Week1 Problem3 :

given $\rightarrow \alpha_p = 3dB$, $\omega_s = 2\pi \times 350 = 700\pi$ rad/sec

$\alpha_s = 10dB$, $\omega_c = \omega_p = 2\pi \times 1000 = 2000\pi$ rad/sec

Digital freq -

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p T}{2} = \frac{2}{2 \times 10^{-4}} \tan \frac{(200\pi \times 2 \times 10^{-4})}{2}$$

$$\Omega_p = 10^4 \tan(0.2\pi) = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s T}{2} = \frac{2}{2 \times 10^{-4}} \tan \frac{(700\pi \times 2 \times 10^{-4})}{2}$$

$$\Omega_s = 10^4 \tan(0.07)\pi = 2235 \text{ rad/sec}$$

Order, $N = \dfrac{\log \sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \dfrac{\Omega_s}{\Omega_p}} = \dfrac{\log \sqrt{\dfrac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log \dfrac{7265}{2235}}$

$$= \frac{0.4771}{0.5118} = 0.932 \approx 1$$

1st order butterworth filter

$\Omega_c = 1$ rad/sec $\Rightarrow H(S) = \dfrac{1}{1+S}$

$\because \Omega_c = \Omega_p = 7265$ rad/sec

$S \rightarrow \dfrac{\Omega_c}{S} \Rightarrow S \rightarrow \dfrac{7265}{S}$

Transfer function of H.P.F :-

$$H(S) = \frac{1}{S+1} \bigg|_{S = \frac{7265}{S}} = \frac{1}{\frac{7265}{S}+1} = \frac{S}{7265+S}$$

Bilinear Transform;

$$H(z) = H(S) \bigg|_{S = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{S}{7265+S} \bigg|_{S = \frac{2}{2 \times 10^{-4}}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{10^4 \left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10^4 \left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265} = \frac{0.5792(1-z^{-1})}{1 - 0.1584 z^{-1}}$$

$\therefore H(z) + 0.1584 z^{-1} H(z) = 0.5792(1-z^{-1})$

$\Rightarrow H(z) + 0.1584 z^{-1} H(z) = 0.5792 - 0.5792 z^{-1}$

We know,

$Y(z) = H(z) \cdot X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)}$

$\therefore \frac{Y(z)}{X(z)} + 0.1584 z^{-1} \frac{Y(z)}{X(z)} = 0.5792 - 0.5792 z^{-1}$

$\Rightarrow Y(z) + 0.1584 z^{-1} Y(z) = 0.5792 X(z) - 0.5792 z^{-1} X(z)$

$\Rightarrow y(n) + 0.1584 y(n-1) = 0.5792 x(n) - 0.5792 x(n-1)$

$\Rightarrow y(n) = 0.5792 x(n) - 0.5792 x(n-1) - 0.1584 y(n-1)$