CPEN 400Q Lecture 23
Quantum channels and noise

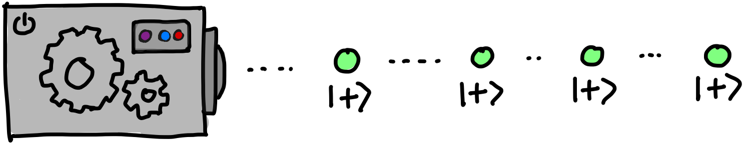
Monday 8 April 2024

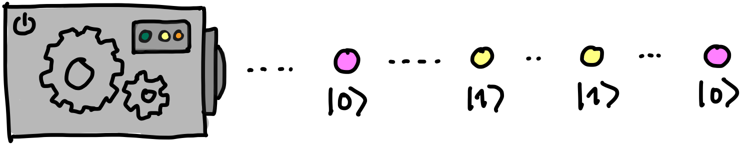
Announcements

- Quiz 10 beginning of class today
- Literacy assignment 3 due Wednesday at 23:59
- Project due Friday at 23:59

Last time

We introduced *mixed states*.





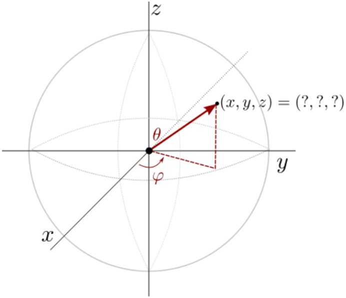
Mixed states are probabilistic mixtures of pure states.

	Pure state	Pure state ρ	Mixed state ρ
States			
Ops.			
Meas.*			

* where $\{\varphi_i\}$ form an orthonormal basis, and $\{P_i\}$ is a set of projectors $P_i = |\varphi_i\rangle \langle\varphi_i|$ or more generally a POVM ($\sum_i P_i = I$).

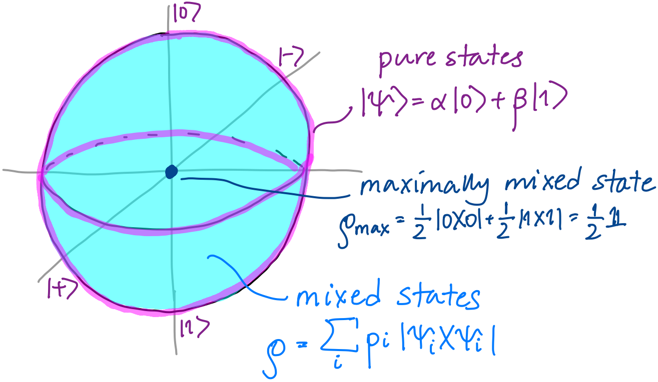
Using the mixed state measurement formalism, we can compute

Write a function that uses quantum measurements to extract its Bloch vector, i.e., its coordinates in 3-dimensional space, as demonstrated by the following graphic:



Last time

I showed that Pauli expectation values for mixed states can produce things *inside the Bloch sphere*, but didn't really explain how...



Learning outcomes

- Define and apply quantum channels to qubit states
- Describe the effects of common noise channels
- Add noise to quantum circuits in PennyLane

Mixed states and the Bloch sphere

Recall ρ is Hermitian; Paulis are a basis.

ρ must have trace 1:

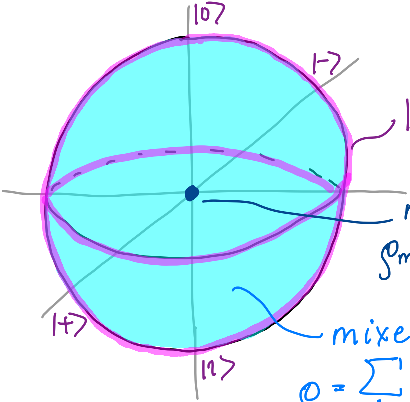
Trace out another Pauli:

Re-express:

Exercise: As ρ is positive semidefinite, its eigvals are ≥ 0. What constraint does this put on $\langle X \rangle, \langle Y \rangle, \langle Z \rangle$? (Hint: look at $\det(\rho)$)

Exercise: How many eigenvalues does ρ have if it is pure? What constraint does this put on $\langle X \rangle, \langle Y \rangle, \langle Z \rangle$?

