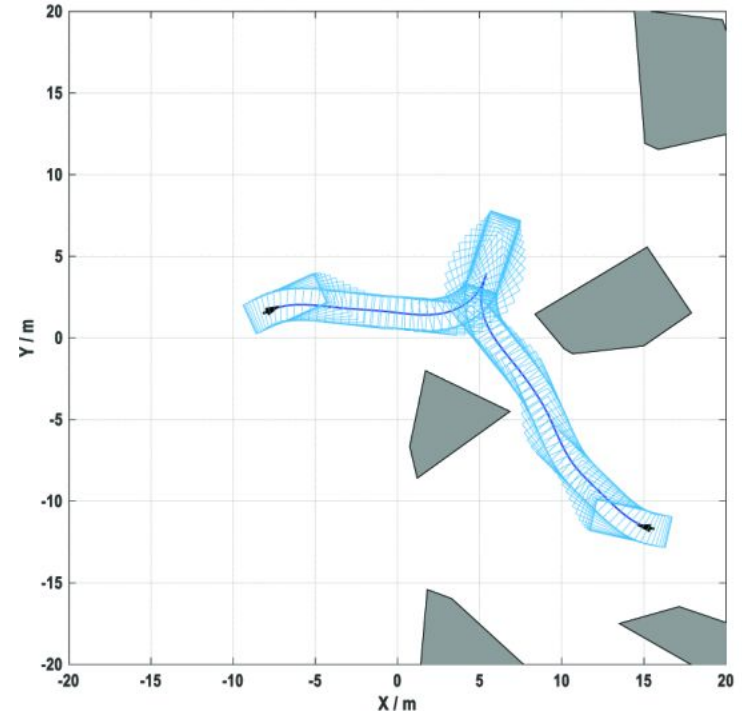


Figure 14: Final path from start to goal.

9. W. Sheng, B. Li and X. Zhong, "Autonomous Parking Trajectory Planning With Tiny Passages: A Combination of Multistage Hybrid A-Star Algorithm and Numerical Optimal Control," in IEEE Access, vol. 9, pp. 102801-102810, 2021, doi: 10.1109/ACCESS.2021.3098676.

Related Work

Random Tree - C*CS Planner

- **Requirements:** *Map representing the environment, vehicle dimensions and kinematics, motion primitives.*
- Uses sampling-based geometric planning and approximation by a topological steering method in configuration space (Triangular cell decomposition).
- Bi RRT is performed where the node is randomly placed anywhere on the neighbouring cells. If there is no clear path, sub branches are created in the current cell.

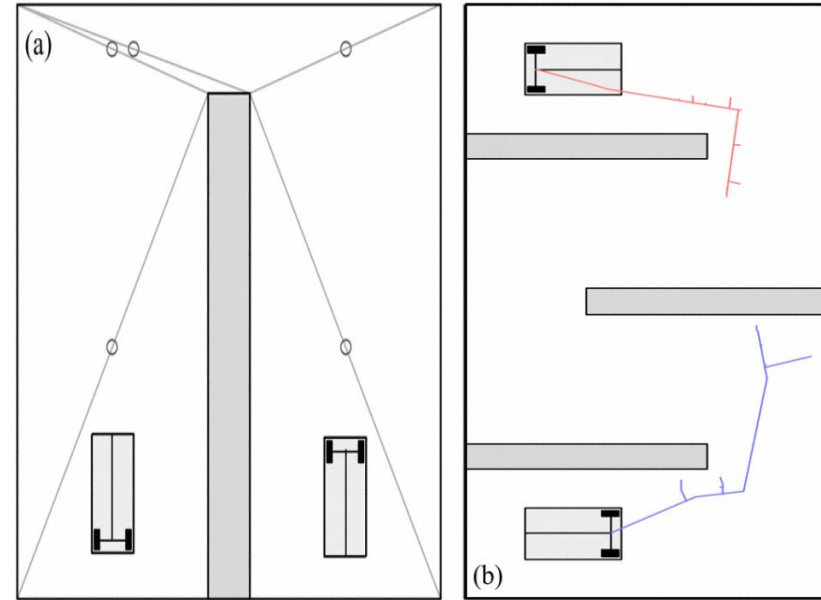


Figure 29: Triangular decomposition and Bi RRTs.

10. Á. Nagy, G. Csorvási and D. Kiss, "Path planning and control of differential and car-like robots in narrow environments," 2015 IEEE 13th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2015, pp. 103-108, doi: 10.1109/SAMI.2015.7061856.

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- **Requirements:** *Map representing the environment, vehicle dimensions and kinematics, motion primitives.*
- Uses sampling-based geometric planning and approximation by a topological steering method in configuration space (Triangular cell decomposition).
- Bi RRT is performed where the node is randomly placed anywhere on the neighbouring cells. If there is no clear path, sub branches are created in the current cell.
- Primary global path planner consists only of straight motion and turning in place primitives.

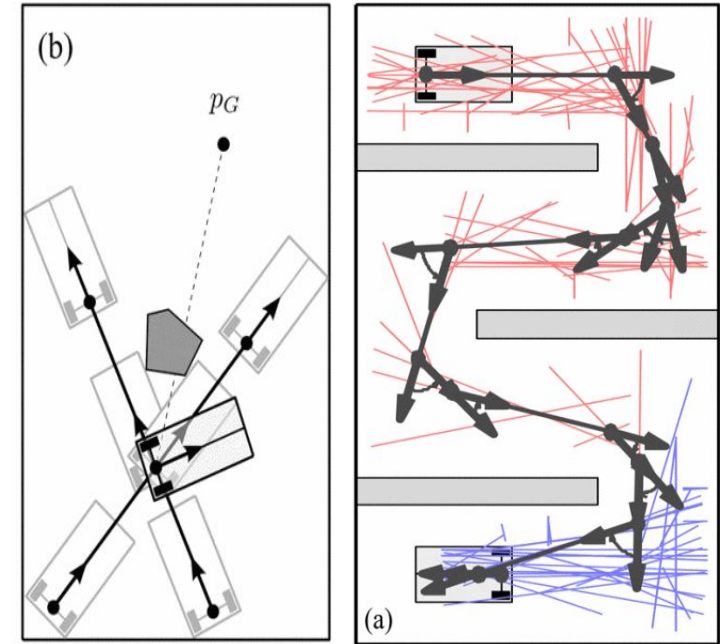


Figure 30: Bi directional RRTs and motion primitives.

10. Á. Nagy, G. Csorvási and D. Kiss, "Path planning and control of differential and car-like robots in narrow environments," 2015 IEEE 13th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2015, pp. 103-108, doi: 10.1109/SAMI.2015.7061856.

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- **Requirements:** *Map representing the environment, vehicle dimensions and kinematics, motion primitives.*
- Uses sampling-based geometric planning and approximation by a topological steering method in configuration space (Triangular cell decomposition).
- Primary global path planner consists only of straight motion and turning in place primitives.
- Bi RRT is performed where the node is randomly placed anywhere on the neighbouring cells. If there is no clear path, sub branches are created in the current cell.
- A local C*CS planner is applied to obtain a secondary path containing straight segments and circular arcs of given lower bounded radii (CCS and SCS paths).

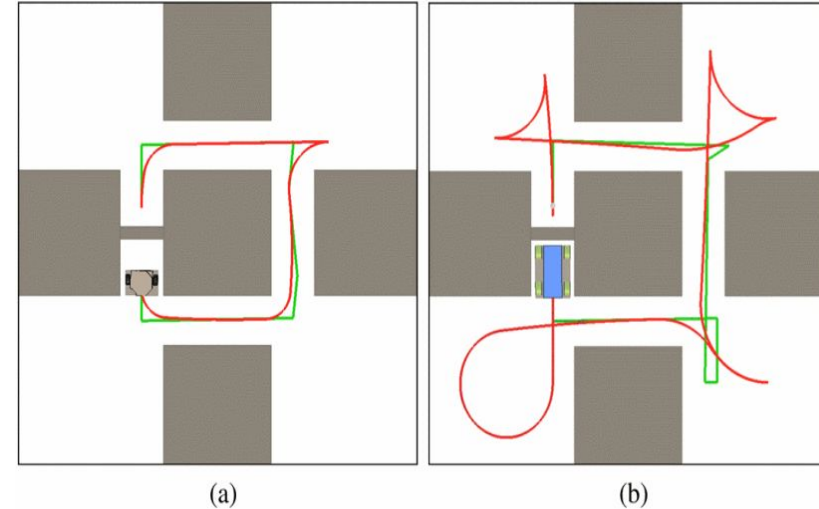


Figure 31: The RTR path (green) and its C*CS approximation (red) for (a) differential and (b) car-like robots.

10. Á. Nagy, G. Csorvási and D. Kiss, "Path planning and control of differential and car-like robots in narrow environments," 2015 IEEE 13th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2015, pp. 103-108, doi: 10.1109/SAMI.2015.7061856.

Related Work

Random Tree - T*TS Planner

- **Requirements:** *Map representing the environment, vehicle dimensions and kinematics, motion primitives.*
- Uses sampling-based geometric planning and approximation by a topological steering method in configuration space (Triangular cell decomposition).
- Primary global path planner consists only of straight motion and turning in place primitives.
- Bi RRT is performed where the node is randomly placed anywhere on the neighbouring cells. If there is no clear path, sub branches are created in the current cell.
- A local T*TS planner is applied to obtain a secondary path (clothoids) containing straight segments 'S' and CC-turn 'T' that can have zero sharpness and curvature (TTS and STS paths).

