Deep Learning

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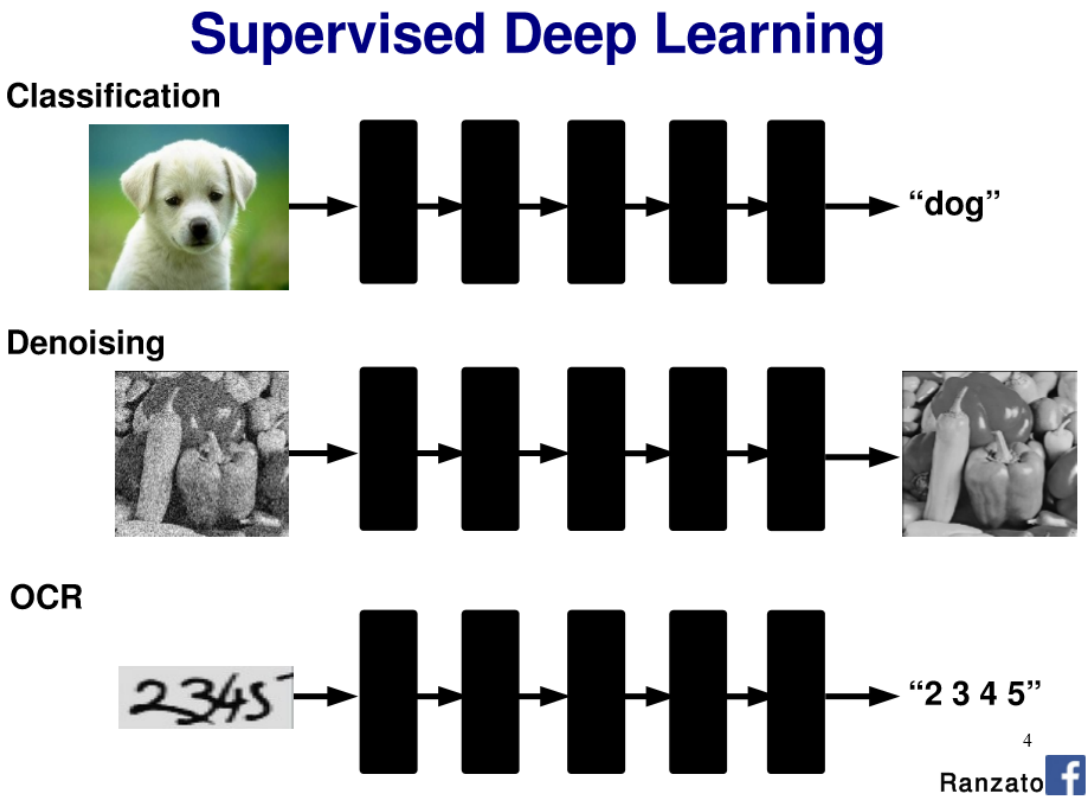
Today's Agenda

