QMM ASSIGNMENT3

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Directions

Heart Start produces automated external defibrillators (AEDs) in each of three different plants (A, B and C). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

	Unit Shipping Cost			*Unit*	*Monthly*
	Warehouse 1	*Warehouse 2*	*Warehouse 3*	*Production Cost*	*Production Capacity*
Plant A	*$20*	*$14*	*$25*	*$400*	*100*
Plant B	*$12*	*$15*	*$14*	*$300*	*125*
Plant C	*$10*	*$12*	*$15*	*$500*	*150*
Monthly Demand	*80*	*90*	*70*		

1. Formulate and solve this transportation problem using R
2. Formulate the dual of this transportation problem
3. Make an economic interpretation of the dual

```r
# Load the lpSolve library
library(lpSolve)

#Install and load the packages
library(dplyr)

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union

library(magrittr)
library(htmltools)

# Define the cost matrix
cost_matrix <- matrix(c(20, 14, 25, 400, 100,
                        12, 15, 14, 300, 125,
                        10, 12, 15, 500, 150,
                        80, 90, 70, NA, NA), nrow = 4, byrow = TRUE)
colnames(cost_matrix) <- c("WAREHOUSE1", "WAREHOUSE2", "WAREHOUSE3",
"PRODUCTIONCOST", "PRODUCTIONCAPACITY")
rownames(cost_matrix) <- c("PlantA", "PLANTB", "PLANTC", "MONTHLYDEMAND")
cost_matrix <- as.table(cost_matrix)
head(cost_matrix)

##                WAREHOUSE1 WAREHOUSE2 WAREHOUSE3 PRODUCTIONCOST
## PlantA                 20         14         25            400
## PLANTB                 12         15         14            300
## PLANTC                 10         12         15            500
## MONTHLYDEMAND          80         90         70
##                PRODUCTIONCAPACITY
## PlantA                        100
## PLANTB                        125
## PLANTC                        150
## MONTHLYDEMAND

#Creating a table
install.packages("kableExtra")
library(kableExtra)

html_table = cost_matrix %>%
kable() %>%
kable_classic() %>%
column_spec(2, border_left = TRUE) %>%
column_spec(5, border_right = TRUE) %>%
```

```
row_spec(3, extra_css = "border-bottom:dotted;")
html_table
```

	WAREHOUSE1	WAREHOUSE2	WAREHOUSE3	PRODUCTIONCOST	PRODUCTIONCAPACITY
PlantA	20	14	25	400	100
PLANTB	12	15	14	300	125
PLANTC	10	12	15	500	150
MONTHLYDEMAND	80	90	70	NA	NA

```
#Install and Load the igraph library
library(igraph)

##
## Attaching package: 'igraph'

## The following objects are masked from 'package:dplyr':
##
##     as_data_frame, groups, union

## The following objects are masked from 'package:stats':
##
##     decompose, spectrum

## The following object is masked from 'package:base':
##
##     union

sources <- c("PlantA", "PlantB", "PlantC")
supply <- c(100, 125, 150)
destinations <- c("WAREHOUSE1", "WAREHOUSE2", "WAREHOUSE3")
demand <- c(80, 90, 70)

#Create an empty graph with the total number of vertices
total_vertices <- length(sources) + length(destinations)
graph <- graph.empty(n = total_vertices, directed = TRUE)
print(graph)
```