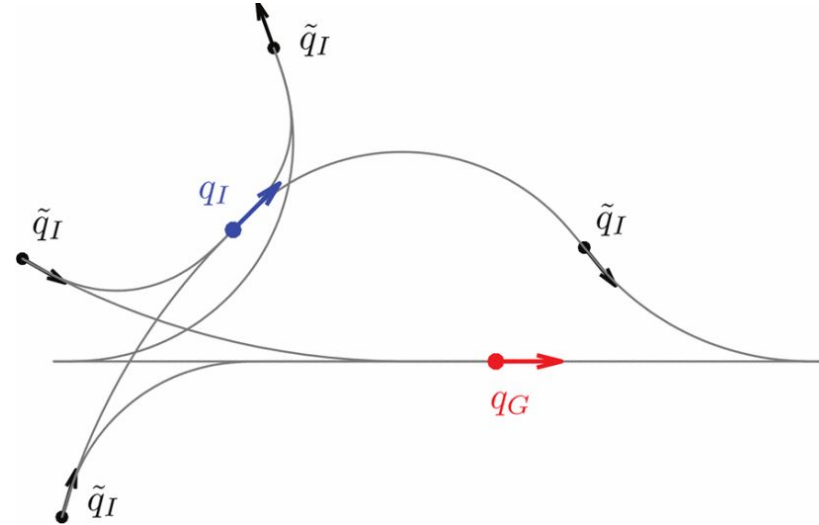


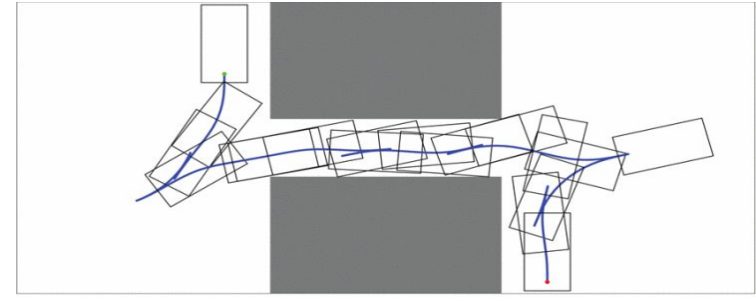
Figure 32: There is a number of T*TS solutions between two configurations.

11. D. Kiss and D. Papp, "Effective navigation in narrow areas: A planning method for autonomous cars," 2017 IEEE 15th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2017, pp. 000423-000430, doi: 10.1109/SAMI.2017.7880346.

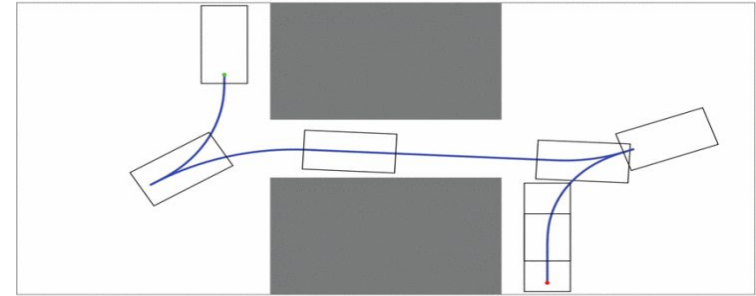
Related Work

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- A local T*TS planner is applied to obtain a secondary path (clothoids) containing straight segments 'S' and CC-turn 'T' that can have zero sharpness and curvature (TTS and STS paths).
- **Drawbacks:** *Cannot handle dynamic environment. Motion primitives should have higher resolution as area gets narrower.*



(a) *RRT-Connect (interp) + CCRS*



(b) *RTR + T*TS*

Figure 33: Crossing a narrow corridor.

11. D. Kiss and D. Papp, "Effective navigation in narrow areas: A planning method for autonomous cars," 2017 IEEE 15th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2017, pp. 000423-000430, doi: 10.1109/SAMI.2017.7880346.

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- **Drawbacks:** *Cannot handle dynamic environment. Motion primitives should have higher resolution as area gets narrower.*

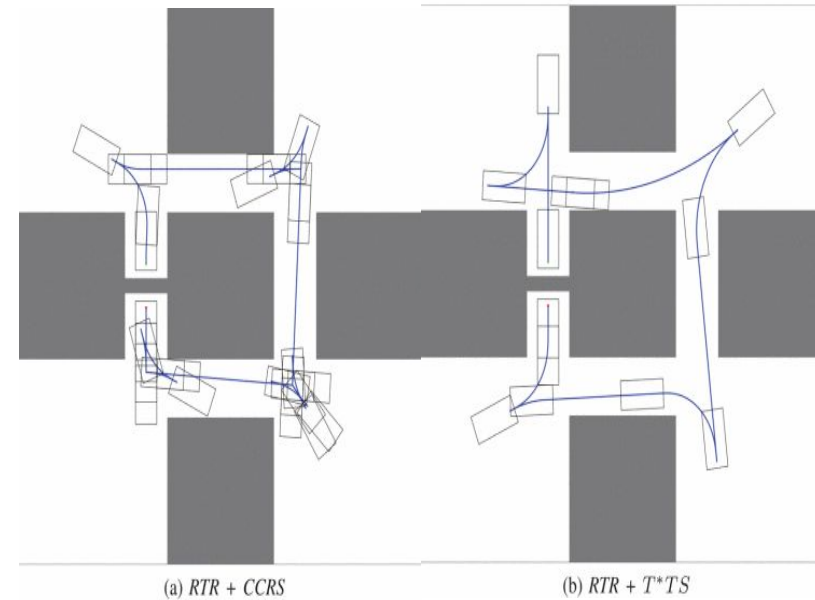


Figure 34: Crossing three narrow corridors.

11. D. Kiss and D. Papp, "Effective navigation in narrow areas: A planning method for autonomous cars," 2017 IEEE 15th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2017, pp. 000423-000430, doi: 10.1109/SAMI.2017.7880346.

Related Work

Shape-Aware Lifelong A* Planning

- **Requirements:** *Configuration space map with obstacles expanded to the width of the robot.*

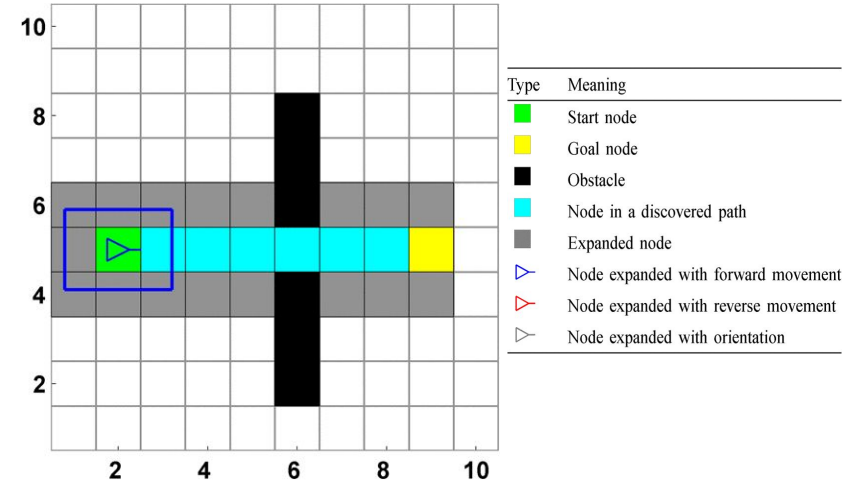


Figure 15: Vehicle unable to pass through a passage that is too narrow

12. S. Yoon and D. H. Shim, "SLPA*: Shape-Aware Lifelong Planning A* for Differential Wheeled Vehicles," in IEEE Transactions on Intelligent Transportation Systems, vol. 16, no. 2, pp. 730-740, April 2015, doi: 10.1109/TITS.2014.2340020.

Related Work

Shape-Aware Lifelong A* Planning

- **Requirements:** *Configuration space map with obstacles expanded to the width of the robot.*
- The child nodes include orientation of the robot and translation (forward and reverse).
- While finding child nodes it checks collision with obstacles from current to child node.

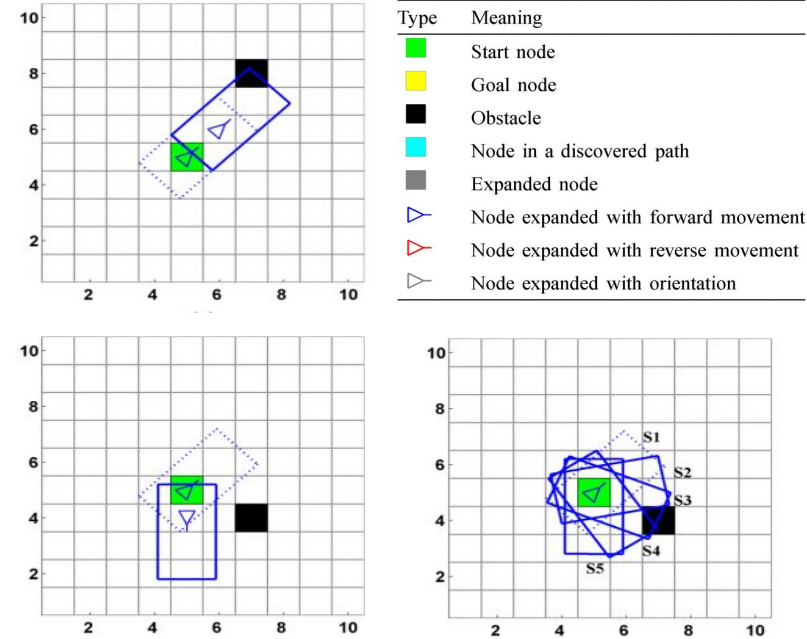


Figure 16: Procedure to check for interference against obstacles.

