Skoltech
Skolkovo Institute of Science and Technology

# Dynamic Programming on Tensors for Solving the Problem of Dependency Parsing in NLP

## Optimization&NLA course project

Sergey Divakov, Anastasia Koloskova, Alfredo De la Fuente and Vladislav Pimanov

Skolkovo Institute of Science and Technology, Data Science Department
Moscow, Russia

22nd December 2017

Skoltech
Skolkovo Institute of Science and Technology

# Outline

Skoltech
Skolkovo Institute of Science and Technology

# Introduction

- Syntactic dependency parsing is the important problem in statistical natural language processing.
- Our goal was to speed up the parsing process.
- Using tensor formulation of the inside-outside algorithm we compared different tensor decompositions.
- Tucker and Tensor Train decompositions were applied.
- We were able to achieve significant performance increase with good accuracy results.

# Setup

## Probabilistic Context-free Grammars

$$G = (\mathcal{N}, \mathcal{L}, \mathcal{R}, \mathcal{P}, \pi),$$

- $\mathcal{N}$ — nonterminal symbols.
- $\mathcal{L}$ — words (lexical tokens).
- $\mathcal{R}$ — set of rules: $a \to bc$ or $a \to x$, $a, b, c \in \mathcal{N}$, $x \in \mathcal{L}$.
- $\mathcal{P}$ — transition probabilities $p(a \to bc|a)$ and $p(a \to x|a)$.
- $\pi_a$ — probability of $a$ being the root symbol.
- All probabilities satisfy normalization conditions.

Skoltech
Skolkovo Institute of Science and Technology

# Setup

Our purpose: given a sentence find the most probable tree of this sentence.

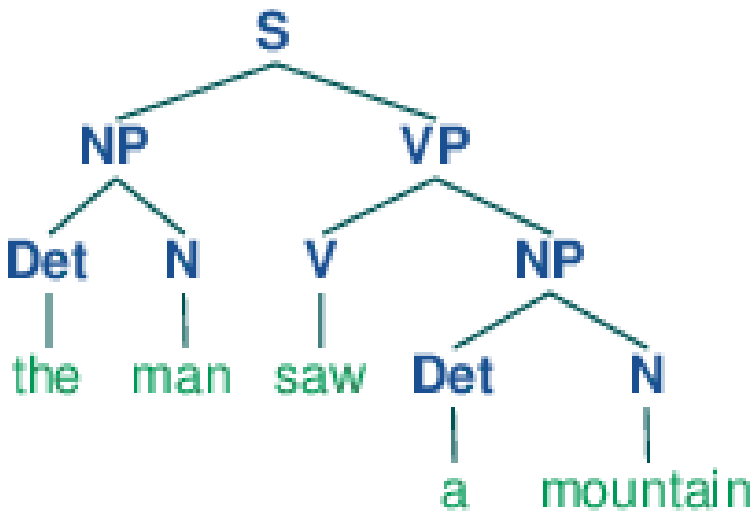
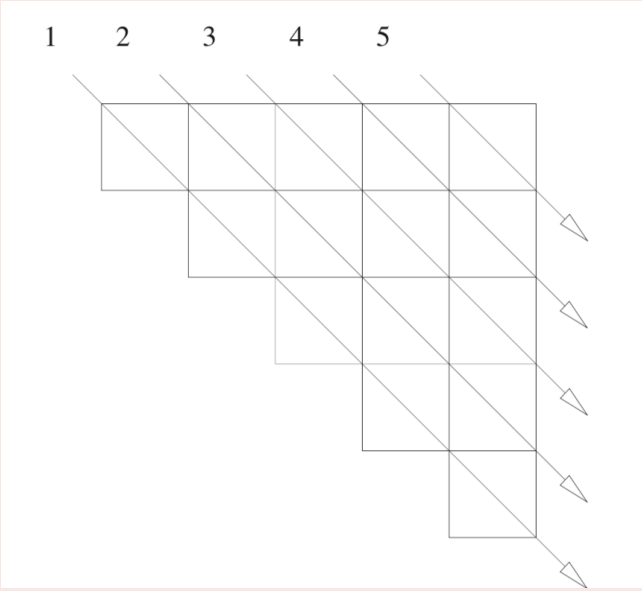



