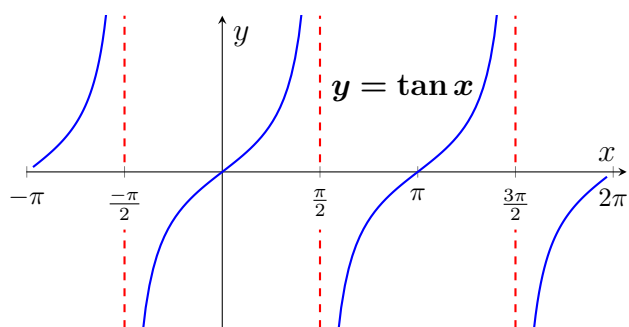
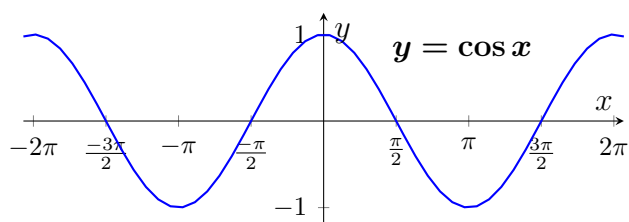
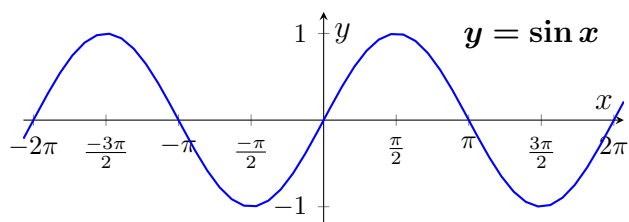


Essential Trigonometry for Calculus

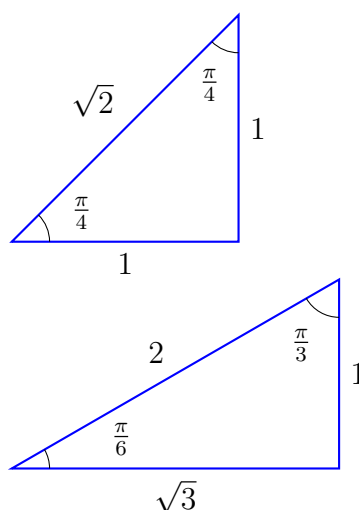
Graphs



Basic Identities

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} & \csc \theta &= \frac{1}{\sin \theta} \\ \cot \theta &= \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} \\ \sin^2 \theta + \cos^2 \theta &= 1\end{aligned}$$

Reference Triangles



SOH-CAH-TOA

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Popular mnemonic: “Some Old Horse Caught Another Horse Taking Oats Away.”

Radians and Degrees

If d is the measure of an angle in degrees and r is its measure in radians, then $r = \frac{\pi}{180}d$ and $d = \frac{180}{\pi}r$.

Degrees	Radians
0°	0
30°	$\frac{\pi}{6}$
45°	$\frac{\pi}{4}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
180°	π
360°	2π

Trigonometric Identities

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta\end{aligned}$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

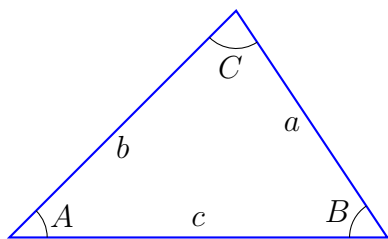
$$\sin \alpha \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos \alpha \cos \beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$



Law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

Trigonometric Limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

Trigonometric Derivatives

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\frac{d(\csc x)}{dx} = -\csc x \cot x$$

$$\frac{d(\cot x)}{dx} = -\csc^2 x$$

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

Trigonometric Integrals

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

The Unit Circle