12. S. Yoon and D. H. Shim, "SLPA*: Shape-Aware Lifelong Planning A* for Differential Wheeled Vehicles," in IEEE Transactions on Intelligent Transportation Systems, vol. 16, no. 2, pp. 730-740, April 2015, doi: 10.1109/TITS.2014.2340020.

Related Work

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- **Requirements:** *Configuration space map with obstacles expanded to the width of the robot.*
- The child nodes include orientation of the robot and translation (forward and reverse).
- While finding child nodes it checks collision with obstacles from current to child node.
- Modified A*, including an algorithm to check for interference against obstacles uses Manhattan distances as heuristic.

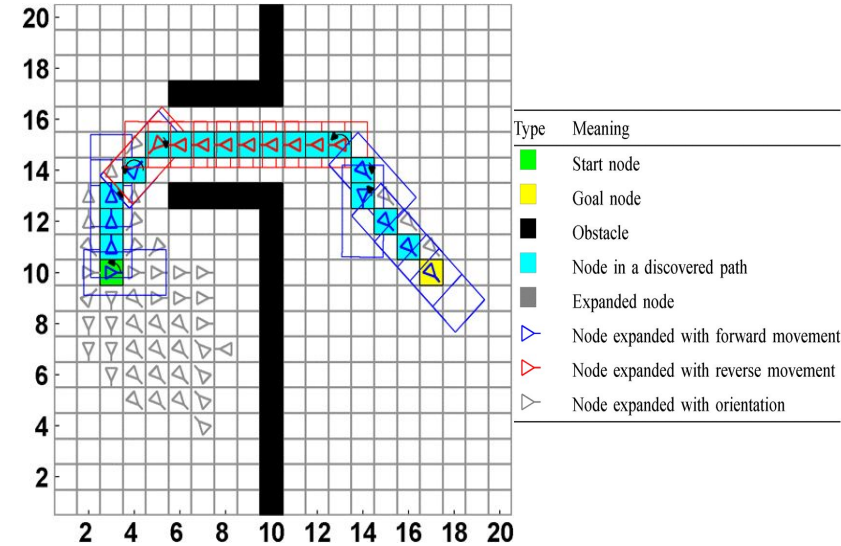


Figure 17: path planner generates a collision-free path, including a reverse movement at coordinates

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Related Work

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- The child nodes include orientation of the robot and translation (forward and reverse).
- While finding child nodes it checks collision with obstacles from current to child node.
- Modified A*, including an algorithm to check for interference against obstacles uses Manhattan distances as heuristic.
- **Drawbacks:** *Complexity increases with increase in resolution and discretization of rotation. Suitable for short range navigation.*

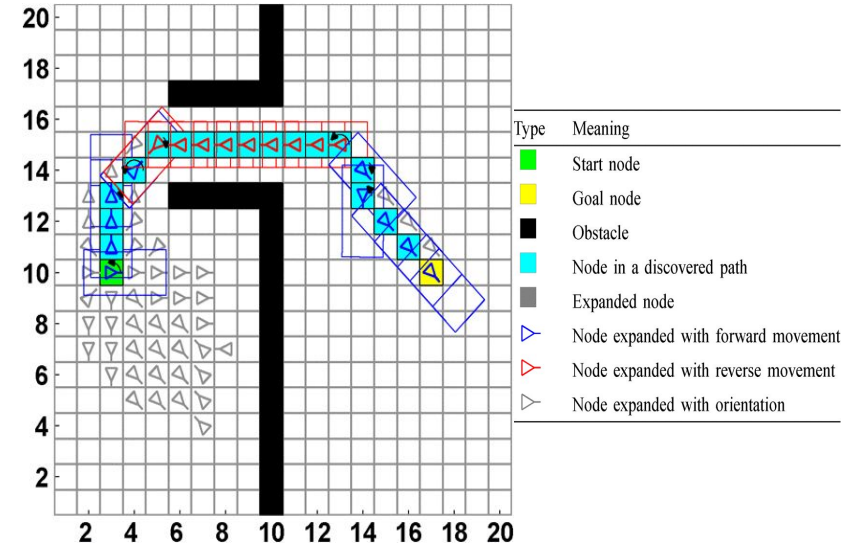


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Related Work

Configuration-Aware Model Predictive Motion Planning

- Convex polygons are used to express the configuration of the robots, obstacles, and walls.
- Farkas' lemma is applied to express the collision avoidance constraints
- Motion Planning based on Receding Horizon. The model of the robot is discretized by the Euler method and Interior Point Optimizer is used to solve the problem (nonlinear MPC)
- The collision check will be done for each obstacle, therefore complexity would increase while working with a point cloud

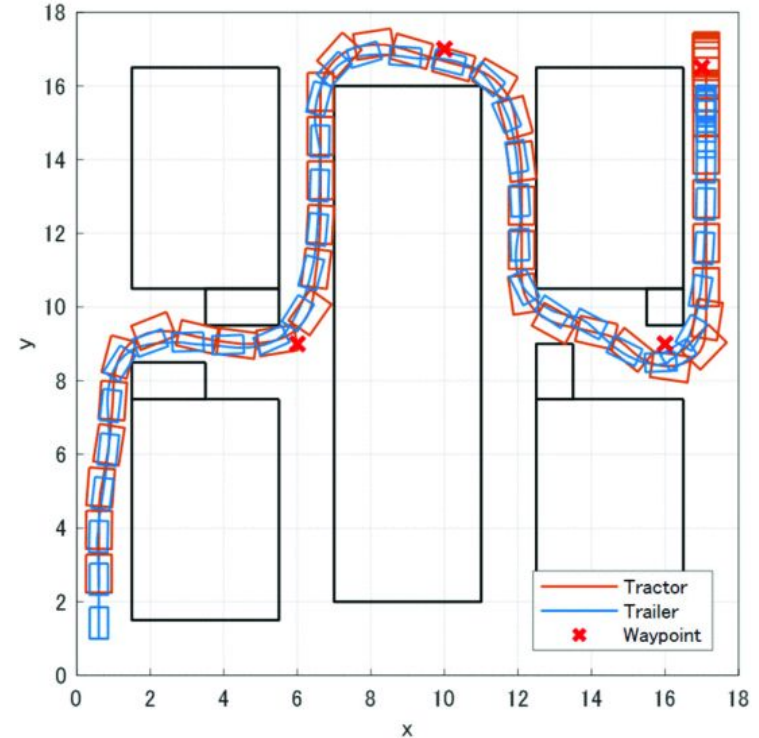


Figure 32: Configuration-Aware Model Predictive Motion Planning

13. N. Ito, H. Okuda, S. Inagaki and T. Suzuki, "Configuration-aware Model Predictive Motion Planning in Narrow Environment for Autonomous Tractor-trailer Mobile Robot," IECON 2021 – 47th Annual Conference of the IEEE Industrial Electronics Society, 2021, pp. 1-7, doi: 10.1109/IECON48115.2021.9589596.

Related Work

Optimization-Based Maneuver Planning

- A* algorithm is deployed to search for a sequence of nodes connecting the initial and terminal locations in the grid map
- The maneuver planning scheme for the vehicle is formulated as an Optimal Control Problem which consists of a cost function and three types of constraints (Kinematic, boundary and collision)
- From the reference path, Safe Travel Corridors are constructed to simplify the collision-avoidance constraints
- Trust-Region-Based Maneuver Optimization (TRMO) is used to solve the optimization problem

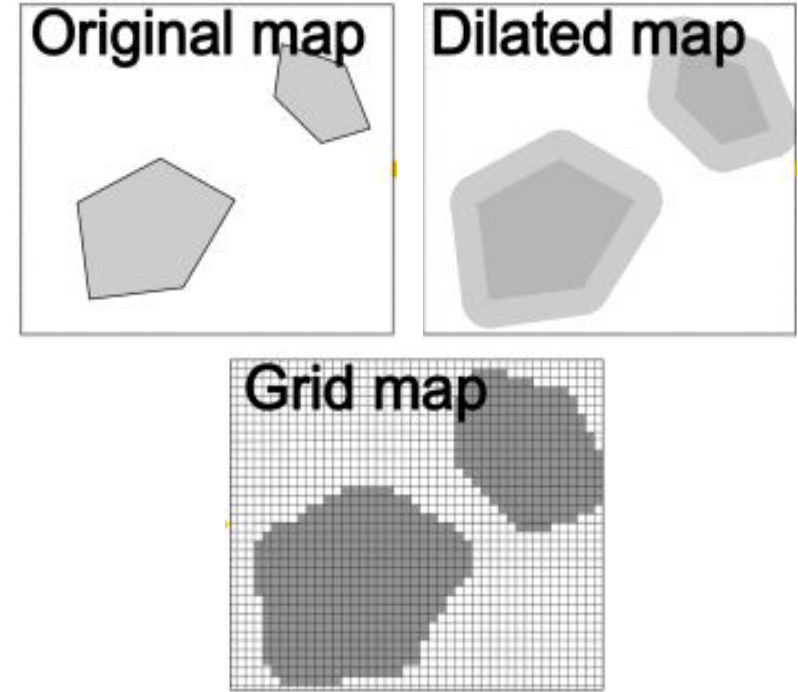


Figure 33: Map representation for Optimization-Based Maneuver Planning

14. B. Li, L. Li, T. Acarman, Z. Shao and M. Yue, "Optimization-Based Maneuver Planning for a Tractor-Trailer Vehicle in a Curvy Tunnel: A Weak Reliance on Sampling and Search," in IEEE Robotics and Automation Letters, vol. 7, no. 2, pp. 706-713, April 2022, doi: 10.1109/LRA.2021.3131693.

