CPEN 400Q Lecture 11
Quantum compilation I

Monday 10 February 2025

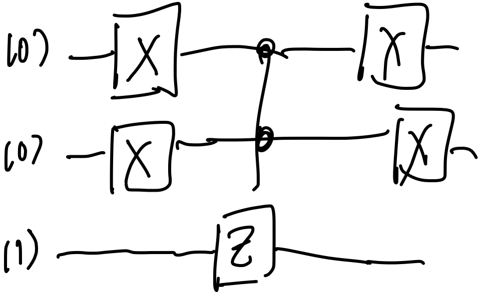
Announcements

- Quiz 4 today
- Literacy Assignment 1 due Wednesday 23:59
- Assignment 2 due Thursday 27 Feb at 23:59
- Project details available soon on PrairieLearn
- Tutorial tomorrow (hands-on assignment about compilation!)

Quiz

Part 1: $|\vec{x}\rangle \longrightarrow (-1)^{f(\vec{x})}|\vec{x}\rangle$

Part 2: $|\vec{x}\rangle|y\rangle \longrightarrow |\vec{x}\rangle|y \oplus f(\vec{x})\rangle$



001

what is the secret string (or, marked element)?

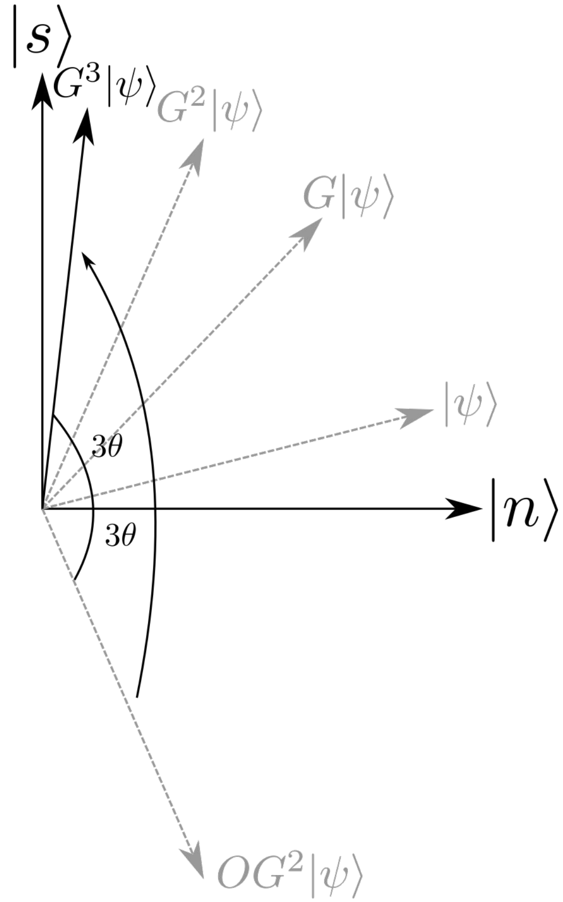
$CC\cdots CZ: \ |11\cdots1\rangle \longrightarrow -|11\cdots1\rangle$

Last time

We discussed how Grover search of an unstructured space has a square-root speedup in **query complexity** over classical methods.

