A Toolkit for Solving Models with a Lower Bound on Interest Rates of Stochastic Duration

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A Appendix

A.1 Proof of Lemma 1

Proof. The system can be written as:

$$
\begin{aligned}
\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} E_t Zt+1 \\ P_t \end{bmatrix} &= \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \begin{bmatrix} Z_t \\ P_{t-1} \end{bmatrix} \\
\begin{bmatrix} A_1 & A_2 & -B_1 & -B_2 \\ A_3 & A_4 & -B_3 & -B_4 \end{bmatrix} \begin{bmatrix} E_t Z_{t+1} \\ P_t \\ Z_t \\ P_{t-1} \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
\begin{bmatrix} A_2 & -B_1 & -B_2 & A_1 \\ A_4 & -B_3 & -B_4 & A_3 \end{bmatrix} \begin{bmatrix} P_t \\ Z_t \\ P_{t-1} \\ E_t Z_{t+1} \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{aligned}
\tag{A.1}
$$

Let the row reduced echelon form of $\begin{bmatrix} A_2 & -B_1 & -B_2 & A_1 \\ A_4 & -B_3 & -B_4 & A_3 \end{bmatrix}$ be $\begin{bmatrix} I & 0 & -C_1 & -C_2 \\ 0 & I & -C_3 & -C_4 \end{bmatrix}$

The system will then be:

$$
\begin{aligned}
\begin{bmatrix} I & 0 & -C_1 & -C_2 \\ 0 & I & -C_3 & -C_4 \end{bmatrix} \begin{bmatrix} P_t \\ Z_t \\ P_{t-1} \\ E_t Z_{t+1} \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
\begin{bmatrix} P_t \\ Z_t \end{bmatrix} &= \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} \begin{bmatrix} P_{t-1} \\ E_t Z_{t+1} \end{bmatrix}
\end{aligned}
\tag{A.2}
$$

At time $t+1$, recall that $j \equiv k_\tau - (t-\tau)$, we know that:

$$
\begin{aligned}
P_{t+1} &= G^{2,j-1} P_t \\
Z_{t+1} &= D^{2,j-1} P_t
\end{aligned}
$$

It follows that $E_t Z_{t+1} = E_t D^{2,j-1} P_t = D^{2,j-1} P_t$. By substituting this result in (A.2) we can solve the system:

$$\begin{bmatrix} P_t \\ Z_t \end{bmatrix} = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} \begin{bmatrix} P_{t-1} \\ E_t Z_{t+1} \end{bmatrix}$$

$$\begin{bmatrix} P_t \\ Z_t \end{bmatrix} = \begin{bmatrix} C_1 P_{t-1} + C_2 D^{2,j-1} P_t \\ C_3 P_{t-1} + C_4 D^{2,j-1} P_t \end{bmatrix}$$

$$\begin{bmatrix} P_t \\ Z_j \end{bmatrix} = \begin{bmatrix} \left(I - C_2 D^{2,j-1}\right)^{-1} C_1 P_{j-1} \\ \left(C_3 + C_4 D^{2,j-1} \left(I - C_2 D^{3}\right)^{-1} C_1\right) P_{t-1} \end{bmatrix}$$

$$\begin{bmatrix} P_j \\ Z_j \end{bmatrix} = \begin{bmatrix} G^{2,j} P_{t-1} \\ \left(C_3 + C_4 D^{2,j-1} G^{2,j}\right) P_{t-1} \end{bmatrix} \tag{A.3}$$

$$\begin{bmatrix} G^{2,j} \\ D^{2,j} \end{bmatrix} = \begin{bmatrix} \left[I - C_2 D^{2,j-1}\right]^{-1} C_1 \\ C_3 + C_4 D^{2,j-1} G^{2,j} \end{bmatrix} \tag{A.4}$$

□

A.2 Data

Output shows deviation of real GDP from the linear trend of real GDP estimated on 2000Q1-2007Q2 sample. Inflation shows chain-type price index of personal consumption expenditures excluding food and energy, percentage change to previous quarter, annualised. The interest rate is the Federal Funds Rate. With the exception of nominal GDP in Figure 8, data series for the price level, NGDP, cumulated NGDP deviations and Dual Mandate index are constructed from output and inflation series.[35]

A.3 Additional welfare metrics

In the Tables providing performance metrics for the policy rules we calculate a volatility index for selected variables z_t as the following.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (z_t - \bar{z})^2 \tag{A.5}$$

A.4 FRBNY DSGE model

The FRBNY DSGE model was developed for policy analysis at the Federal Reserve Bank of New York and builds on several milestone papers in the DSGE literature.[36] The model features nominal wage and price rigidities, variable capital utilisation, costs of adjusting investment, habit formation in consumption and credit frictions. In total, the equilibrium conditions of the model include 17 equations.[37]

The equilibrium conditions are taken one for one from Del Negro, Giannoni and Patterson (2013). To implement the model in our toolkit, we make different assumptions on the shock structure. Del Negro,

[35]Series identifiers are GDPC1 (output), GDP (nominal output), PCEPILFE (price level), DPCCRV1Q225SBEA (inflation) and FEDFUNDS (Federal Funds rate). Data retrieved from FRED (2020), on January 29, 2020, except data for inflation, which was retrieved on October 1, 2020.

[36]The FRBNY DSGE Model is explained in detail in Del Negro et al. (2013). We implement a slightly different version of the model which is presented in Del Negro, Giannoni and Patterson (2013). We decide in favor of this version because it features a preference shock.

[37]The model includes several lagged terms. Setting up the full model with our toolkit, we count 15 state variables.

Giannoni and Patterson (2013) include eight structural shocks, each following an *AR(1)* process, and two i.i.d. monetary policy shocks. We assume that the shocks are perfectly correlated and follow a two-state Markov Process with absorbing state. The applications in this paper feature two structural shocks: a preference shock, $\hat{b}_t$ and a cost push shock, $\tilde{\lambda}_{f,t}$. The preference shock scales the overall per period utility and acts as a negative shock to the natural rate of interest in our experiment. The cost push shock enters the Phillips curve and is used to target a specific drop in inflation.

The policy rule proposed in Del Negro, Giannoni and Patterson (2013) is the following:[38]

$$R_t = \max\left\{1; \rho_R R_{t-1} + (1-\rho_R)\left(\varphi_\pi \sum_{j=0}^{3} \hat{\pi}_{t-j} + \varphi_y \sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1}) + \ln \bar{R}\right)\right\} \tag{A.6}$$

were R_t is the (gross) nominal interest rate, π_t inflation rate, y_t output gap and the remaining are parameters. All hatted variables are in log-deviation from steady state, while steady state variables are denoted by a bar. The policy rule in this model has a standard form: The central bank sets the interest rate according to a function of (lagged) terms of inflation and output as well as the nominal interest rate of the previous period. If this number turns out to be negative, the nominal interest rate is equal to zero, the lower bound. In regimes 1 and 2, the lower bound part of the policy rule will be in effect. In regimes 0 and 3, the endogenous part of the policy rule is part of the equilibrium conditions and has to hold at a candidate solution.

The model includes several predetermined variables introducing inertial dynamics into the model. A consequence of this is that we can no longer impose the monotonicity of k: As the shock is on for longer, the state variables, like capital, approach their new 'steady state'. As they do this, it can turn out to be optimal to keep the interest rate at the lower bound for a shorter period as contingencies get higher, i.e. a hump shaped k.[39]

A.4.1 NYFRB Model description

We refer the reader to the Appendix in Del Negro, Giannoni and Patterson (2013).

A.4.2 Calibration

To parametrise our model, we choose the posterior means as reported in Del Negro, Giannoni and Patterson (2013) as parameter values.

There are only five exceptions: the transition probability of the two-state Markov shock, μ, the values of the two shocks in the low state, $\hat{b}_L$ and $\tilde{\lambda}_{f,L}$, the discount factor β and the steady state inflation rate $\bar{\pi}$. The first three values are chosen to minimise the quadratic distance to three targets: maximum drop of output of 8.5%, a maximum drop of inflation when at the ELB to a value of 1.5% and an expected duration at the ELB at the point in time of hitting the ELB of 4 quarters. The first two are motivated by observed values during the Great Recession. Expected duration at the ELB of 4 quarters we take from Blue Chip survey of forecaster, see Aspen Publishers 2008-12.

We use the model with the FRBNY policy rule, Equation (A.6), to do the calibration. Importantly, the targets have to be matched for a realisation of the shock that implies 28 quarters at the ELB, as observed in the data. In our calibration this corresponds to the two-state Markov shock switching to the high state

[38] We will commonly refer to this rule as the "FRBNY rule".

[39] It should also be mentioned that we do not prove analytically that there is a unique k and henceforth a unique equilibrium in the model. We have not encountered any case of multiple equilibria when experimenting with the model.

in period 32, i.e. contingency 32.[40] The discount factor β is chosen to match a steady state natural rate of real interest of 0%. A real rate of 0% corresponds to the lower bound of December 2019 FOMC long run projections.[41] Finally, the steady state inflation rate is set to 2%. Table A.1 shows the values of the variables we target in the data and the model, respectively, and the implied values for the parameters. Furthermore, the shock variables $\hat{\mu}$, $\hat{z}$, $\hat{\chi}$, $\hat{\psi}$, $\hat{g}$, ϵ^R, ϵ^R_k, $\tilde{\sigma}_\omega$ are all set to zero.

Since the model has non-zero steady state inflation, we adjust the welfare loss function used to compare the performance of the policy rules to:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [(\pi_t - \bar{\pi})^2 + \lambda \hat{Y}_t^2] \tag{A.7}$$

The Fixed Length Forward Guidance Rule (FLFG) in the FRBNY model is the following:

$$R_t = \begin{cases} 1 & \text{for } t \leq \nu \\ \text{Equation (A.6, } \textit{FRBNY Rule}) & \text{otherwise} \end{cases} \tag{A.8}$$

Note that R_t is the gross nominal interest rate, so the rule imposes the ZLB in the case of $t \leq \nu$. We choose $\nu = 6$ as it minimises welfare loss according to the loss function A.7 with equal weights. We impose $\tilde{T} = 1$ in this rule, meaning that the interest rate goes to zero immediately in this rule.

Target	Data	Model	Parameter	Value
$\min \pi_t\|_{ELB}$	1.5	1.486	$\tilde{\lambda}_{f,L}$	0.0015
$\min \hat{Y}_t$	-8.5	-8.49	$\hat{b}_L$	-0.105
$\mathbb{E}(ELB)\|_{ELB}$	4	3.95	μ	0.736
$\bar{r}$	0.0	0.0	β	1.0042
$\bar{\pi}$	2.0	2.0		

Table A.1: Calibration results. See text for details. $\mathbb{E}(ELB)|_{ELB}$ is in quarters. Note: First three parameters are calibrated simultaneously to hit targets. Changing one of the three parameters will affect all three targets. Last two parameters have one-to-one relation with respective target. $\bar{\pi}$ is set directly.

[40]The solution features 4 periods in regime 0, i.e. $\tilde{T} = 5$ and $k_{32} = 1$.

[41]See Board of Governors of the Federal Reserve System (2019).

Acronym	Source	Standard Values	Optimal Values
TTR	Nakov (2008)	$\phi_\pi = 1.5$ $\phi_y = 0.5$	N/A
TTR-1	Nakov (2008)	$\phi_\pi = 1.5$ $\phi_y = 0.5$	$\phi_\pi = 1.14$ $\phi_y = 0.28$
TTR+1	Nakov (2008)	$\phi_\pi = 1.5$ $\phi_y = 0.5$	N/A
TTRS	Nakov (2008)	$\phi_\pi = 1.5$ $\phi_y = 0.5$ $\phi_i = 0.8$	$\phi_\pi = 51.50$ $\phi_y = 48.77$ $\phi_i = 0.98$
TTRP	Wolman (2005)	$\phi_p = 1.5$ $\phi_y = 0.5$	$\phi_p = 1.23$ $\phi_y = 1.23$
ATR	Reifschneider and Williams (2000)	$\alpha = 1$ $\phi_\pi = 1.5$ $\phi_y = 0.5$	$\alpha = 92.09$ $\phi_\pi = 100$ $\phi_y = 34.23$
SUP	Rotemberg and Woodford (1999)	$\phi^{SUP} = 1.28$	$\phi^{SUP} = 1.48$
AIT	Reifschneider and Wilcox (2019)	$\phi^{AIT} = 5$	$\phi^{AIT} = 18.7$

Table A.2: Policy rules, standard parametrisation, and optimal parametrisation. The Table reports, for each Taylor-type policy rule, the parameter values used for the simulations. The second column reports values used in the literature as well as the source. The last column reports optimal values, that minimise the welfare loss (25).In TTR and TTR+1 the standard values are already optimal.

