CPEN 400Q Lecture 19
Intro to quantum error correction

Wednesday 19 March 2025

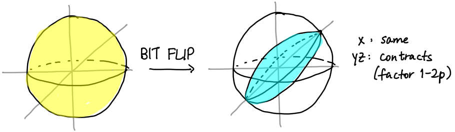
Announcements

- Quiz 9 Monday
- Sign up for project presentations, and final oral interview (Canvas calendar)
- A3 due Tuesday 25 March 23:59

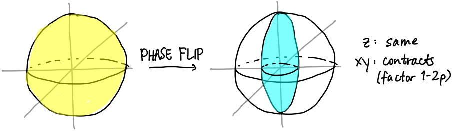
Last time

We expressed a noisy processes as quantum channels.

Bit flip channel

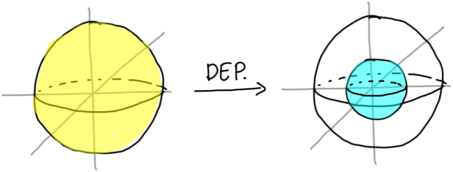


Phase flip channel



Last time

Depolarizing channel



Amplitude damping channel

Last time

We compared density matrices using two distance measures.

Trace distance

Fidelity

We simulated these processes and computed distance measures using PennyLane's ``default.mixed'' device.

Learning outcomes

Module 5 (NEW!): Introduction to quantum error correction

Today:

1. Design circuits to correct bit flips using a simple quantum error correcting code (repetition code)
2. Apply logical operations to encoded qubits
3. Correct bit and phase flip errors with the 9-qubit code

Quantum error correction

Our current picture of noise:

We can protect qubits against noise using error correcting codes.

Repetition codes

Imagine sending a bit string through a classical channel that flips each bit (individually) with probability p.

Idea: add redundant information to enable *detection* and *correction* of bit flips.

Define *encoding* and *decoding* operations, $\mathcal{E}$ and $\mathcal{D}$,

Repetition codes

Devise a procedure, $\mathcal{R}$, to recover from an error: majority voting.

Operation	Outcome
	0 0 1 0 1
$\mathcal{E}$	
ϕ	
$\mathcal{R}$	
$\mathcal{D}$	

Is this better? When will this fail?

Quantum repetition code (bit flip code)

Idea:

Why won't this work?

-
-
-

Alternative:

Bit flip code: encoding circuit

Exercise: (a) How does $\mathcal{E}$ affect a general state,

$$\mathcal{E}(\alpha|0\rangle + \beta|1\rangle)$$

(b) What does the corresponding circuit look like?

Bit flip code: recovery operation

Let's design a circuit to detect (a) *whether* an error occurs, and (b) *which* error occurs.

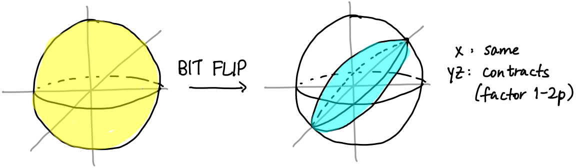
Bit flip code: recovery operation

Design a circuit to *recover* the state of the first qubit

Bit flip code: full circuit for bit flip code

Bit flip code: analysis

Imagine any possible state that can go through the channel:



Exercise (from last time): Determine $F(|\psi\rangle, \Phi(|\psi\rangle\langle\psi|))$ where Φ is the *bit flip channel* with parameter p.

Bit flip code: analysis

Compare overall fidelity of the state *after the error*, vs. fidelity *after recovery*.

After the error

After recovery, have the state

Designing logical operations

Logical operations are specific to codes.

They should act on the logical states $|0\rangle_L$, $|1\rangle_L$ the same way the physical operations act.

Exercise: design circuits for logical X, Z, H, S, and $CNOT$.

Phase flip errors

With our encoding

and appropriate circuitry, we can correct *bit flip errors* but not phase flip errors.

Phase flip code: encoding circuit

Main idea: make phase flip errors “look like” bit flip errors.

Shor code

To correct a combination of one bit flip and/or phase flip error, we can *concatenate* codes: use logical qubit of a phase flip code as the "physical" qubits in a bit flip code.

Next time

Next class:

- Properties of errors and error correcting codes
- Stabilizers and stabilizer codes

Action items:

1. A3 (due 25 March 23:59)
2. Work on project

Recommended reading:

- From this class: Codebook EC; N&C 10.1-10.2
- For next class: Codebook EC; 10.3, 10.5