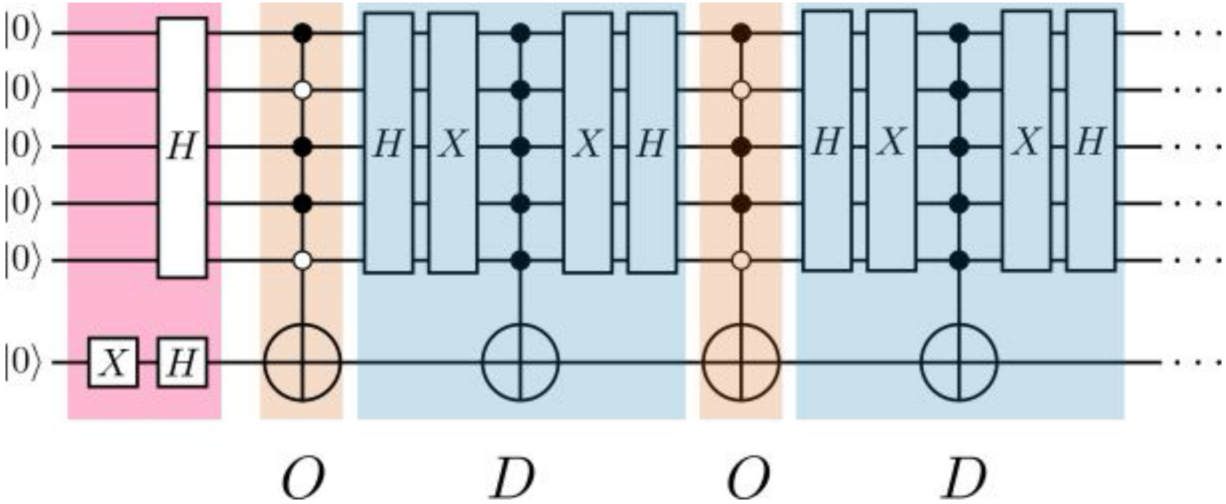
classically: $O(2^n)$

quantum: $O(\sqrt{2^n})$



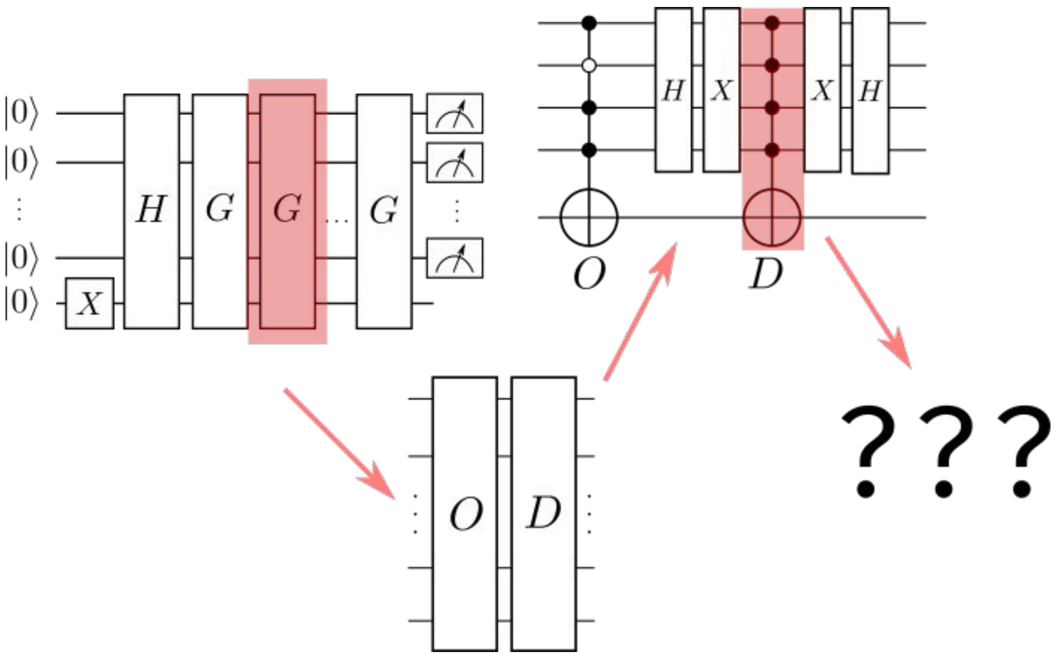
Last time

We implemented Grover’s algorithm in PennyLane



Last time

We finished off with this picture



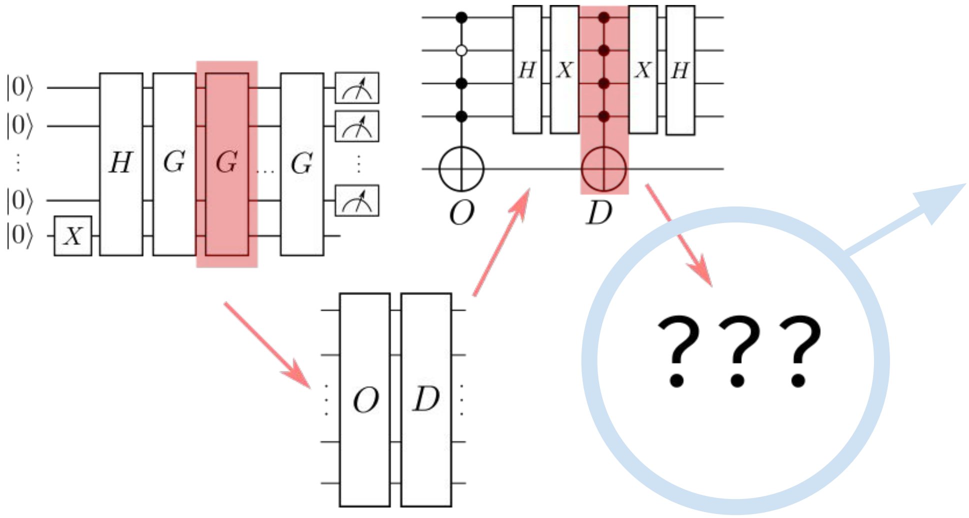
Today

Learning outcomes:

- explain the limitations of query complexity as a metric
- define and calculate **cost metrics** of quantum circuits
- define and list key **universal** gate sets
- perform local **optimizations** on quantum circuits
- **decompose** multi-controlled operations

In tomorrow’s tutorial assignment and Wednesday’s lecture, we will put this into context within the 💗**quantum compilation stack**💗

Quantum circuit synthesis and optimization

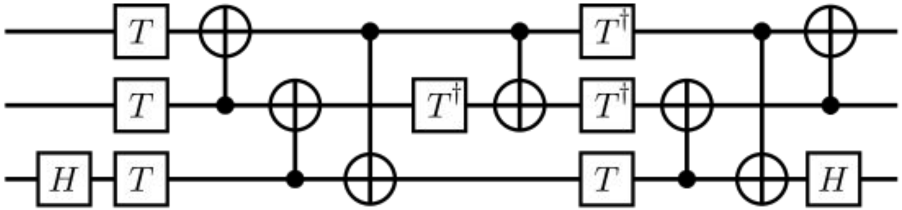


- How do we decide what goes here?
- What criteria determine whether our choice is “good”?
- How do we improve on our choices?

Metrics: width, depth, gate count

Circuit width

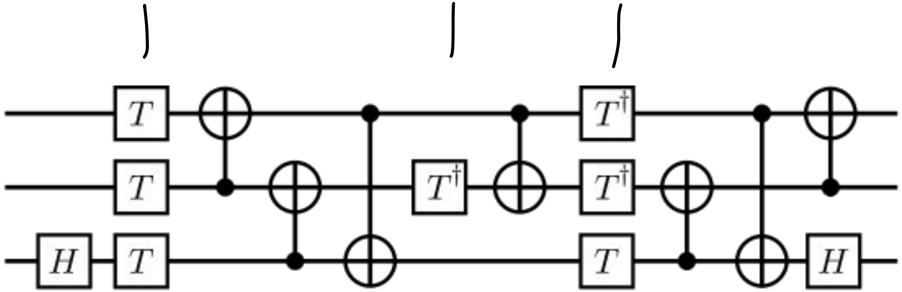
width



number of qubits.