1. Introduction to Deep Learning
2. Neural Networks
3. Modifications
4. Regularization

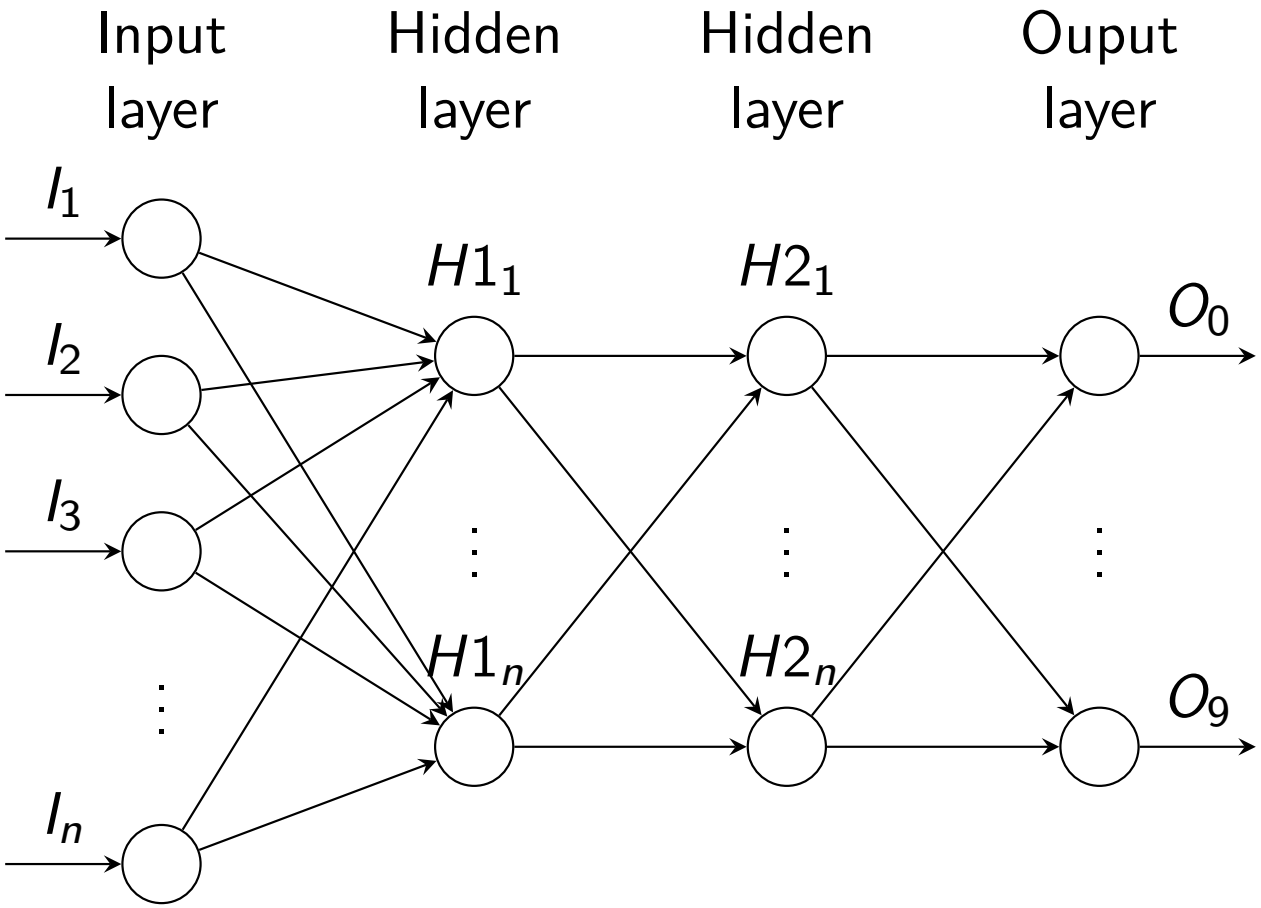
Outline

Recent developments

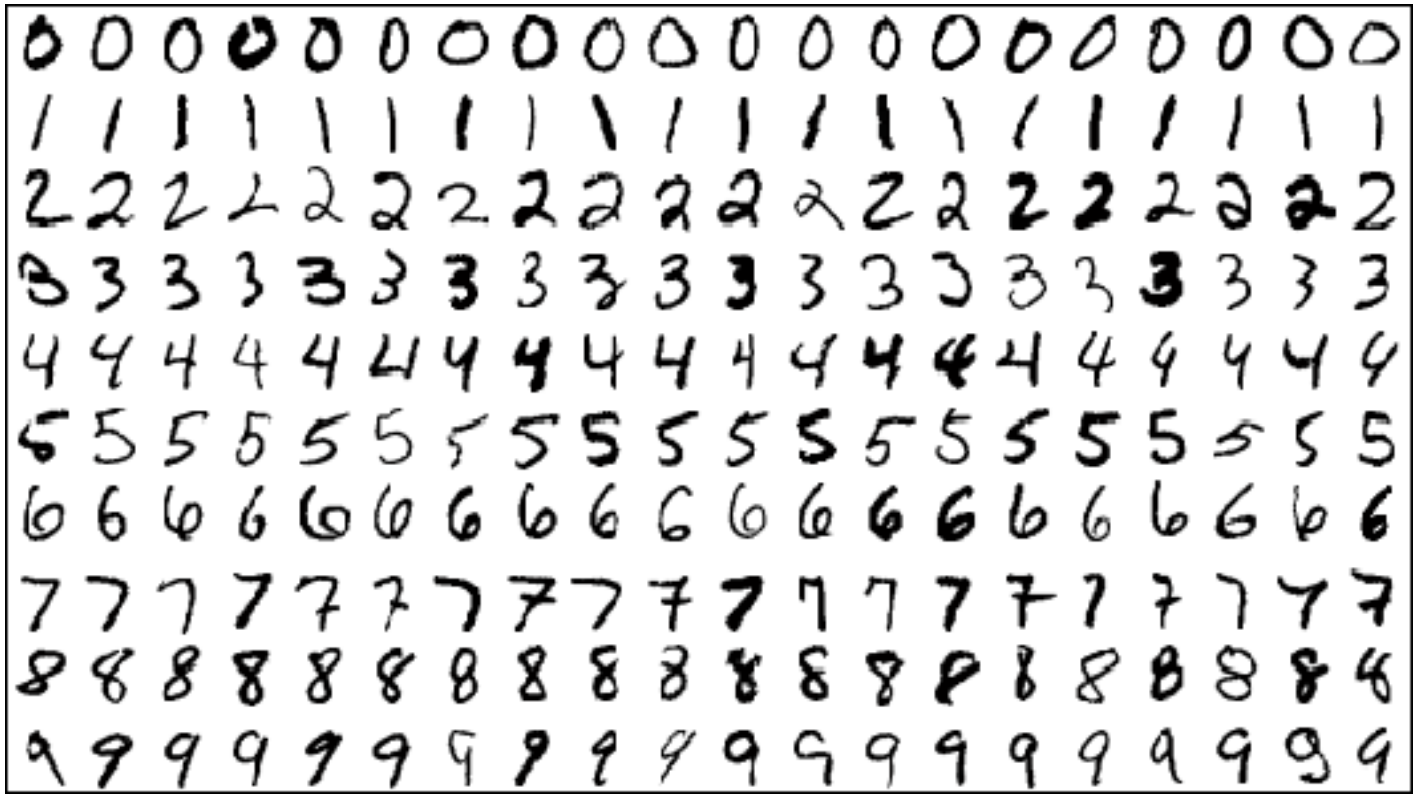


Outline

Overview


Input layer
Hidden layer
Hidden layer
Ouput layer
I_1
I_2
I_3
I_n
$H1_1$
$H1_n$
$H2_1$
$H2_n$
O_0
O_9

MNIST Data



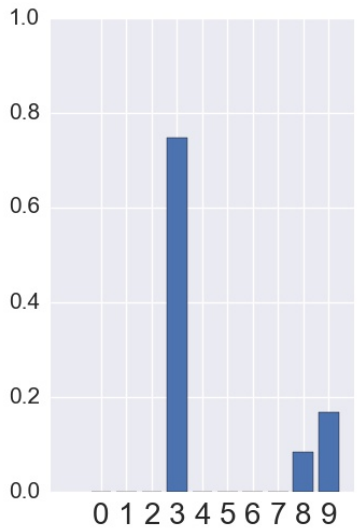
MNIST Handwritten Database

50000 Training Data

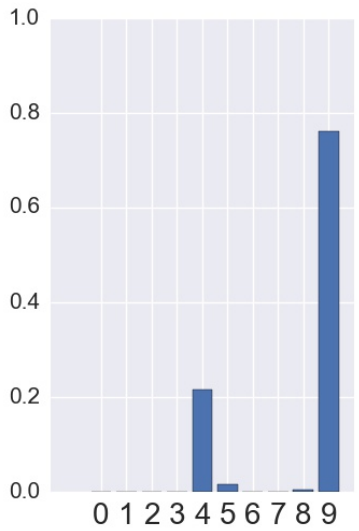
10000 Validation Data

10000 Test Data

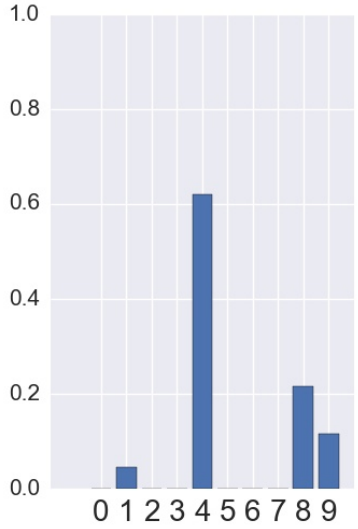
Overview



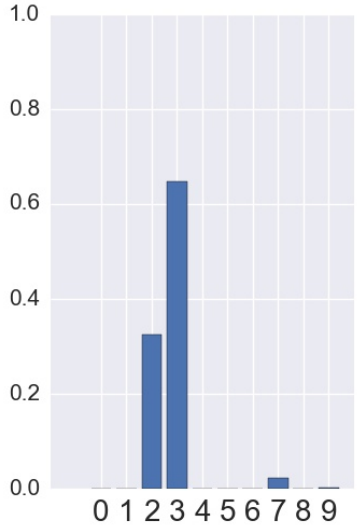
True Label: 3



True Label: 4



True Label: 9



True Label: 2

Overview

Architecture	**Activation**	**Cost**	**Algorithm**
Dense	Sigmoid	Quadratic	Gradient Descent (GD)
Convolutional	Softmax	Cross Entropy	Stochastic GD
	Tanh	Log-likelihood	Backpropagation
	ReLU		

Improvements

Regularization

Dropout

Initialisation

Overview

Session 1
Activations
Cost
Stochastic Gradient Descent
Backpropagation
Overfitting
Regularization

Session 2
Initialisation
Hyper-parameters
Variants of SGD
More Regularisers
Intro to ConvNets

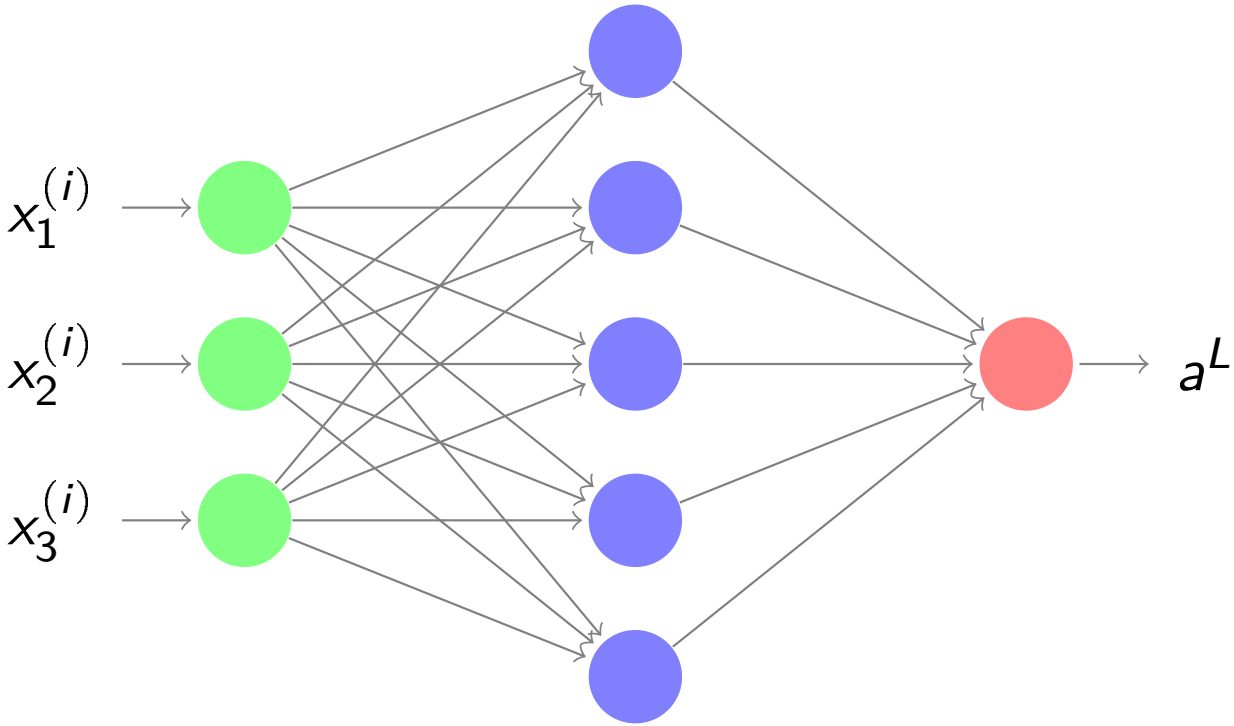
Session 3
Popular ConvNet Architectures
AlexNet
ZFNet
VGG
GoogLeNet
ResNet
Denoising

Session 4/5
Autoencoders
Transposed Convolution
Generative Network
Generative Adversarial Network

Outline

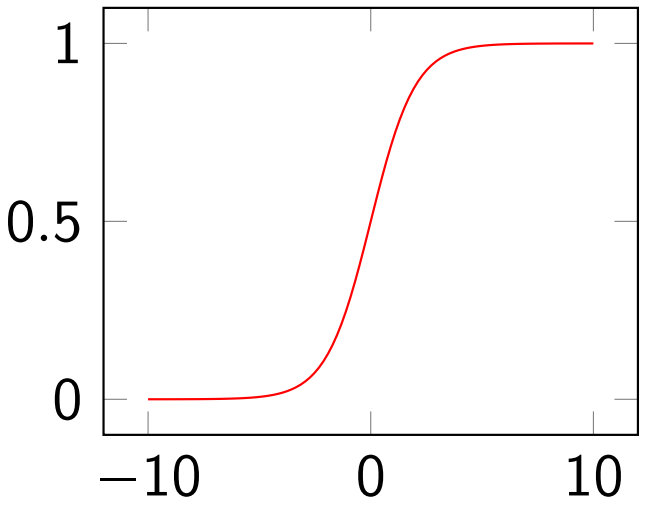
Outline

Perceptron



$$a^l = \begin{cases} 1, & W^l a^{l-1} + b^l \geqslant 0 \\ 0, & W^l a^{l-1} + b^l < 0 \end{cases}$$