Figure: Parse tree representation of *the man saw a mountain*

Skoltech
Skolkovo Institute of Science and Technology

# Algorithms

## Inside-Outside



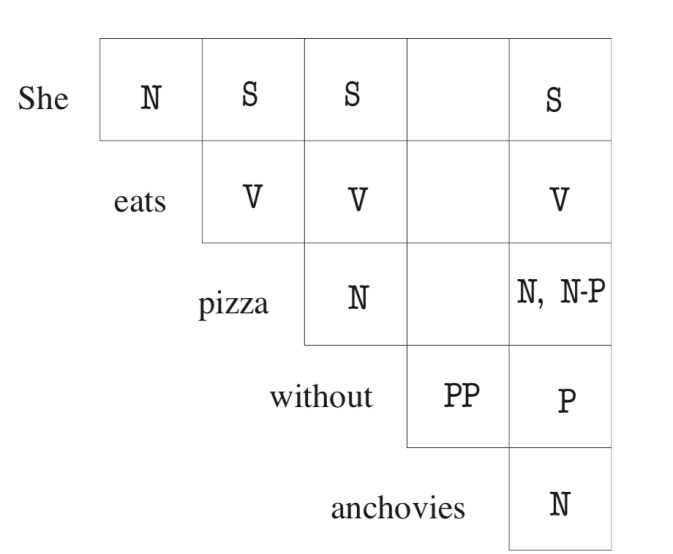
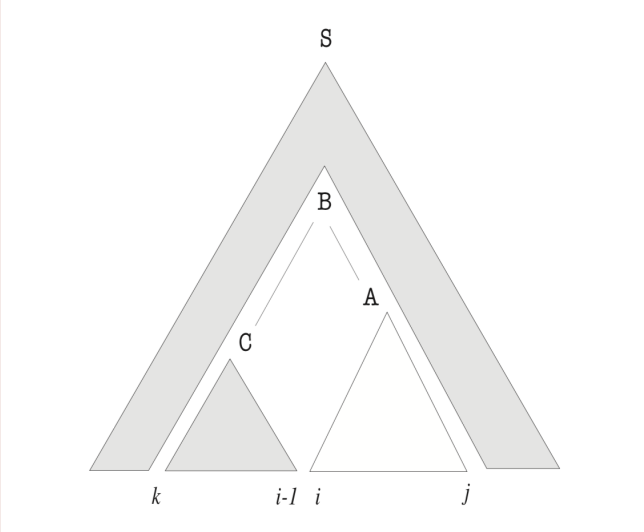




Figure: Inside Recursion

Skoltech
Skolkovo Institute of Science and Technology

# Algorithms

## Inside-Outside



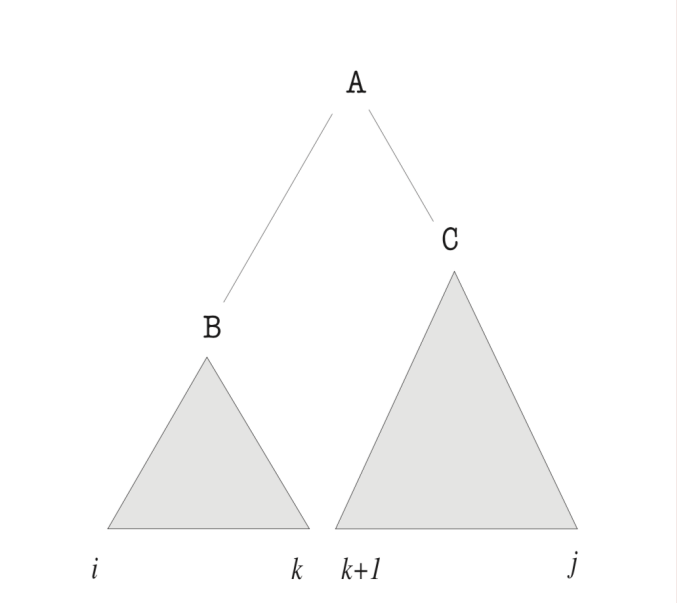




Figure: Inside Recursion

# Algorithms

**Algorithm 1** Inside-Outside Algorithm in the Tensor Form

1: **(Inside base case):** $\forall (a \to x_i) \in \mathcal{R}\ [\alpha^{i,i}]_a = p(a \to x|a)$