width = 3

Gate count



1-qubit gate count: 9

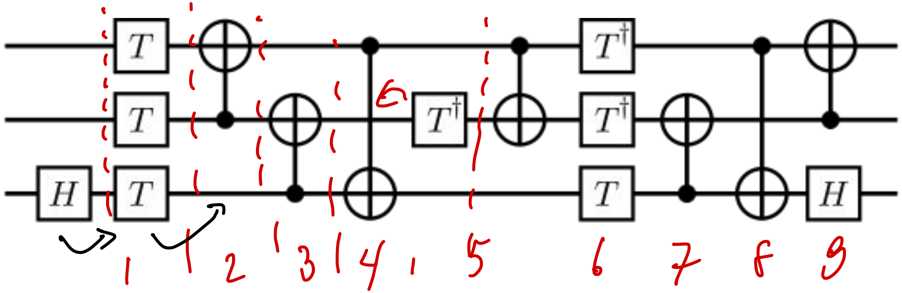
2-qubit gate count: 7

<u>T gates</u>: T count 7

T depth 3

Circuit depth

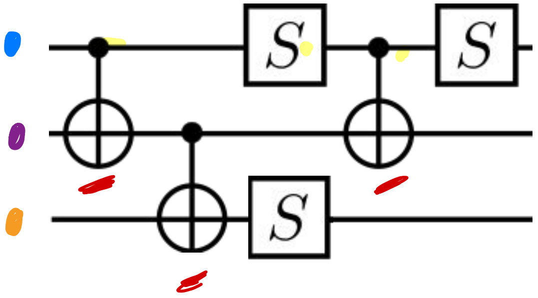
timesteps



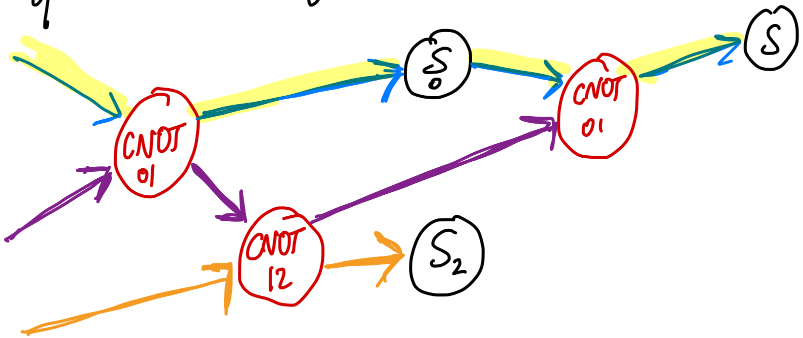
Circuit depth

The circuit depth is the length of the longest path in its **directed acyclic graph (DAG)** representation.

gates: nodes
qubits: edges



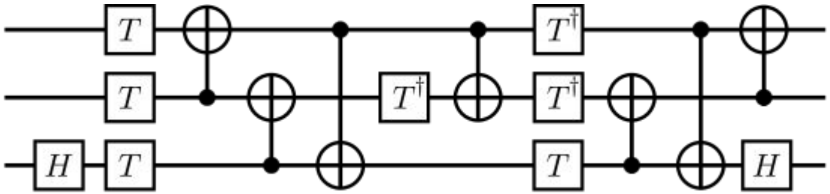
depth = 4

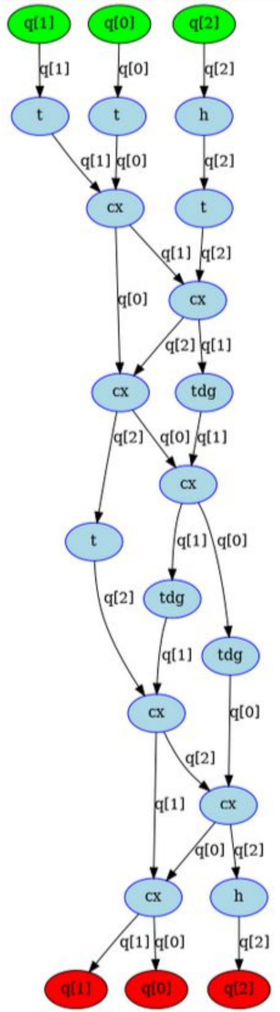


Circuit depth

Generally don't do by hand; NetworkX is your best friend.

https://networkx.org/

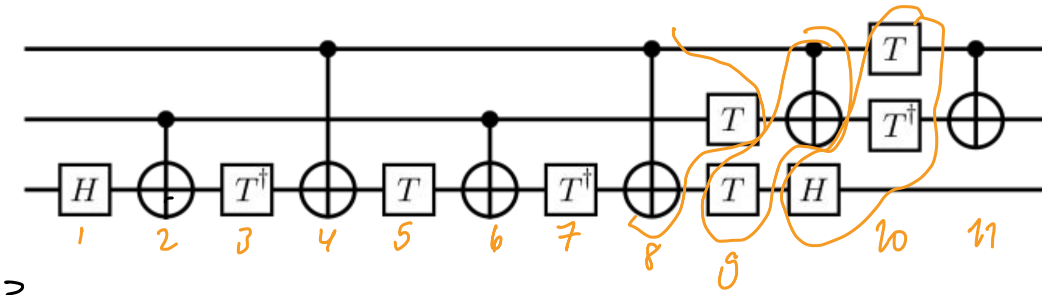




(DAG generated in Qiskit)

Exercise

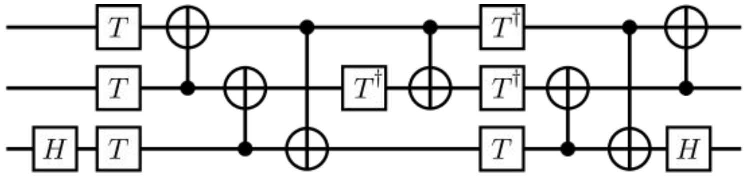
Determine resource counts of the following circuit.