Sigmoid

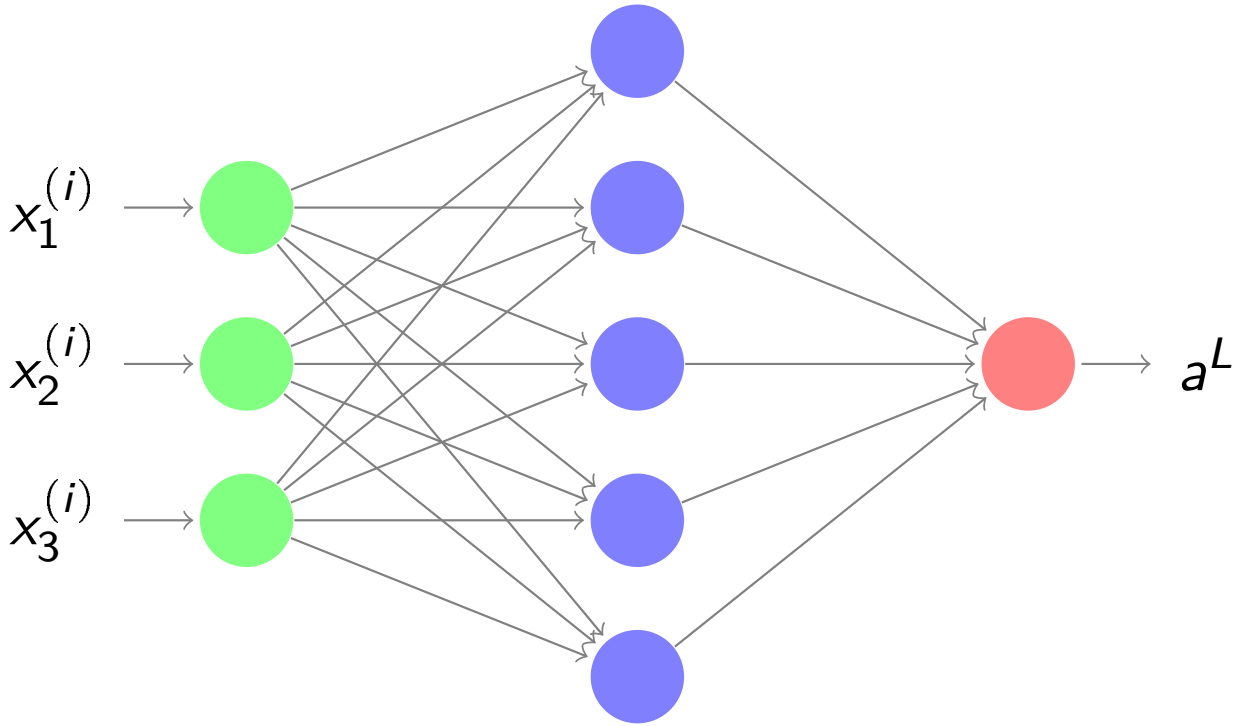


$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$a^l = \sigma(z^l) \approx \begin{cases} 1, & z \to \infty \\ 0, & z \to -\infty \end{cases}$$

Outline

Quadratic Cost



$$C(\mathbf{x};\Theta) = C(\mathbf{x},\mathbf{w},\mathbf{b}) = \frac{1}{2n}\sum_{i=1}^{n}\left\|y(\mathbf{x}^{(i)}) - a^L(\mathbf{x}^{(i)})\right\|_2^2,$$

$$\text{where } \mathbf{x} = \left\{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, ..., \mathbf{x}^{(n)}\right\}$$

Outline

Outline

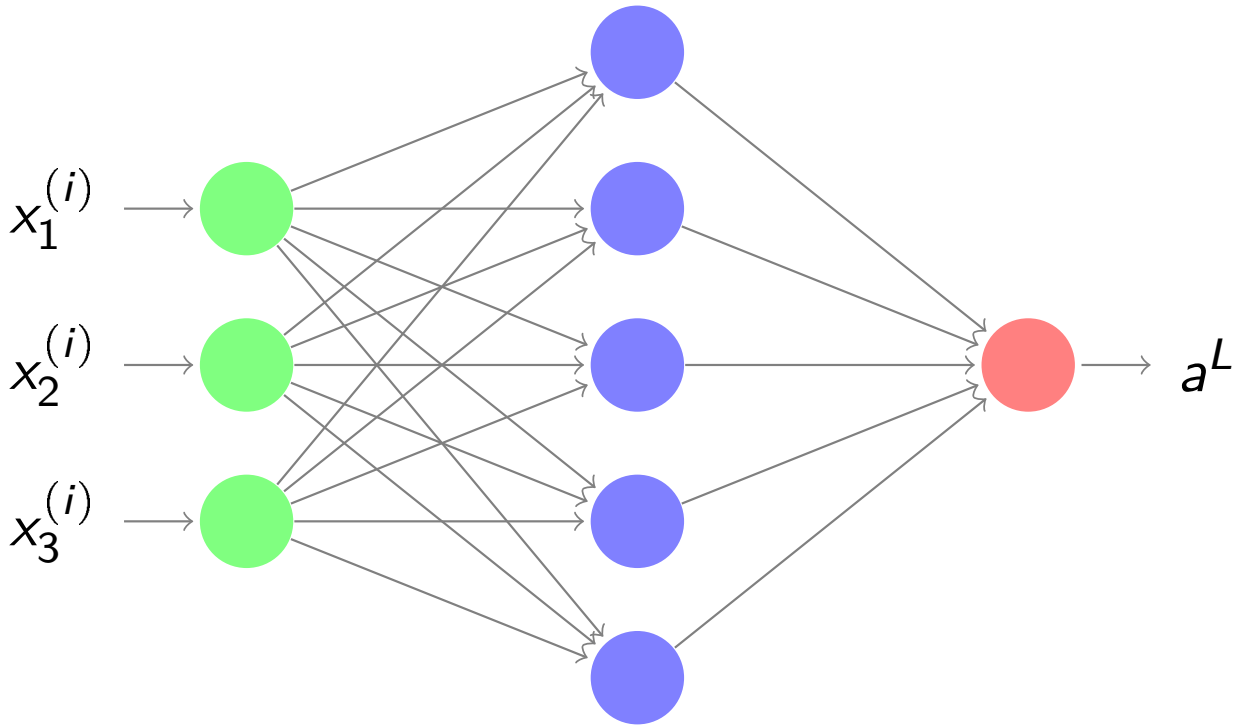
Gradient descent

For each $\mathbf{x}^{(i)}$,

$$
\begin{aligned}
C(\mathbf{x}^{(i)}, w, b) &= \frac{1}{2}\left[y(\mathbf{x}^{(i)}) - a^2(\mathbf{x}^{(i)})\right]^2 \\
\frac{\partial C}{\partial w^{(2)}} &= \left(\frac{\partial C}{\partial a^{(2)}}\frac{\partial a^{(2)}}{\partial z^{(2)}}\right)\frac{\partial z^{(2)}}{\partial w^{(2)}} \\
\frac{\partial C}{\partial b^{(2)}} &= \left(\frac{\partial C}{\partial a^{(2)}}\frac{\partial a^{(2)}}{\partial z^{(2)}}\right)\frac{\partial z^{(2)}}{\partial b^{(2)}} \qquad (1) \\
\frac{\partial C}{\partial w^{(1)}} &= \left(\frac{\partial C}{\partial a^{(2)}}\frac{\partial a^{(2)}}{\partial z^{(2)}}\right)\left(\frac{\partial z^{(2)}}{\partial a^{(1)}}\frac{\partial a^{(1)}}{\partial z^{(1)}}\right)\frac{\partial z^{(1)}}{\partial w^{(1)}} \\
\frac{\partial C}{\partial b^{(1)}} &= \left(\frac{\partial C}{\partial a^{(2)}}\frac{\partial a^{(2)}}{\partial z^{(2)}}\right)\left(\frac{\partial z^{(2)}}{\partial a^{(1)}}\frac{\partial a^{(1)}}{\partial z^{(1)}}\right)\frac{\partial z^{(1)}}{\partial b^{(1)}}
\end{aligned}
$$

Backpropagation (3).