2: **(Inside recursion):**

$$[\alpha^{i,j}]_a = \sum_{k=i}^{j-1} \sum_{a \to bc} T^{a \to bc}([\alpha^{i,k}]_b, [\alpha^{k+1,j}]_c)$$

3: **(Outside base case):** $\forall a \in \mathcal{N}\ [\beta^{1,N}]_a = \pi_a$

4: **(Outside recursion):**

$$[\beta^{i,j}]_a = \sum_{k=1}^{i-1} \sum_{b \to ca} T^{b \to ca}_{(1,2)}([\beta^{k,j}]_b, [\alpha^{k,i-1}]_c) + \\ + \sum_{k=j+1}^{N} \sum_{b \to ac} T^{b \to ac}_{(1,3)}([\beta^{ik}]_b, [\alpha^{j+1,k}]_c)$$

5: **(Marginals):** $\mu(a, i, j) = [\alpha^{ij}]_a \cdot [\beta^{ij}]_a$

# Tensor Decompositions (3D case)

## Canonical

$$\mathcal{A} = \sum_{i=1}^{r} \lambda_i \mathbf{a}_i^1 \otimes \mathbf{a}_i^2 \otimes \mathbf{a}_i^3,$$

$\mathcal{A} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, $\lambda_i \in \mathbb{R}$ and $\mathbf{a}_i^j \in \mathbb{R}^{n_j}$

**Time of multiplying by vector (of size $n_3$):** $\mathcal{O}(rn_3)$

## Tensor Train

$$\mathcal{A}(i_1, i_2, i_3) = \sum_{k_1=1}^{r_1} \sum_{k_2=1}^{r_2} G_1(i_1, k_1) G_2(k_1, i_2, k_2) G_3(k_2, i_3),$$

$\mathcal{A} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, $G_1 \in \mathbb{R}^{n_1 \times r_1}$, $G_2 \in \mathbb{R}^{r_1 \times n_2 \times r_2}$, $G_3 \in R^{r_2 \times n_3}$.

**Time of multiplying by vector (of size $n_3$):** $\mathcal{O}(r_1 r_2 n_3)$

# Tensor Decompositions (3D case)

## Tucker

$$\text{vec}(\mathcal{A}) = (W \otimes V \otimes U) \cdot \text{vec}(\mathcal{C}),$$

$\mathcal{A} \in \mathbb{R}^{n_1 \times n_2 \times n_3}, U \in \mathbb{R}^{n_1 \times r_1}$, $V \in \mathbb{R}^{n_2 \times r_2}$, $W \in \mathbb{R}^{n_3 \times r_3}$ and $\mathcal{C} \in \mathbb{R}^{r_1 \times r_2 \times r_3}$

**Time of multiplying by vector (of size $n_3$):**

$\mathcal{O}\left(n_3 r_3 + r_1 r_2 r_3 + r_1 r_2 n_1 + r_2 n_2\right) = \mathcal{O}(nr^2 + r^3)$.