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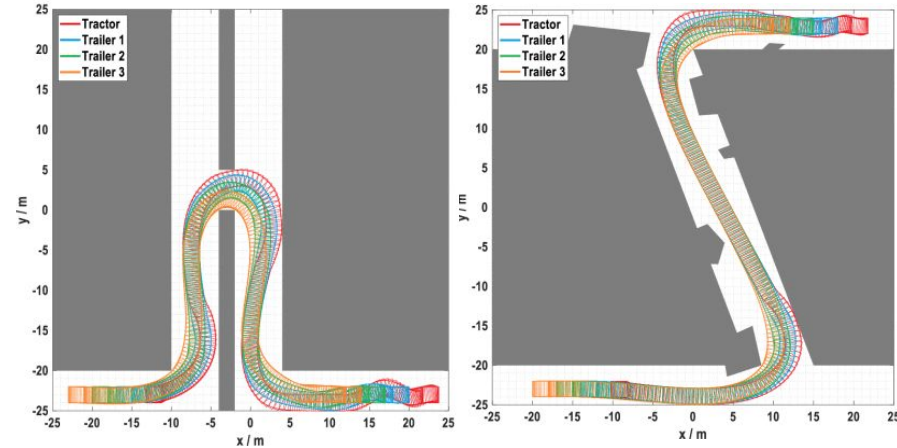


Figure 34: Optimization-Based Maneuver Planning in narrow spaces

14. B. Li, L. Li, T. Acarman, Z. Shao and M. Yue, "Optimization-Based Maneuver Planning for a Tractor-Trailer Vehicle in a Curvy Tunnel: A Weak Reliance on Sampling and Search," in IEEE Robotics and Automation Letters, vol. 7, no. 2, pp. 706-713, April 2022, doi: 10.1109/LRA.2021.3131693.

Related Work

Modified A* Path-Planning

- Traditional A* algorithm doesn't account for the size of the robot
- Obstacles are enlarged to the equivalent size i.e. $(2n+1)$ size of the cell
- Can only apply to square shaped robots
- Computationally costly if resolution of the map is increased

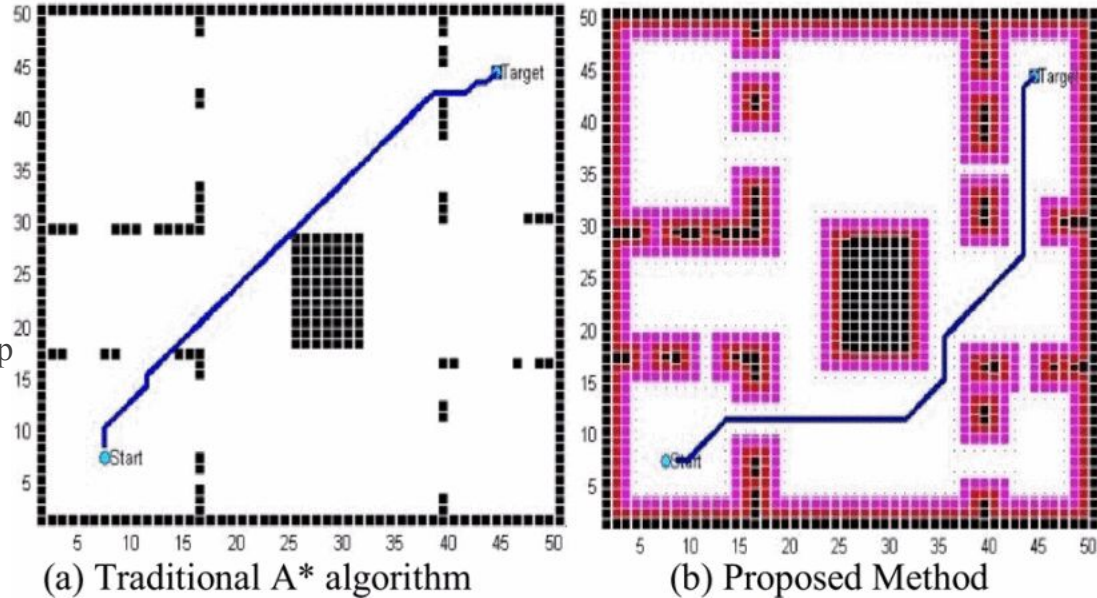


Figure 46: Modified A* Path-Planning

15. J. K. Goyal and K. S. Nagla, "A new approach of path planning for mobile robots," 2014 International Conference on Advances in Computing, Communications and Informatics (ICACCI), 2014, pp. 863-867, doi: 10.1109/ICACCI.2014.6968200.

Related Work

Improved A* Algorithm

- Traditional A* algorithm creates a rough and uneven path

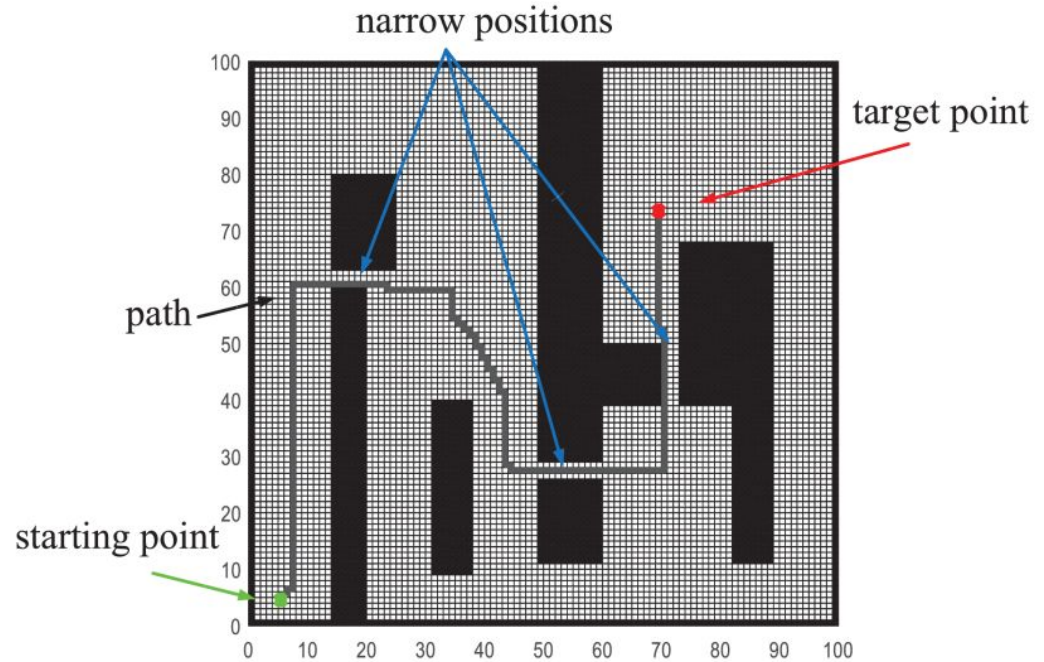


Figure 47: Traditional A*

16. L. Chang, L. Shan, J. Li and Y. Dai, "The Path Planning of Mobile Robots Based on an Improved A* Algorithm," 2019 IEEE 16th International Conference on Networking, Sensing and Control (ICNSC), 2019, pp. 257-262, doi: 10.1109/ICNSC.2019.8743249.