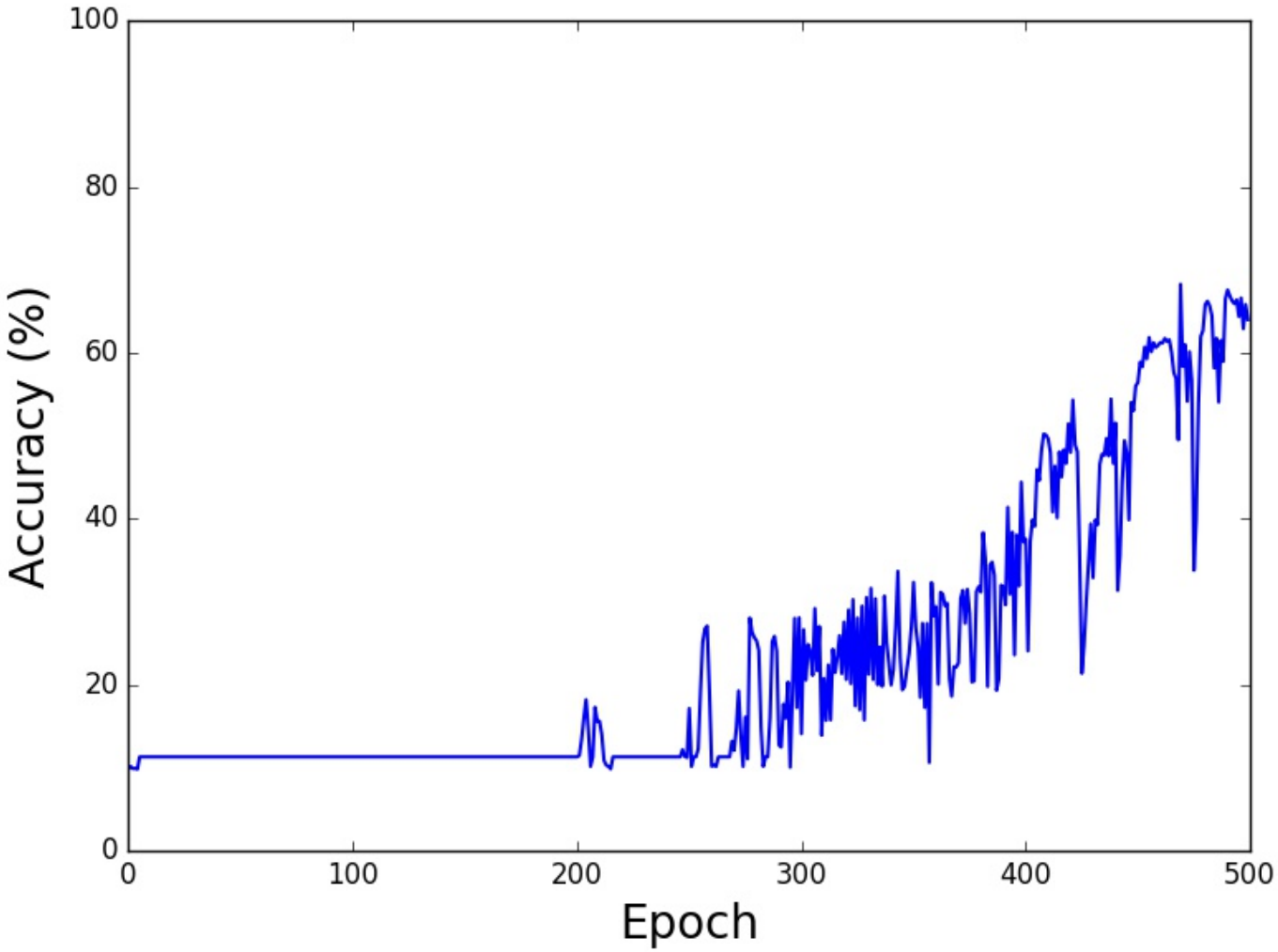
Gradient descent



$$C = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}|y - a^L|^2 \qquad w^{(k+1)} = w^{(k)} - \eta\nabla_{w^{(k)}}C(\mathbf{x})$$

$$\nabla C(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}\nabla C(\mathbf{x}^{(i)}) \qquad b^{(k+1)} = b^{(k)} - \eta\nabla_{b^{(k)}}C(\mathbf{x})$$

Gradient Descent



Slow Learning!

Outline

Stochastic Gradient Descent

Randomly sample into m mini-batches of size k:

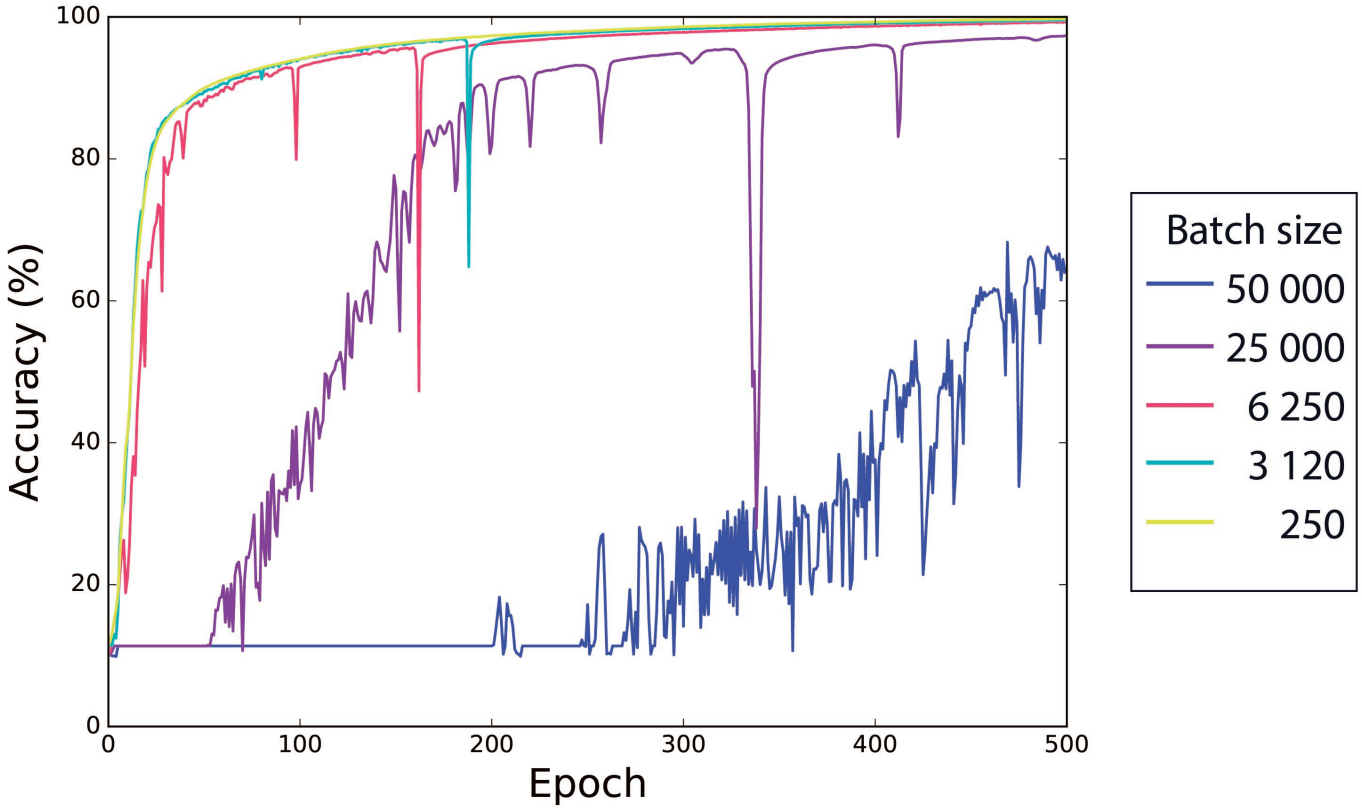
$$\begin{aligned}\mathbf{x} &= \left\{\{x^{(1)}, x^{(2)}, ..., x^{(k)}\}, \{x^{(k+1)}, x^{(k+2)}, ..., x^{(2k)}\}, ..., \{x^{(n-k+1)}, ..., x^{(n)}\}\right\} \\ &= \{M_1, M_2, ..., M_m\}\end{aligned}$$

$$\nabla C(\mathbf{x}) \approx \frac{1}{m}\sum_{i=1}^{m} \nabla C(\mathbf{x}_{M_j}) \text{ , for each mini-batch } M_j$$

Parameters update for each mini-batch M_i:

$$\begin{aligned} w &\leftarrow w - \frac{\eta}{m}\frac{\partial C}{\partial w} \\ b &\leftarrow b - \frac{\eta}{m}\frac{\partial C}{\partial b} \end{aligned} \tag{2}$$