A.5 Results if ELB constraint not imposed

In this section we present results for both the Simple New Keynesian model of Section 4.5 and the FRBNY DSGE model of Section 5 for the case of not imposing the ELB constraint. This means the nominal interest rate can go arbitrarily negative. Apart from dropping the ELB assumption, the parametrisation will be identical to the baseline experiments in the main text. Due to the linearity of the model (if the ELB is not imposed), the results also tell us how the rules perform in response to small enough shocks that do not make the ELB bind.[42]

A.5.1 Simple New Keynesian model without imposing the ELB

Table A.3 shows results if we replicate the rule comparison of Section 4.5 but do not impose the ELB. The parametrisation is exactly the same as the one used to derive the results shown in Table 2, except that the ELB constraint is not imposed.

First let us note that under optimal commitment, deeply negative interest rates are implemented. The nominal rate drops to almost -5%. Comparing optimal commitment with and without imposing the ELB, we get that in the latter case the welfare loss is about 40% lower.

When it comes to the relative ranking of the rules, we see that our proposed rules HD-NGDPT and SDTR perform very well. The only rule outperforming both is a price level targeting rule (PLT), which coincides with optimal commitment in the case of no ELB imposed.[43] NGDPT and HD-NGDPT overlap, which is due to the ability of the central bank to keep nominal GDP on target at all times if it is not constrained by the ELB. Finally, rules ATR and SUP lose some appeal once we do not impose the ELB.

[42] Figures for this section are available from the authors upon request.
[43] This result is shown in Eggertsson and Woodford (2003).

	Welfare Loss (1)	Volatility x (2)	Volatility π (3)	Volatility i (4)	Impact x (5)	Impact π (6)
OCP	$5.196\,10^{-4}$	$3.497\,10^{-3}$	$3.011\,10^{-4}$	$4.697\,10^{-3}$	−0.233	2.914
PANEL A: baseline rules						
OCP	1.000	1.000	1.000	1.000	1.000	1.000
TTR	2.156	3.923	0.873	0.250	16.673	0.739
HD-NGDPT	1.258	2.404	0.427	1.283	2.401	0.730
SDTR	1.821	2.165	1.571	0.264	12.406	0.991
ATR	2.156	3.923	0.873	0.250	16.673	0.739
SUP	2.147	2.904	1.597	0.128	17.239	0.941
PANEL B: additional rules						
PLT	1.000	1.000	1.000	1.000	1.000	1.000
NGDPT	1.258	2.404	0.427	1.283	2.401	0.730
TTRP	6.643	15.777	0.012	0.060	36.881	−0.143
TTRS-1	2.414	4.091	1.196	0.109	20.730	0.797
TTR-1	2.155	3.807	0.955	0.220	18.079	0.764
AIT	2.532	5.779	0.175	0.451	17.672	0.426

Table A.3: Some metrics for selected interest rate rules in the simple two-equation NK model in the presence of a correlated cost push shock without imposing the ELB. All rows except the first show values normalised with respect to the optimal commitment policy (OCP, first row). Column (1) reports the welfare loss computed from a quadratic loss function for the central bank with equal weights; Columns (2)-(4) report a summary measure of deviations of the endogenous variables from target, computed according to Equation (A.5); Finally, Columns (5) and (6) show the response on impact, in annual percentage points, of the output gap and inflation to a natural interest rate shock and a correlated cost push shock of the same size as in Table 2. Rule calibration reported in Table (A.2). The model is calibrated with the standard EW (2003) parameter values reported in footnote 21. FLFG coincides with TTR if the ELB constraint is not imposed.

A.5.2 FRBNY DSGE model without imposing the ELB

Table A.4 shows results if we replicate the rule comparison of Section 5 but do not impose the ELB. The parametrisation is exactly the same as the one used to derive the results shown in Table 3, except that the ELB constraint is not imposed.

As in the simple two-equation NK model, we can get deep negative rates in this case. While the nominal rate does not drop far below zero under the FRBNY Rule, rule PLT lowers the nominal rate to almost -15% for a period of more than 7 years. The welfare loss is not much lower under the FRBNY Rule if the ELB is not imposed, but drops by more than 96% if one compares the best performing rule when the ELB is not imposed (rule PLT) to the FRBNY Rule with the ELB constraint active.

When it comes to the relative ranking of the rules, we see that our proposed rules HD-NGDPT and SDTR perform very well. The only rule outperforming both is again a price level targeting rule (PLT), which does not come as a surprise given the result in the simple NK model of the previous section.[44] NGDPT and HD-NGDPT again overlap, for the same reasons stated in Section A.5.1.

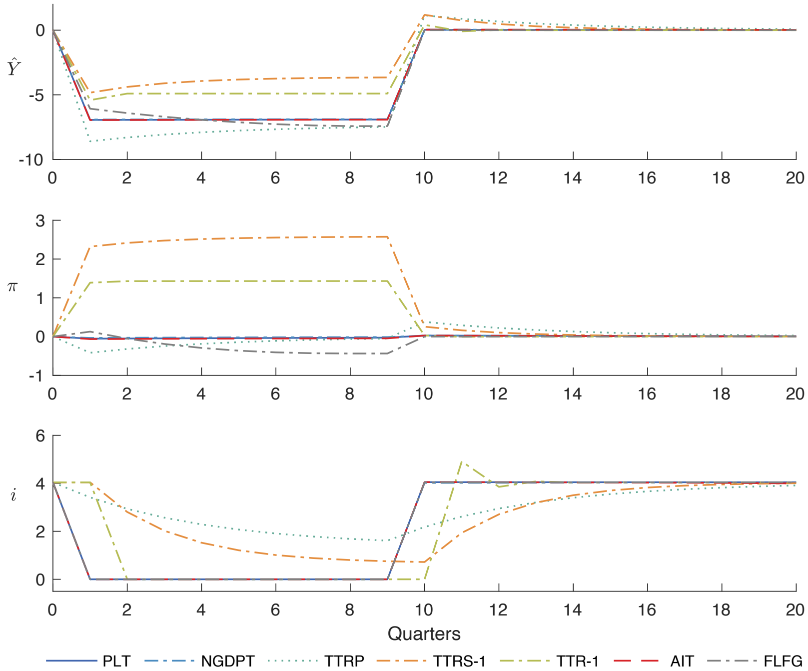
[44] Note however that we are not aware of a derivation that shows that PLT is equivalent to optimal commitment in the FRBNY DSGE model.

	Welfare Loss (1)	Volatility $\hat{Y}$ (2)	Volatility π (3)	Volatility i (4)	Impact $\hat{Y}$ (5)	Impact π (6)
FRBNY Rule	$8.216\ 10^{-4}$	0.013	$1.3433\ 10^{-5}$	$7.2973\ 10^{-5}$	−2.523	2.809
PANEL A: baseline rules						
FRBNY Rule	1.000	1.000	1.000	1.000	1.000	1.000
HD-NGDPT	0.058	0.025	2.052	68.381	0.116	1.130
SDTR	0.082	0.042	2.498	322.130	0.495	1.157
ATR	0.392	0.372	1.570	5.250	0.717	1.059
SUP	0.288	0.254	2.336	5.129	0.598	1.157
PANEL B: additional rules						
PLT	0.040	0.003	2.256	77.405	0.030	1.136
NGDPT	0.058	0.025	2.052	68.381	0.116	1.130
TTRP	0.871	0.869	1.003	1.100	0.955	1.035
TTRS-1	0.406	0.386	1.564	2.598	0.798	1.076
TTR	0.392	0.372	1.570	5.250	0.717	1.059
TTR-1	0.368	0.348	1.600	4.590	0.735	1.066
AIT	0.440	0.424	1.354	4.156	0.753	1.064
TTR+1	0.436	0.418	1.527	6.192	0.710	1.051

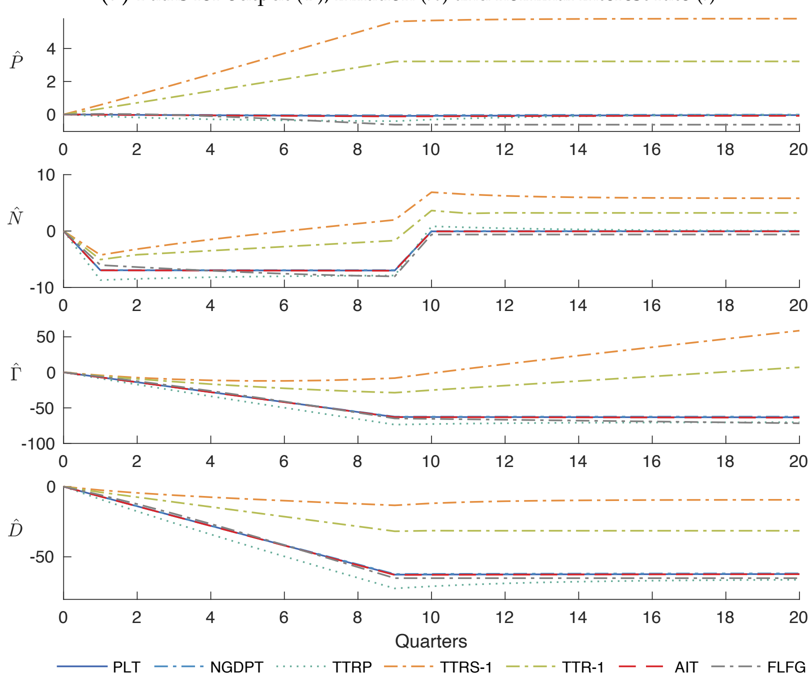
Table A.4: Some metrics for interest rate rules in the FRBNY model in the presence of a preference shock and correlated cost push shock without imposing the ELB. All rows except the first show values normalised with respect to the modified Taylor rule in Del Negro, Giannoni and Patterson (2013) (FRBNY Rule, first row). Column (1) reports the welfare loss computed from a quadratic loss function for the central bank with equal weights and an inflation target, see Equation (A.7); Columns (2)-(4) report a summary measure of deviations of the endogenous variables from target, computed according to Equation (A.5); Finally, Columns (5) and (6) show the response on impact, in annual percentage points, of the output gap and inflation to a preference shock and a correlated cost push shock of the same size as in Table 3. See Section A.4.2 for details on calibration. The list of acronyms is detailed in Table 1. FLFG coincides with FRBNY Rule if the ELB constraint is not imposed.

A.6 Robustness checks

A.6.1 Simple New Keynesian model with -0.5% inflation drop – Additional policy rules and impulse responses

