The University of British Columbia: CPEN 400Q Midterm

Wednesday 31 January 2024 15:00-16:20
Instructor: Olivia Di Matteo

Name:

Student ID:

Duration: 80 minutes
Resources: formula sheet only (provided on the last page)
Pages: 13 (including this cover page and formula sheet)
Total possible score: 23 points

Student conduct during examinations:

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, their UBCcard for identification.

2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.

4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

    (a) speaking or communicating with other examination candidates, unless otherwise authorized

    (b) purposely exposing written papers to the view of other examination candidates or imaging devices;

    (c) purposely viewing the written papers of other examination candidates;

    (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

    (e) using or operating electronic devices including but not limited to telephone, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

This midterm is divided into two sections. Every question is tied to one or more course learning outcomes. The problems in Part 1 are procedural and/or technical questions. Part 2 tests your ability to generalize your knowledge to new situations.

Question	**Value**	**Score**
1.1	3	
1.2	5	
1.3	2	
1.4	2	
1.5	2	
1.6	2	
2.1	3	
2.2	4	
Total	23	

1 Part 1 (16 points)

1.1 (3 points)

(a) Indicate whether the following quantum states are valid (0.5 points each).

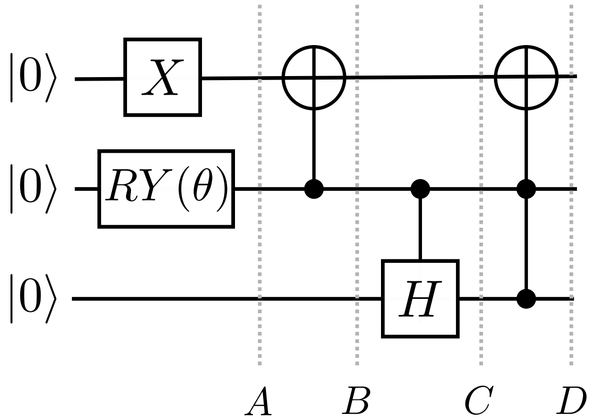
	Valid state?
$\|\psi\rangle = \frac{1}{\sqrt{3}}(\|100\rangle + \|010\rangle + \|001\rangle)$	
$\|\psi\rangle = \frac{1}{\sqrt{2}}(\|00\rangle + e^{-i\frac{\pi}{6}}\|11\rangle)$	

(b) Indicate whether the following matrices correspond to valid single-qubit operations, or measurable observables (0.5 point per case).

	Valid operation?	Valid observable?
$\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$		
$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{3}} \end{pmatrix}$		

1.2 (5 points)

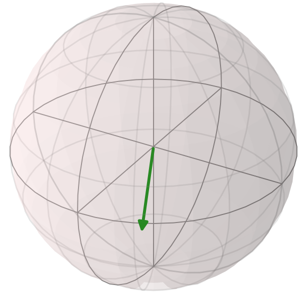
Consider the following quantum circuit. Determine the quantum state at the points marked A-D (1 point each). Then, compute the measurement outcome probability of all 3 qubits being in state $|111\rangle$ at the end of the circuit as a function of θ (1 point). Write your answers in the provided table.



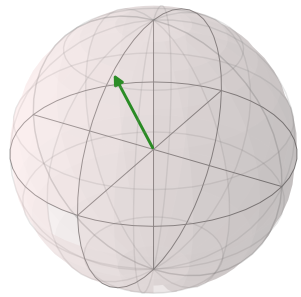
State at A	
State at B	
State at C	
State at D	
Prob. of meas. $\|111\rangle$	

1.3 (2 points)

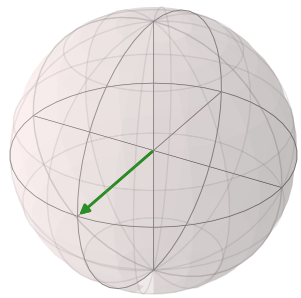
Draw lines to match the quantum states below with their Bloch sphere representations (0.5 points each).



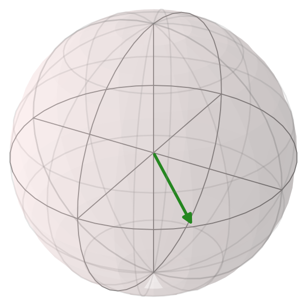
$|+\rangle$



$\frac{1}{\sqrt{2}}(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle)$



$\frac{\sqrt{2}}{8}|0\rangle - e^{i\frac{\pi}{4}}\frac{\sqrt{62}}{8}|1\rangle$



$\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$

1.4 (2 points)

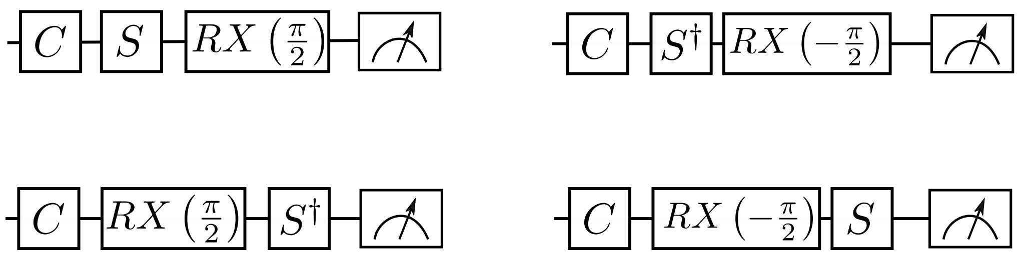
Suppose Alice wishes to apply a circuit C and measure her qubit in a different basis. However, her quantum computer can only perform computational basis measurements.

