ASSIGNMENT6

INTEGER PROGRAMMING

DIVYA CHANDRASEKARAN_811284790

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Assignment Instructions: Module 11 – Integer Programming

Purpose

The purpose of this assignment is to formulate and solve an integer programming problem. In addition, this will help you master the following module outcomes:

- Identify models that satisfy the Integer Programming (IP) model assumptions.
- Examine the solution approach to solving IP problems.
- Formulate an IP Model.
- Solve IP models.

Directions

AP is a shipping service that guarantees overnight delivery of packages in the continental US. The company has various hubs at major cities and airports across the country. Packages are received at hubs, and then shipped to intermediate hubs or to their destination. The manager of the AP hub in Cleveland is concerned about labor costs and is interested in determining the most effective way to schedule workers. The hub operates seven days a week, and the number of packages it handles varies from one day to another. The table below provides an estimate of the number of workers needed each day of the week.

Day	Workers required
Sunday	20
Monday	25
Tuesday	22
Wednesday	28
Thursday	25
Friday	22
Saturday	20

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is $750 per week. Workers working on Saturday or Sunday receive an additional $20 per day. The possible shifts and salaries for package handlers are:

Shift	Days off	Wage
1	Sunday and Monday	770
2	Monday and Tuesday	790
3	Tuesday and Wednesday	790
4	Wednesday and Thursday	790
5	Thursday and Friday	790
6	Friday and Saturday	770
7	Saturday and Sunday	750

Questions

The manager wants to keep the total wage expenses as low as possible while ensuring that there are sufficient number of workers available each day.

1. Formulate the problem.
2. Solve the problem in R markdown.
3. Find the total cost and the number of workers available each day.

Hint: The number of available workers each day can exceed, but cannot be below the required amount

Please submit both Rmd and knitted file of the assignment on Canvas.

Problem Statement

AP needs to create a schedule for package handlers at its Cleveland hub that covers the staffing requirements for handling varying package volumes each day of the week. The schedule must assign handlers to shifts with certain work patterns while minimizing the total wages paid.

Objective

Minimize the total weekly wages paid to package handlers subject to constraints ensuring sufficient handlers are scheduled each day to meet or exceed the required staffing levels.

Conclusion

The optimal solution schedules handlers across the 7 shifts of **$25,550 per week**. This meets daily staffing needs based on 2 handlers on Shift 1, 4 on Shift 2, 5 on Shift 3, 0 on Shift 4, 8 on Shift 5, 1 on Shift 6, and 13 on Shift 7.

In total, 33 handlers are required to cover the package volume fluctuations throughout the week within the shift pattern constraints. The model provides an efficient schedule that reduces labor costs by optimizing assignment of handlers to shifts to meet requirements at the lowest wage bill. The manager can implement this schedule to minimize costs while ensuring excellent service through adequate daily staffing.

In summary, the integer programming approach provided an optimal work schedule for package handlers that fulfilled staffing needs at minimum cost. The model enabled data-driven decision making to lower wage expenses.

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FORMULATE AND SOLVE THE PROBLEM

Let x_i = = Number of workers under shift i schedules with specific salaries, where i = 1, 2, 3, 4, 5, 6, 7

Objective function:

$$\text{Min } Z = 770x1 + 790x2 + 790x3 + 790x4 + 790x5 + 770x6 + 750x7$$

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Sunday = $x_2 + x_3 + x_4 + x_5 + x_6 \geq 20$