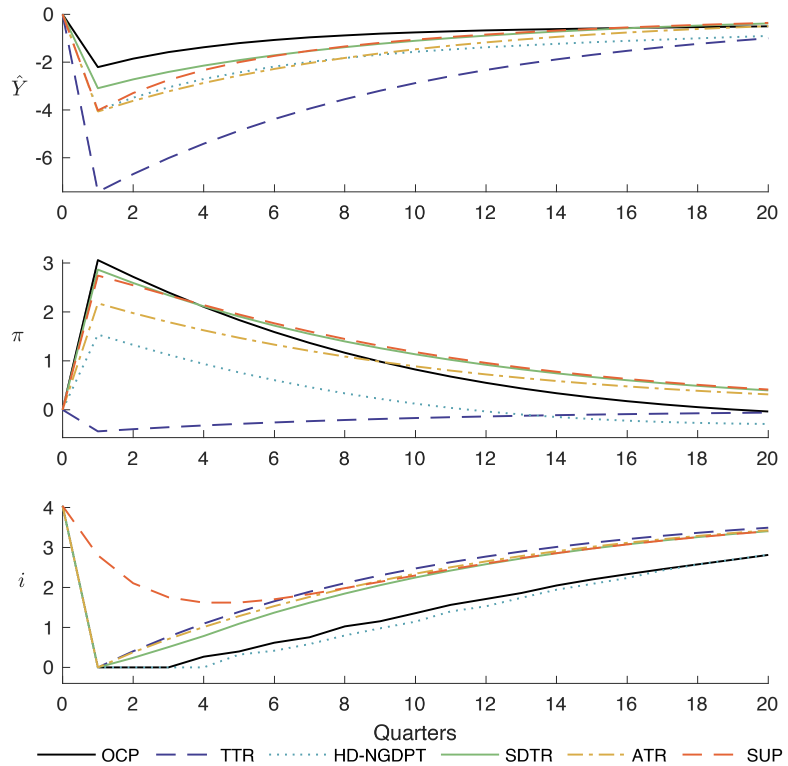


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

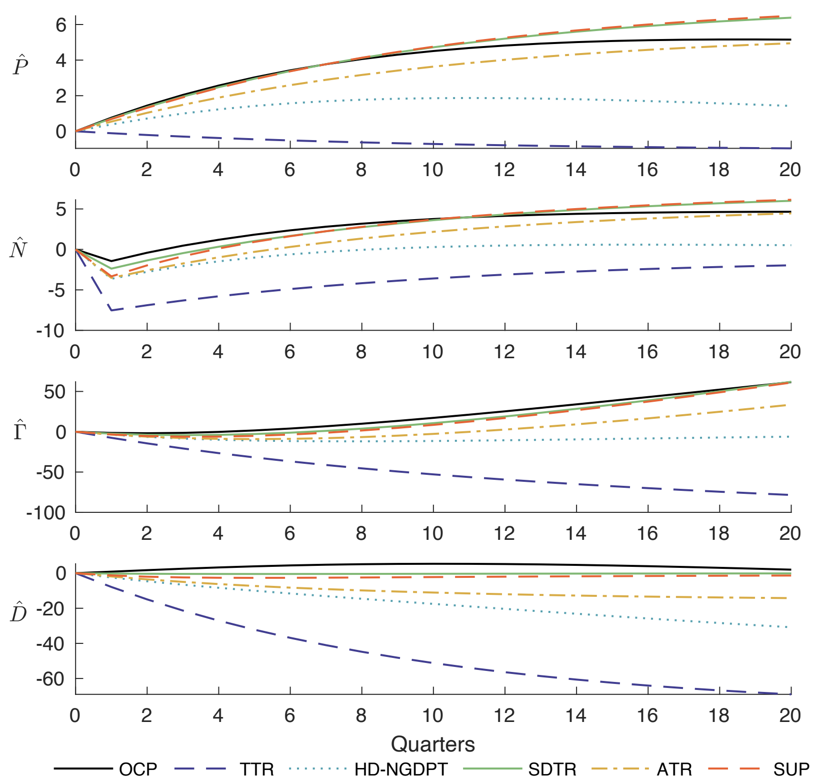


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.1: Dynamic response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). The natural interest rate reverts to the absorbing state after 10 quarters (10th contingency). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

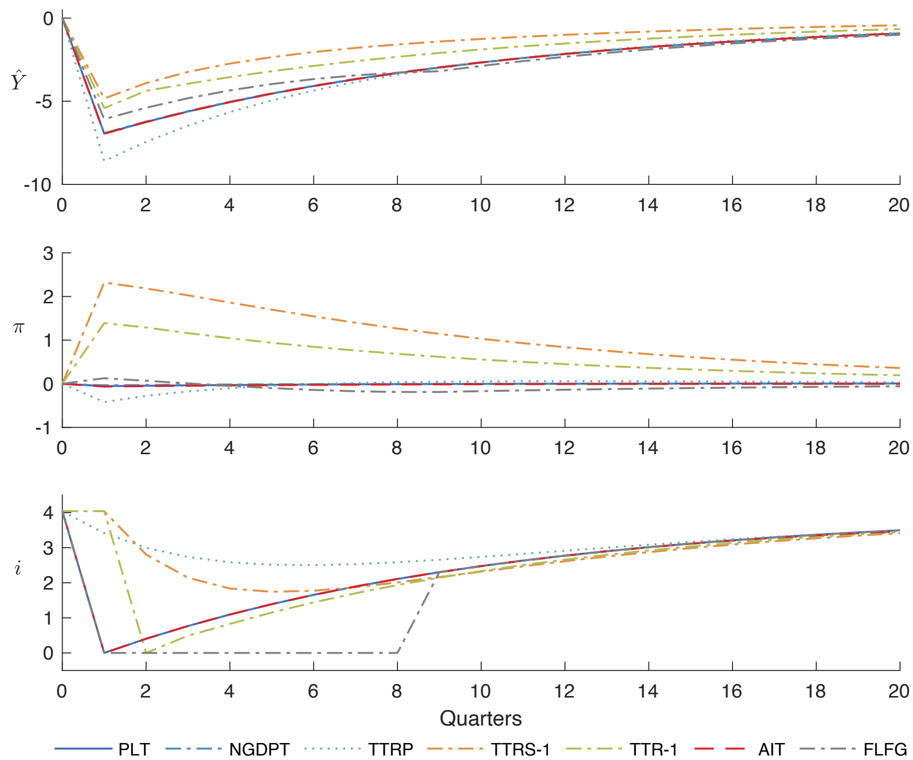


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

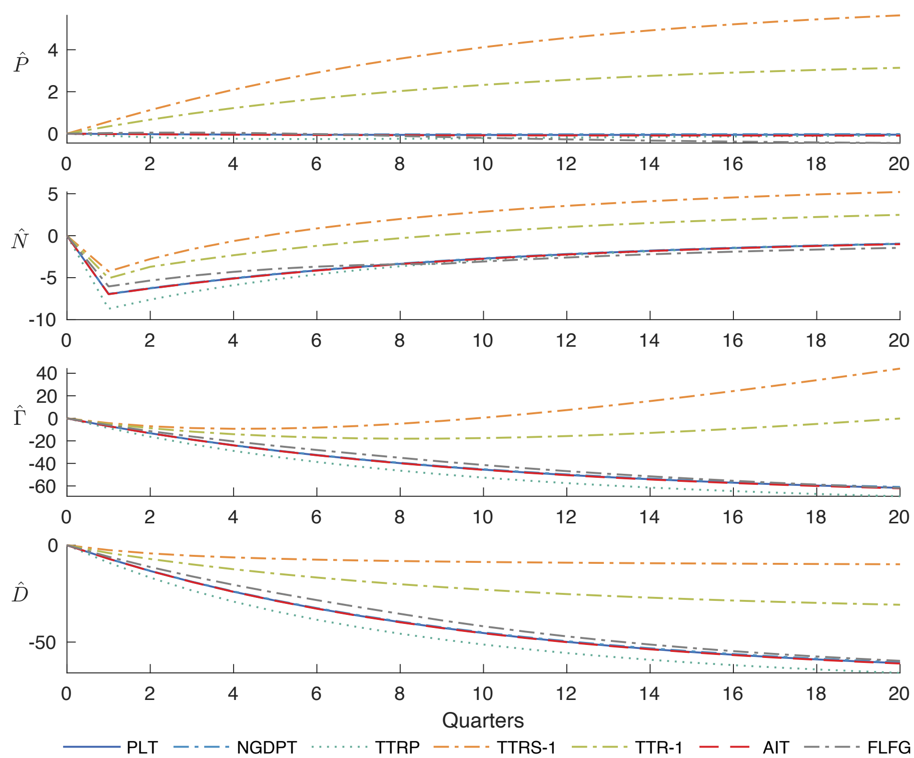


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.2: Average impulse response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for *i* reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.



(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)



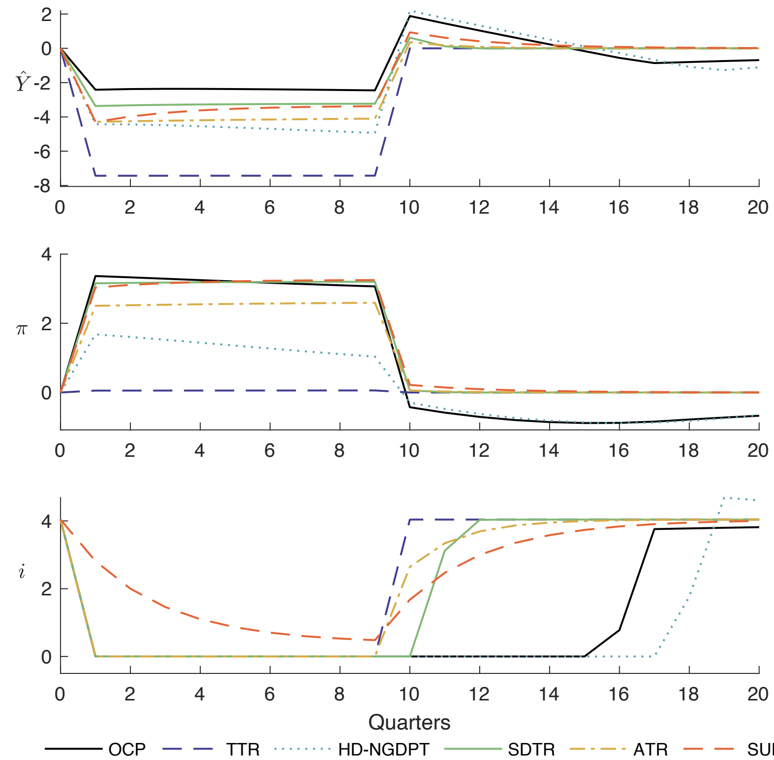
(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.3: Average impulse response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

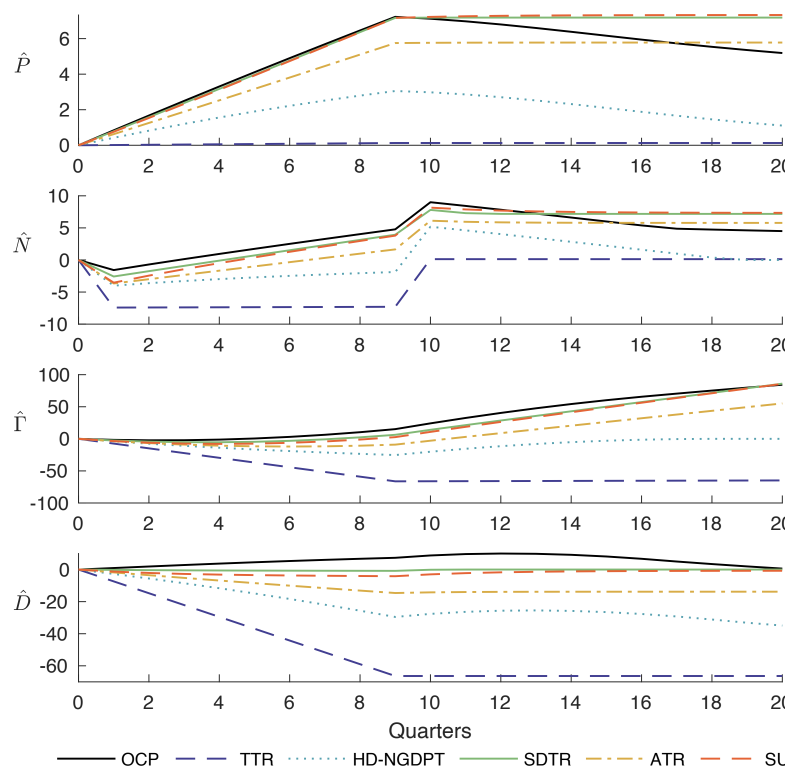
A.6.2 Simple New Keynesian model with 0% inflation drop

	Welfare Loss (1)	$\mathbb{E}_0[\tau + k_\tau - \tilde{T}]$ (2)	Volatility x (3)	Volatility π (4)	Volatility i (5)	Impact x (6)	Impact π (7)
OCP	$9.938\,10^{-4}$	15.471	$6.409\,10^{-3}$	$5.932\,10^{-4}$	$1.438\,10^{-3}$	−2.415	3.366
Panel A: baseline rules							
OCP	1.000	1.000	1.000	1.000	1.000	1.000	1.000
TTR	3.150	0.646	7.814	0.000	0.645	3.077	0.016
HD-NGDPT	1.577	1.108	3.613	0.203	1.098	1.832	0.499
SDTR	1.191	0.693	1.512	0.974	0.700	1.393	0.937
ATR	1.361	0.646	2.437	0.634	0.653	1.775	0.744
SUP	1.335	0.000	1.860	0.980	0.411	1.778	0.902
Panel B: additional rules							
PLT	3.213	0.646	7.971	0.000	0.645	3.105	0.000
NGDPT	3.213	0.646	7.971	0.000	0.645	3.105	0.000
TTRP	4.082	0.000	10.119	0.006	0.186	3.824	−0.121
TTRS-1	1.469	0.000	2.496	0.775	0.352	2.094	0.787
TTR-1	1.622	0.640	3.500	0.354	0.641	2.243	0.560
AIT	3.213	0.646	7.971	0.000	0.645	3.105	0.000
FRBNY Rule	67.154	0.782	124.781	28.239	1.700	12.431	−5.177
FLFG	2.908	0.795	7.196	0.012	0.799	2.518	0.185

Table A.5: Some metrics for selected interest rate rules in the simple two-equation NK model in the presence of a correlated cost push shock. All rows except the first show values normalised with respect to the optimal commitment policy (OCP, first row). Column (1) reports the welfare loss computed from a quadratic loss function for the central bank with equal weights; Column (2) displays the unconditional expected duration of the Zero Lower Bound (regimes 1 and 2); Columns (3)-(5) report a summary measure of deviations of the endogenous variables from target, computed according to Equation A.5; Finally, Columns (6) and (7) show the response on impact, in annual percentage points, of the output gap and inflation to a natural interest rate shock and a correlated cost push shock such that output falls by 7.5% and inflation remains constant under a Truncated Taylor Rule (TTR). Rule calibration reported in Table (A.2). The model is calibrated with the standard EW (2003) parameter values reported in footnote 21.