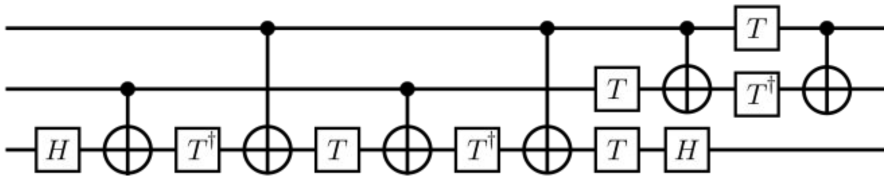


width: 3
depth: 11
1q gates: 9
2q gates: 6

Circuit synthesis and decomposition

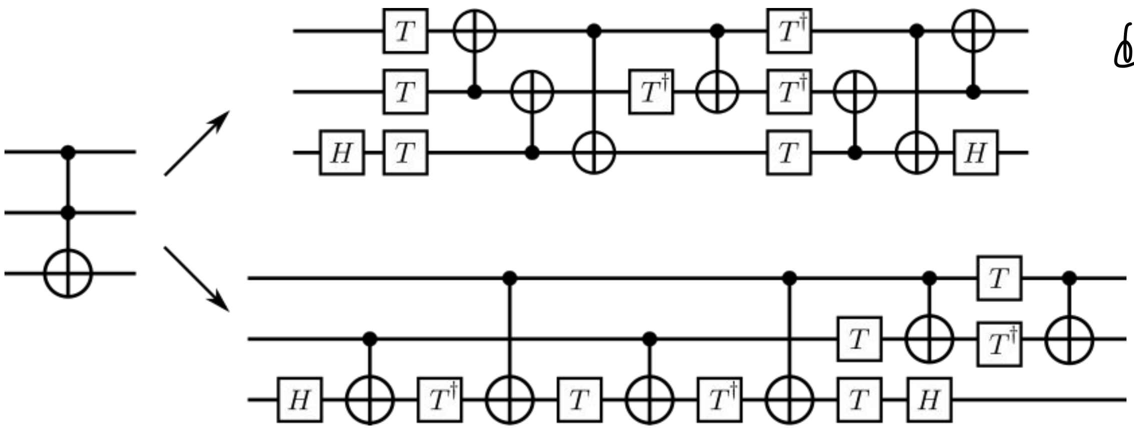
What do these circuits do?





Circuit synthesis and decomposition

What do these circuits do?



7 CNOTs
depth 3

6 CNOTs
depth 12

Circuit synthesis

Given:

- arbitrary unitary matrix U
- a finite set of gates $\{G_k\}$
- a target precision, ϵ

Find

$$U' = C_1 C_2 \cdots C_m$$

s.t.

$$||U - U'|| < \epsilon, C_i \in \{G_k\}$$

Universal gate sets

A $\{G_k\}$ for which this can be done is called a **universal gate set.**

Examples for a single qubit:

- Any two Pauli rotations (exact)
- Hadamard and T (approximate)

Theoretical bound: $O(\log^c(1/\epsilon))$ gates, c varies from 1-4 depending on assumptions.

Universal gate sets

Any single-qubit unitary can be expressed using a sequence of 3 Pauli rotations around a minimum of 2 unique axes

- ZYZ
- XYX
- XYZ
- 9 other combinations *(Euler angles)*

qml.Rot

***class* Rot(*phi, theta, omega, wires, id=None*)**

Bases: `pennylane.operation.Operation`

Arbitrary single qubit rotation

$$R(\phi, \theta, \omega) = RZ(\omega)RY(\theta)RZ(\phi) = \begin{bmatrix} e^{-i(\phi+\omega)/2}\cos(\theta/2) & -e^{i(\phi-\omega)/2}\sin(\theta/2) \\ e^{-i(\phi-\omega)/2}\sin(\theta/2) & e^{i(\phi+\omega)/2}\cos(\theta/2) \end{bmatrix}.$$

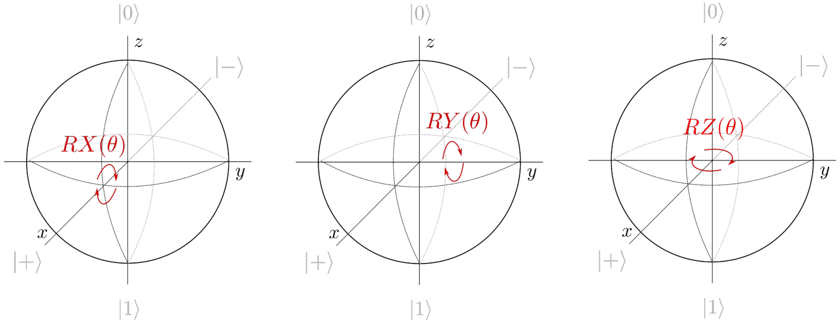


Image: Xanadu Quantum Codebook Node I.6

Universal gate sets

Using a different gate set is like *speaking a different language*.

I would like to use a real quantum computer.

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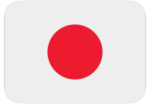
J’aimerais utiliser un vrai ordinateur quantique.

I would like to use a real computer that is quantum.



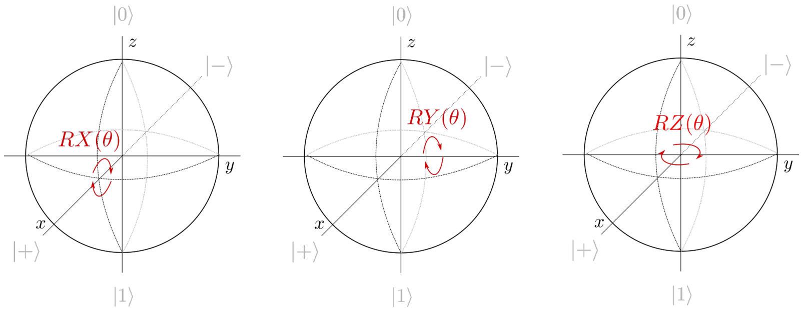
本当の量子コンピュータを作りたい。

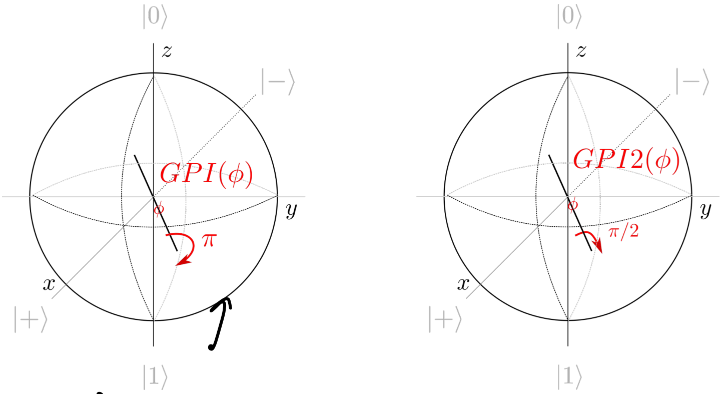
A real quantum computer, I would like to use.