Skoltech
Skolkovo Institute of Science and Technology

# Experiments

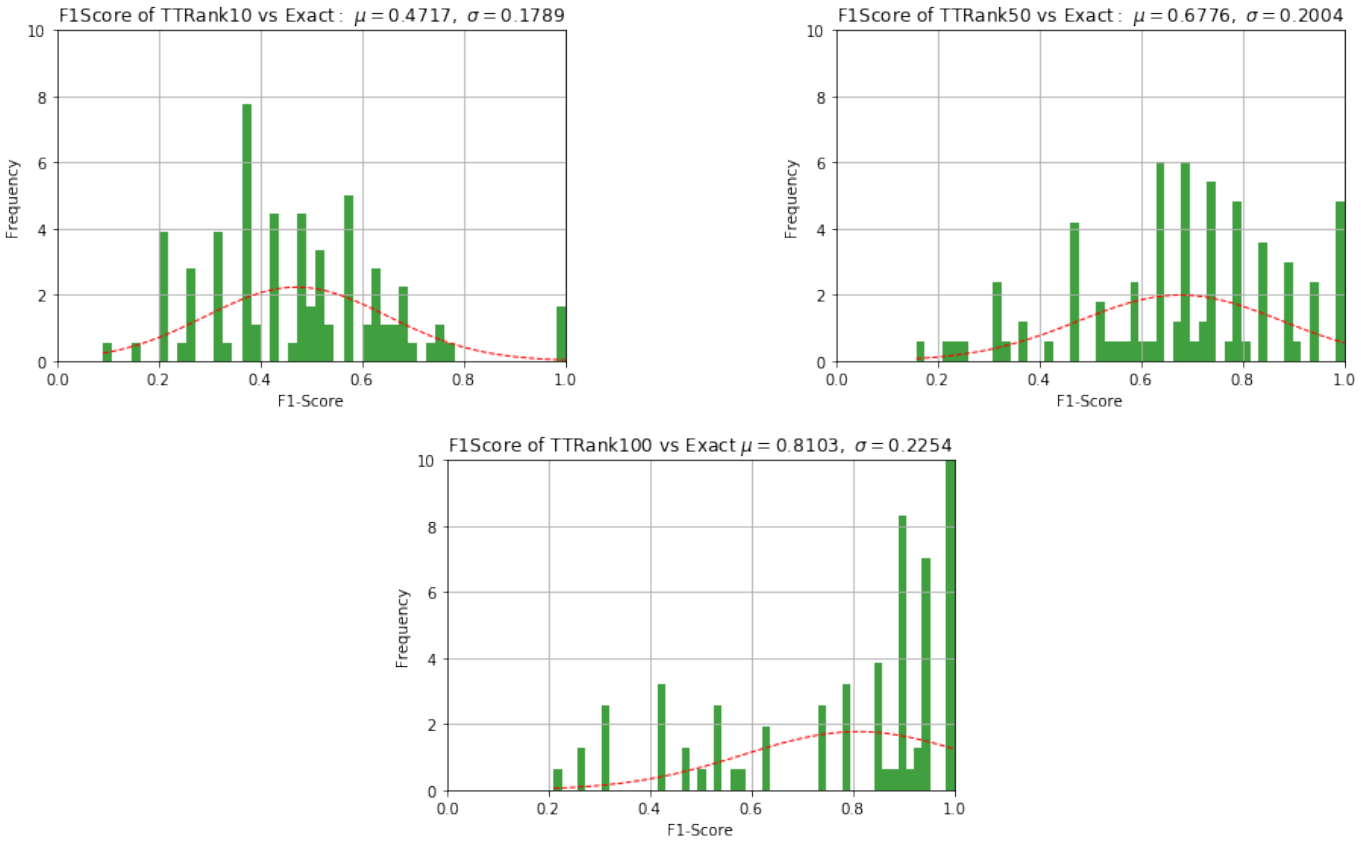



Figure: F1 scores TT

Skoltech
Skolkovo Institute of Science and Technology

# Experiments

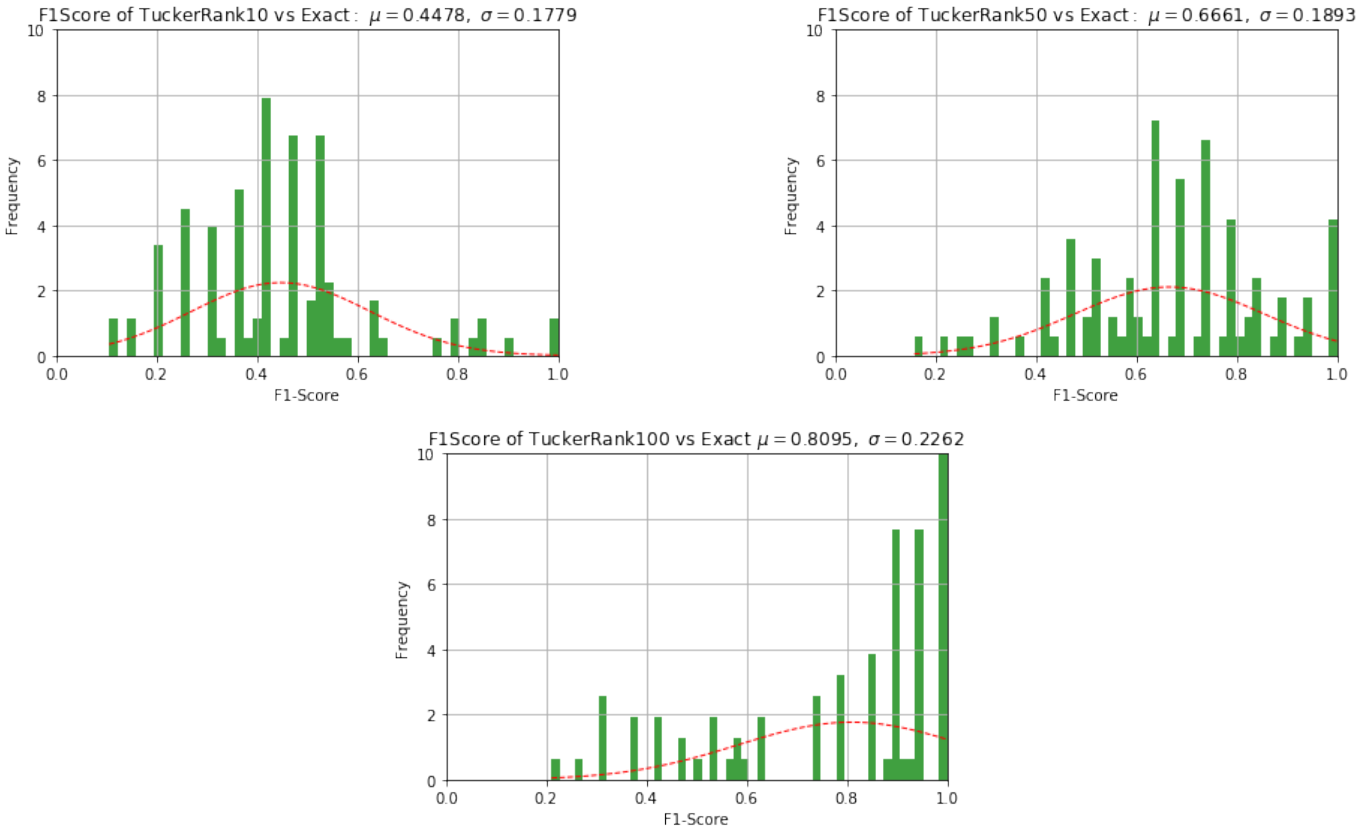



Figure: F1 scores Tucker

Skoltech
Skolkovo Institute of Science and Technology