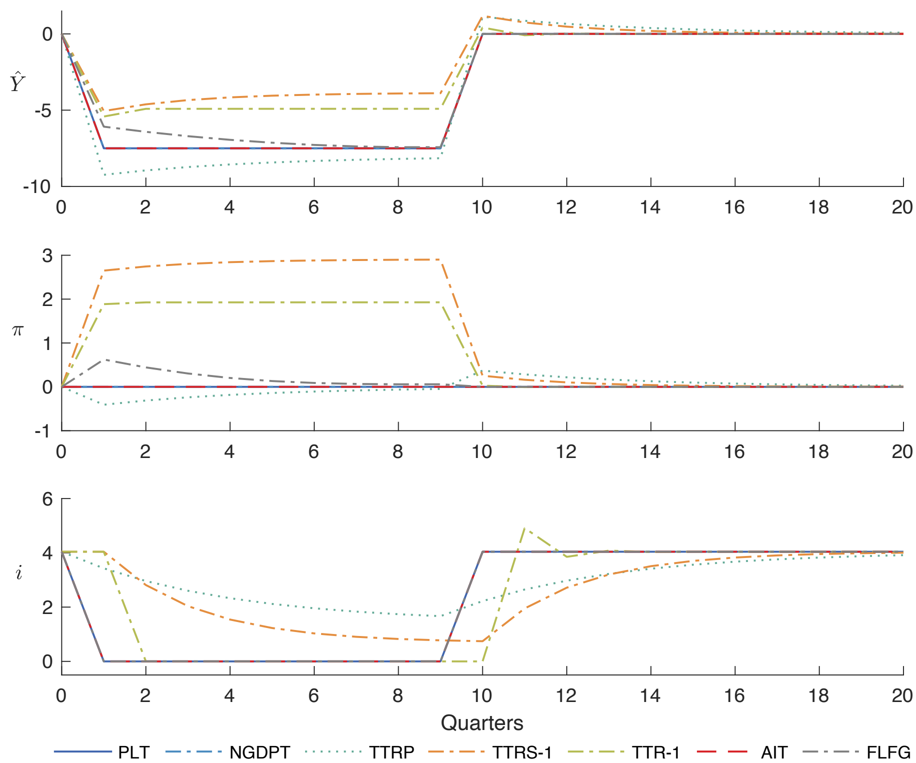


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

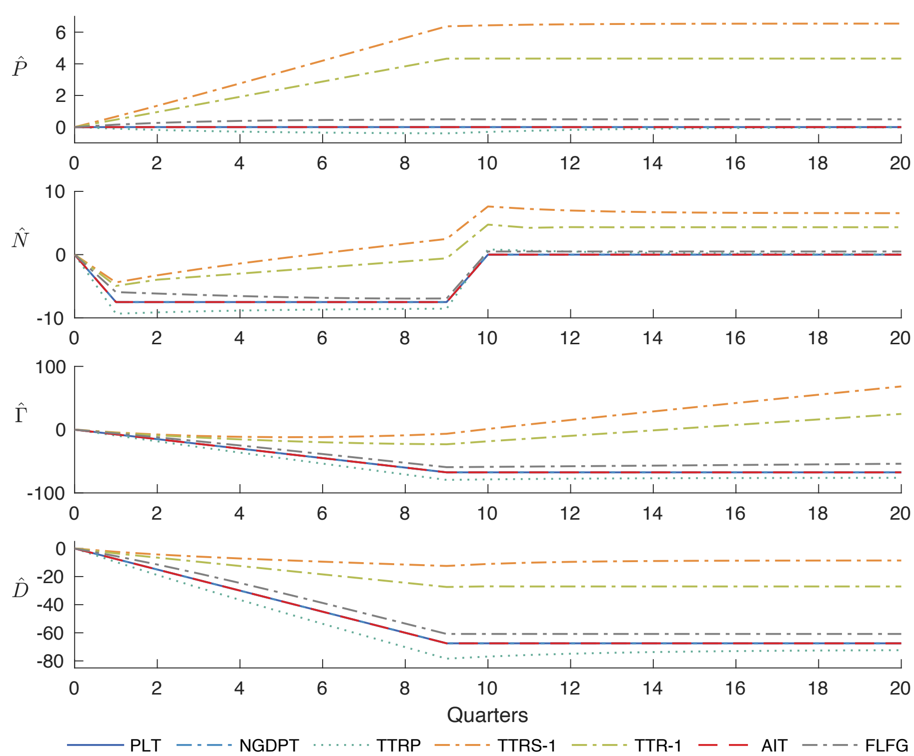


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.4: Dynamic response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation remains constant under a Truncated Taylor Rule (TTR). The natural interest rate reverts to the absorbing state after 10 quarters (10th contingency). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

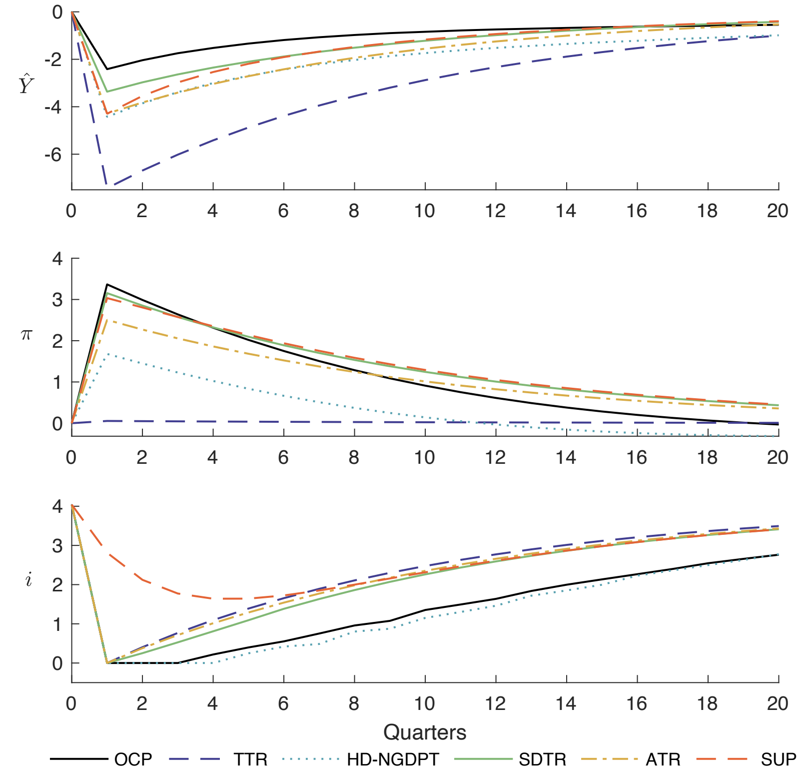


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

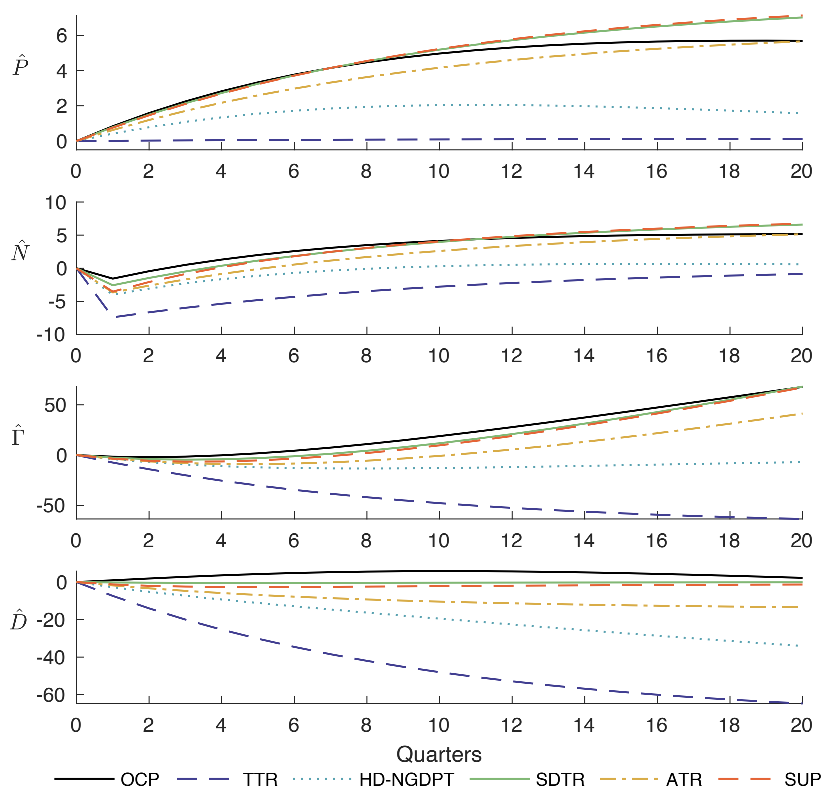


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.5: Dynamic response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation remains constant under a Truncated Taylor Rule (TTR). The natural interest rate reverts to the absorbing state after 10 quarters (10th contingency). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

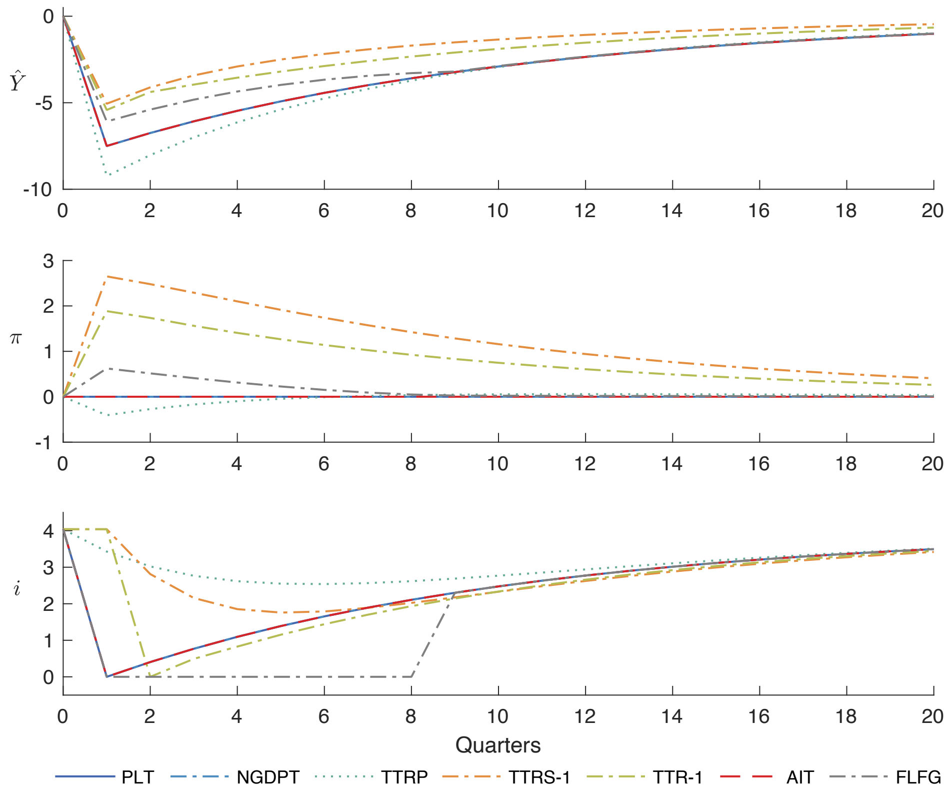


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

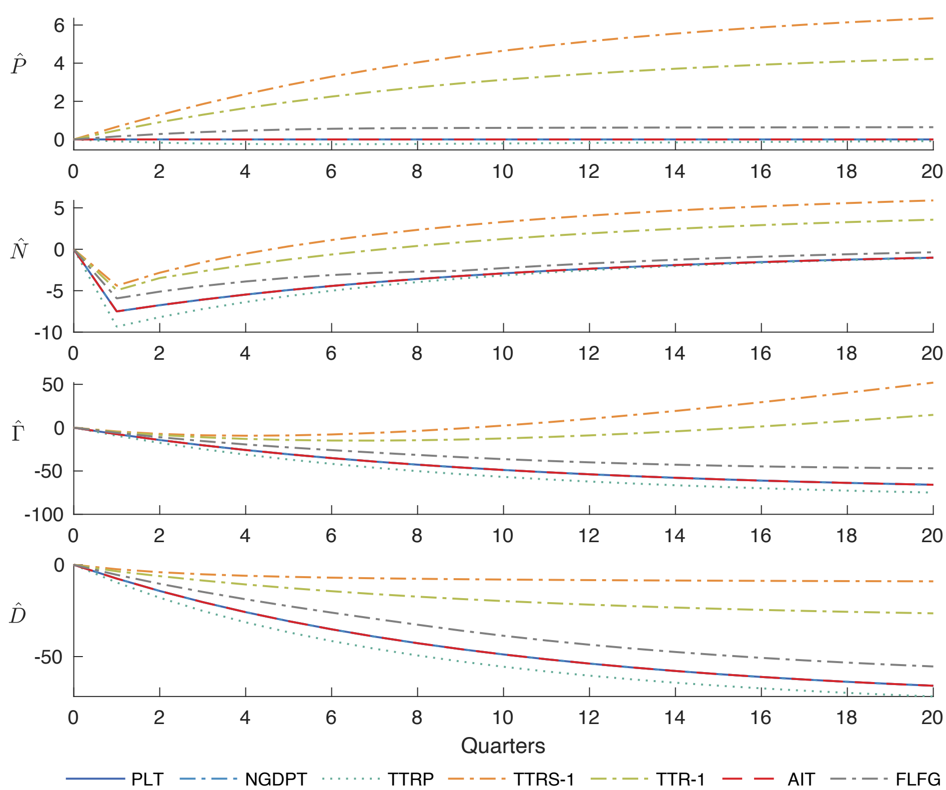


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.6: Average impulse response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation remains constant under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for *i* reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.