Hardware-specific example

RX/RY/RZ are **arbitrary-angle** rotations about **fixed axes**

IonQ's trapped ion hardware native gates: *GPI/GPI2*, **fixed-angle** rotations about **arbitrary axes** in *xy*-plane





transpilation

Image adapted from Xanadu Quantum Codebook Node I.6
See blog post: https://pennylane.ai/blog/2023/06/the-ionizer-building-a-hardware-specific-transpiler-using-pennylane

Exercise

Express Pauli X in terms of H and T.

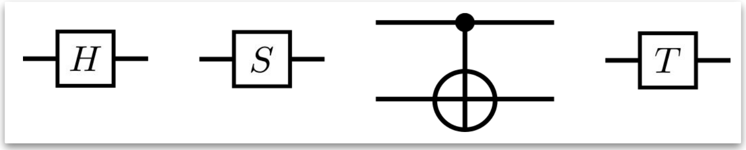
Hint: start by expressing it using H and Z.

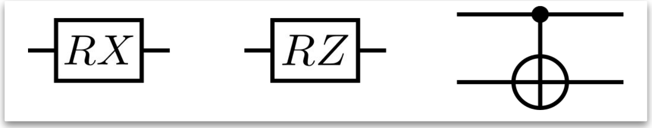
$$X = H\,T\,T\,T\,T\,H = H\,Z\,H$$

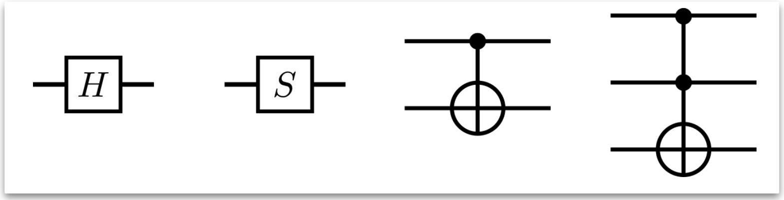
Universal gate sets

Multi-qubit gate sets

Clifford + T







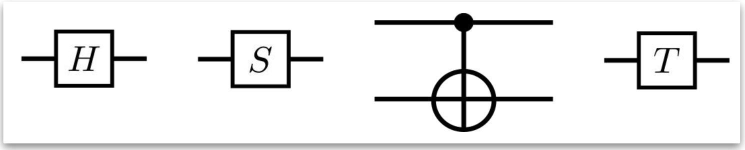
Exercise

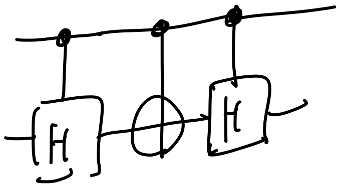
$X = HZH$

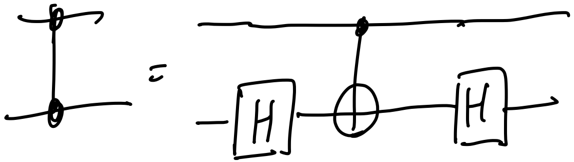
$\downarrow$

$Z = HXH$

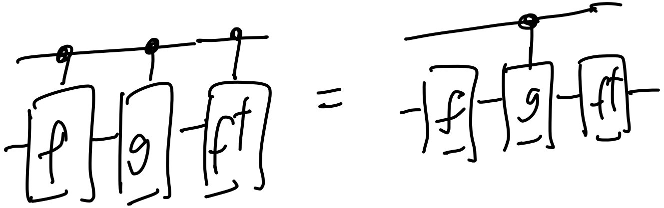
Express *CZ* using gates from





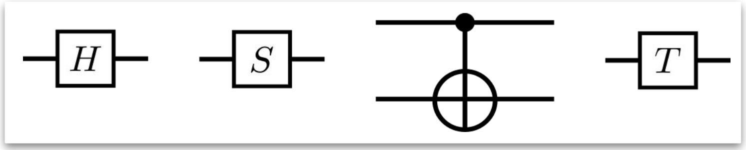


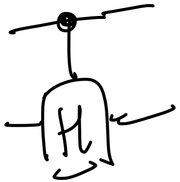
generally useful optimization:



Take-home exercise

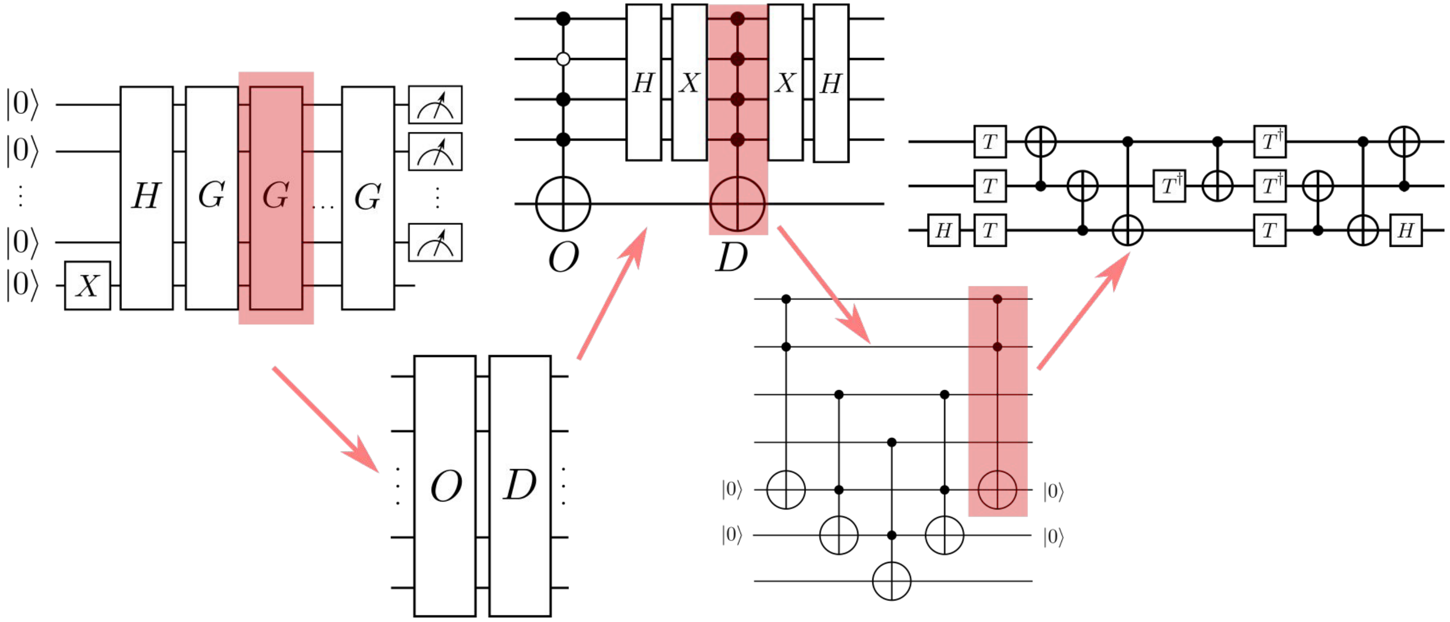
Express controlled Hadamard using gates from





Circuit decomposition

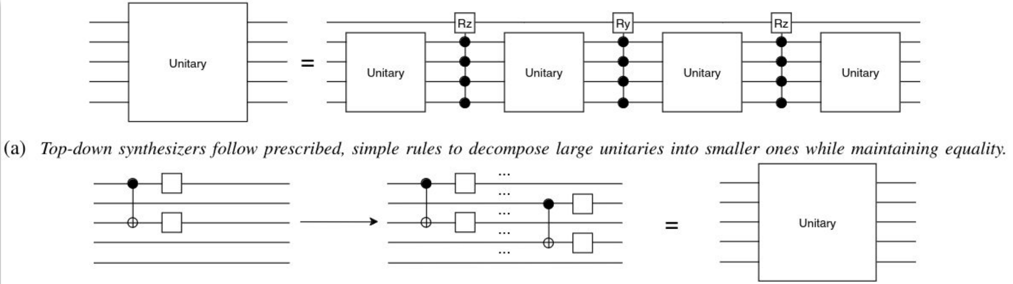
Decompositions of many common operations are **known**.



Synthesis and decomposition techniques

For under-specified or arbitrary operations, or for special gate sets, we may need to ***find*** a decomposition.

- search problem
- number-theoretic techniques
- numerical methods



(a) *Top-down synthesizers follow prescribed, simple rules to decompose large unitaries into smaller ones while maintaining equality.*

(b) *Bottom-up synthesizers start with an empty circuit and build up to equality.*

Figure 1: *Quantum synthesizers are either top-down or bottom-up.*

Image: E. Younis, K. Sen, K. Yelick, and C. Iancu. *QFAST: Conflating Search and Numerical Optimization for Scalable Quantum Circuit Synthesis.* arXiv:2103.07093 [quant-ph]

Challenges

Synthesis is hard!

- algorithm complexity and computational resources

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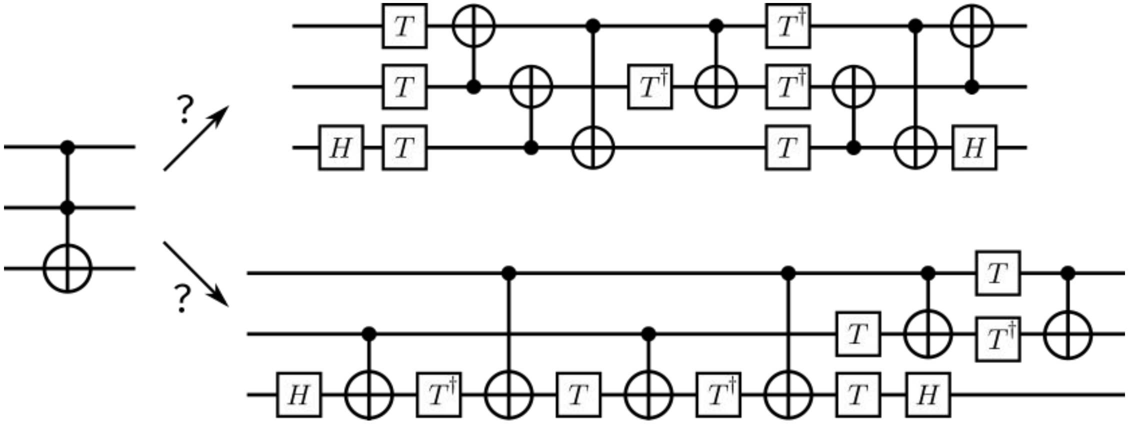
- how to choose the best decomposition to *enable further optimization* down the line?



Challenges

Synthesis is hard!