# Experiments

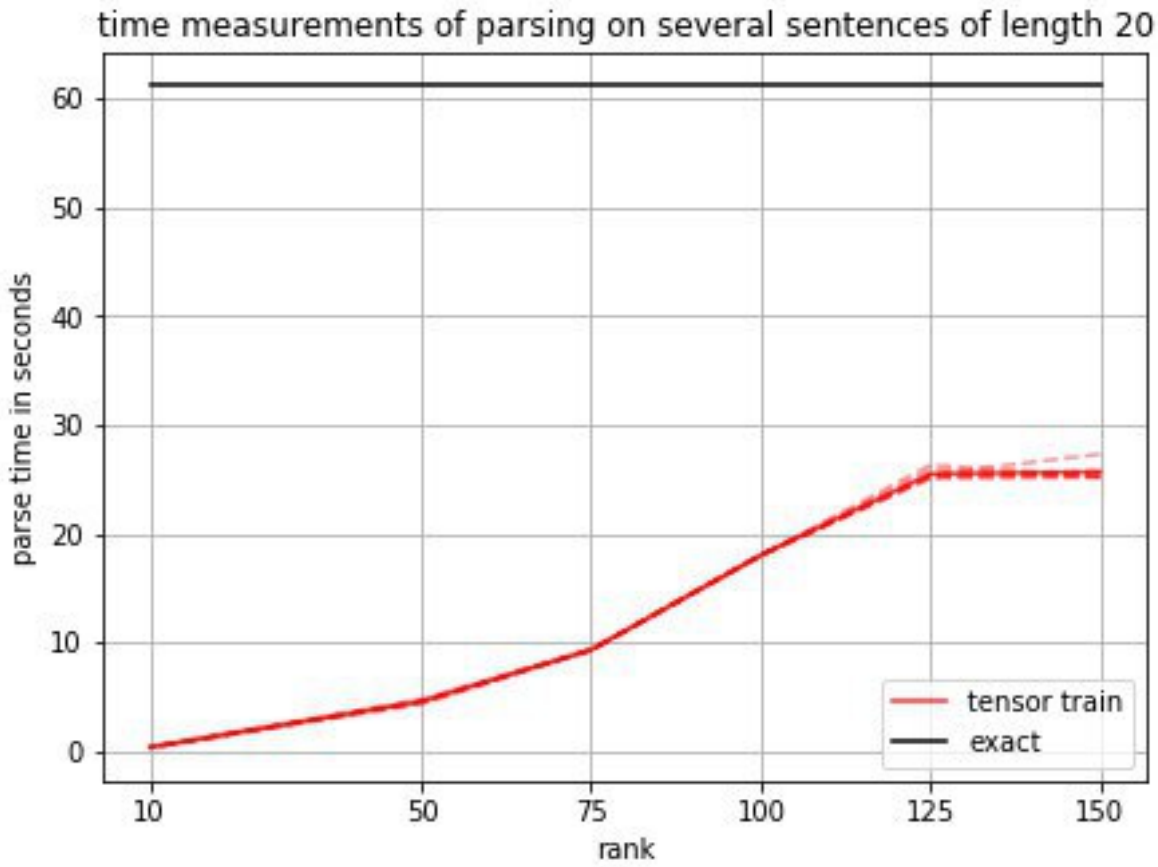



Figure: Time efficiency

Skoltech
Skolkovo Institute of Science and Technology

# Experiments

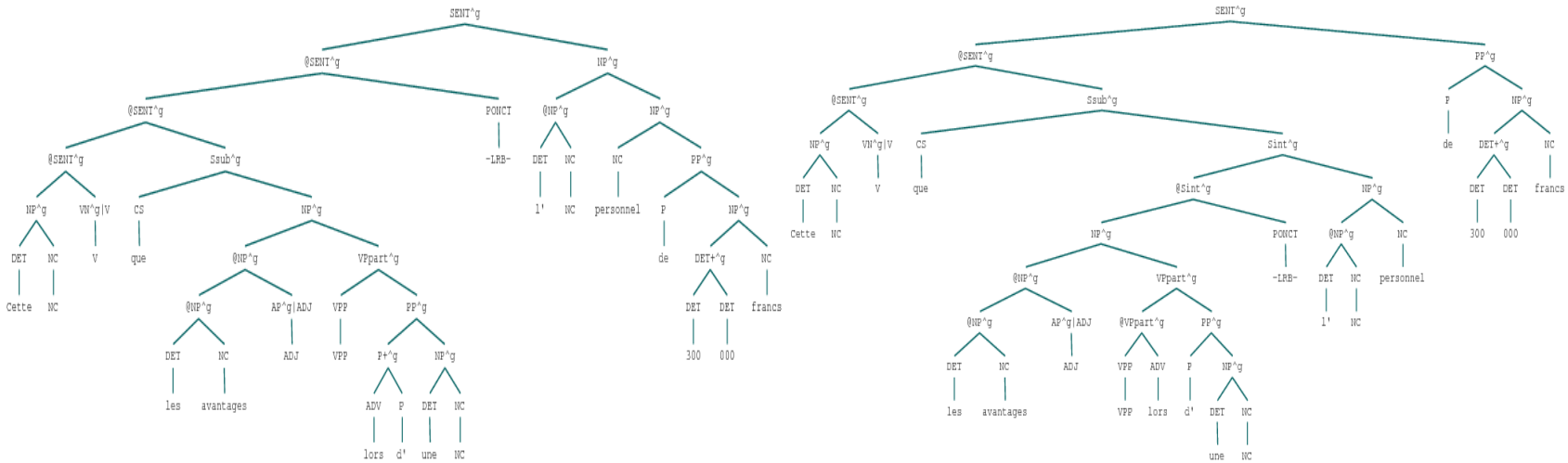



Figure: Tree comparison

Skoltech
Skolkovo Institute of Science and Technology

# Conclusion

- Using tensor formulation of the inside-outside algorithm we tried different tensor decompositions to speed up parsing process.
- Tucker and Tensor Train decompositions were applied.
- Experiments showed that using tensor decompositions we can achieve significant speed up with the good accuracy of parsing.