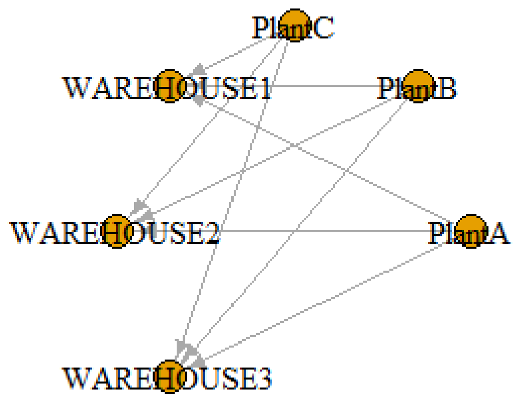
```
## 6 x 6 sparse Matrix of class "dgCMatrix"
##
## [1,] . . . . . .
## [2,] . . . . . .
## [3,] . . . . . .
## [4,] . . . . . .
## [5,] . . . . . .
## [6,] . . . . . .

#Add vertices for sources and destinations
V(graph)$name <- c(sources, destinations)

#Create edges from each source to each destination
for (i in 1:length(sources)) {
 for (j in 1:length(destinations)) {
 weight <- min(supply[i], demand[j])
 if (weight > 0) {
 graph <- add_edges(graph, edges = c(sources[i], destinations[j]), weight =
weight)
 }
 }
}

#Graphical representation of all possible routes to supply from three PLANTS
to THREE WAREHOUSES.
library(igraph)
#Define a layout with x-coordinates for sources and destinations
layout <- layout_in_circle(graph, order = c(1, 2, 3, 4, 5, 6, 7, 8))

#Plot the graph with directed edges and the custom layout
plot(graph, layout = layout, vertex.label.color = "black", vertex.size = 20,
edge.arrow.size = 0.5)
```




```r
#NOW FORMULATING THE TRANSPORTATION MODEL, AND IT CONSISTS OF THREE PARTS:
#1. OBJECTIVE FUNCTION:
#Min TC = 20x_11 + 14x_12 + 25x_13 + 2x_21 + 15x_22 + 14x_23 +10x_31 + 12x_32
+ 15x_33
#Where x_ij is AED’s produced from plant i to warehouse j; i=1,2,3 and
j=1,2,3

#Production Constraints:
#Plant A : x_11 + x_12 + x_(13 ) = 100
#Plant B:  x_21 + x_22 + x_(23 ) = 125
#Plant C:  x_31+ x_32 + x_(33 ) = 150

#Demand Constraints:
#Warehouse1: x_11 + x_12 + x_(13 ) = 80
#Warehouse2: x_21 + x_22 + x_(23 ) = 90
#Warehouse3: x_31 + x_32 + x_(33 ) = 70
#Non-negativity decision variables: x_ij ≥ 0 where i=1,2,3 and j=1,2,3

#Now solving the above formulated transportation model in R.
#The first chunk calls the library.
library(lpSolve)

#The second chunk sets the objective function.
costs <- matrix(c(20, 14, 25,
                  12, 15, 14,
                  10, 12, 15), nrow = 3)
```

```
#Set up constraint signs and right-hand sides (supply side)
row.signs <- rep("<=", 3)
row.rhs <- c(100, 125,150)

#Demand (sinks) side constraints
col.signs <- rep(">=", 3)
col.rhs <- c(80,90,70)

#Run the model
lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)


#Getting the values of 12 decision variables
lptrans$solution

##      [,1] [,2] [,3]
## [1,]    0   75   25
## [2,]   80    0   45
## [3,]    0   15    0

#Out of 9 decision variables, we got 4 of them non-zero. From the solution of
the decision variables, we can deduce that PlantA allocates its monthly
production capacity 100 into two different warehouses: Warehouses 2 and 3.
Similarly, PlantB supplies its shipping costs into warehouse 1 and 3.
Similarly, plantC shipping costs to warehouse 2 only.

#QUESTION2
#Here, let and be the dual variables associated to the two classes of
constraints: supply and demand constraints.

#Constants of the right hand side of the primal will be the coefficients of
the objective function of the dual. RHS constants of the primal are 100, 125,
150, 80, 90, 70 . So, these numbers serve as the coefficients of the
objective function of the dual.

#The dual objective function is the value addition (VA) for the production
capacity. The positive part is what the monthly production gets by shipping
goods to the demanders and the negative part is the cost of producing goods.
The difference is the VA or profit. Thus the dual problem is to maximize VA.

#The objective function of the dual is the
#MAX Z = 80y_1 + 90y_2 + 70y_3 - 100y_1 - 125y_2 - 150y_3

#constraints= y_j-y_i≥c_(ij )

#The PLANT A transports goods to three possible destinations. That is
#y_1 - y_1 ≥ c_(11 ) = 20
#y_2 - y_1 ≥ c_(12 ) = 14
#y_3 - y_1 ≥ c_(13 ) = 25
```

```
#The PLANT B transports goods to three possible destinations. That is
#y_1 - u_1 ≥ c_(11 ) = 12
#y_2 - u_1 ≥ c_(12 ) = 15
#y_3 - u_1 ≥ c_(13 ) = 14

#The PLANT C transports goods to three possible destinations. That is
#v_1 - u_1 ≥ c_(11 ) = 10
#v_2 - u_1 ≥ c_(12 ) = 12
#v_3 - u_1 ≥ c_(13 ) = 15

#Where,
#v_j ≥ 0 for j = 1, 2, 3, and u_(i ) ≥ 0 for i = 1, 2, 3
```