Mixed states and the Bloch sphere



pure states

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

maximally mixed state

$\rho_{max} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}\mathbb{1}$

mixed states

$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

Exercise: What happens if we apply a unitary U?

$$\det(\rho) = \frac{1}{2}(1 - \langle X \rangle^2 - \langle Y \rangle^2 - \langle Z \rangle^2)$$

To "get inside" the Bloch sphere, we have to apply a different kind of operation: a quantum channel.

A quantum channel Φ *maps* states to other states.

Channels are linear CPTP (completely positive, trace-preserving) maps characterized by a set of **Kraus operators** $\{K_i\}$,

where

Quantum channels

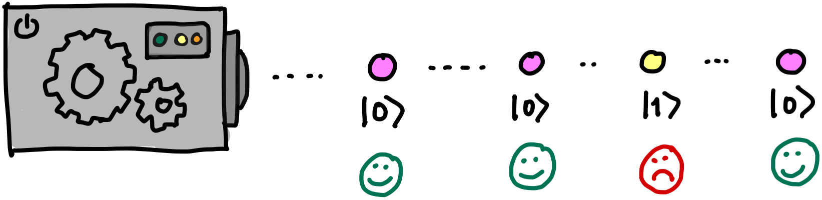
Example: a channel with a single Kraus operator is a unitary operation ("unitary channel"): $\mathcal{U}$.

A channel's Kraus operators represent, loosely, a set of possible things that can happen to a system, including *errors*. We can use them to model noise in a system.

The bit flip channel

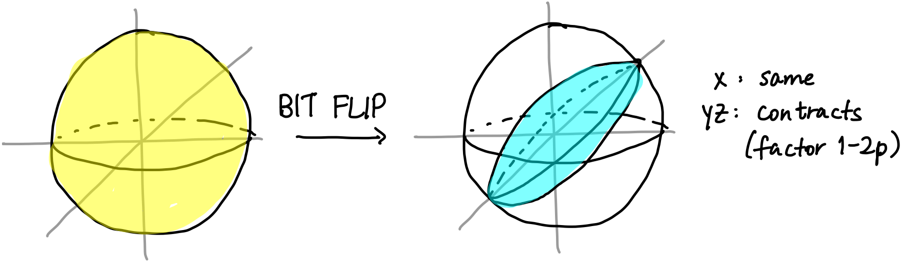
Suppose a "bit flip" (Pauli X) error occurs with probability p.

$$\mathcal{E}(\rho) = (1-p)\cdot\rho + p\cdot X\rho X$$



The bit flip channel

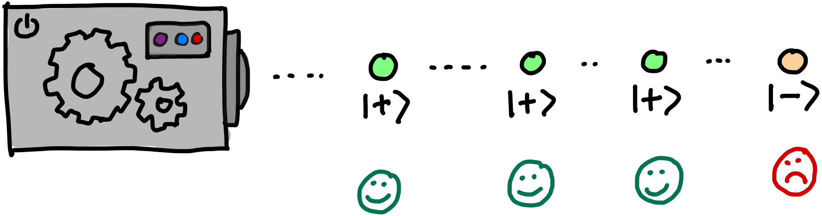
We can visualize the effects of such a channel by observing how it deforms the Bloch sphere.



