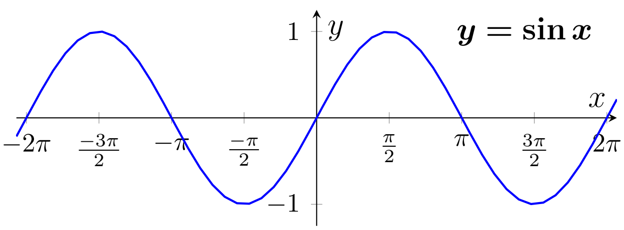
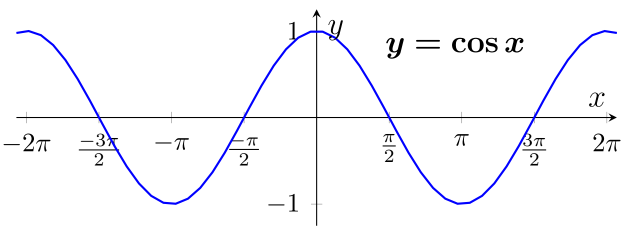
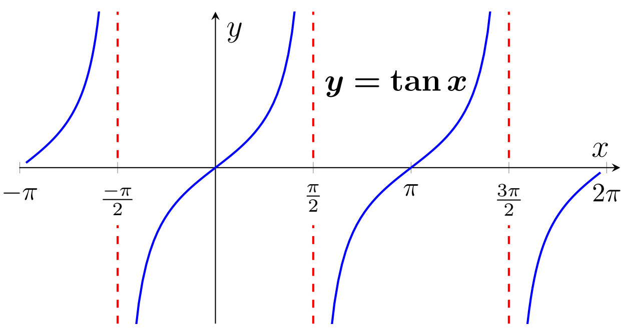
Essential Trigonometry for Calculus

Graphs







SOH–CAH–TOA

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Popular mnemonic: "Some Old Horse Caught Another Horse Taking Oats Away."

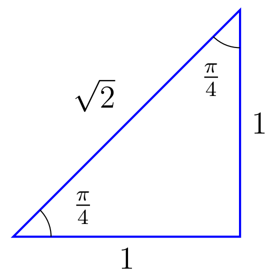
Basic Identities

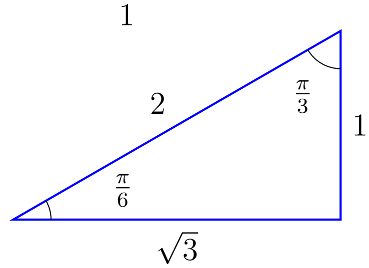
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Reference Triangles





Radians and Degrees

If d is the measure of an angle in degrees and r is its measure in radians, then $r = \frac{\pi}{180}d$ and $d = \frac{180}{\pi}r$.

Degrees	Radians
0°	0
30°	$\frac{\pi}{6}$
45°	$\frac{\pi}{4}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
180°	π
360°	2π

Trigonometric Identities

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$

$\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$

$\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$

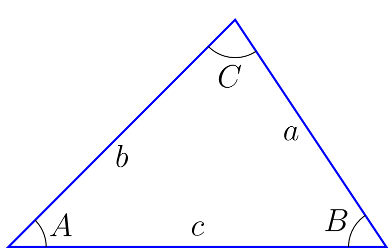
$\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$

$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha-\beta) + \cos(\alpha+\beta))$

$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$



Law of sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Trigonometric Limits

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$

Trigonometric Derivatives

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\sec x)}{dx} = \sec x\tan x$$

$$\frac{d(\csc x)}{dx} = -\csc x\cot x$$

$$\frac{d(\cot x)}{dx} = -\csc^2 x$$

$$\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1}x\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$$

Trigonometric Integrals

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x\tan x\,dx = \sec x + C$$

$$\int \csc x\cot x\,dx = -\csc x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

The Unit Circle