Related Work

Improved A* Algorithm

- Traditional A* algorithm creates a rough and uneven path
- Compressing the map resolution avoids narrow gaps

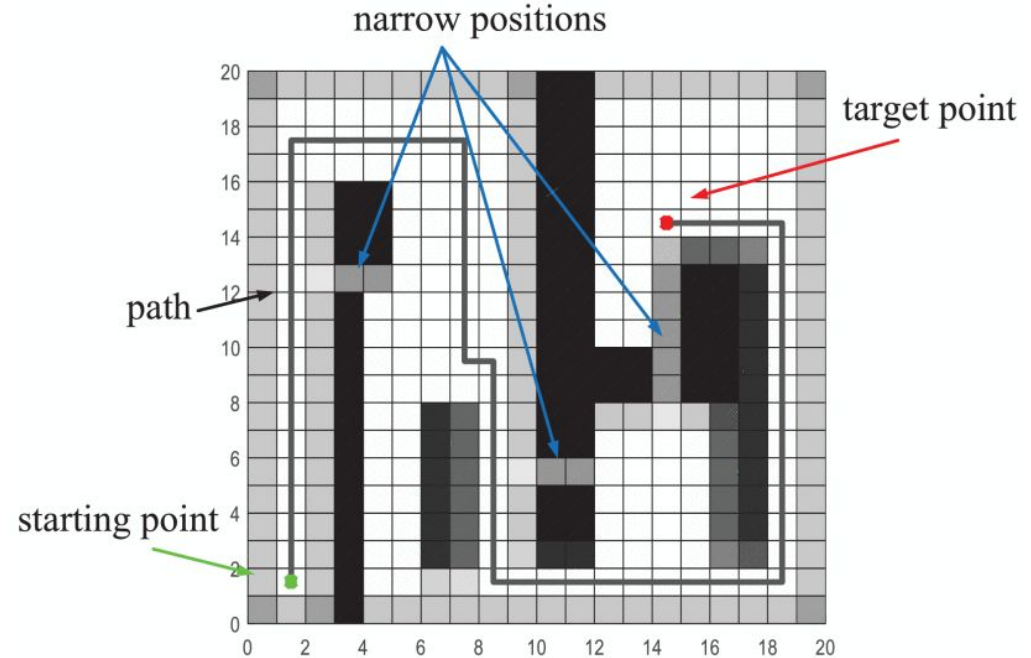


Figure 48: Path-Planning with reduced resolution

16. L. Chang, L. Shan, J. Li and Y. Dai, "The Path Planning of Mobile Robots Based on an Improved A* Algorithm," 2019 IEEE 16th International Conference on Networking, Sensing and Control (ICNSC), 2019, pp. 257-262, doi: 10.1109/ICNSC.2019.8743249.

Related Work

Improved A* Algorithm

- Traditional A* algorithm creates a rough and uneven path
- Compressing the map resolution avoids narrow gaps
- Path is then smoothened by recursively finding nodes on each line that shortens the path

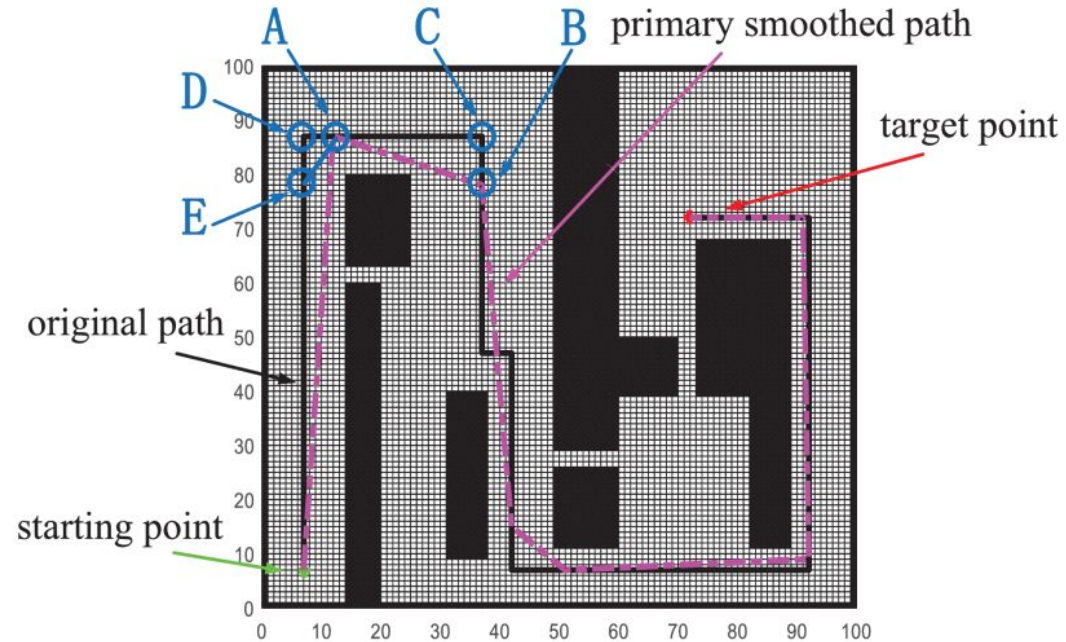


Figure 49: Path smoothening at original resolution

16. L. Chang, L. Shan, J. Li and Y. Dai, "The Path Planning of Mobile Robots Based on an Improved A* Algorithm," 2019 IEEE 16th International Conference on Networking, Sensing and Control (ICNSC), 2019, pp. 257-262, doi: 10.1109/ICNSC.2019.8743249.

Related Work

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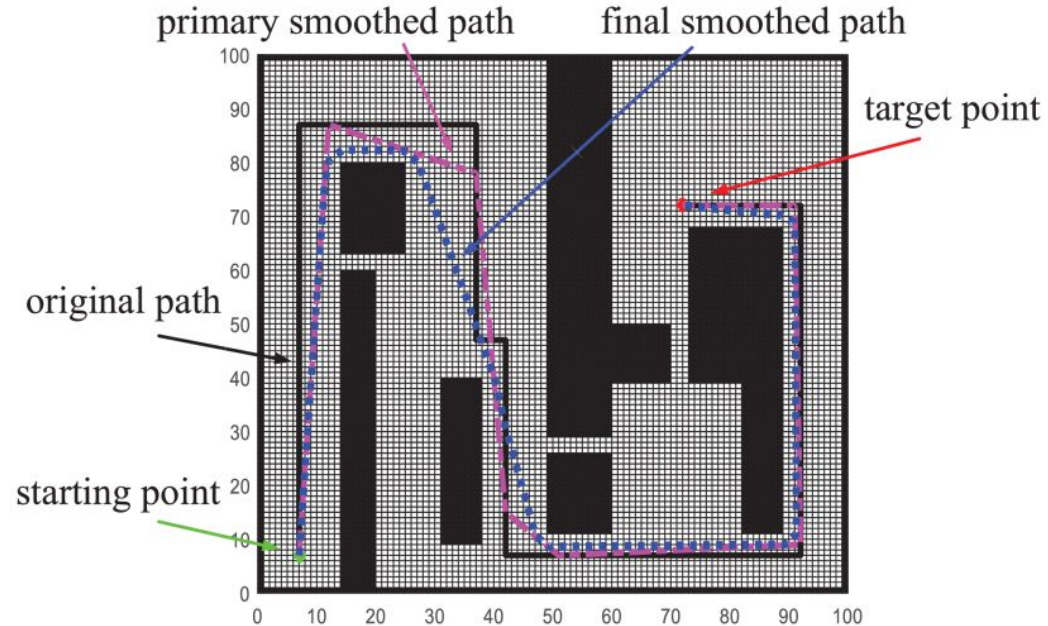


Figure 50: Path smoothening at original resolution

16. L. Chang, L. Shan, J. Li and Y. Dai, "The Path Planning of Mobile Robots Based on an Improved A* Algorithm," 2019 IEEE 16th International Conference on Networking, Sensing and Control (ICNSC), 2019, pp. 257-262, doi: 10.1109/ICNSC.2019.8743249.

Related Work

Improved A* Algorithm

- Traditional A* algorithm creates a rough and uneven path
- Compressing the map resolution avoids narrow gaps
- Path is then smoothened by recursively finding nodes on each line that shortens the path
- Computationally expensive for long range planning

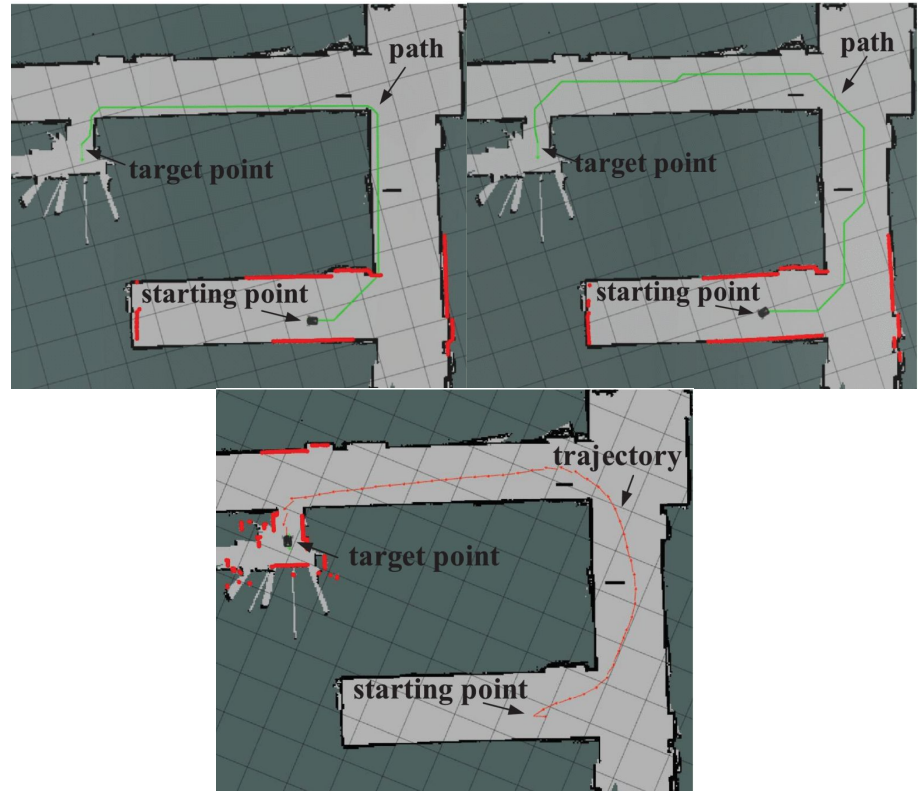


Figure 51: Path planning with Improved A* Algorithm

16. L. Chang, L. Shan, J. Li and Y. Dai, "The Path Planning of Mobile Robots Based on an Improved A* Algorithm," 2019 IEEE 16th International Conference on Networking, Sensing and Control (ICNSC), 2019, pp. 257-262, doi: 10.1109/ICNSC.2019.8743249.