OBJECTIVE FUNCTION OF THE DUAL

$$MAX\ Z = 80v_1 + 90v_2 + 70v_3 - 100u_1 - 125u_2 - 150u_3$$

$$constraints = v_j - u_i \geq c_{ij}$$

Plant A:

$$v_1 - u_1 \geq c_{11} = 20$$

$$v_2 - u_1 \geq c_{12} = 14$$

$$v_3 - u_1 \geq c_{13} = 25$$

Plant B

$$v_1 - u_1 \geq c_{11} = 12$$

$$v_2 - u_1 \geq c_{12} = 15$$

$$v_3 - u_1 \geq c_{13} = 14$$

Plant C:

$$v_1 - u_1 \geq c_{11} = 10$$

$$v_2 - u_1 \geq c_{12} = 12$$

$$v_3 - u_1 \geq c_{13} = 15$$

Where,
$v_j \geq 0\ for\ j = 1,2,3, and\ u_i\ \geq 0\ for\ i = 1,2,3$

```
#QUESTION3
# The dual minimizes demands*dual costs subject to dual costs being >= costs
# This is the dual linear programming formulation.

# Economic interpretation:
# The dual variables represent the marginal costs of supplying an additional
unit of demand from each warehouse. The optimal dual variables tell you the
minimum marginal cost needed to supply another unit to each warehouse while
still meeting all demands. This helps determine which supply routes have
unused capacity that could be utilized if demands changed.

#PART1

##MR = MC rule
#v_j - u_i ≥ c_ij  This means, v_j ≥  u_(i ) +  c_ij

#To be more specific,
#v_3 ≥  u_1 + 420
#The left side is the per unit revenue received by selling one unit of the
product. This is what we call MR (marginal revenue) in economics. The right
side is the per unit cost of making and transporting good. This is called MC
(marginal cost). Production Capacity keeps on increasing production and
shipping to the warehouse3 as long as v_3 ≥  u_1 + 420, that is as long as MR
≥ MC.

#On the opposite, production capacity reduces production and shipping if v_3
≤  u_1 + 420, that is, MR ≤ MC .
#These both are dynamic situations where either production increases or
decreases. When, v_3 = u_1 + 420 that is, MR = MC


#2. hiring or not hiring shipping company for shipping goods


#if the difference, between v,{ j} and u,{ i} is lesser than or equal to c,{
ij}. still, if another shipping company can meet the demand of v,{ j}. u,{ i}
being lower than or equal to c,{ ij} the supplier chooses to hire them to
handle the transportation themselves. The supplier opts for the shipping
company only if they can transport particulars within the specified limits
rather than those that fall outside of it. thus, when v,{ j}. u,{ i} is
lesser than or equal, to c,{ ij} both the patron( supplier) and the shipper
are one and the same. However, in the event that v,{ j}- u,{ i}< =  c,{ ij},
the patron(force) will simply manufacture the particulars and contract with a
different shipping business to deliver them.
```