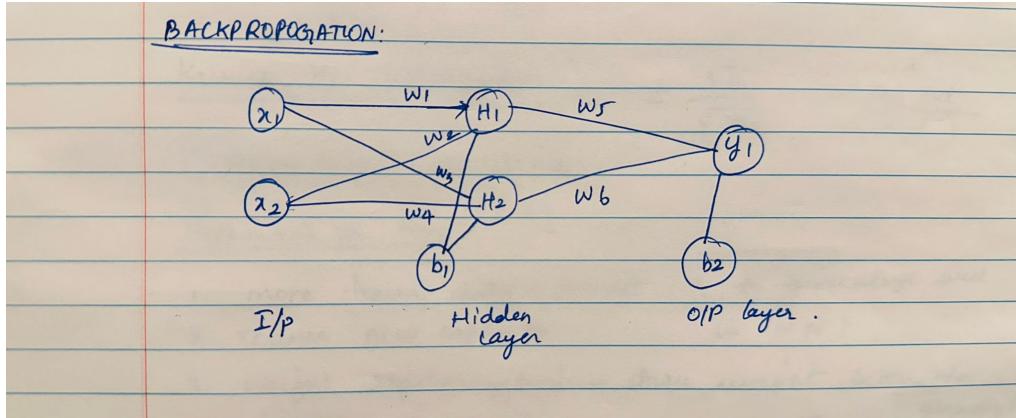


## BACKPROPAGATION:



$$\text{activation function} = \boxed{\text{Sigmoid} = \frac{1}{1+e^{-x}}}$$

FORWARD Example

$$x_1 = 0.35 \quad w_1 = 0.1$$

$$w_5 = 0.3$$

$$\text{Target} \\ y_1 = 0.9$$

$$x_2 = 0.9 \quad w_2 = 0.4$$

$$w_6 = 0.9$$

$$w_3 = 0.8$$

$$b_1 = 0.2 \quad w_4 = 0.6$$

$$b_2 = 0.4$$

Forward Pass :

$$\begin{aligned} H_1 &= x_1 w_1 + x_2 w_2 + b_1 \\ &= 0.35 \times 0.1 + 0.9 \times 0.4 + 0.2 \\ &= 0.035 + 0.36 + 0.2 = 0.595 \end{aligned}$$

$$\text{Out } H_1 = \frac{1}{1+e^{-H_1}} = \frac{1}{1+e^{-0.595}} = 0.64451155$$

$$\begin{aligned} H_2 &= x_1 w_3 + x_2 w_4 + b_2 \\ &= 0.35 \times 0.8 + 0.9 \times 0.6 + 0.2 \\ &= 0.28 + 0.54 + 0.2 = 0.94 \end{aligned}$$

$$\text{Out } H_2 = \frac{1}{1+e^{-0.94}} = 0.719099657$$

$$\begin{aligned} y_1 &= \text{out } H_1 \times w_5 + \text{out } H_2 \times w_6 + b_2 \\ &= 0.64451155 \times 0.3 + \cancel{0.719099657} \times 0.9 \\ &\quad + 0.4 \\ &= 0.19335346 + 0.64718968 + 0.4 \\ &= 1.24054314 \end{aligned}$$

$$\text{out } y_1 = \frac{1}{1+e^{-1.24054314}} = 0.7756585$$

Error calculation:

$$\begin{aligned} E_{\text{total}} &= \sum \frac{1}{2} (\text{target} - \text{output})^2 \\ &= \frac{1}{2} (y_t - \text{out } y_1)^2 = \frac{1}{2} (0.9 - 0.7756585)^2 \\ &= \frac{1}{2} (0.1243415)^2 = 0.0077304 \end{aligned}$$

$$E_{\text{total}} = E_1 = 0.0077304$$

Backpropagation:

Output Layer

$$\text{Error at } E_5 = \frac{\partial E_{\text{total}}}{\partial w_5}$$

$$\frac{\partial E_{\text{total}}}{\partial w_5} = \frac{\partial E_{\text{total}}}{\partial \text{out } y_1} \times \frac{\partial \text{out } y_1}{\partial y_1} \times \frac{\partial y_1}{\partial w_5}$$

$$\frac{\partial E_{\text{total}}}{\partial \text{out } y_1} = \cancel{x} \times \frac{1}{\cancel{x}} (y_t - \text{out } y_1) = 0.1243415$$

$$\frac{\partial \text{out } y_1}{\partial y_1} = \text{out } y_1 (1 - \text{out } y_1)$$

$$\frac{\partial y_1}{\partial w_5} = 0.715658 (1 - 0.715658)$$

$$= 0.17401267$$

$$\frac{\partial y_1}{\partial w_5} = 1 \times \text{out } H_1 \times w_5^{(1-1)}$$

$$= \text{out } H_1 = 0.64451155$$

$$\frac{\partial E_{\text{total}}}{\partial w_5} = 0.1243415 \times 0.17401267 \times 0.64451155$$

$$= 0.01394529$$

Updating weight  $w_5$ :  $w_5 = w_5 - \eta \frac{\partial E_{\text{total}}}{\partial w_5}$

