CPEN 400Q Lecture 18
Noise and error channels

Monday 17 March 2025

Announcements

- Quiz 8 today
- MT checkpoints this week
- Sign up for final oral interview (Canvas calendar)
- A3 due Tuesday 25 March 23:59
- New slide format - feedback welcome

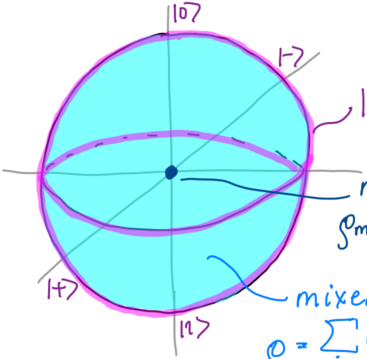
Last time

We expressed density matrices (for pure or mixed states) as

Since ρ is positive semidefinite,

Last time

The set $(\langle X \rangle, \langle Y \rangle, \langle Z \rangle)$ form the Bloch vector, $\langle P \rangle = \mathrm{Tr}(P\rho)$.



pure states

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

maximally mixed state

$\rho_{max} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}\mathbb{1}$

mixed states

$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

Last time

Unitary operations preserve length of Bloch vector

To take pure states to mixed states, apply a *quantum channel*:

Quantum channels are CPTP linear maps.

- **T**race-**P**reserving:
- Positive:
- **C**ompletely **P**ositive:

Last time

Quantum channels are characterized by **Kraus operators** $\{K_i\}$,

Example: unitary channel, $\mathcal{U}$, $\{K_i\} = \{U\}$.

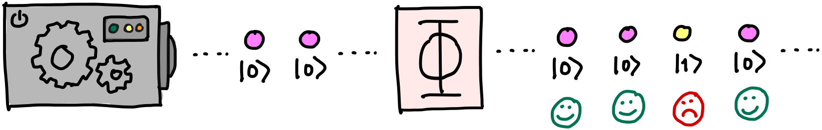
Example: a projective measurement $\mathcal{M}$, $\{K_i\} = \{\Pi_i\}$

Learning outcomes

1. Express noise in quantum systems using quantum channels
2. Compare density matrices with fidelity and trace distance
3. Apply noise to quantum circuits in PennyLane

The bit flip channel

Suppose a “bit flip” (Pauli X) error occurs with probability p.



How do we write this as a channel? What are the Kraus operators?

The bit flip channel

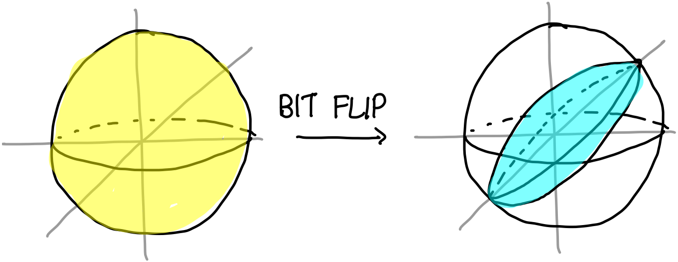
We can visualize the effects of the channel by observing how it deforms the Bloch sphere.

Consider:

- Which states are not affected by a bit flip channel?
- Which states are *most* affected, and where do they go?

Then look at the density matrices of these states.

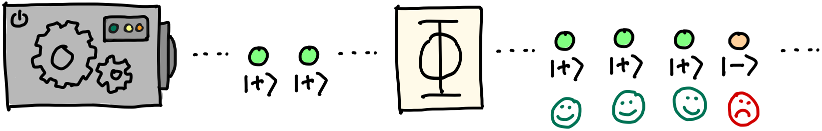
The bit flip channel



x : same
yz : contracts
(factor 1-2p)

The phase flip channel

Suppose a "phase flip" (Pauli Z) error occurs with probability p.



How do we write this as a channel? What are the Kraus operators?

