

CPEN 400Q Lecture 18

Noise and error channels

Monday 17 March 2025

Announcements

- Quiz 8 today
- MT checkpoints this week
- Sign up for final oral interview (Canvas calendar)
- A3 due Tuesday 25 March 23:59
- New slide format - feedback welcome

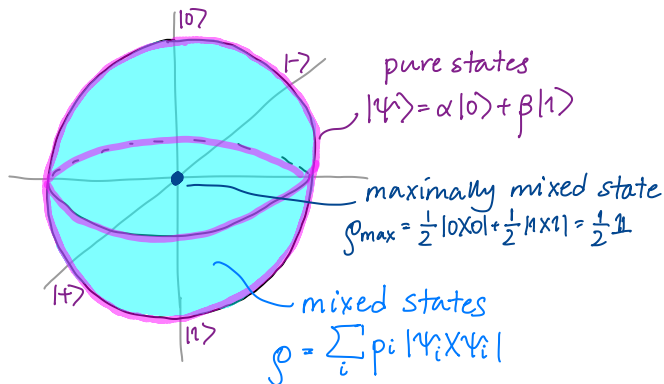
Last time

We expressed density matrices (for pure or mixed states) as

Since ρ is positive semidefinite,

Last time

The set $(\langle X \rangle, \langle Y \rangle, \langle Z \rangle)$ form the Bloch vector, $\langle P \rangle = \text{Tr}(P\rho)$.



Last time

Unitary operations preserve length of Bloch vector

To take pure states to mixed states, apply a *quantum channel*:

Quantum channels are CPTP linear maps.

- **T**race-**P**reserving:
- **P**ositive:
- **C**ompletely **P**ositive:

Last time

Quantum channels are characterized by **Kraus operators** $\{K_i\}$,

Example: unitary channel, \mathcal{U} , $\{K_i\} = \{U\}$.

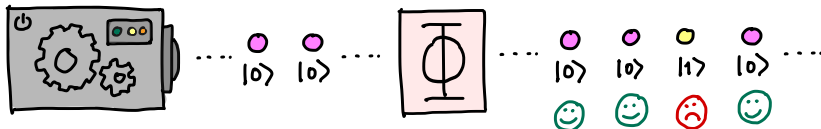
Example: a projective measurement \mathcal{M} , $\{K_i\} = \{\Pi_i\}$

Learning outcomes

- 1 Express noise in quantum systems using quantum channels
- 2 Compare density matrices with fidelity and trace distance
- 3 Apply noise to quantum circuits in PennyLane

The bit flip channel

Suppose a “bit flip” (Pauli X) error occurs with probability p .



How do we write this as a channel? What are the Kraus operators?

The bit flip channel

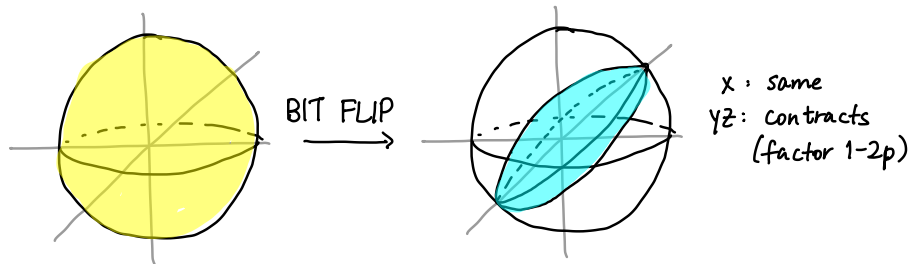
We can visualize the effects of the channel by observing how it deforms the Bloch sphere.

Consider:

- Which states are not affected by a bit flip channel?
- Which states are *most* affected, and where do they go?

Then look at the density matrices of these states.

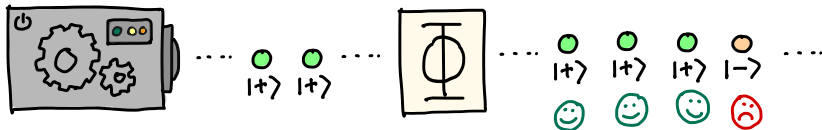
The bit flip channel



x : same
 y, z : contracts
(factor $1-2p$)

The phase flip channel

Suppose a “phase flip” (Pauli Z) error occurs with probability p .



How do we write this as a channel? What are the Kraus operators?