(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)



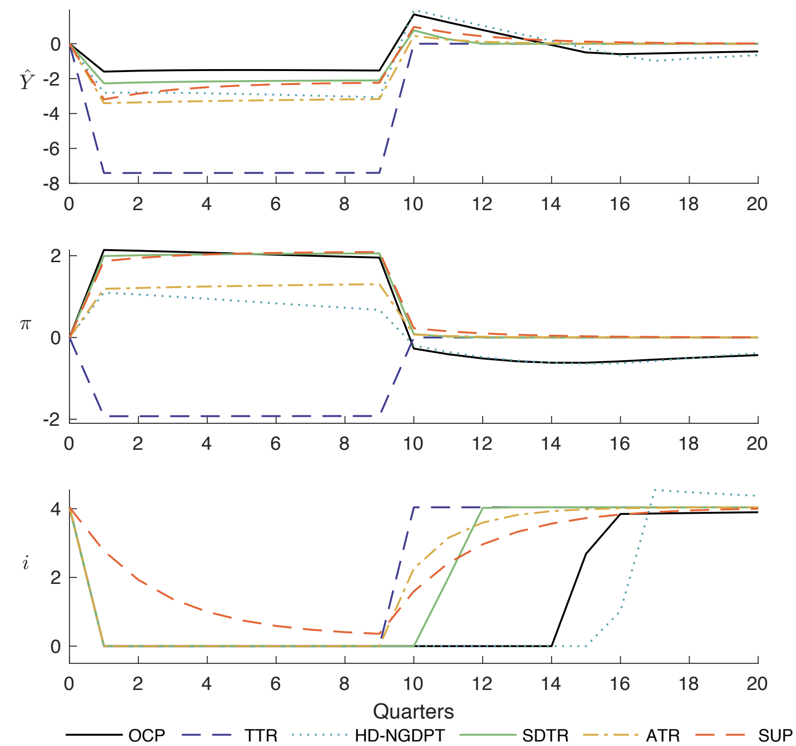
(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.7: Average impulse response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation remains constant under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

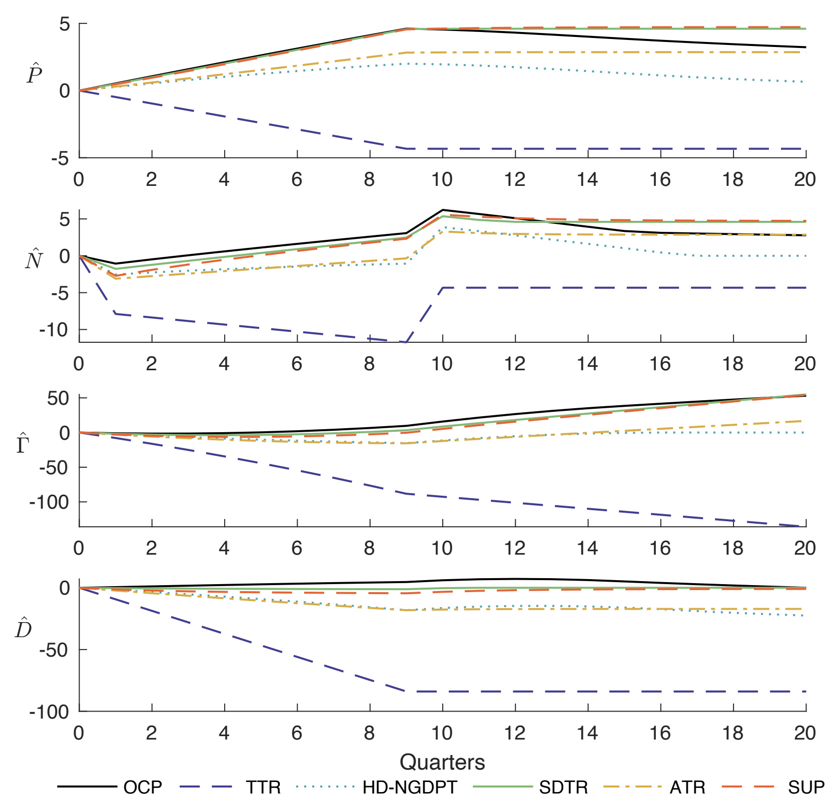
A.6.3 Simple New Keynesian model with -2% inflation drop

	Welfare Loss (1)	$\mathbb{E}_0[\tau + k_\tau - \tilde{T}]$ (2)	Volatility x (3)	Volatility π (4)	Volatility i (5)	Impact x (6)	Impact π (7)
OCP	$4.162\,10^{-4}$	14.073	$2.784\,10^{-3}$	$2.422\,10^{-4}$	$1.326\,10^{-3}$	−1.597	2.141
PANEL A: baseline rules							
OCP	1.000	1.000	1.000	1.000	1.000	1.000	1.000
TTR	7.975	0.711	17.869	0.865	0.699	4.638	−0.900
HD-NGDPT	1.537	1.096	3.372	0.217	1.083	1.762	0.512
SDTR	1.202	0.768	1.514	0.978	0.774	1.422	0.931
ATR	1.647	0.711	3.405	0.383	0.714	2.133	0.556
SUP	1.428	0.000	2.056	0.977	0.477	1.997	0.873
PANEL B: additional rules							
PLT	3.660	0.711	8.745	0.006	0.699	3.303	−0.101
NGDPT	3.504	0.711	8.377	0.002	0.699	3.229	−0.064
TTRP	4.645	0.000	11.085	0.017	0.242	4.179	−0.208
TTRS-1	1.796	0.000	3.523	0.555	0.403	2.603	0.623
TTR-1	3.336	0.703	7.976	0.001	0.695	3.377	−0.045
AIT	3.737	0.711	8.923	0.011	0.700	3.327	−0.121
FRBNY Rule	15.284	0.729	30.660	4.234	1.157	5.849	−1.549
FLFG	7.323	0.874	16.451	0.764	0.866	3.793	−0.635

Table A.6: Some metrics for selected interest rate rules in the simple two-equation NK model in the presence of a correlated cost push shock. All rows except the first show values normalised with respect to the optimal commitment policy (OCP, first row). Column (1) reports the welfare loss computed from a quadratic loss function for the central bank with equal weights; Column (2) displays the unconditional expected duration of the Zero Lower Bound (regimes 1 and 2); Columns (3)-(5) report a summary measure of deviations of the endogenous variables from target, computed according to Equation A.5; Finally, Columns (6) and (7) show the response on impact, in annual percentage points, of the output gap and inflation to a natural interest rate shock and a correlated cost push shock such that output falls by 7.5% and inflation by 2% under a Truncated Taylor Rule (TTR). Rule calibration reported in Table (A.2). The model is calibrated with the standard EW (2003) parameter values reported in footnote 21.

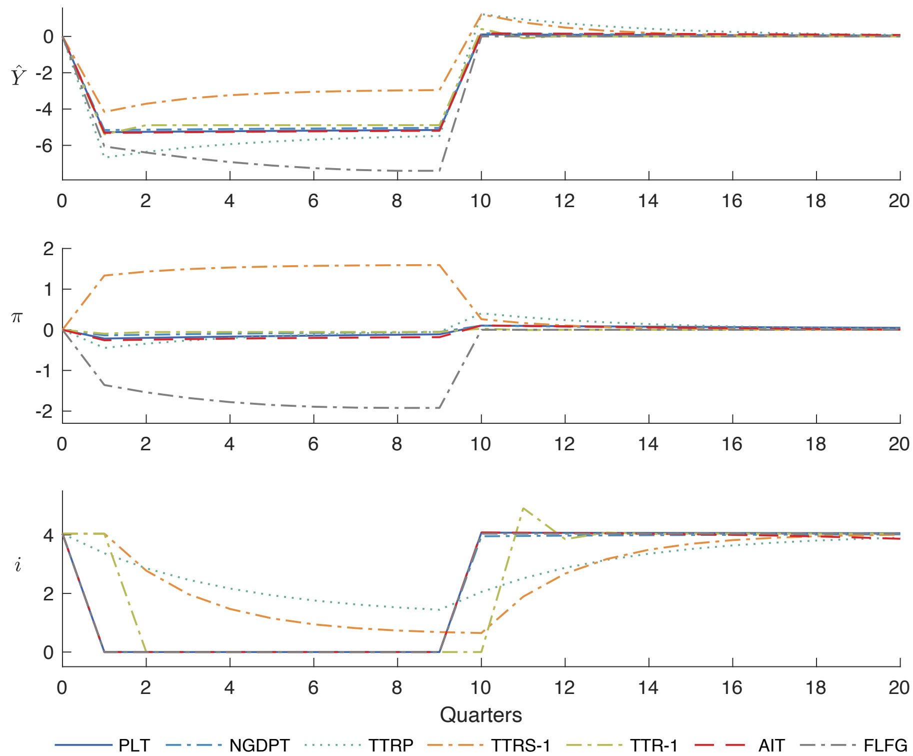


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

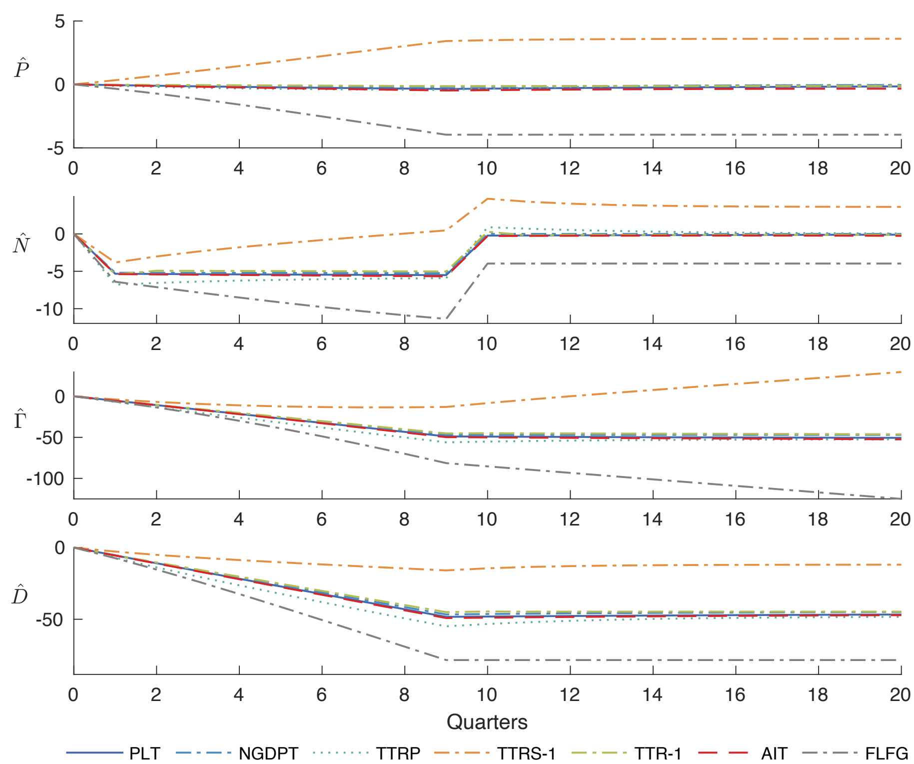


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.8: Dynamic response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 2% constant under a Truncated Taylor Rule (TTR). The natural interest rate reverts to the absorbing state after 10 quarters (10th contingency). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