(a) If the first vector of her basis is

$$|\psi_1\rangle = \frac{1+i}{2}|0\rangle + \frac{i-1}{2}|1\rangle,$$

what is the second vector? (1 point)

(b) Which of the following four gadgets should she append to her circuit in order to perform the appropriate measurement (circle one)? (1 point)



1.5 (2 points)

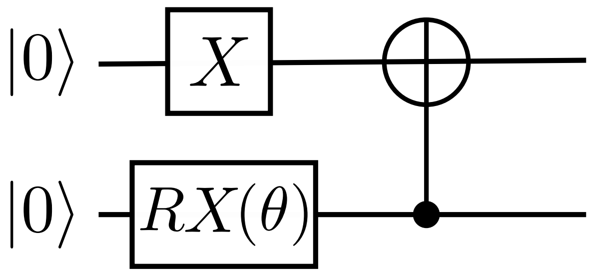
While the CNOT is our go-to two-qubit gate for most algorithms, it is not always the most natural for physical quantum computers to implement. For instance, trapped-ion platforms generally implement the Mølmer-Sørensen (MS) gate,

$$MS = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix}$$

instead of CNOT. Show, mathematically, that the MS gate is an entangling gate.

1.6 (2 points)

(a) Analytically compute the expectation value of the observable $X \otimes X$ for the output state of the circuit below. (1 point)



(b) Suppose you wish to use this circuit to implement a variational quantum classifier. Is the observable $X \otimes X$ well-suited for the task? If yes, explain why. If no, suggest an alternative. (1 point)

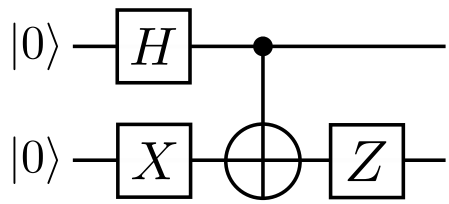
2 Part 2 (7 points)

2.1 Double trouble (3 points)

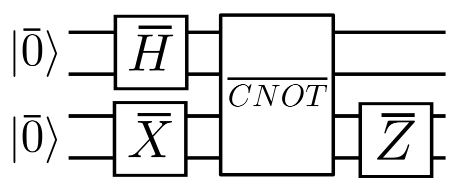
While we typically consider a qubit as a single physical quantum system, there are some settings where it is more convenient to have multiple physical qubits work together to form a *logical qubit.* For instance, we can use two qubits, and define two logical basis states as

$$|\bar{0}\rangle = |10\rangle, \quad |\bar{1}\rangle = |01\rangle.$$

This is called the *dual rail encoding.* As a physical example, this could correspond to a photonic system with two lines where a photon present in the first line but not the second line represents a $|\bar{0}\rangle$, and vice versa. Suppose you have such a system and wish to run the following quantum algorithm:



In the dual rail encoding, each qubit is now two qubits, so the gates don't work as normal! Using standard 1- and 2-qubit gates, develop an implementation for each of the four logical operations, $\bar{X}$, $\bar{Z}$ (0.5 point each), $\bar{H}$ and $\overline{CNOT}$ (1 point each) such that the circuit below has the equivalent effect on $|\bar{0}\rangle$ and $|\bar{1}\rangle$.



2.2 Managing expectations (4 points)

An important application of expectation values is in stabilizer codes for error correction. Given a state $|\psi\rangle$, we say that a set of Pauli observables $\mathcal{S} = \{P_i\}$ is a stabilizer of $|\psi\rangle$ if

$$P_i |\psi\rangle = |\psi\rangle \quad \forall P_i \in \mathcal{S},$$

i.e., $|\psi\rangle$ is an eigenstate with eigenvalue $+1$ for all the observables in $\mathcal{S}$. Error correction codes can be designed around this principle by using stabilized states as *codewords*, or "valid" states. Operations are specially designed to map codewords to other codewords, and after every operation, the expectation values of all elements of $\mathcal{S}$ are measured. If we ever see an expectation value that is not $+1$, we can detect that an error has occurred.

Suppose we have an error correcting code that defines one logical qubit using three physical qubits under the following mapping:

$$|\bar{0}\rangle = |000\rangle, \quad |\bar{1}\rangle = |111\rangle.$$

(a) Find a set of Pauli observables that are stabilizers for both $|\bar{0}\rangle$ and $|\bar{1}\rangle$. *Hint: there are 3, but you may choose any 2.* (1 point)

(b) Suppose that only one type of error, a bit flip, can occur on any of the three physical qubits (e.g., a bit flip on the first qubit leads to $|100\rangle$ and $|011\rangle$, neither of which are valid codewords). For each of the 3 possible error cases, what are the expectation values of the stabilizers of part (a)? (1.5 points)

(c) For each set of observed expectation values in part (b), suggest a correction that can be applied to fix the error (1.5 points).

Quantum gate reference

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \qquad S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix} \qquad T = \begin{pmatrix}1 & 0\\ 0 & e^{i\frac{\pi}{4}}\end{pmatrix}$$

$$X = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad Y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad \sqrt{X} = \frac{1}{2}\begin{pmatrix}1+i & 1-i\\ 1-i & 1+i\end{pmatrix}$$

$$RX(\theta) = \begin{pmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix} \qquad RY(\theta) = \begin{pmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix} \qquad RZ(\theta) = \begin{pmatrix}e^{-i\frac{\theta}{2}} & 0\\ 0 & e^{i\frac{\theta}{2}}\end{pmatrix}$$

$$CRX(\theta) = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ 0 & 0 & -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix} \quad CRY(\theta) = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ 0 & 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix} \quad CRZ(\theta) = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & e^{-i\frac{\theta}{2}} & 0\\ 0 & 0 & 0 & e^{i\frac{\theta}{2}}\end{pmatrix}$$

$$CNOT = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix} \qquad CZ = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} \qquad SWAP = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$TOF = \begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{pmatrix}$$