$$\boxed{\eta = 0.5} \quad w_5 = 0.3 - 0.5 \times 0.01394529$$

$$\boxed{w_5 = 0.29302736}$$

Error at  $w_6$  =  $\frac{\partial E_{\text{total}}}{\partial \text{out } y_1} \times \frac{\partial \text{out } y_1}{\partial y_1} \times \frac{\partial y_1}{\partial w_6}$

$$\frac{\partial y_1}{\partial w_6} = 1 \times \text{out } H_2 \times w_6^{(1-1)}$$

$$= \text{out } H_2 = 0.719099657$$

$$\frac{\partial E_{\text{total}}}{\partial w_6} = 0.1243415 \times 0.17401267 \times 0.719099657$$

$$= 0.01555916$$

Updating weight  $w_6$  =  $0.9 - 0.5 \times 0.01555916$

$$\boxed{w_6 = 0.89222042}$$

$$\text{DFF } H_2 = \frac{1+6-0.64}{1} = 0.1104492$$

Output layer	$w_5 = 0.29302736$
updated weight	$w_6 = 0.89222042$

Hidden Layer Backpropagation:

$$\frac{\partial E_{\text{total}}}{\partial w_1} = \left( \frac{\partial E_1}{\partial \text{out } y_1} \times \frac{\partial \text{out } y_1}{\partial y_1} \times \frac{\partial y_1}{\partial \text{out } H_1} \right) \times \frac{\partial \text{out } H_1}{\partial H_1} \times \frac{\partial H_1}{\partial w_1}$$

$w_1$ :

$$\begin{aligned} \frac{\partial E_{\text{total}}}{\partial w_1} &= \left[ \left( \frac{\partial E_1}{\partial \text{out } y_1} \times \frac{\partial \text{out } y_1}{\partial y_1} \times \frac{\partial y_1}{\partial \text{out } H_1} \right) + \frac{\partial E_2}{\partial \text{out } H_1} \right] \\ &\quad \times \frac{\partial \text{out } H_1}{\partial H_1} \times \frac{\partial H_1}{\partial w_1} \\ &= (0.1243415 \times 0.17401267 \times 0.3 + 0) \times \\ &\quad (\text{out } H_1 \cdot (1 - \text{out } H_1)) \times x_1 \\ &= 6.0064911 \times [0.64451155 (1 - 0.64451155)] \times 0.35 \\ &= 0.00052053 \end{aligned}$$

$$\begin{aligned} \text{Updating } w_1 &= w_1 - \eta \times \frac{\partial E_{\text{total}}}{\partial w_1} = 0.9 - (0.5 \times 0.00052053) \\ &= 0.9973974 \end{aligned}$$

$w_2$ :

$$\begin{aligned}\frac{\partial E_{\text{total}}}{\partial w_2} &= \frac{\partial E_i}{\partial \text{out}y_1} \times \frac{\partial \text{out}y_1}{\partial y_1} \times \frac{\partial y_1}{\partial \text{out}H_1} \times \frac{\partial \text{out}H_1}{\partial H_1} \times \frac{\partial H_1}{\partial w_2} \\ &= 0.00148722 \times 0.9 \\ &= 0.0013385\end{aligned}$$

updating  $w_2 = 0.4 - (0.5 \times 0.0013385)$

$$= 0.4 - 0.00066925$$

$$w_2 = 0.39933075.$$

$w_3$ :

$$\begin{aligned}\frac{\partial E_{\text{total}}}{\partial w_3} &= \frac{\partial E_i}{\partial \text{out}y_1} \times \frac{\partial \text{out}y_1}{\partial y_1} \times \frac{\partial y_1}{\partial \text{out}H_2} \times \frac{\partial \text{out}H_2}{\partial H_2} \times \frac{\partial H_2}{\partial w_3} \\ &= 0.1243415 \times 0.17401267 \times 0.9 \times [0.719099657(1 - 0.719099657)] \times 0.8 \\ &= 0.00393352 \times 0.8 \\ &= 0.00314681\end{aligned}$$

updating  $w_3 = 0.8 - (0.5 \times 0.00314681)$

$$= 0.79842659$$

$w_4$ :

$$\begin{aligned}\frac{\partial E_{\text{total}}}{\partial w_4} &= \frac{\partial E_i}{\partial \text{out}y_1} \times \frac{\partial \text{out}y_1}{\partial y_1} \times \frac{\partial y_1}{\partial \text{out}H_2} \times \frac{\partial \text{out}H_2}{\partial H_2} \times \frac{\partial H_2}{\partial w_4} \\ &= 0.00393352 \times 0.6 \\ &= 0.00236011\end{aligned}$$

updating  $w_4 = 0.6 - (0.5 \times 0.00236011)$

$$= 0.59881994.$$

updated weights in hidden layer  
after backpropagation:

$$w_1 = 0.09973974$$

$$w_2 = 0.39933075$$

$$w_3 = 0.79842659$$

$$w_4 = 0.59881994$$