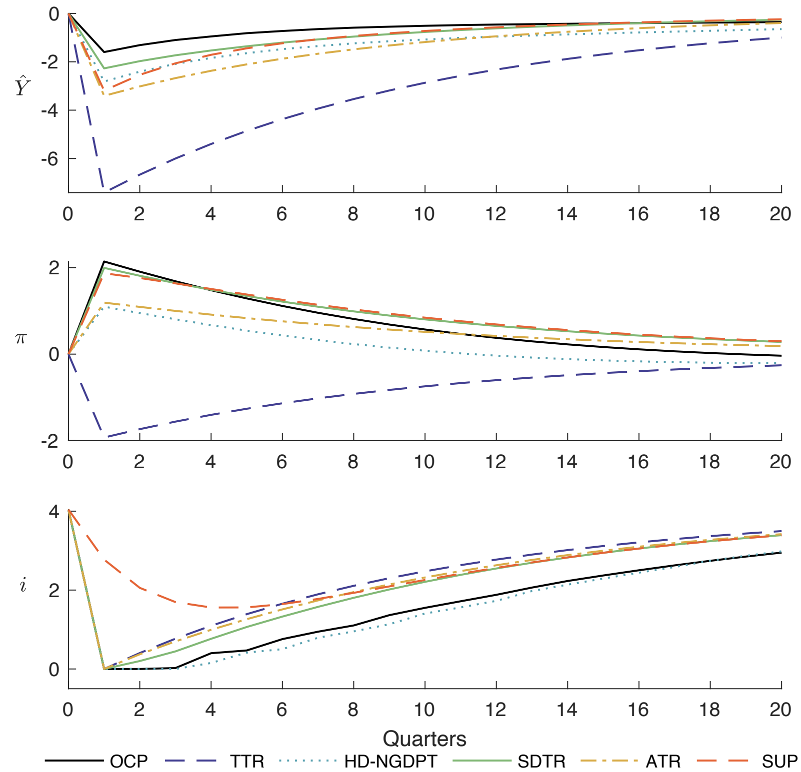


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

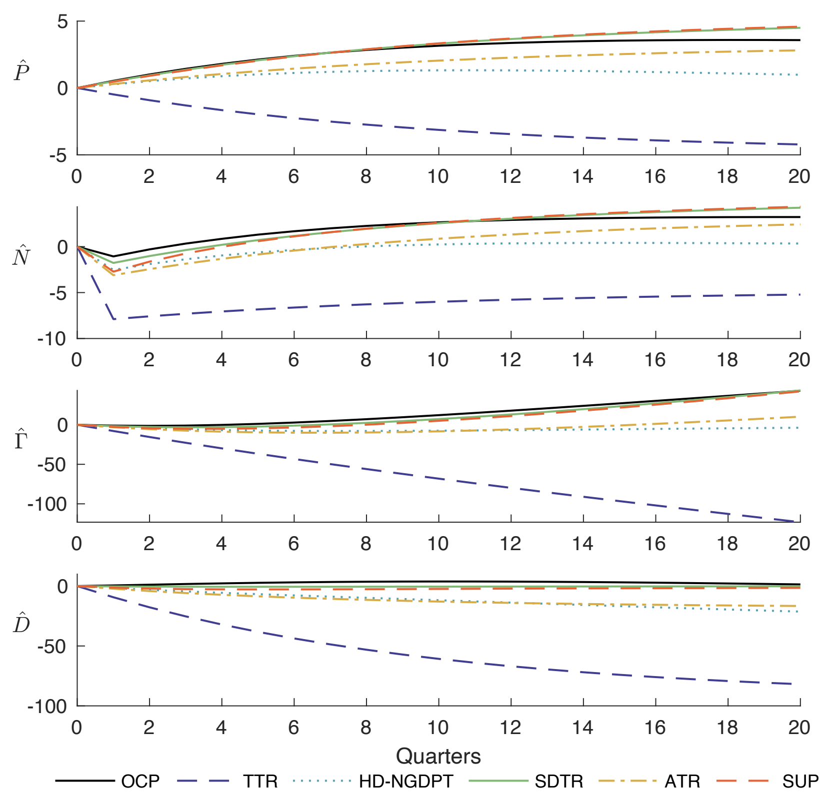


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.9: Dynamic response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 2% under a Truncated Taylor Rule (TTR). The natural interest rate reverts to the absorbing state after 10 quarters (10th contingency). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

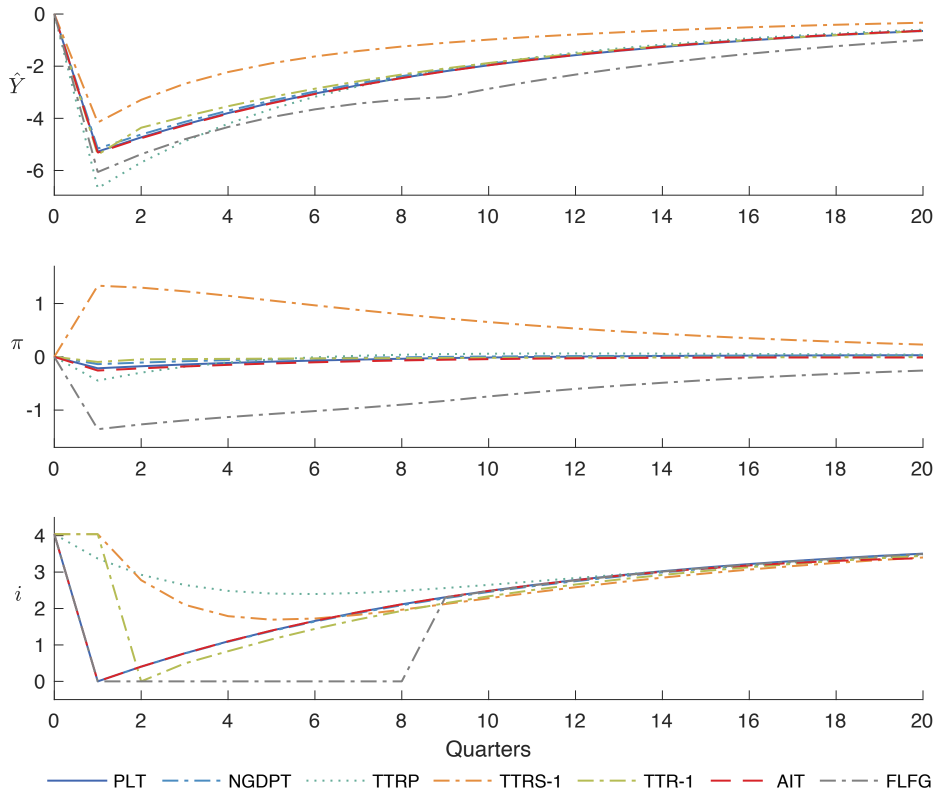


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

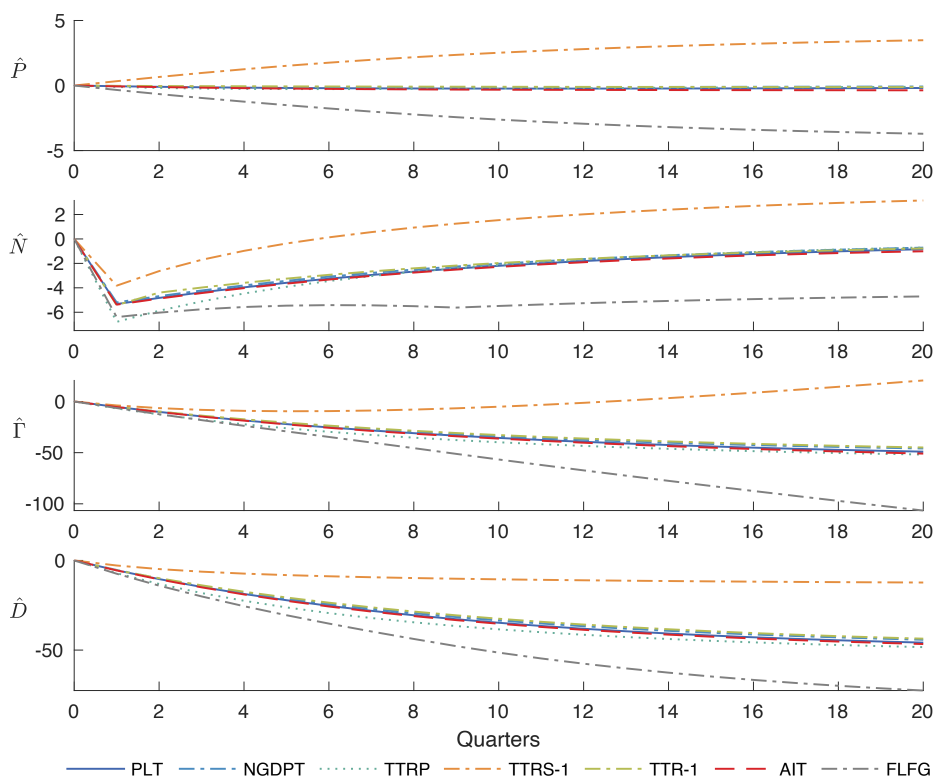


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.10: Average impulse response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 2% under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.



(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)



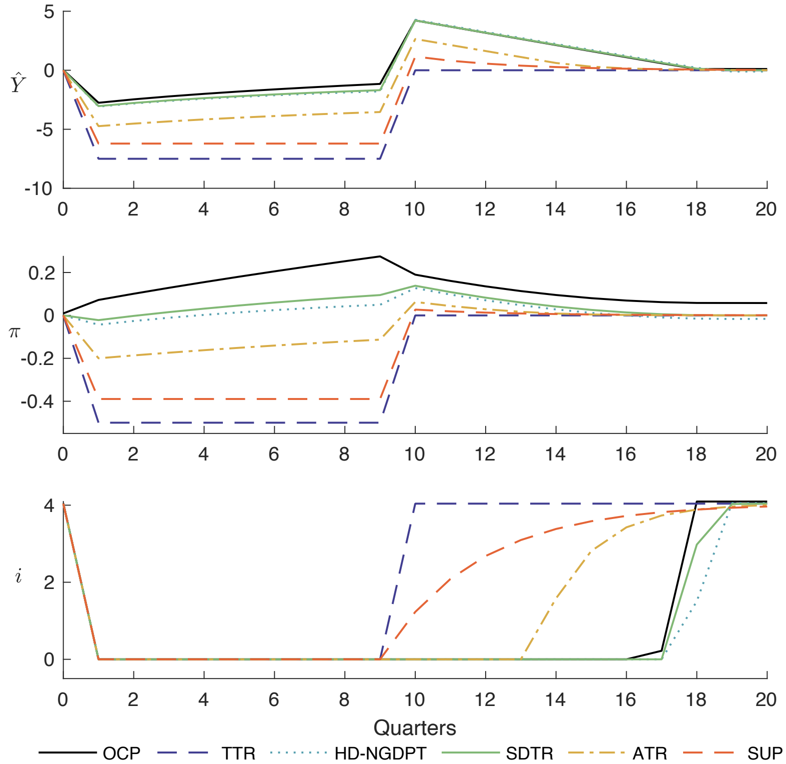
(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.11: Average impulse response of a natural interest rate shock and a correlated cost push shock in a simple two-equation NK model, under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 2% under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

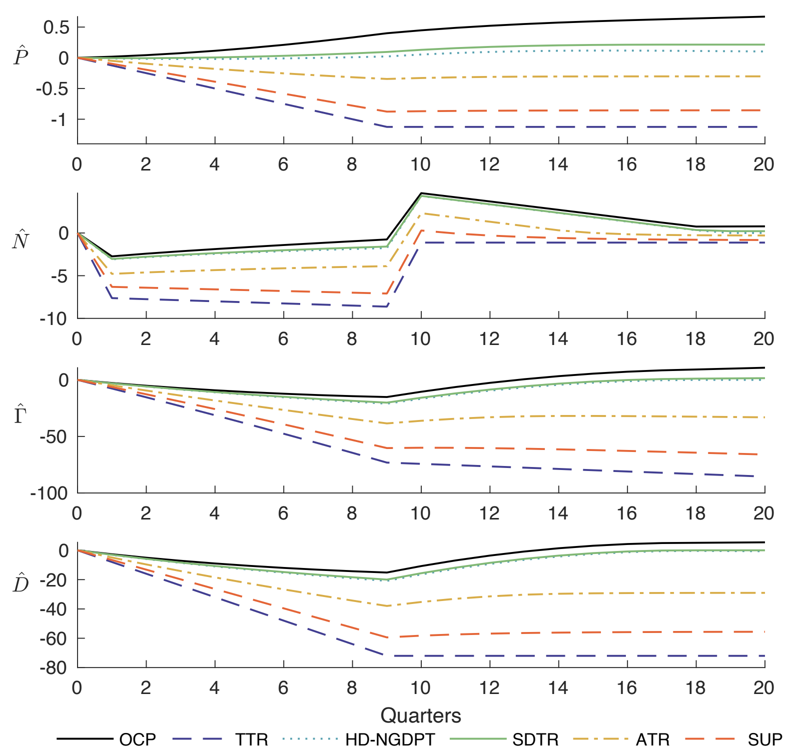
A.6.4 Simple New Keynesian model with -0.5% inflation drop – Price rigidity

