MANIPAL UNIVERSITY JAIPUR

Faculty of Engineering
School of Information Technology
B.Tech.-Information Technology-VI Semester
Second Sessional Examination: 2022-23
IT3202-Automata Theory & Compiler Design
(CLOSE BOOK)

Duration: 1 Hour **Max. Marks: 20**

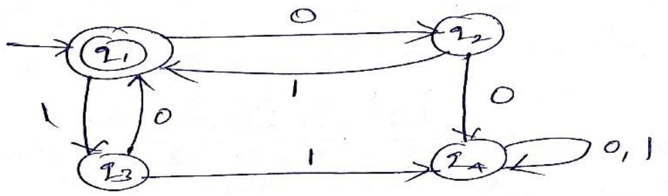
Solution and Solution Scheme

1 (a) Construct a Regular Expression using Arden's Theorem for the following FA which are shown in transition table. Here q1 is initial and final state. [3]

	0	1
→*q1	q2	q3
q2	q4	q1
q3	q1	q4
q4	q4	q4

Complete solution--full marks; final state equation with solution-1.5 marks; non final state equation each-0.5 marks

Soln 1(a) Arden's Theorem



$q_1 = q_2 \cdot 1 + q_3 \cdot 0 + \lambda$ —— (1)

$q_2 = q_1 \cdot 0$ —— (2)

$q_3 = q_1 \cdot 1$ —— (3)

$q_4 = q_2 \cdot 0 + q_3 \cdot 1 + q_4(0+1)$ —— (4)

Since q_1 is only final state; so as per arden's theorem we ~~will~~ will solve only q_1 using (2) & (3)

$q_1 = q_1 \cdot 0 \cdot 1 + q_1 \cdot 1 \cdot 0 + \lambda$

$\Rightarrow$ ~~q_1~~ $\underbrace{q_1}_{R} = \underbrace{\lambda}_{Q} + \underbrace{q_1}_{R}\underbrace{(01+10)}_{P}$

↳ $R = QP$

$\therefore \boxed{q_1 = \lambda(01+10)}$

Ans

(b) Construct FA from following Regular Grammar [2]

$A \rightarrow aB \mid bA \mid b$ $\qquad B \rightarrow aC \mid bB$

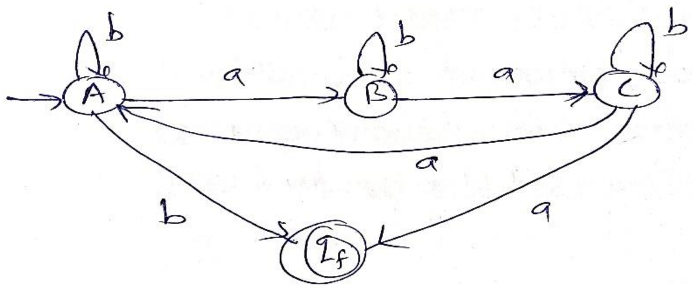
$C \rightarrow aA \mid bC \mid a$

Full Solution: 03 marks otherwise 0 marks

Sol^n 1(b) RG to FA

$A \rightarrow aB \mid bA \mid b$
$B \rightarrow aC \mid bB$
$C \rightarrow aA \mid bC \mid a$

Let A, B, C are 3 states of desired FA



2 (a) Construct a CFG for the following language [3]

$L = \{a^i b^j c^k \mid i = j \text{ or } j = k;\ i, j, k \geq 1\}$

Grammar for L1 & L2: 1.25 marks each and its union: 0.5 marks

Sol^n 2(a) CFG Construction

we split L into two parts L_1 & L_2

$L_1 \Rightarrow$ if $i = j = n$

$L_1 = a^i b^j c^k = \underbrace{a^n b^n}_{A_1} \underbrace{c^k}_{B_1}$

Grammar for L_1 : $S_1 \rightarrow A_1 B_1$
$A_1 \rightarrow a A_1 b \mid ab$
$B_1 \rightarrow B_1 c \mid c$

Similarly $L_2 \Rightarrow$ if $j = k = m$ $\quad L_2 = \underbrace{a^i}_{A_2} \underbrace{b^m c^m}_{B_2}$

Grammar for L_2 : $S_2 \rightarrow A_2 B_2$
$A_2 \rightarrow a A_2 \mid a$
$B_2 \rightarrow b B_2 c \mid bc$