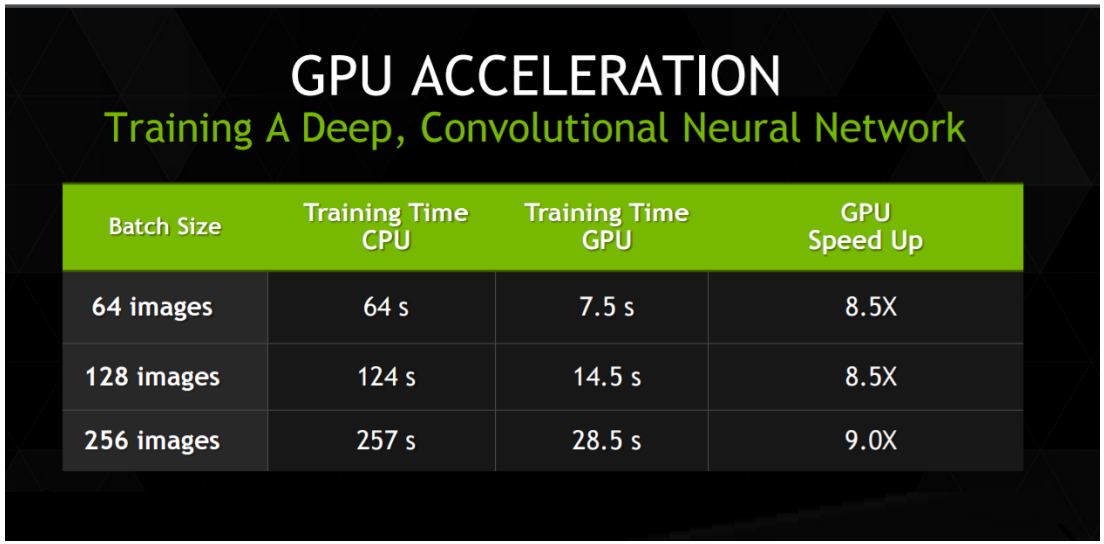
Stochastic Gradient Descent



1 Epoch - All $\{M_1, M_2, ...\}$ are used.

Learning accelerates with more minibatches(smaller minibatch size).

Stochastic Gradient Descent



http://www.nvidia.com
Deep Learning For Image Classification (Slide 28)

Remark:

- $w^{(k+1)} = w^{(k)} - \frac{\eta}{m} \frac{\partial C}{\partial w^{(k)}}$,
  need to adjust the learning rate η accordingly.
- Batch-size vs GPU computation

Outline

Backpropagation

Recall the gradient descent in Eqn (1).

Definition (Error term δ_j^l)

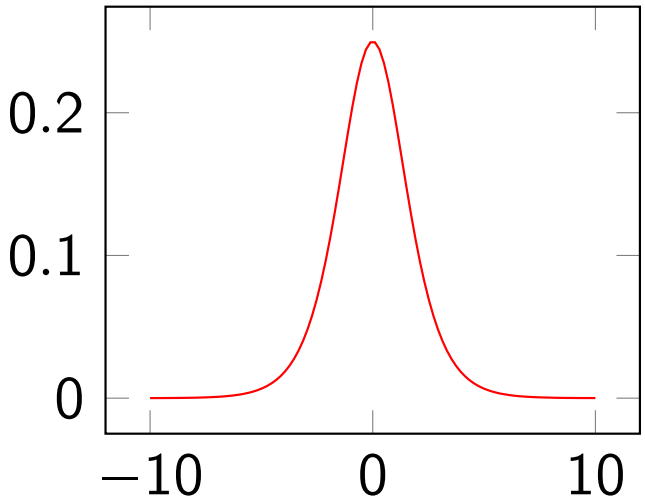
$$\delta_j^l = \frac{\partial C}{\partial z_j^l}.$$

We have the four following Backpropagation equations

$$\begin{aligned}
\delta_j^l &= \frac{\partial C}{\partial a_j^l} g'(z_j^l). \\
\delta^l &= \left([W^{l+1}]^T \delta^{l+1} \right) \odot g'(z^l). \\
\frac{\partial C}{\partial b_i^l} &= \delta_i^l. \\
\frac{\partial C}{\partial w_{ij}^l} &= a_j^{l-1} \delta_i^l.
\end{aligned} \tag{3}$$

Outline

Saturation



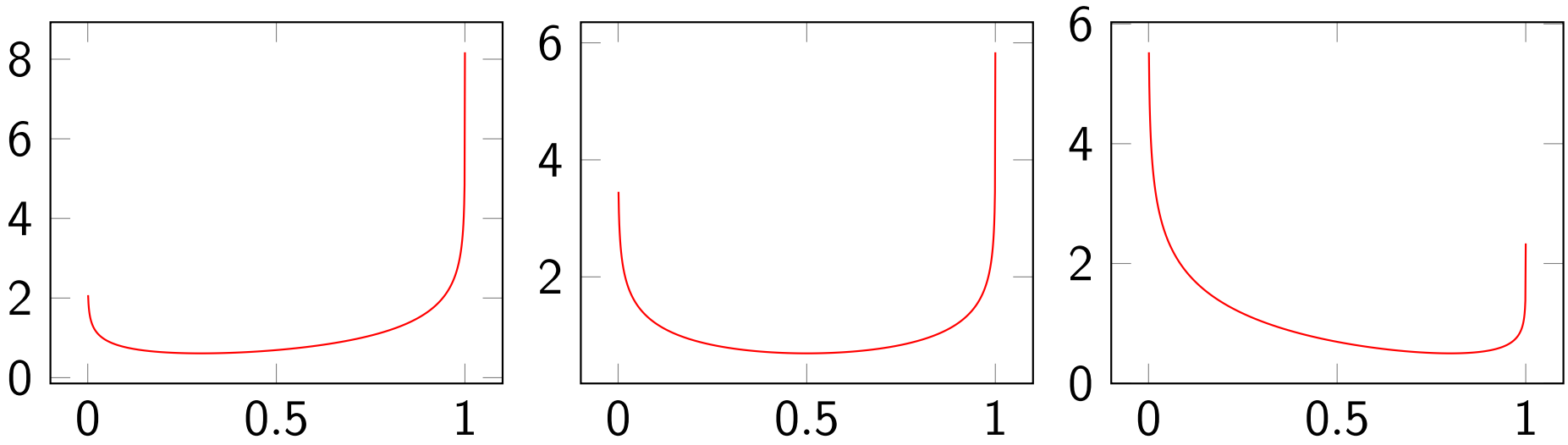
$$C(\mathbf{x}, \mathbf{w}, \mathbf{b}) = \frac{1}{2n} \left\| \mathbf{y} - \mathbf{a}^L \right\|_2^2$$

$$\frac{\partial C}{\partial w^L} = a^{L-1}(a^L - y)\sigma'(z) \to 0$$

$$\frac{\partial C}{\partial b^L} = (a^L - y)\sigma'(z) \to 0$$

Outline

Cross Entropy Cost



$$C(\mathbf{x}, \mathbf{w}, \mathbf{b}) = -\left[\left(y \log a^L\right) + (1-y) \log\left(1 - a^L\right)\right]$$

$$\nabla C(\mathbf{x}, \mathbf{w}, \mathbf{b}) = -\left[\frac{y}{a^L} - \frac{1-y}{1-a^L}\right]$$

$$\frac{\partial C}{\partial w^L} = a^{L-1}(a^L - y)$$

$$\frac{\partial C}{\partial b^L} = a^L - y$$

Other Activations and Cost Functions

Softmax function:

$$\text{softmax}(z^l) = \frac{e^{z^l_j}}{\sum_k e^{z^l_k}}$$

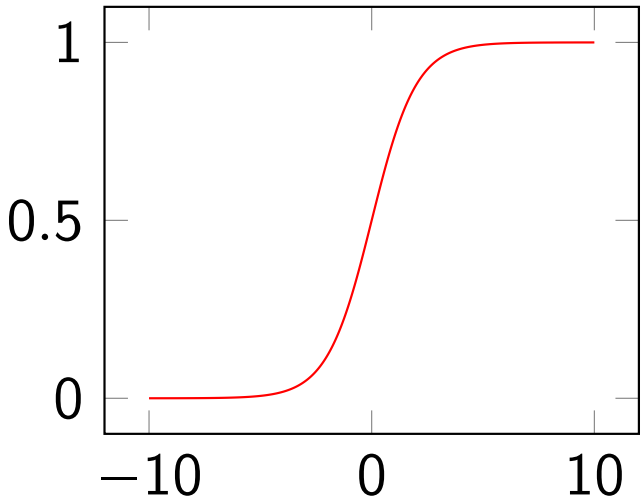
Log-likelihood Cost:

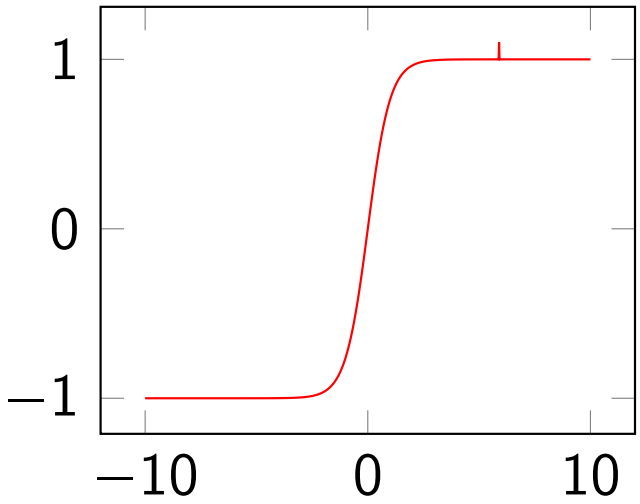
$$C = -\frac{1}{n}\sum_x \sum_k y_k \log a^L_k(x), \text{ for k classes}$$

$$\frac{\partial C}{\partial w^L} = a^{L-1}(a^L - y)$$
$$\frac{\partial C}{\partial b^L} = a^L - y$$

Outline

tanh





$$\sigma(z) = \frac{1+\tanh(\frac{z}{2})}{2}$$

$$\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Pros

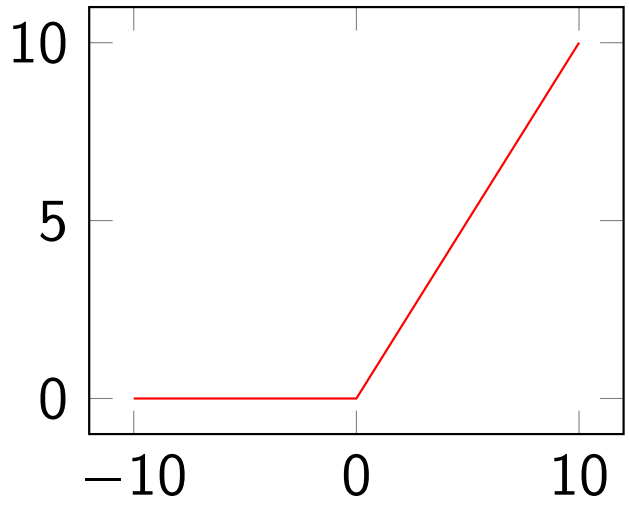
$\tanh(z) \in (-1, 1)$

weights can increase/decrease

Cons

Possibly saturate greater

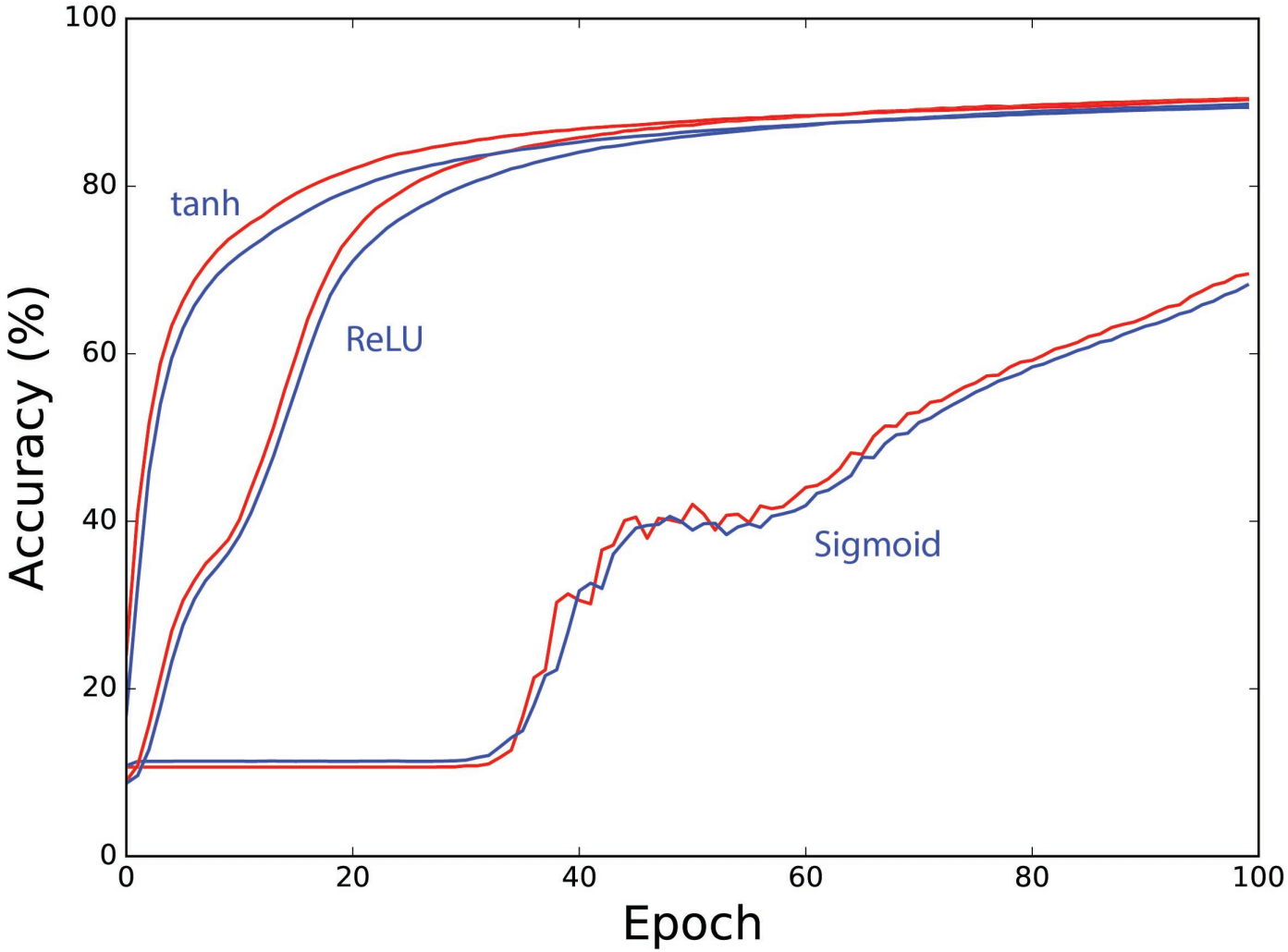
ReLU



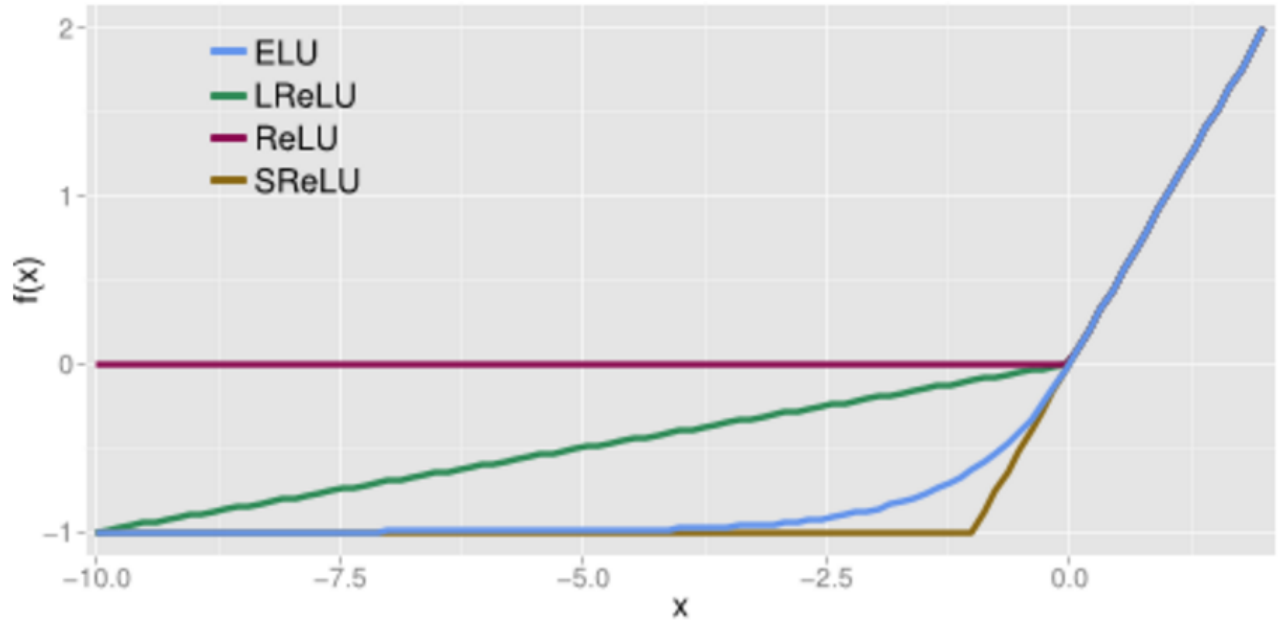
$$\text{ReLU}(z) = \max\{0, z\}$$

Pros	Cons
Does not saturate at large z	gradient and value is 0 at negative z
Perform well in image processing	
More computationally efficient	