	Welfare Loss (1)	$\mathbb{E}_0[\tau + k_\tau - \tilde{T}]$ (2)	Volatility x (3)	Volatility π (4)	Volatility i (5)	Impact x (6)	Impact π (7)
OCP	$4.896\,10^{-4}$	16.549	$7.661\,10^{-3}$	$1.081\,10^{-5}$	$1.524\,10^{-3}$	−2.755	0.072
PANEL A: baseline rules							
OCP	1.000	1.000	1.000	1.000	1.000	1.000	1.000
TTR	6.550	0.604	6.668	1.312	0.608	2.722	−6.929
HD-NGDPT	1.137	1.048	1.162	0.034	1.036	1.116	−0.600
SDTR	1.085	1.020	1.108	0.079	1.018	1.102	−0.304
ATR	1.876	0.794	1.916	0.105	0.824	1.719	−2.769
SUP	4.520	0.604	4.604	0.797	0.664	2.255	−5.398
PANEL B: additional rules							
PLT	6.328	0.604	6.445	1.133	0.608	2.692	−6.608
NGDPT	4.492	0.604	4.584	0.439	0.608	2.394	−4.589
TTRP	4.133	0.000	4.222	0.183	0.227	2.716	−3.517
TTRS-1	1.522	0.000	1.552	0.173	1.010	1.820	−2.890
TTR-1	5.874	0.598	5.981	1.149	0.605	2.740	−6.528
AIT	5.431	0.604	5.533	0.911	0.611	2.547	−5.984
FRBNY Rule	23.603	0.620	23.399	32.647	1.449	4.404	−8.159
FLFG	6.084	0.744	6.193	1.253	0.753	2.294	−6.257

Table A.7: Some metrics for selected interest rate rules in the simple two-equation NK model with increased price rigidity (lower κ). All rows except the first show values normalised with respect to the optimal commitment policy (OCP, first row). Column (1) reports the welfare loss computed from a quadratic loss function for the central bank with equal weights; Column (2) displays the unconditional expected duration of the Zero Lower Bound (regimes 1 and 2); Columns (3)-(5) report a summary measure of deviations of the endogenous variables from target, computed according to Equation A.5; Finally, Columns (6) and (7) show the response on impact, in annual percentage points, of the output gap and inflation to a natural interest rate shock such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). Rule calibration reported in Table (A.2). Remaining parameters are calibrated according to standard EW (2003) values reported in footnote 21.

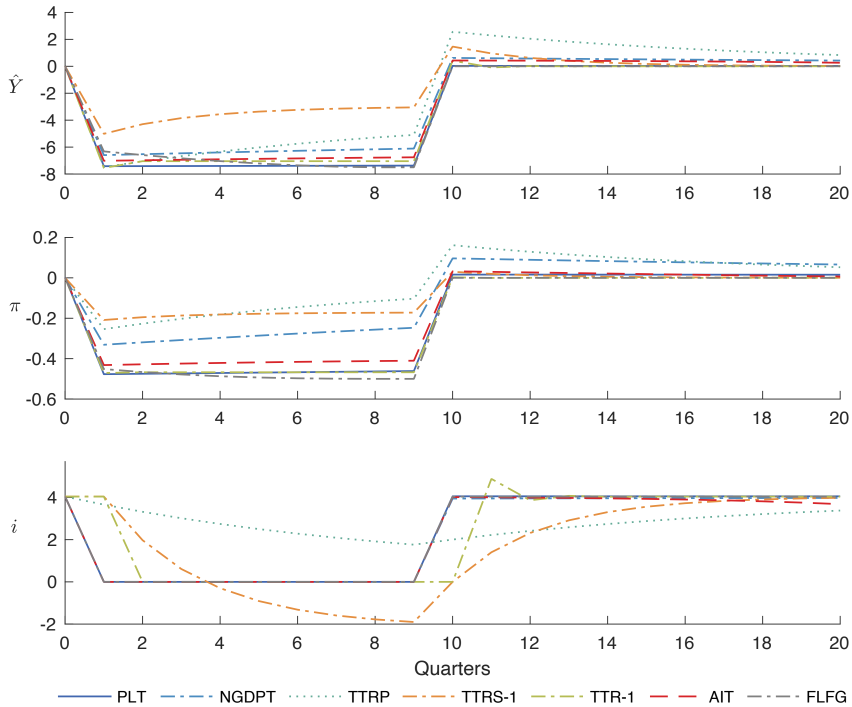


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

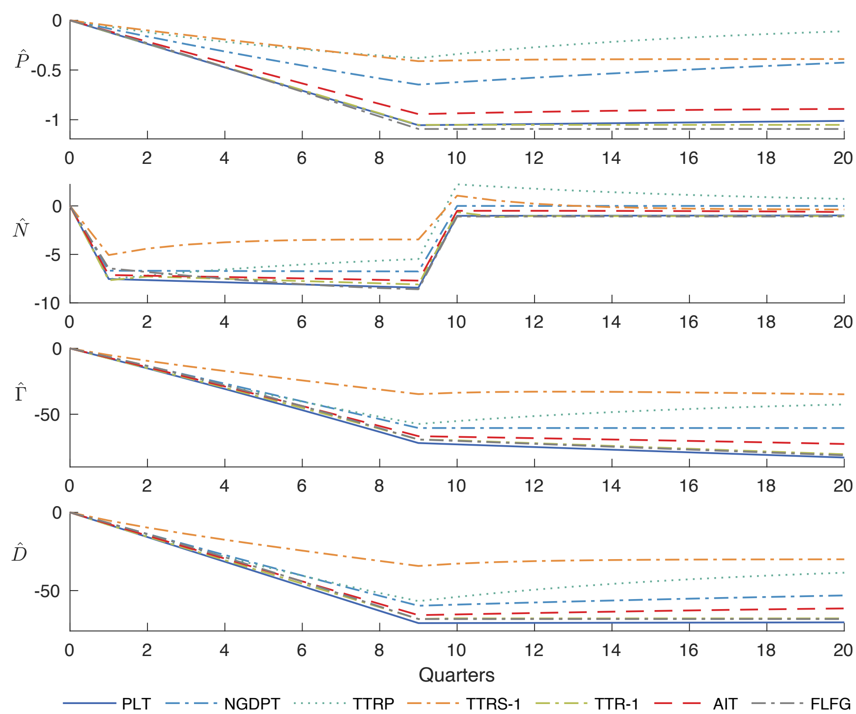


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.12: Dynamic response of a natural interest rate shock in a simple two-equation NK model with increased price rigidity (lower κ), under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). The natural interest rate reverts to the absorbing state after 10 quarters (10th contingency). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. Remaining parameters are calibrated according to standard EW (2003) values reported in footnote 21.

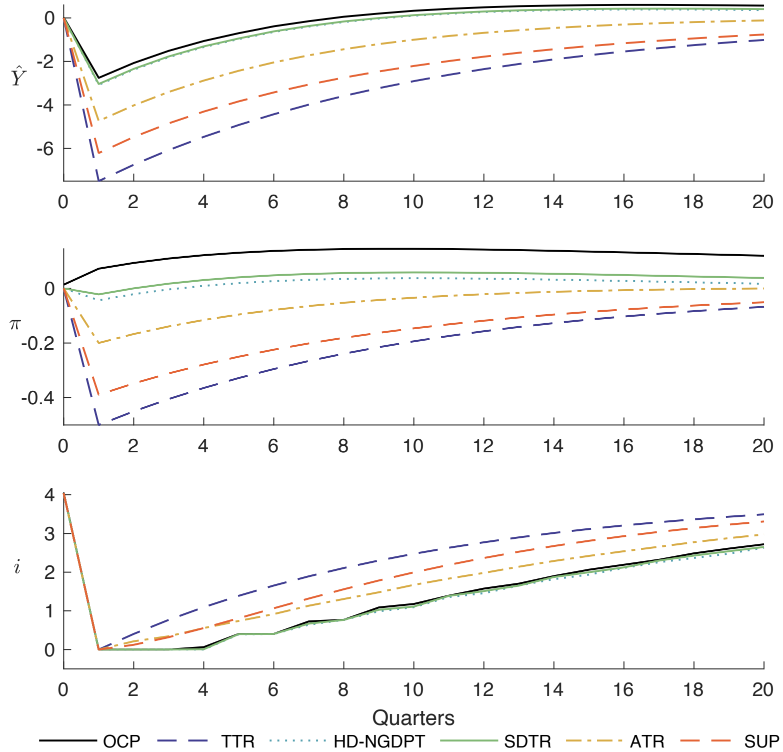


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

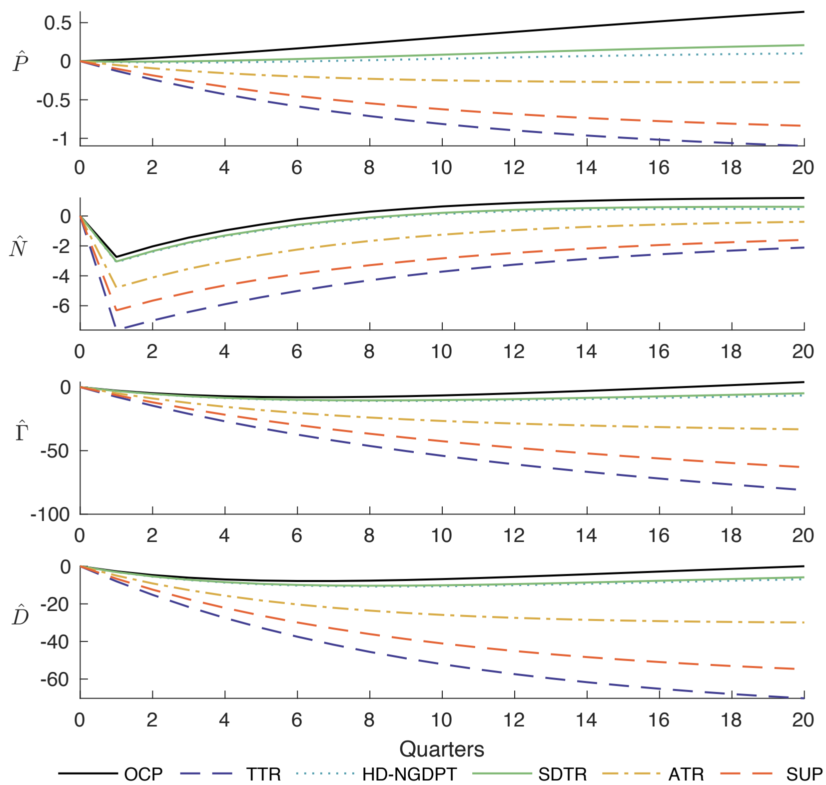


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.13: Dynamic response of a natural interest rate shock in a simple two-equation NK model with increased price rigidity (lower κ), under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). The natural interest rate reverts to the absorbing state after 10 quarters (10th contingency). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. Remaining parameters are calibrated according to standard EW (2003) values reported in footnote 21.