Related Work

Hybrid A* Potential Field Method

- Takes the best of both methods to reduce the overall path cost
- To avoid oscillations in narrow spaces an Adaptive Moment Estimation (ADAM) which is a variation of gradient descent method is used to optimize the potential field method

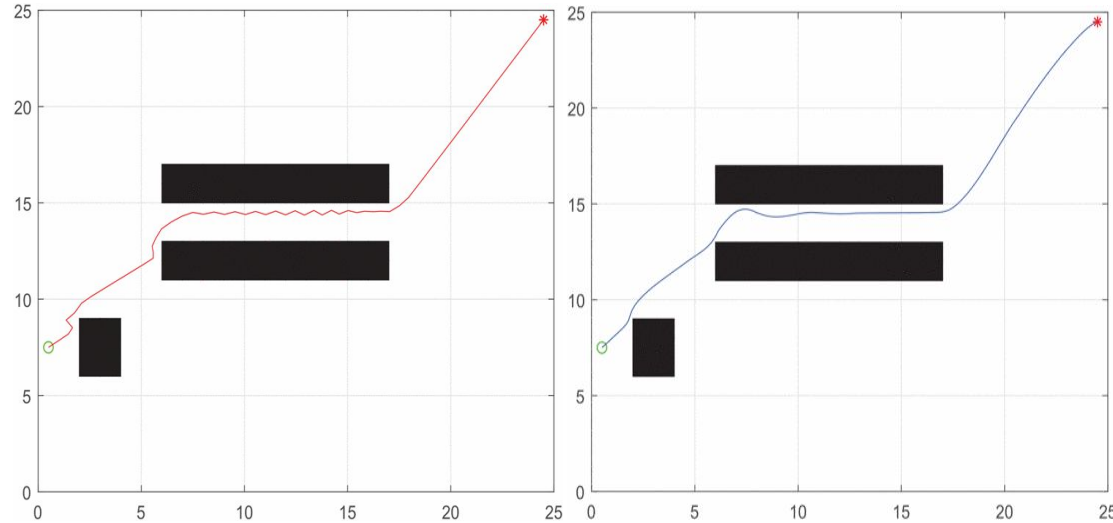


Figure 52: Oscillations at narrow spaces using potential fields

17. H. Wang, Z. Wang, L. Yu, Q. Wang and C. Liu, "A Hybrid Algorithm For Robot Path Planning," 2018 IEEE International Conference on Mechatronics and Automation (ICMA), 2018, pp. 986-990, doi: 10.1109/ICMA.2018.8484297.

Related Work

Hybrid A* Potential Field Method

- Takes the best of both methods to reduce the overall path cost
- To avoid oscillations in narrow spaces an Adaptive Moment Estimation (ADAM) which is a variation of gradient descent method is used to optimize the potential field method
- A* is first applied to a pixelated map to get an initial path after which it is smoothed using the potential field method

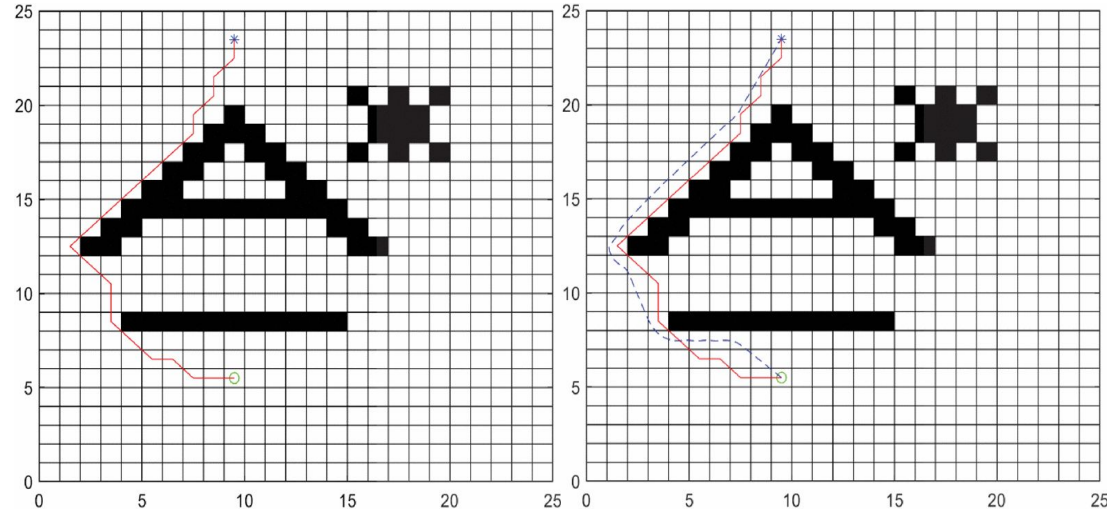


Figure 53: Hybrid A* Potential Field Method applied to grid map

17. H. Wang, Z. Wang, L. Yu, Q. Wang and C. Liu, "A Hybrid Algorithm For Robot Path Planning," 2018 IEEE International Conference on Mechatronics and Automation (ICMA), 2018, pp. 986-990, doi: 10.1109/ICMA.2018.8484297.

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- To avoid oscillations in narrow spaces an Adaptive Moment Estimation (ADAM) which is a variation of gradient descent method is used to optimize the potential field method
- A* is first applied to a pixelated map to get an initial path after which it is smoothed using the potential field method
- Discretization and map resolution still affects the complexity

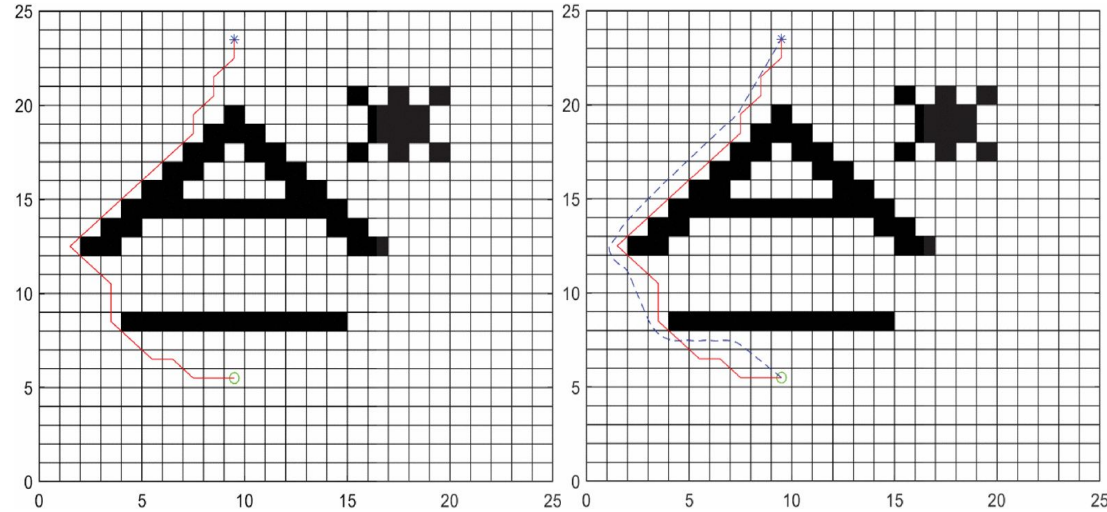


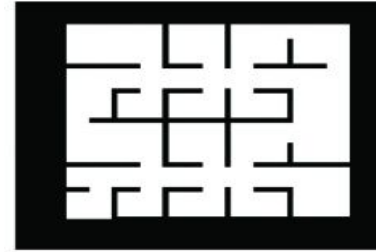
Figure 53: Hybrid A* Potential Field Method applied to grid map

17. H. Wang, Z. Wang, L. Yu, Q. Wang and C. Liu, "A Hybrid Algorithm For Robot Path Planning," 2018 IEEE International Conference on Mechatronics and Automation (ICMA), 2018, pp. 986-990, doi: 10.1109/ICMA.2018.8484297.

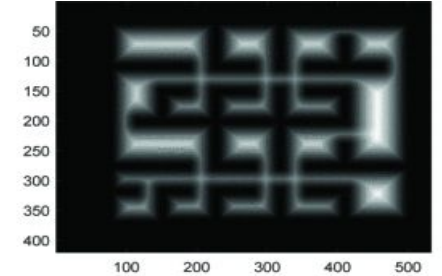
Related Work

New Global Path Planning Strategy

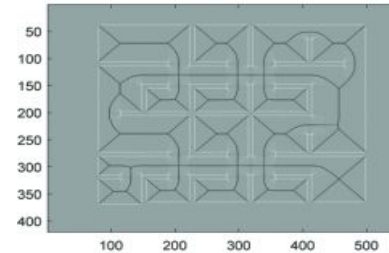
- Skeletonization is performed on a pixel map using a distance field map combined with thinning



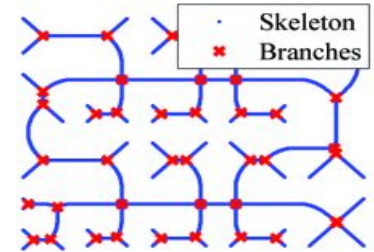
(a) Original map



(b) Distance field map from boundary (DFB)



(c) Ridges of DFB



(d) Skeleton with branching-points obtained from image thinning

Figure 54: Skeletonization is performed on a pixel map

18. J. -w. Han, S. Jeon and H. -J. Kwon, "A New Global Path Planning Strategy for Mobile Robots Using Hierarchical Topology Map and Safety-Aware Navigation Speed," 2019 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), 2019, pp. 1586-1591, doi: 10.1109/AIM.2019.8868423.