- algorithm complexity and computational resources
- how to choose the best decomposition to *enable further optimization* down the line?
- manage tradeoffs between various resources

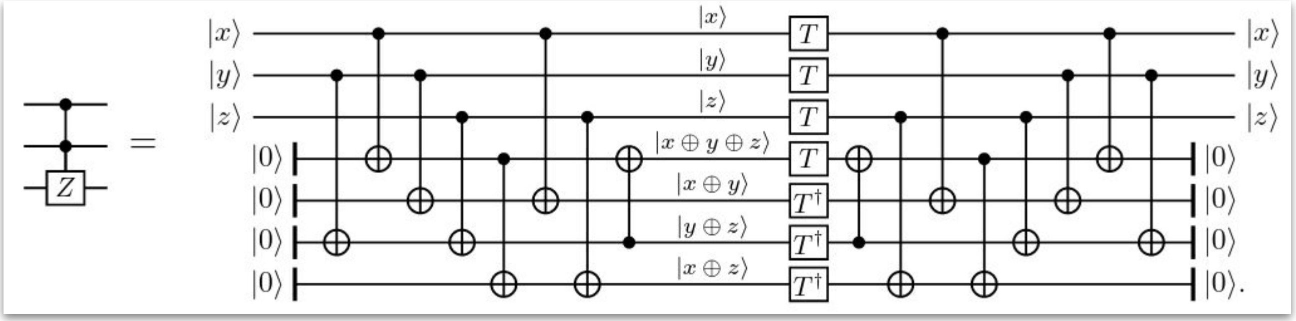
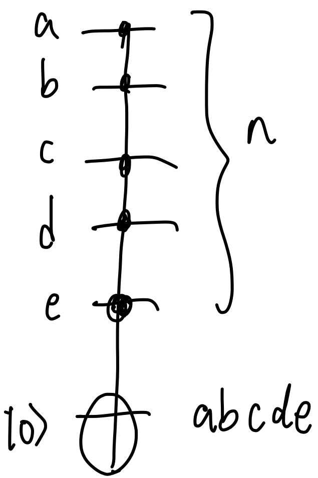
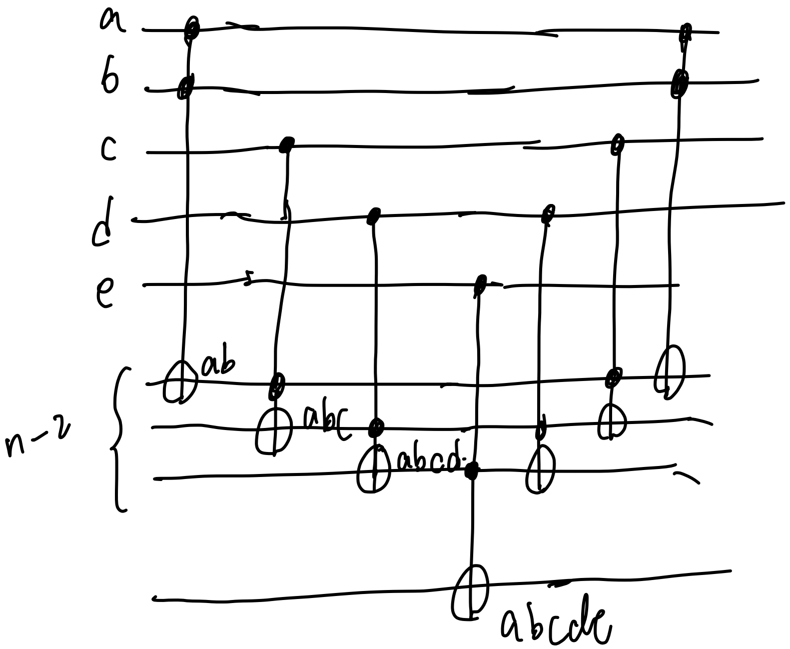


Image: P. Selinger (2013) *Quantum circuits of T -depth one*. Phys. Rev. A 87, 042302 (2013)

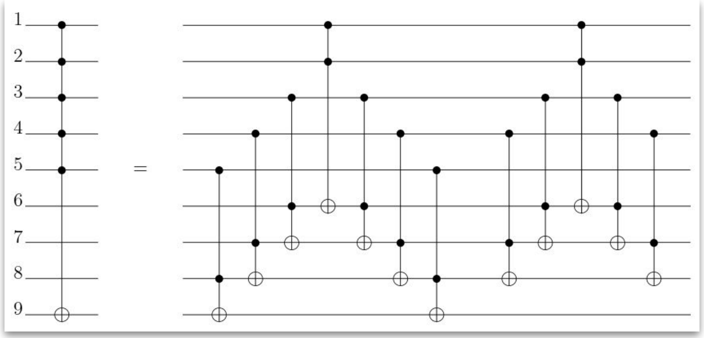
Example

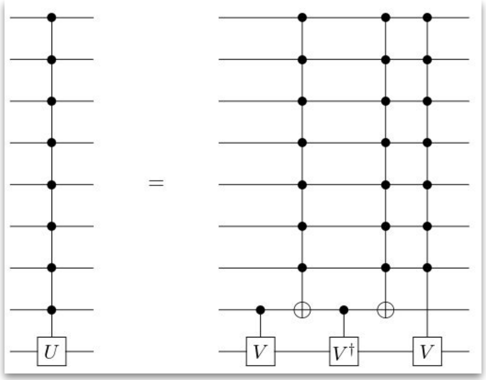
Let’s decompose a multi-controlled NOT.