Now we combine L_1 & L_2 by grammar $S \rightarrow S_1 \mid S_2$

	(b) Show that L = {ww | w ∈ $(0, 1)^*$} is not a Context Free Language using Pumping Lemma. For two pumping: 1.5 marks and for three pumping: 02 marks or any two-rule violating pumping: 02 marks	[2]

Soln 2(b) Pumping Lemma

Let given L is a context free Language,
L must have a pumping length say P; here we take P=3

Let w = 01 so, one of the string for this language will be ww = 0101

Now, we take a string $s = 0^P 1^P 0^P 1^P$

$s = 0^3 1^3 0^3 1^3$

$s = \underbrace{000}_{u}\ \underbrace{1}_{v}\ \underbrace{1}_{x}\ \underbrace{1}_{y}\ \underbrace{000\ 111}_{z}$

We divid s into 5 parts say u, v, x, y, z such that
$|vy| > 0$
$|vxy| \le P$

Now we pump for some value of i in the following way $uv^i xy^i z$

for $i=1 \Rightarrow uvxyz$
$\Rightarrow 000111000111 \in L$

for $i=2 \Rightarrow uv^2xy^2z$
$\Rightarrow 000\ 11111\ 000111$
$\Rightarrow 0^3 1^5 0^3 1^3 \notin L$

for $i=3 \Rightarrow uv^3xy^3z$
$\Rightarrow 000\ 1111111\ 000111$
$\Rightarrow 0^3 1^7 0^3 1^3 \notin L$

~~Therefor~~ Here, rule violets for some values of i,
therefore we can say that given Language is not a context free language.

3 (a) Design a PDA for the following language [3]

$L = \{a^n b^m c^m d^n \mid m, n \geq 1 \text{ and } m \neq n\}$

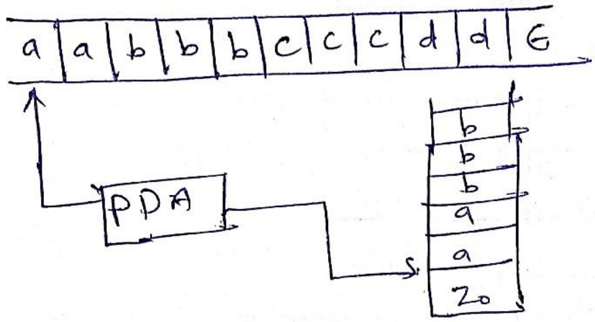
Full Solution: 03 marks; only graphical diagram or IDs without showing through stack: 2.5 marks

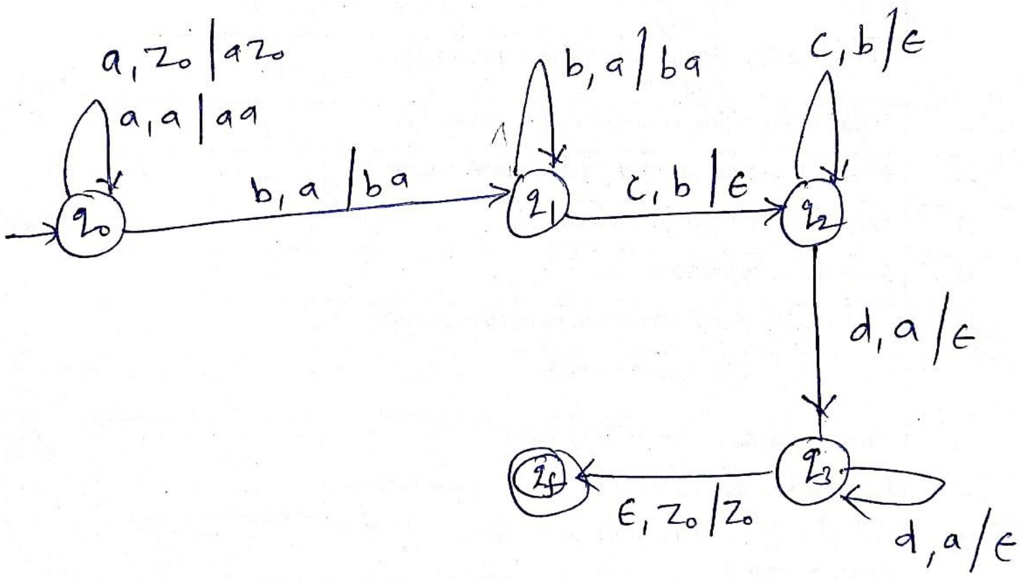
Sol^n 3 (a) PDA Designing

$L = \{a^n b^m c^m d^n \mid m, n \geq 1 \ \& \ m \neq n\}$

Let $n = 2$ & $m = 3$

$L = aabbbcccdd$





	(b)Write the Regular Expression for the following Regular Language $L = \{a^n b^m \mid n + m \text{ is even}\}$ 01 marks for each condition	[2]