Related Work

New Global Path Planning Strategy

- Skeletonization is performed on a pixel map using a distance field map combined with thinning
- Allowable speed and Cost-to-Go parameters are used to defined nodes and edges

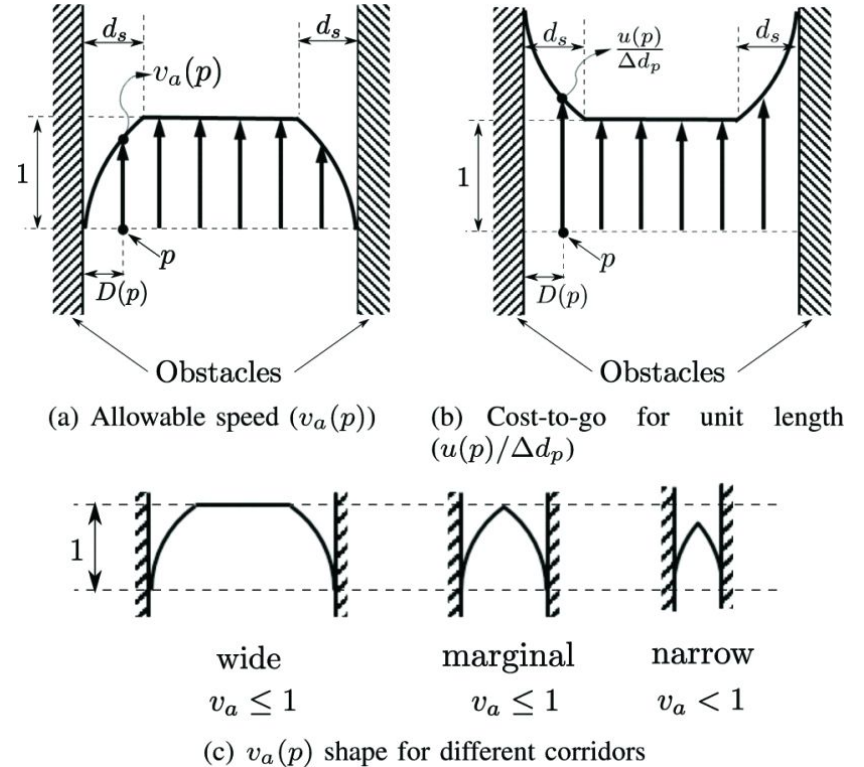


Figure 55: Cost assigned to nodes and edges

18. J. -w. Han, S. Jeon and H. -J. Kwon, "A New Global Path Planning Strategy for Mobile Robots Using Hierarchical Topology Map and Safety-Aware Navigation Speed," 2019 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), 2019, pp. 1586-1591, doi: 10.1109/AIM.2019.8868423.

Related Work

New Global Path Planning Strategy

- Skeletonization is performed on a pixel map using a distance field map combined with thinning
- Allowable speed and Cost-to-Go parameters are used to defined nodes and edges
- The map is then represented as a hierarchical graph

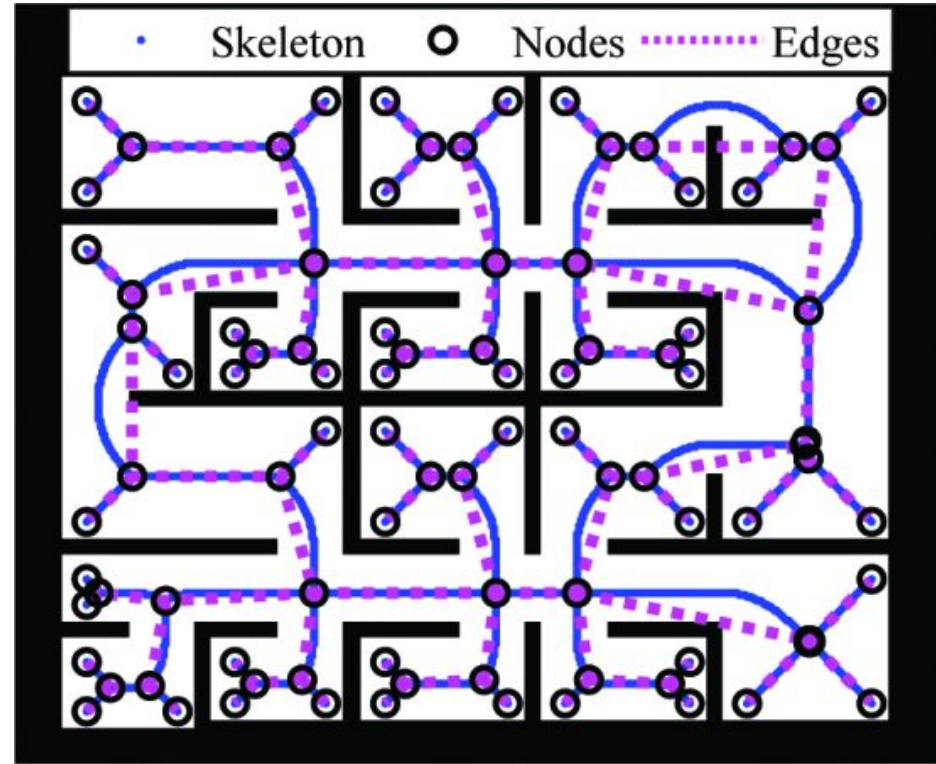


Figure 56: Map represented as a hierarchical graph

18. J. -w. Han, S. Jeon and H. -J. Kwon, "A New Global Path Planning Strategy for Mobile Robots Using Hierarchical Topology Map and Safety-Aware Navigation Speed," 2019 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), 2019, pp. 1586-1591, doi: 10.1109/AIM.2019.8868423.

Related Work

New Global Path Planning Strategy

- Skeletonization is performed on a pixel map using a distance field map combined with thinning
- Allowable speed and Cost-to-Go parameters are used to defined nodes and edges
- The map is then represented as a hierarchical graph
- Finally the path planner takes the time taken by the robot to safely reach the goal for optimization



Figure 57: Global Path Planning Strategy applied in real environment

18. J. -w. Han, S. Jeon and H. -J. Kwon, "A New Global Path Planning Strategy for Mobile Robots Using Hierarchical Topology Map and Safety-Aware Navigation Speed," 2019 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), 2019, pp. 1586-1591, doi: 10.1109/AIM.2019.8868423.

Related Work

Rapid Random Tree - C*CS Planner

- **Requirements:** *Geometric map, vehicle dimensions and kinematics, motion primitives*
- **Triangular cell decomposition** applied on geometric map
- **Bi RRT** performed with steps to neighbouring cells and motion primitives are used to connect nodes

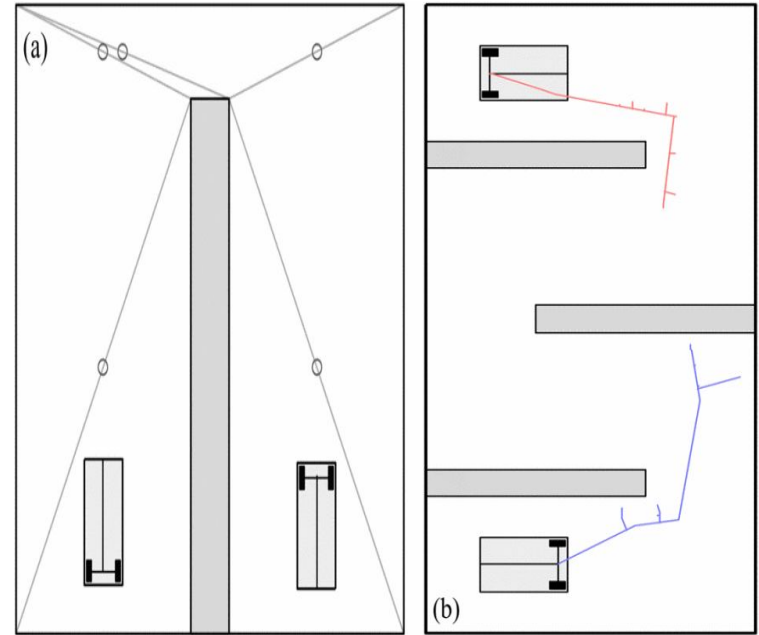


Figure 27: Triangular decomposition and Bi RRTs.

8. Á. Nagy, G. Csorvási and D. Kiss, "Path planning and control of differential and car-like robots in narrow environments," 2015 IEEE 13th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2015, pp. 103-108, doi: 10.1109/SAMI.2015.7061856.