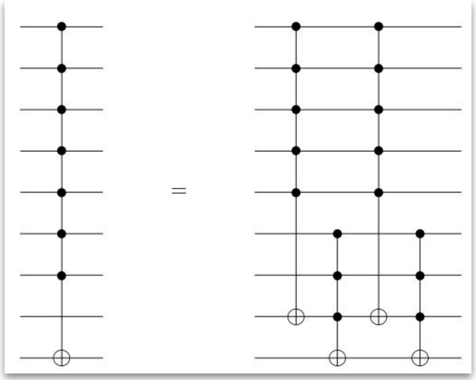




Multi-controlled NOT decomposition strategies





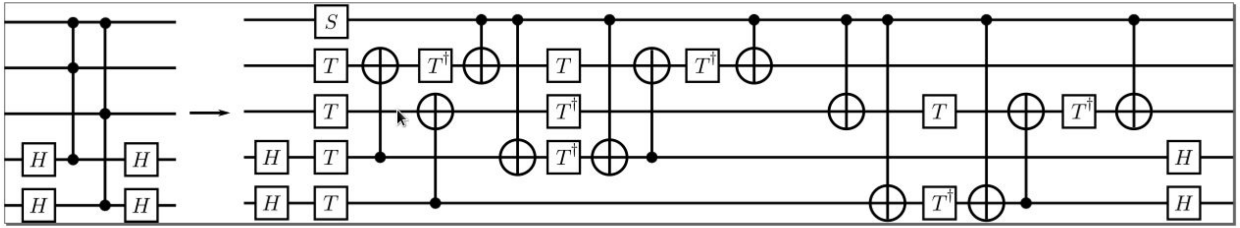


Images: Barenco et al., *Elementary gates for quantum computation* Phys.Rev. A 52 (1995) 3457

Circuit optimization

Machine-independent (or dependent) sequence of *passes* to reduce:

- gate count (overall, or for particular gate)
- circuit depth
- qubit count



Optimization passes

Passes have varying levels of complexity:

- inverse cancellation

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- template matching

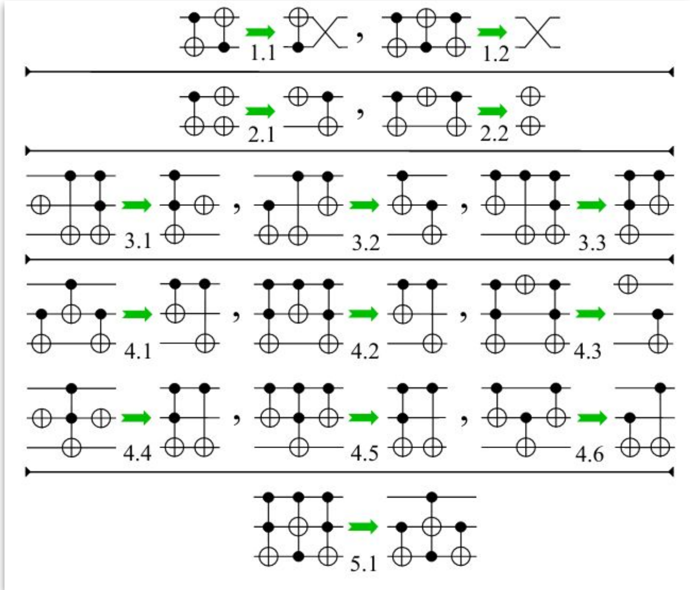
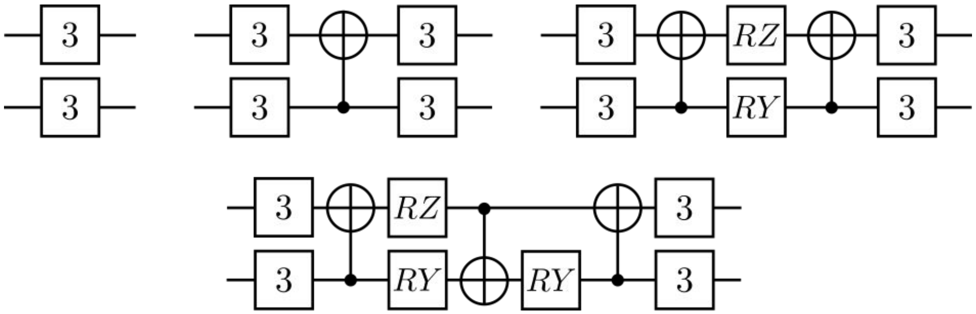


Image: D. M. Miller, D. Maslov, G. W. Dueck. *A Transformation Based Algorithm for Reversible Logic synthesis*. DAC 2003.

Optimization passes

Passes have varying levels of complexity:

- inverse cancellation
- rotation merging
- template matching
- two-qubit block *resynthesis*

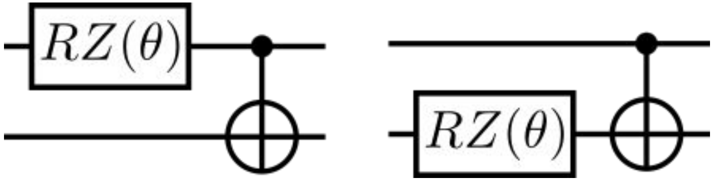


Reference: V. V. Shende, I. L. Markov, and S. S. Bullock (2004) *Minimal universal two-qubit controlled-NOT-based circuits*. Phys. Rev. A 69 062321.

Commutation

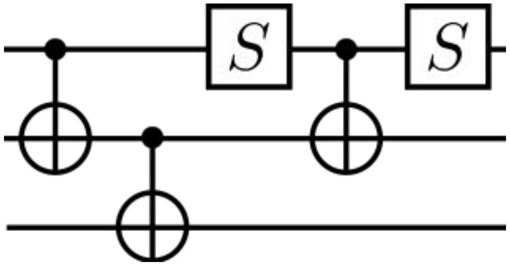
Commutation relations can help us move gates “through” each other to enable optimizations.

Exercise: do the following gates commute?



Exercise

Determine this circuit’s resource counts, then optimize it.
What is the minimum depth?



Next time

Content:

- The quantum compilation stack (from top to bottom)

Action items:

- A2 and Lit A1 (last question from A2 will be posted this week)
- Tutorial Assignment 2 tomorrow

Recommended reading:

- Nielsen and Chuang 4.5
- Transforms blog post (*syntax is out-of-date, but idea holds*)
- Explore the **qml.transforms** module