Comparison


100
80
60
40
20
0
Accuracy (%)
tanh
ReLU
Sigmoid
0
20
40
60
80
100
Epoch

Even More Activations



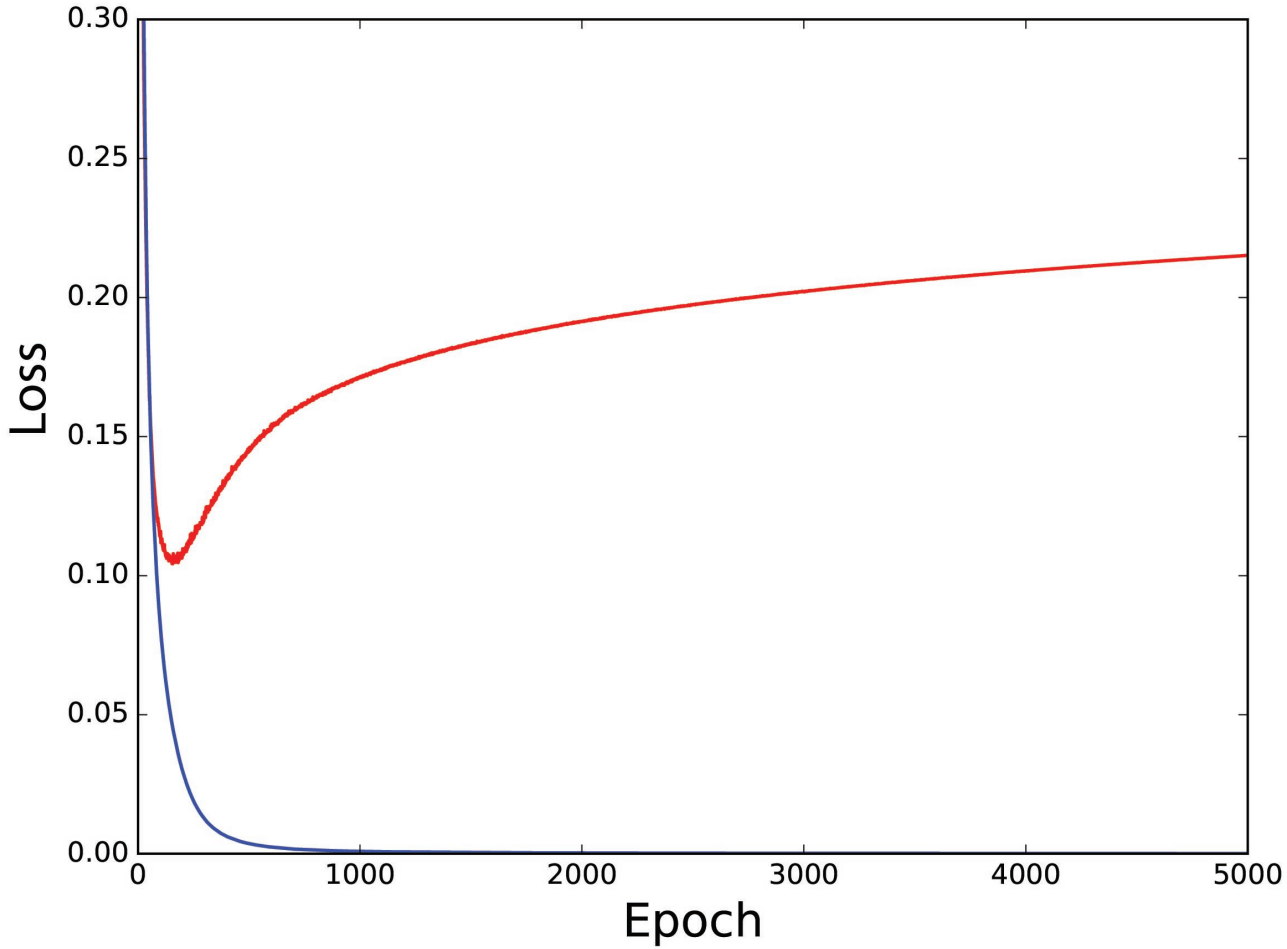
$$\text{LReLu}(z) = \max(\alpha z, z)$$

$$\text{ELU}(z) = \max(\alpha(e^z - 1), z)$$

Outline

Overfitting

- Too many parameters $\Longrightarrow$ Overfitting
- Regularize Weights



Outline

Outline

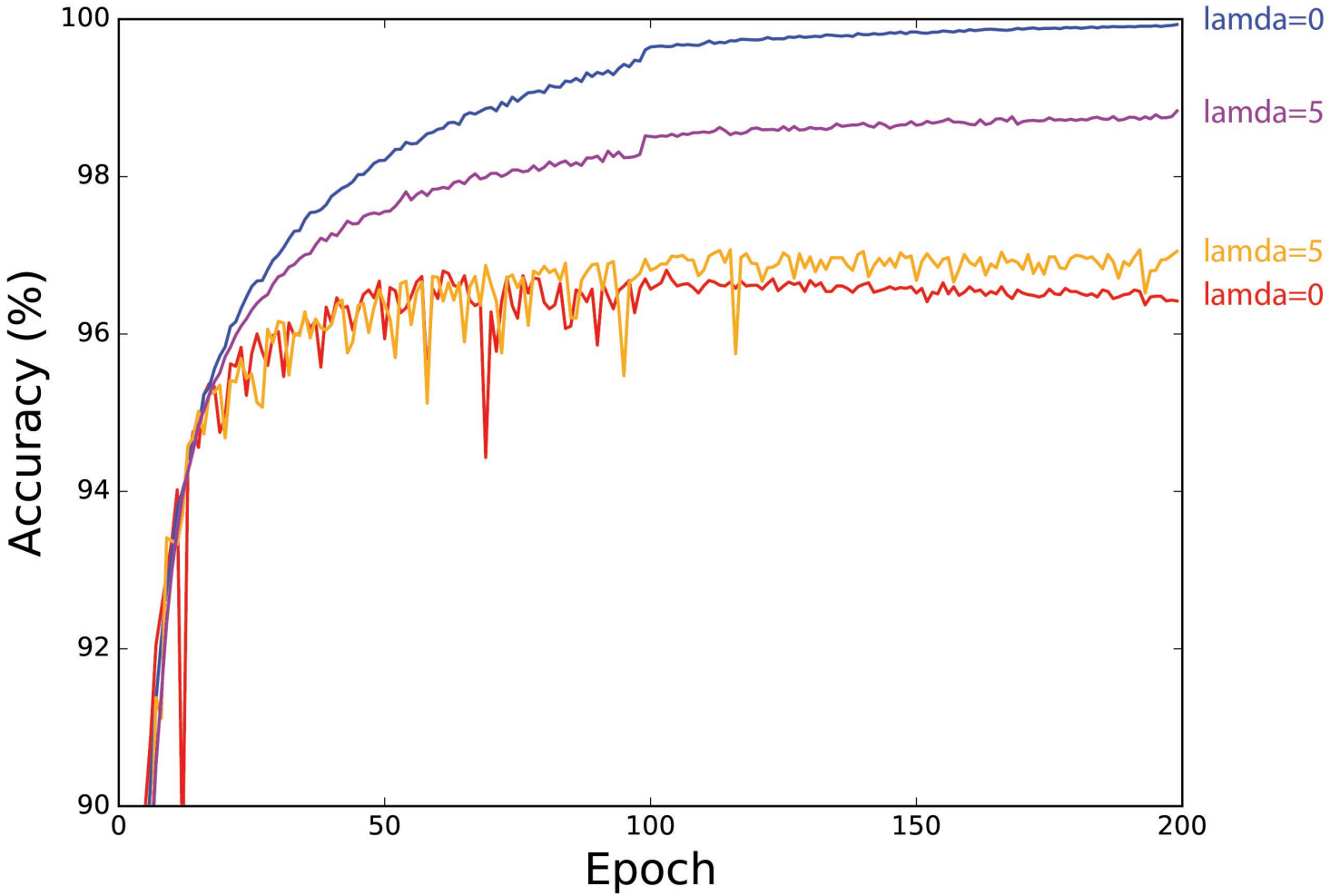
L2 Regularization

$$C = C_0 + \frac{1}{n}\frac{\lambda}{2}||w||_2^2 \tag{4}$$

$$\begin{aligned} \frac{\partial C}{\partial w} &= \frac{\partial C_0}{\partial w} + \frac{\lambda}{n} w \\ \frac{\partial C}{\partial b} &= \frac{\partial C_0}{\partial b} \end{aligned} \tag{5}$$

$$\begin{aligned} w_{k+1} &= \left(1 - \frac{\eta\lambda}{n}\right) w_k - \frac{\eta}{m}\frac{\partial C_0}{\partial w} \\ b_{k+1} &= b_k - \frac{\eta}{m}\frac{\partial C_0}{\partial b} \end{aligned} \tag{6}$$