Related Work

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- **Requirements:** *Geometric map, vehicle dimensions and kinematics, motion primitives*
- **Triangular cell decomposition** applied on geometric map
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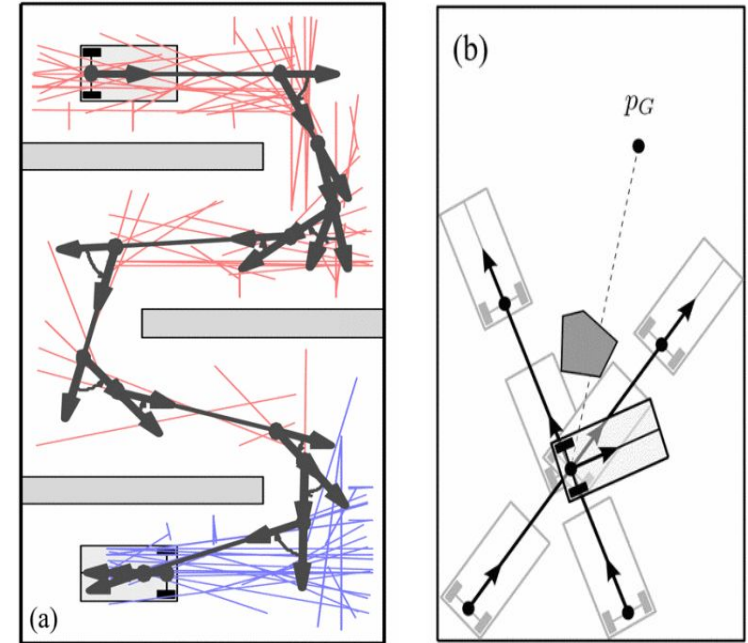


Figure 28: Bi directional RRTs and motion primitives.

8. Á. Nagy, G. Csorvási and D. Kiss, "Path planning and control of differential and car-like robots in narrow environments," 2015 IEEE 13th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2015, pp. 103-108, doi: 10.1109/SAMI.2015.7061856.

Related Work

Rapid Random Tree - C*CS Planner

- **Requirements:** *Geometric map, vehicle dimensions and kinematics, motion primitives*
- **Triangular cell decomposition** applied on geometric map
- **Bi RRT** performed with steps to neighbouring cells and motion primitives are used to connect nodes
- **C*CS planner** applied to obtain a secondary path containing straight segments and circular arcs (CCS and SCS paths)

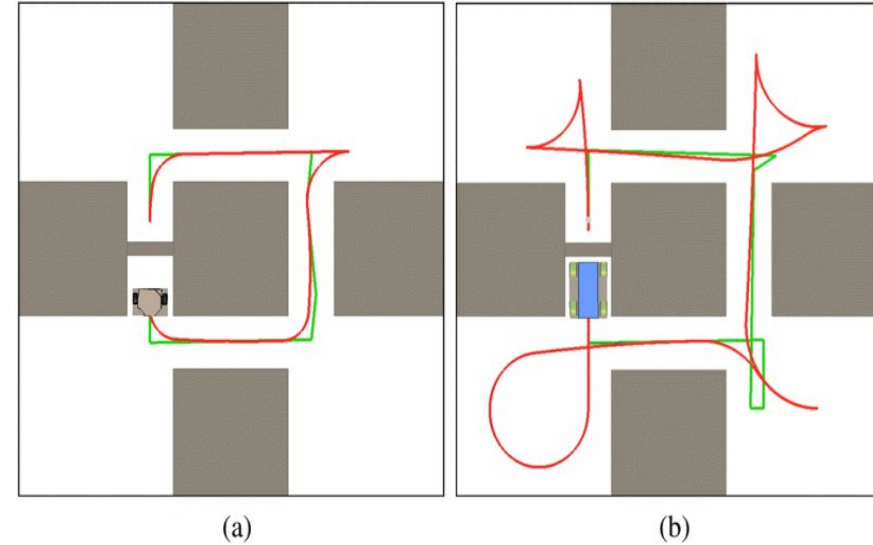


Figure 29: The RTR path (green) and its C*CS approximation (red) for (a) differential and (b) car-like robots.

8. Á. Nagy, G. Csorvási and D. Kiss, "Path planning and control of differential and car-like robots in narrow environments," 2015 IEEE 13th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2015, pp. 103-108, doi: 10.1109/SAMI.2015.7061856.

Related Work

Rapid Random Tree - T*TS Planner

- **Requirements:** *Geometric map, vehicle dimensions and kinematics, motion primitives*
- **Triangular cell decomposition** applied on geometric map
- **Bi RRT** performed with steps to neighbouring cells and motion primitives are used to connect nodes
- **T*TS planner** applied to obtain a secondary path (clothoids) containing straight segments 'S' and CC-turn 'T' that can have zero sharpness and curvature (TTS and STS paths)

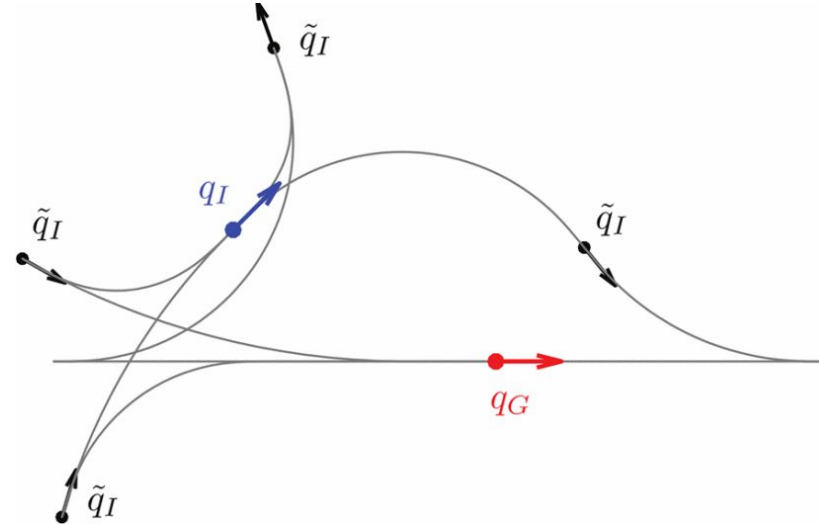


Figure 30: There is a number of T*TS solutions between two configurations.

9. D. Kiss and D. Papp, "Effective navigation in narrow areas: A planning method for autonomous cars," 2017 IEEE 15th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2017, pp. 000423-000430, doi: 10.1109/SAMI.2017.7880346.

Related Work

Rapid Random Tree - T*TS Planner

- **Requirements:** *Geometric map, vehicle dimensions and kinematics, motion primitives*
- **Triangular cell decomposition** applied on geometric map
- **Bi RRT** performed with steps to neighbouring cells and motion primitives are used to connect nodes
- **T*TS planner** applied to obtain a secondary path (clothoids) containing straight segments 'S' and CC-turn 'T' that can have zero sharpness and curvature (TTS and STS paths)
- **Drawbacks:** *Requires geometric information of environment. More motion primitives needed for narrower areas.*

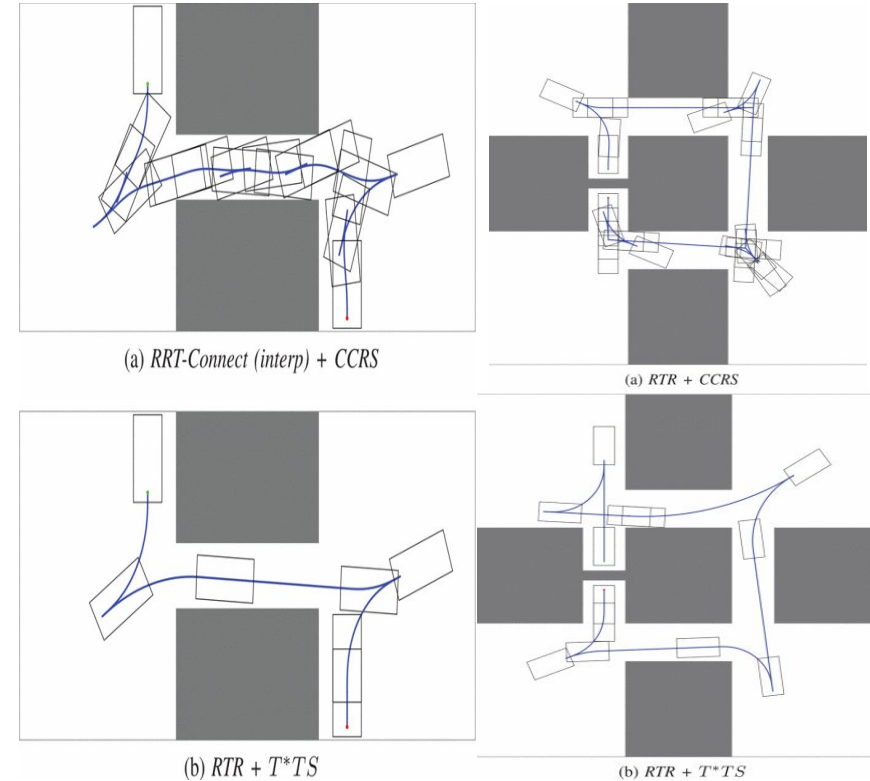


Figure 31: Crossing a narrow corridor and three narrow corridors.

9. D. Kiss and D. Papp, "Effective navigation in narrow areas: A planning method for autonomous cars," 2017 IEEE 15th International Symposium on Applied Machine Intelligence and Informatics (SAMI), 2017, pp. 000423-000430, doi: 10.1109/SAMI.2017.7880346.