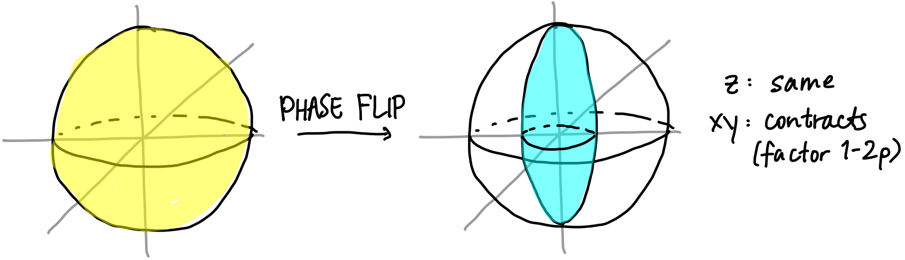
The phase flip channel

Same questions:

- Which states are not affected by a phase flip channel?
- Which states are *most* affected, and where do they go?

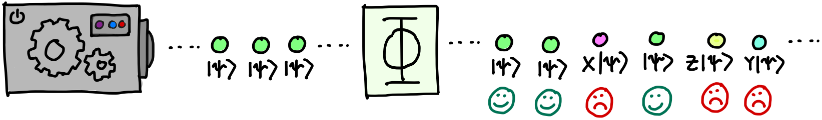
Then look at the density matrices of these states.

The phase flip channel



The depolarizing channel

Suppose each Pauli error occurs with probability $p/3$.



How do we write this as a channel? What are the Kraus operators?

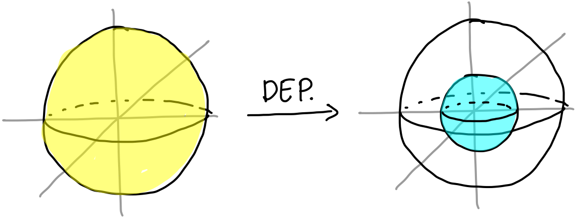
Q: What will happen to the Bloch sphere?

The depolarizing channel

The depolarizing channel

$$\Phi(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$$

can also be written as



Amplitude damping channel

Suppose $|1\rangle$ relaxes to $|0\rangle$ with probability p.

How do you think this affects the Bloch sphere?

Comparing density matrices

The inner product tells us how close two pure states are:

How close are two mixed states ρ and σ?

One common metric is the **trace distance**:

Bounded by $0 \leq T(\rho, \sigma) \leq 1$; *lower* trace distance is better.

Comparing density matrices

Can also compare using **fidelity**,

Bounded by $0 \leq F(\rho, \sigma) \leq 1$. *Higher* fidelity is better.

Example: Determine $F(\rho, \sigma)$ if both ρ, σ are pure.

Comparing density matrices

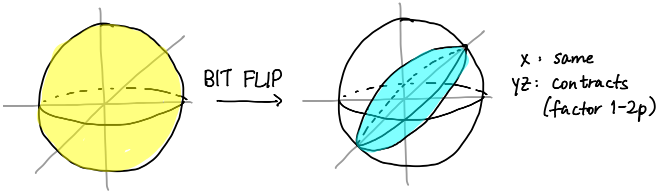
Exercise: Determine $F(\rho, \sigma)$ if ρ is pure but σ is mixed.

Comparing density matrices

Exercise: What is the fidelity of any pure ρ with the maximally mixed state, $\sigma = \frac{1}{2}I$?

Comparing density matrices

Exercise: Determine $F(\rho, \sigma)$ if ρ is pure and and $\sigma = \Phi(\rho)$, where Φ is the *bit flip channel* with parameter p.



Simulating noisy systems

Exercise: Suppose we prepare a system in the state

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

If a depolarizing channel with $p = 0.02$ is applied, what's the probability of observing $|0\rangle$?
Solution 1: solve by hand. Tedious, but can evaluate

Simulating noisy systems

Solution 2: use PennyLane's ``default.mixed'' device!

Resources:

- https://docs.pennylane.ai/en/stable/introduction/operations.html#noisy-channels
- https://docs.pennylane.ai/en/stable/code/qml_noise.html

Next time

Last class:

- Intro to quantum error correction

Action items:

1. MT checkpoint meetings
2. A3 (due 25 March 23:59)
3. Work on project

Recommended reading:

- From this class: Codebook NT, DM; N&C 8.2-8.3, 9.1-9.2
- For next class: Codebook EC; N&C 10.1-10.2