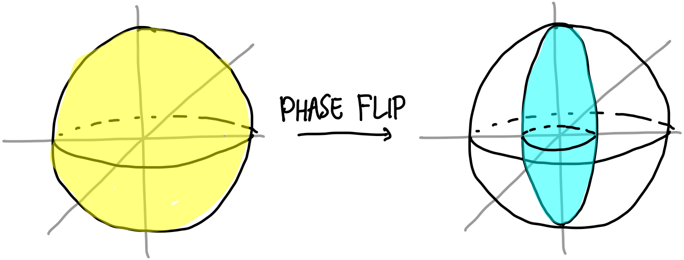
The phase flip channel

Suppose a "phase flip" (Pauli Z) error occurs with probability p.

$$\mathcal{E}(\rho) = (1-p)\cdot\rho + p\cdot Z\rho Z$$



The phase flip channel



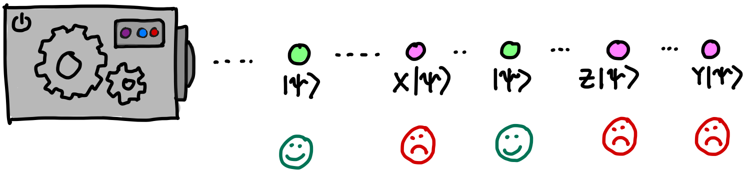
z : same

xy : contracts
(factor $1-2p$)

The depolarizing channel

Suppose each Pauli error occurs with probability $p/3$. This is called the *depolarizing channel*.

$$\mathcal{E}(\rho) = (1-p)\cdot\rho + \frac{p}{3}\cdot X\rho X + \frac{p}{3} Y\rho Y + \frac{p}{3} Z\rho Z$$



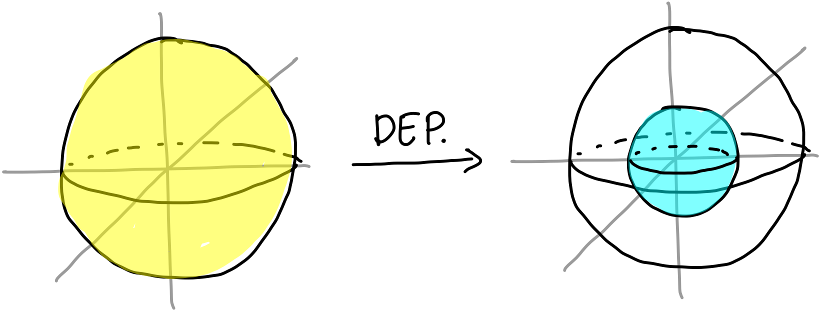
The depolarizing channel

$$\mathcal{E}(\rho) = (1 - p)\rho + \frac{p}{3} X\rho X + \frac{p}{3} Y\rho Y + \frac{p}{3} Z\rho Z$$

can also be written as

Think of this as outputting ρ w/probability $1 - p$, and maximally mixed state with probability p.

The depolarizing channel



The depolarizing channel

Exercise: Suppose we prepare a system in the state

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

However, depolarization with strength $p = 0.02$ occurs. What is the probability of measuring (in the computational basis) and obtaining the $|0\rangle$ state as output?

The depolarizing channel

Solution 1: solve by hand.

... too tedious, but you can evaluate

Solution 2: solve with PennyLane's ‘‘`default.mixed`’’ device!

Amplitude damping channel

Example: amplitude damping. $|1\rangle$ relaxes to $|0\rangle$ with probability p.

Comparing density matrices

We use the inner product to tell us how close two pure states $|\psi\rangle$ and $|\phi\rangle$ were:

What can we do for mixed states?

Comparing density matrices

How close are two mixed states σ, ρ?

One common metric is the **trace distance**:

Value of trace distance is bounded by $0 \leq T(\rho, \sigma) \leq 1$, and *lower* trace distance is better.

Comparing density matrices

Another is the **fidelity**:

Value of fidelity is bounded by $0 \leq F(\rho, \sigma) \leq 1$, and *higher* fidelity is better.

Exercise: Suppose both σ and ρ are pure states. What does the expression for fidelity reduce to?

Comparing density matrices

Exercise: Suppose ρ is pure but σ is not. What does the expression for fidelity reduce to?

Example

Let's apply some simulated noise to the VQE problem from hands-on 4.

Next time

Last class:

- Discussion about current state of quantum computers

Action items:

1. Literacy assignment 3
2. Project code and report

Recommended reading:

- Quantum volume demo `https://pennylane.ai/qml/demos/quantum_volume.html`