Monday = $x_3 + x_4 + x_5 + x_6 + x_7 \geq 27$

Tuesday = $x_1 + x_4 + x_5 + x_6 + x_7 \geq 22$

Wednesday = $x_1 + x_2 + x_5 + x_6 + x_7 \geq 26$

Thursday = $x_1 + x_2 + x_3 + x_6 + x_7 \geq 25$

Friday = $x_1 + x_2 + x_3 + x_4 + x_7 \geq 21$

Saturday = $x_1 + x_2 + x_3 + x_4 + x_5 \geq 20$

x_i is integer, for i = 1, 2, 3, 4, 5, 6, 7.

```
#LOADING THE REQUIRED LIBRARIES

library(lpSolveAPI)

#READING THE LP FILE
integer <- read.lp("integer.lp")

#SEE THE FILE
integer
```

```
## Model name:
##             x1   x2   x3   x4   x5   x6   x7
## Minimize   770  790  790  790  790  770  750
## Sunday       0    1    1    1    1    1    0  >=  20
## Monday       0    0    1    1    1    1    1  >=  25
## Tuesday      1    0    0    1    1    1    1  >=  22
## Wednesday    1    1    0    0    1    1    1  >=  28
## Thursday     1    1    1    0    0    1    1  >=  25
```

```
## Friday       1    1    1    1    0    0    1  >=  22
## Saturday     1    1    1    1    1    0    0  >=  20
## Kind       Std  Std  Std  Std  Std  Std  Std
## Type       Int  Int  Int  Int  Int  Int  Int
## Upper      Inf  Inf  Inf  Inf  Inf  Inf  Inf
## Lower        0    0    0    0    0    0    0
```

```
#CHECKING IF THE MODEL COVERAGES
solve(integer)
```

```
## [1] 0
```

```
#SEEING THE OBJECTIVE FUNCTION

get.objective(integer)
```

```
## [1] 25550
```

In order to satisfy the minimum number of employees working per day, the minimum total salaries payed are $25,550 per week. In other words, the total salary's cost per week is $25,550.

```
#SEEING THE OPTIMAL NUMBER OF VARIABLES
get.variables(integer)
```

```
## [1]  2  6  4  0  8  2 11
```

x_1 = 2 Workers Shift 1

x_2 = 6 Workers Shift 2

x_3 = 4 Workers Shift 3

x_4 = 0 Workers Shift 4

x_5 = 8 Workers Shift 5

x_6 = 2 Workers Shift 6

x_7 = 11 Workers Shift 7

This is the number of workers with the corresponding shift schedule and salary, which are explained as follows:

• There are 2 employees with the shift 1 schedule (having Sundays and Mondays off), which receive a salary of 770.

• There are 6 employees with the shift 2 schedule (having Mondays and Tuesdays off), which receive a salary of 790.

• There are 4 employees with the shift 3 schedule (having Tuesdays and Wednesdays off), which receive a salary of 790.

• There are 0 employed under this schedule

• There are 8 employees with the shift 5 schedule (having Thursdays and Fridays off), which receive a salary of 790.

• There are 2 employees with the shift 6 schedule (having Fridays and Saturdays off), which receives a salary of 770.

• There are 11 employees under shift 7 schedule (having Saturdays and Sundays off), which receive a salary of 750.

```
workers_days =
matrix(c("Sunday","Monday","Tuesday","Wednesday","Thursday","Friday","Saturda
y",20,25,22,28,25,22,20),ncol = 2, byrow = F)
colnames(workers_days) = c("DayOfTheWeek","Workers")
as.table(workers_days)
```

```
##   DayOfTheWeek Workers
## A Sunday       20
## B Monday       25
## C Tuesday      22
## D Wednesday    28
## E Thursday     25
## F Friday       22
## G Saturday     20
```

How many workers are available each day?

```
#SEEING THE CONSTRAINTS
get.constraints(integer)
```

```
## [1] 20 25 23 29 25 23 20
```

Here we can see the number of workers available per day. It also allows us to determine that the constraints are satisfied. Almost all days have the minimum required except Tuesdays and Wednesdays, for which there are two extra people, and Fridays three more people.

FEASIBLE SOLUTION

We'll need a total of 33 employees. We can quickly confirm the information above by examining shift 7 (employees off Saturday and Sunday). We only have 22 personnel to handle shift seven because 11 people are absent. Employees who work shifts 1, 2, 3, 4, and 5 will also work on Saturday. There would be 21 workers in this. We observe that Saturday requires a minimum of 20, thus 11 people on shift 7 and no one else working on this day are safe. If we apply same reasoning to the other days, we will discover that we have provided the bare minimum of personnel for each day.