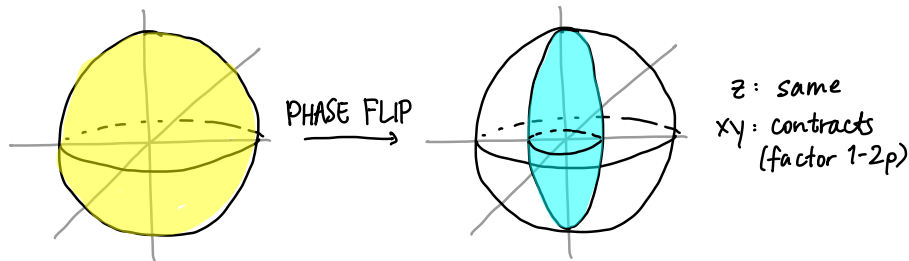
The phase flip channel

Same questions:

- Which states are not affected by a phase flip channel?
- Which states are *most* affected, and where do they go?

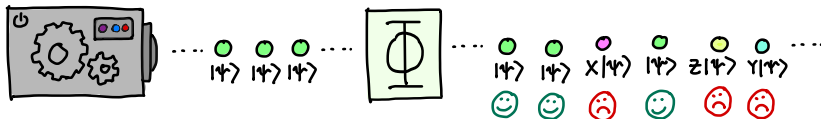
Then look at the density matrices of these states.

The phase flip channel



The depolarizing channel

Suppose each Pauli error occurs with probability $p/3$.



How do we write this as a channel? What are the Kraus operators?

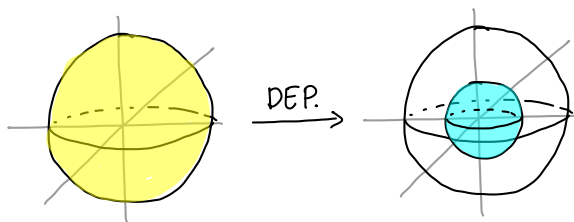
Q: What will happen to the Bloch sphere?

The depolarizing channel

The depolarizing channel

$$\Phi(\rho) = (1 - p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$$

can also be written as



Amplitude damping channel

Suppose $|1\rangle$ relaxes to $|0\rangle$ with probability p .

How do you think this affects the Bloch sphere?

Comparing density matrices

The inner product tells us how close two pure states are:

How close are two mixed states ρ and σ ?

One common metric is the **trace distance**:

Bounded by $0 \leq T(\rho, \sigma) \leq 1$; *lower* trace distance is better.

Comparing density matrices

Can also compare using **fidelity**,

Bounded by $0 \leq F(\rho, \sigma) \leq 1$. *Higher* fidelity is better.

Example: Determine $F(\rho, \sigma)$ if both ρ, σ are pure.

Comparing density matrices

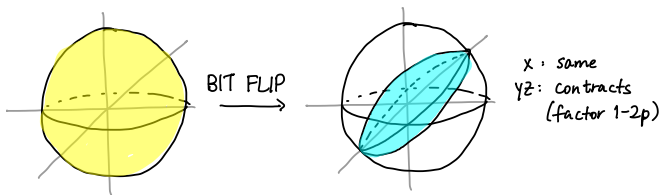
Exercise: Determine $F(\rho, \sigma)$ if ρ is pure but σ is mixed.

Comparing density matrices

Exercise: What is the fidelity of any pure ρ with the maximally mixed state, $\sigma = \frac{1}{2}I$?

Comparing density matrices

Exercise: Determine $F(\rho, \sigma)$ if ρ is pure and $\sigma = \Phi(\rho)$, where Φ is the *bit flip channel* with parameter p .



Simulating noisy systems

Exercise: Suppose we prepare a system in the state

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

If a depolarizing channel with $p = 0.02$ is applied, what's the probability of observing $|0\rangle$?

Solution 1: solve by hand. Tedious, but can evaluate

Simulating noisy systems

Solution 2: use PennyLane's ‘‘default.mixed’’ device!

Resources:

- <https://docs.pennylane.ai/en/stable/introduction/operations.html#noisy-channels>
- https://docs.pennylane.ai/en/stable/code/qml_noise.html

Next time

Last class:

- Intro to quantum error correction

Action items:

- ① MT checkpoint meetings
- ② A3 (due 25 March 23:59)
- ③ Work on project

Recommended reading:

- From this class: Codebook NT, DM; N&C 2.2.6, 2.4
- For next class: Codebook EC; N&C 8.2-8.3,