Entanglement using coupled superconducting qubit-qutrit system

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The main objective of our project is to demonstrate the preparation of Bell States with the help of superconducting qutrit in order to reduce decoherence.

One of the most powerful features that qubits exhibit is entanglement; using two entangled qubits one can build circuits and algorithms that find extensive applications in the field of quantum information processing, security analysis of Quantum Key Distribution protocols, remote state preparation, teleportation and so on. One of the most used quantum states of two qubits entangled is the Bell State because it's the simplest state with maximum entanglement. In fact, we have four different Bell States:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \tag{1}$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B) \tag{2}$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B) \tag{3}$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B) \tag{4}$$

A big issue in quantum computing is decoherence, if qubits interact with the environment it will disturb their state and consequently the final state we measure won't be as expected. In order to reduce decoherence as much as possible, we should find the way to run our algorithms with less number of manipulations and in the least amount of time possible.

A way of achieving this is using an additional qutrit besides the two qubits we want to entangle. Since we have higher energy states ($|2\rangle$), instead of using the typical quantum gates we will have to program our circuit with microwave pulses (using Qiskit Pulse).

- **Effective Hamiltonian:** Here, we consider a circuit with four connected superconducting islands (Ref 1) and the schematic of the process is shown in Fig (1). The qutrit can encode a qubit in the states ($|0\rangle, |1\rangle$) or states ($|0\rangle, |2\rangle$). Toggle between these two encodings is done by applying a π-pulse on the $|1\rangle \leftrightarrow |2\rangle$ transition. The effective hamiltonian is given as:

$$H = \frac{1}{2}\Delta_L\sigma_L^z + \Delta_M|1\rangle\langle 1| + (\Delta_M + \delta_M)|2\rangle\langle 2| + \frac{1}{2}\Delta_R\sigma_R^z$$
$$+ J_{LM_{01}}(\sigma_L^-|1\rangle\langle 0| + \sigma_L^+|0\rangle\langle 1|)$$
$$+ J_{RM_{01}}(\sigma_R^-|1\rangle\langle 0| + \sigma_R^+|0\rangle\langle 1|)$$

$$
\begin{aligned}
&+J_{LM_{12}}(\sigma_L^-|2\rangle\langle 1|+\sigma_L^+|1\rangle\langle 2|)\\
&+J_{RM_{12}}(\sigma_R^-|2\rangle\langle 1|+\sigma_R^+|1\rangle\langle 2|)\\
&+J_{LM}^{(z)}\sigma_L^z(D_1|1\rangle\langle 1|+D_2|2\rangle\langle 2|)\\
&+J_{RM}^{(z)}\sigma_R^z(D_1|1\rangle\langle 1|+D_2|2\rangle\langle 2|)
\end{aligned}
\tag{5}
$$

where σ_α^+ and σ_α^+ are the spin-$\frac{1}{2}$ raising and lowering operators for the left ($\alpha = L$) and right ($\alpha = R$) qubits, σ_α^z is the Pauli Z operator, and $\Delta_{L,R}$ is the energy differences between the spin-up and spin-down states of the corresponding qubit. The states of the qutrit are denoted by $|j\rangle$ ($j = 0,1,2$), Δ_M is the energy of state $|1\rangle$ and $\Delta_M + \delta_M$ is the energy of state $|2\rangle$, making the anharmonicity equal to $\Delta_M - \delta_M$, with the energy of the ground state $|0\rangle$ set to zero.
Also, an external microwave field with frequency ω used to drive the system induces transitions between the states of the qubit and the qutrit according to the Hamiltonian

$$
H_{mw} = cos(\omega t)(\Omega_L\sigma_L^+ + \Omega_R\sigma_R^+ + \Omega_1|0\rangle\langle 1| + \Omega_2|1\rangle\langle 2| + H.c) \tag{6}
$$

where Ω's are corrsponding Rabi frequencies.

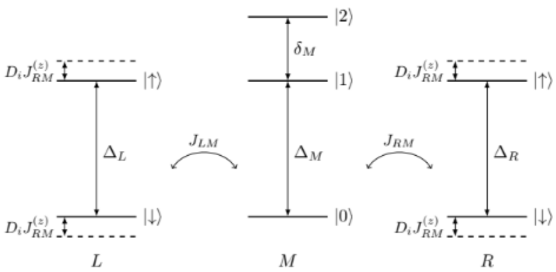

Figure 1: *Energy diagram of the system of two qubits (left, L, and right, R) and a qutrit (middle, M) described by the Hamiltonian in Equation*

- Entangled state preparation:We will start in the state $|000\rangle$ and first, we have to get the qutrit into the state $\frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$ by applying two pulses that bring the qutrit partially from $|0\rangle$ to $|1\rangle$ and then from $|1\rangle$ to $|2\rangle$, ending without the state $|1\rangle$ in the final superposition. These pulses aren't resonant with the $|0\rangle$ to $|1\rangle$ transition of the qubits, so they will stay in the state $|0\rangle$.

  Now we have the state $\frac{1}{\sqrt{2}}(|00\rangle(|0\rangle + |2\rangle)$ that comes to the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$ when the qutrit de-excitation of state $|2\rangle$ to $|0\rangle$ converts into two qubit excitations $|11\rangle$.
  The operators for the transformations are given in the jpg files.

Reference:

- *"Realization of efficient quantum gates with a superconducting qubit-qutrit circuit"* T. Bakkegaard, L.B. Kristensen, N.J.S. Loft, C.K. Andersen, D. Petrosyan, and N.T. Zinner