L2 Regularization



Outline

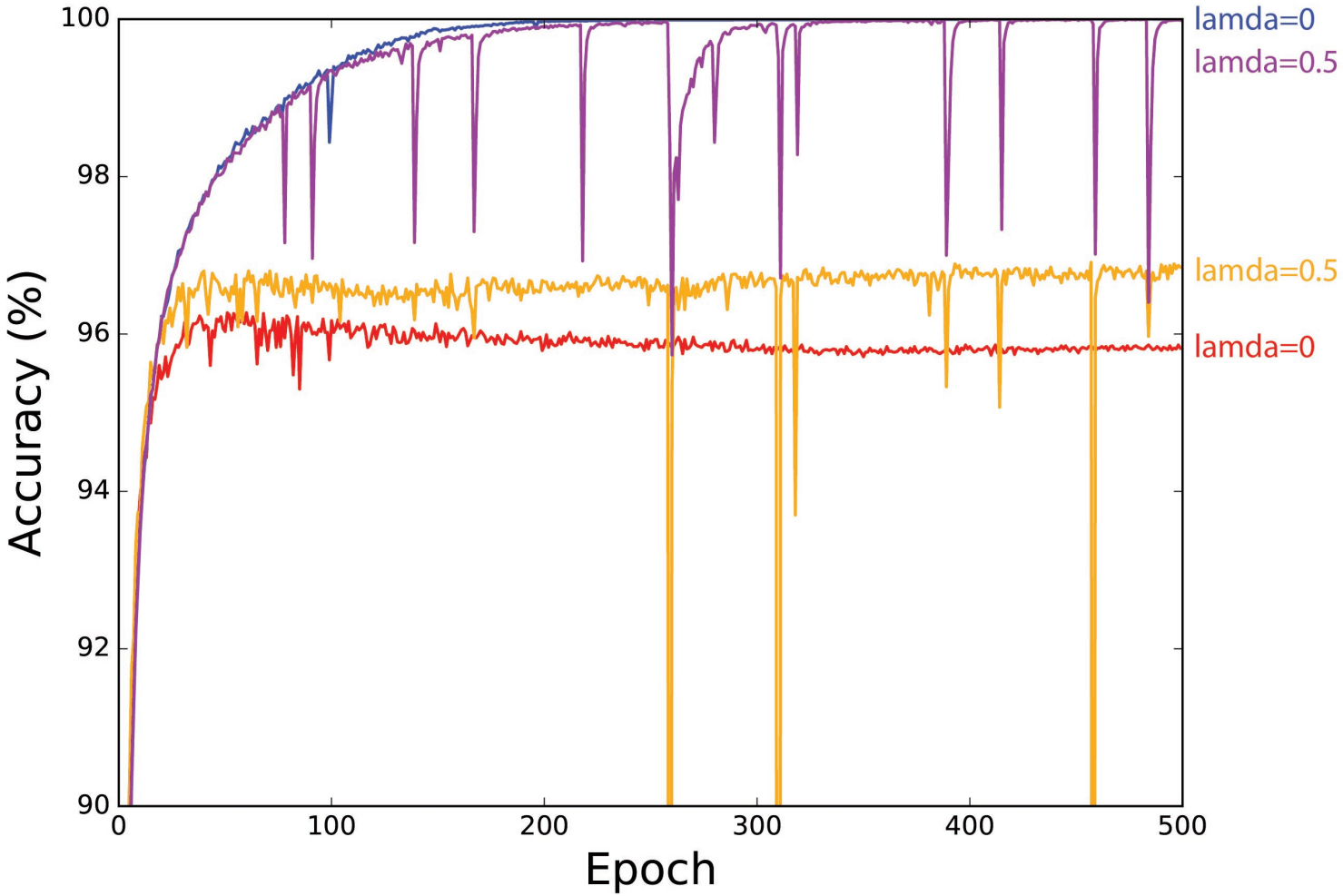
L1 Regularization

$$C = C_0 + \frac{\lambda}{n}||w||_1 \tag{7}$$

$$\begin{aligned} \frac{\partial C}{\partial w} &= \frac{\partial C_0}{\partial w} + \frac{\lambda}{n}\text{sgn}(w) \\ \frac{\partial C}{\partial b} &= \frac{\partial C_0}{\partial b} \end{aligned} \tag{8}$$

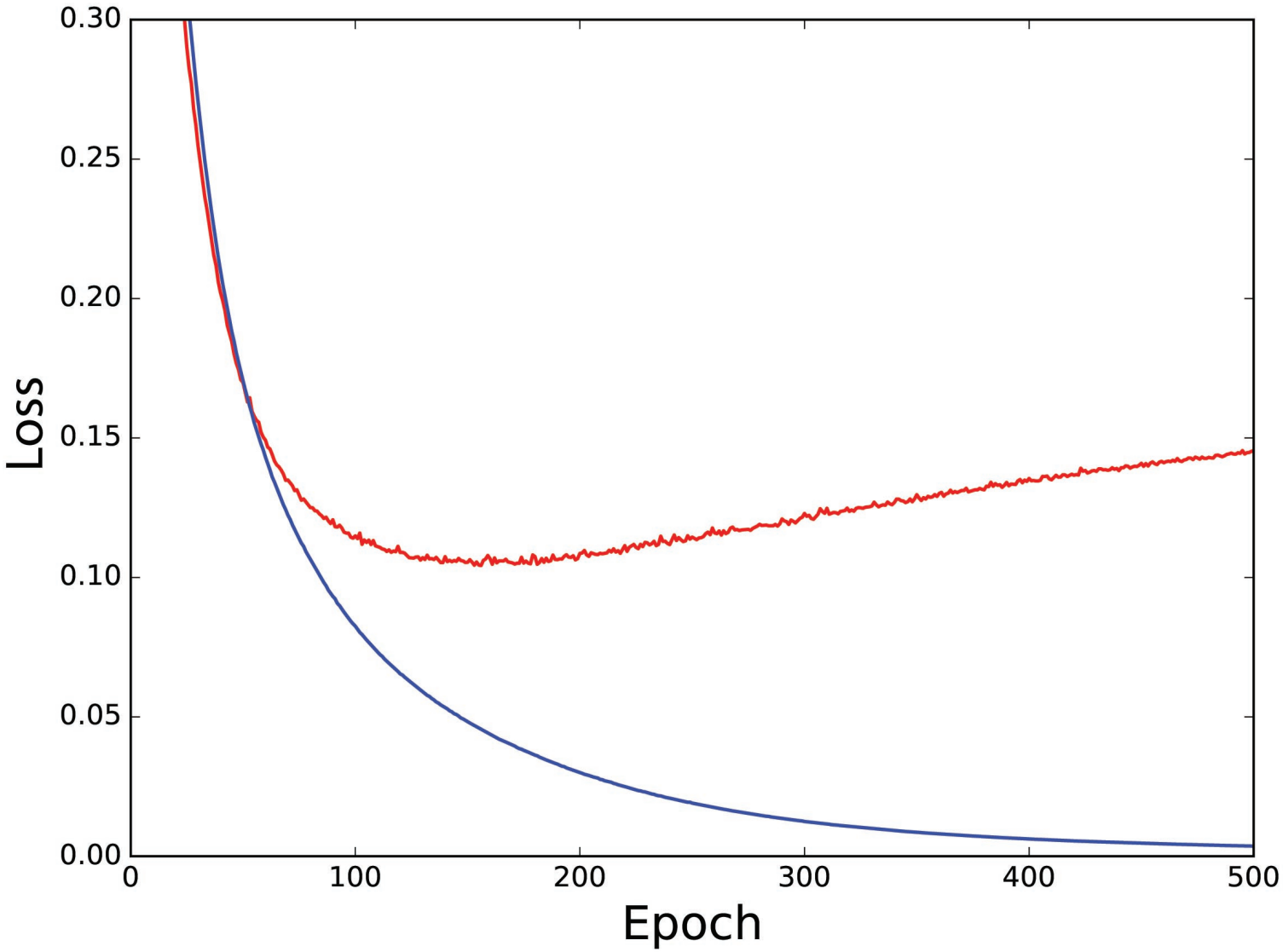
$$\begin{aligned} w_{k+1} &= \left(w_k - \frac{\eta\lambda}{n}\text{sgn}(w_k)\right) - \frac{\eta}{m}\frac{\partial C_0}{\partial w} \\ b_{k+1} &= b_k - \frac{\eta}{m}\frac{\partial C_0}{\partial b} \end{aligned} \tag{9}$$

L1 Regularization



Outline

Early Stopping



Thank You

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www.cs.toronto.edu/∼ranzato


neuralnetworksanddeeplearning.com/


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