Sol^n 3(b) RE for RL

Conditions for even number

(i) n, m both should be even $(aa)^*(bb)^*$

(ii) n, m both should be odd

$(aa)^*.a\,(bb)^*.b$

finally we combine both the condition (i) & (ii)

$$\boxed{(aa)^*(bb)^* + (aa)^*.a\,(bb)^*.b}$$

4	(a) Convert the following Context Free Grammar into Chomsky Normal Form S→ AACD A→ aAb / λ C→ aC / a D→ aDa / bDb / λ where λ is null string. Elimination of Null and Unit Production: 01 marks each; Elimination of useless productions: 0.5 marks; Conversion of CFG to CNF: 1.5 marks	[4]

Solution 4(a) CFG to CNF

① Elimination of Null productions

$S \to AACD$
$A \to aAb/\lambda$
$C \to aC/a$
$D \to aDa/bDb/\lambda$

$\xrightarrow{\lambda\text{-removal}}$

$S \to AACD/ACD/CD/AAC$
$\quad /C/AC$
$A \to aAb/ab$
$C \to aC/a$
$D \to aDa/aa/bDb/bb$

② Elimination of Unit Production

$S \to AACD/ACD/CD/AAC/aC/a/AC$
$A \to aAb/ab$
$C \to aC/a$
$D \to aDa/bDb/aa/bb$

③ Elimination of Useless Production :→ In above grammar, all the productions are useful; so no need to remove anyone.

Now, grammar is simplified, we can start conversion from CFG to CNF as it requires two types of productions $\boxed{NT \rightarrow NT\ NT \quad \text{or } NT \rightarrow T}$

where NT = Non terminal & T = Terminal

~~Therefore~~, Let $J \rightarrow a$ & $K \rightarrow b$

replace J & K in appropriate position of grammar

$S \rightarrow AACD \mid ACD \mid CD \mid AAC \mid JC \mid a \mid AC$

$A \rightarrow JAK \mid JK$

$C \rightarrow JC \mid a$

$D \rightarrow JDJ \mid KDK \mid JJ \mid KK$

$J \rightarrow a$
$K \rightarrow b$

Above grammar is still not in CNF; so need some more replacement. Let $L \rightarrow AAC$; $M \rightarrow AC$; $N \rightarrow AA$; $O \rightarrow JA$; $P \rightarrow JD$ & $Q \rightarrow KD$

$S \rightarrow LD \mid MD \mid CD \mid NC \mid JC \mid a \mid AC$

$A \rightarrow OK \mid JK$

$C \rightarrow JC \mid a$

$D \rightarrow PJ \mid QK \mid JJ \mid KK$

$J \rightarrow a$
$K \rightarrow b$
$L \rightarrow AAC$
$M \rightarrow AC$

$N \rightarrow AA$
$O \rightarrow JA$
$P \rightarrow JD$
$Q \rightarrow KD$

Now this grammar is in CNF

(b) If following grammar has left recursion, then eliminate it and rewrite the grammar [1]

$S \rightarrow Aa \mid b$
$A \rightarrow Sc \mid d$

Assign marks either 0 or 1

Soln 4 (b) Left Recursion

$S \rightarrow Aa \mid b$
$A \rightarrow Sc \mid d$

This grammar having indirect left recursion.

$\Rightarrow$ $S \rightarrow Aa \mid b$
$A \rightarrow (Aa \mid b)c \mid d$

Concept of Left Recursion
if $A \rightarrow A\alpha \mid \beta$
then
$A \rightarrow \beta A'$
$A' \rightarrow \alpha A' \mid \epsilon$

$\Rightarrow$ $S \rightarrow Aa \mid b$

$$A \rightarrow A\underbrace{ac}_{\alpha} \mid \underbrace{bc \mid d}_{\beta}$$

$A \rightarrow (bc \mid d)A'$
$A' \rightarrow acA' \mid \epsilon$

$A \rightarrow bcA' \mid dA'$
$A' \rightarrow acA' \mid \epsilon$

Original grammar can be written after removal of left recursion as follows

$S \rightarrow Aa \mid b$
$A \rightarrow bcA' \mid dA'$
$A' \rightarrow acA' \mid \epsilon$