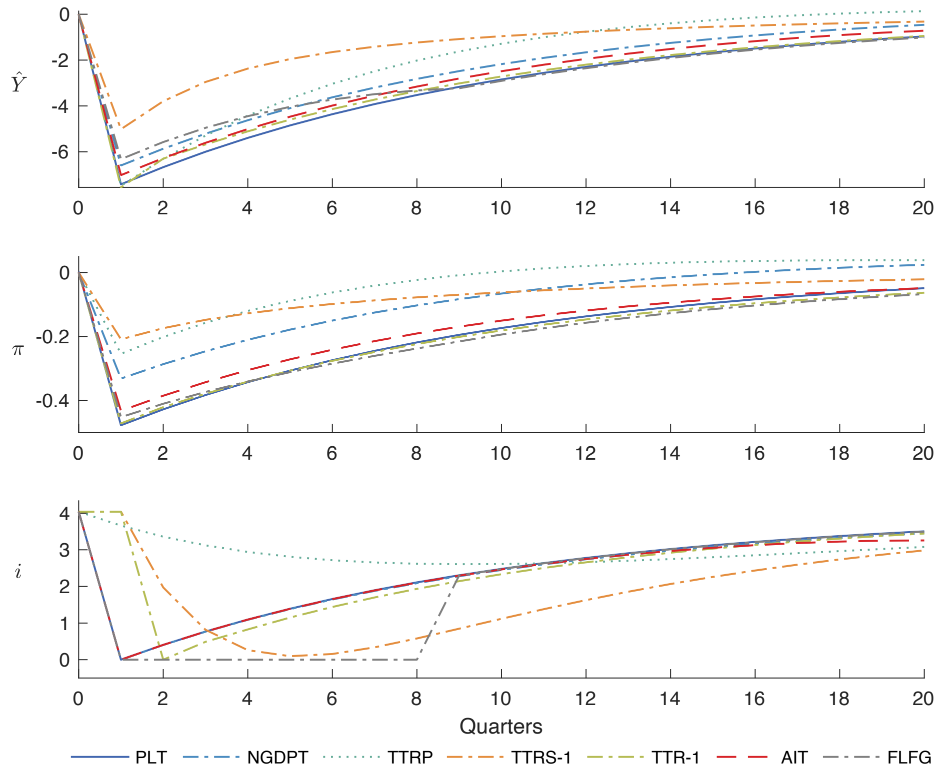


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

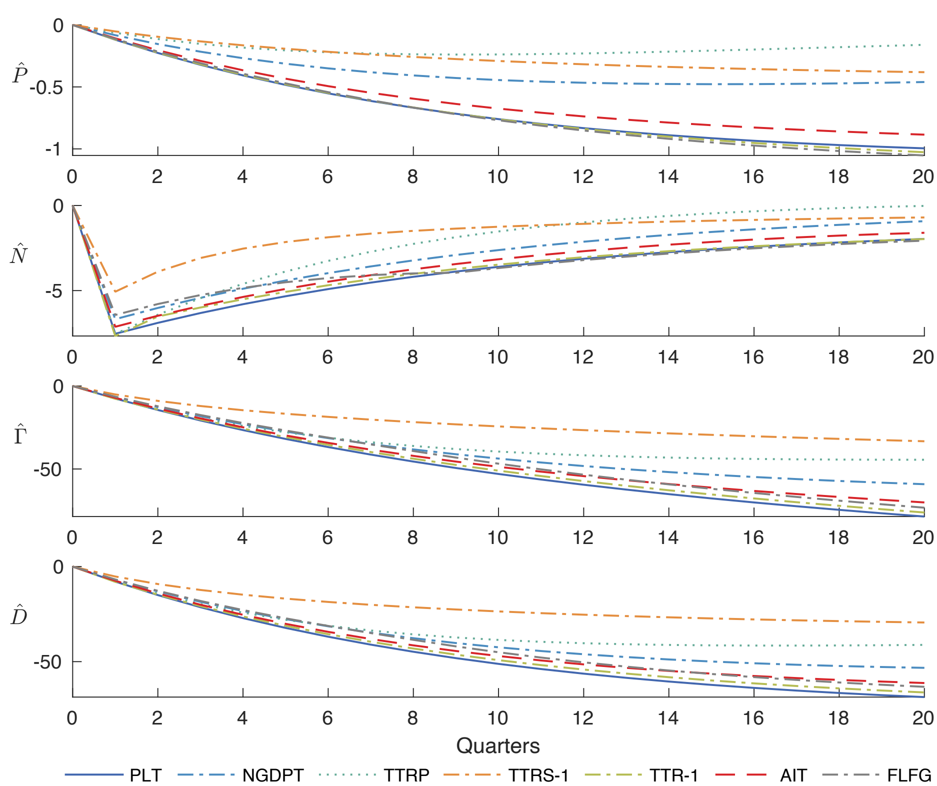


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.14: Average impulse response of a natural interest rate shock in a simple two-equation NK model with increased price rigidity (lower κ), under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.



(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)



(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.15: Average impulse response of a natural interest rate shock in a simple two-equation NK model with increased price rigidity (lower κ), under different policy rules. Shocks are calibrated such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). The vertical axes report percentage deviations from steady state (annualised figures). The vertical axis for i reports annualised percentage points. The list of acronyms is detailed in Table 1. The parametrisation is reported in Table A.2.

A.6.5 Simple New Keynesian model with -0.5% inflation drop – optimised policy rules

	Welfare Loss (1)	$\mathbb{E}_0[\tau + k_\tau - \tilde{T}]$ (2)	Volatility x (3)	Volatility π (4)	Volatility i (5)	Impact x (6)	Impact π (7)
OCP	$8.252\,10^{-4}$	15.257	$5.356\,10^{-3}$	$4.904\,10^{-4}$	$1.411\,10^{-3}$	−2.208	3.059
PANEL A: baseline rules							
OCP	1.000	1.000	1.000	1.000	1.000	1.000	1.000
TTR	3.800	0.655	9.335	0.022	0.657	3.364	−0.144
HD-NGDPT	1.568	1.099	3.563	0.207	1.094	1.818	0.502
SDTR	1.194	0.703	1.514	0.975	0.716	1.400	0.936
ATR	1.169	0.709	1.070	1.237	0.721	1.191	1.053
SUP	1.181	0.590	1.101	1.235	0.689	1.147	1.068
PANEL B: additional rules							
PLT	3.294	0.655	8.118	0.000	0.657	3.145	−0.018
NGDPT	3.267	0.655	8.054	0.000	0.657	3.132	−0.011
TTRP	3.269	0.655	8.058	0.000	0.657	3.133	−0.012
TTRS-1	1.353	0.000	0.295	2.076	0.647	0.882	1.358
TTR-1	1.732	0.649	3.845	0.289	0.651	2.360	0.504
AIT	3.278	0.655	8.079	0.000	0.657	3.138	−0.014
FLFG	3.105	1.019	7.391	0.180	1.023	1.558	0.597

Table A.8: Some metrics for selected interest rate rules in the simple two-equation NK model with optimised policy rules. All rows except the first show values normalised with respect to the optimal commitment policy (OCP, first row). Column (1) reports the welfare loss computed from a quadratic loss function for the central bank with equal weights; Column (2) displays the unconditional expected duration of the Zero Lower Bound (regimes 1 and 2); Columns (3)-(5) report a summary measure of deviations of the endogenous variables from target, computed according to Equation A.5; Finally, Columns (6) and (7) show the response on impact, in annual percentage points, of the output gap and inflation to a natural interest rate shock such that output falls by 7.5% and inflation by 0.5% under a Truncated Taylor Rule (TTR). Rule calibration reported in Table (A.2). The model is calibrated with the standard EW (2003) parameter values reported in footnote 21.

A.6.6 FRBNY DSGE Model

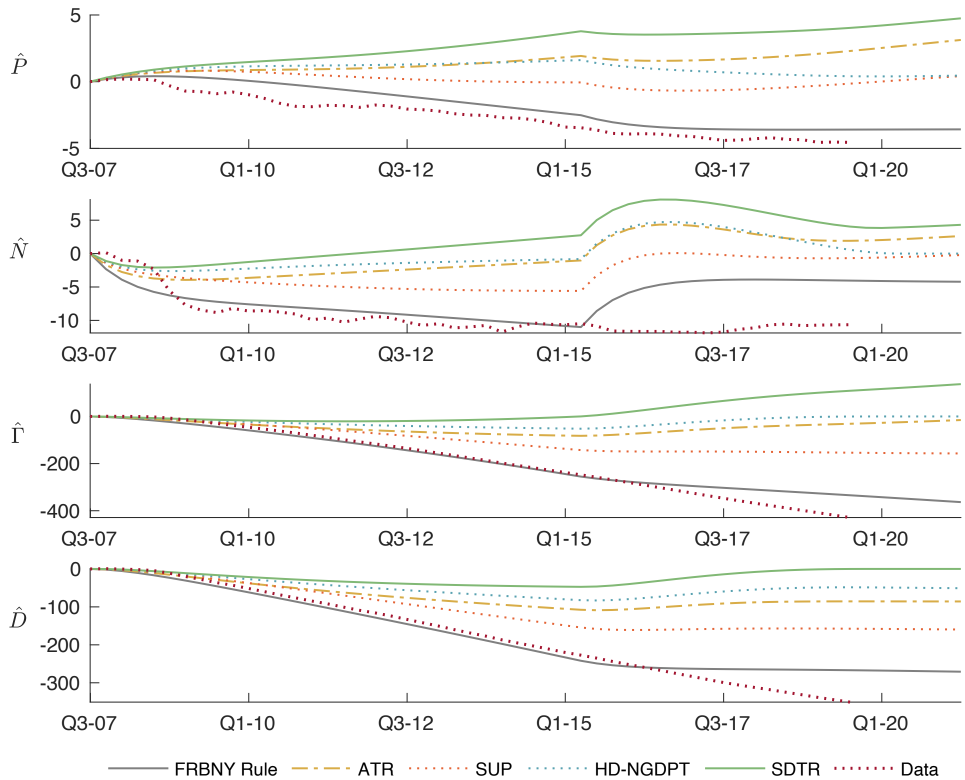


Figure A.16: Dynamic response to a preference shock and a correlated cost push shock in FRBNY model, under baseline policy rules. Lines show paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$). The two-state Markov shocks switch to low state in Q4-07 and revert to the absorbing state after 32 quarters (32nd contingency). The vertical axis for P reports percent deviations from its trend. The vertical axis for $\hat{N}$ reports deviations from detrended steady state, in percentage points (annualised figures). The vertical axes for $\hat{\Gamma}$ and $\hat{D}$ report deviations from initial levels (Q3-2007 = 0). The horizontal axis shows quarter and calendar year. See Section A.4.2 for calibration. The list of acronyms is detailed in Table 1. FRBNY Rule refers to Equation (A.6).

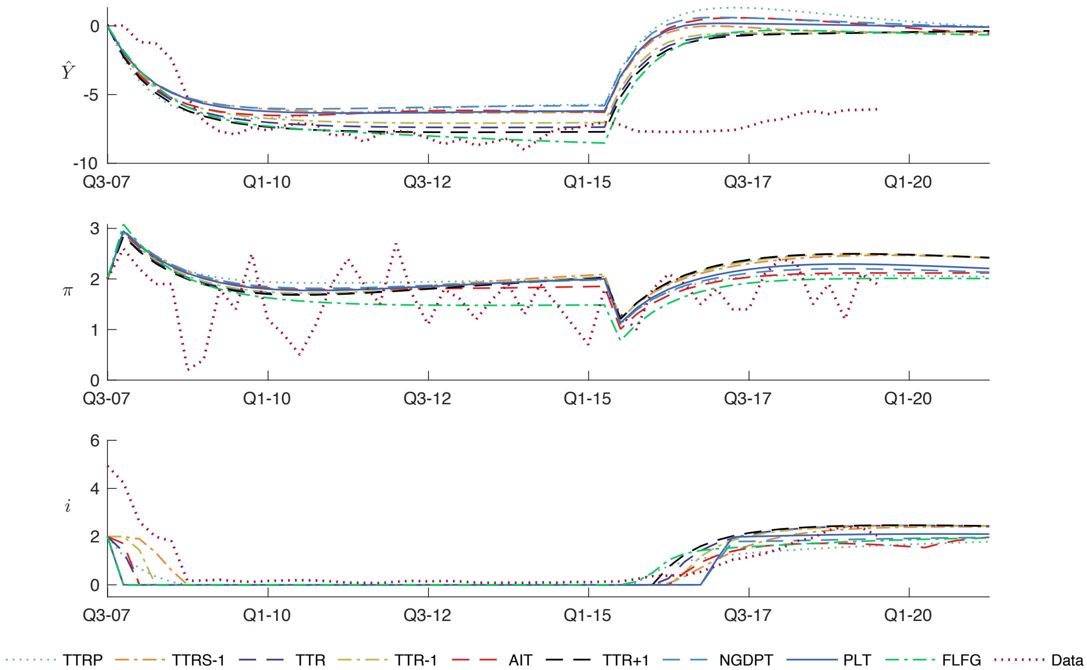


Figure A.17: Dynamic response to a preference shock and a correlated cost push shock in FRBNY model, under additional policy rules. Lines show paths for output ($\hat{Y}_t$), inflation (π), the nominal interest rate (i), and nominal GDP ($\hat{N}$). Dotted red line is data. The two-state Markov shocks switch to low state in Q4-07 and reverts to the absorbing state after 32 quarters (32nd contingency). The vertical axes for $\hat{Y}_t$ and $\hat{N}$ report deviations from detrended steady state, in percentage points (annualised figures). The vertical axes for π and i report annualised percentage points. The horizontal axis shows quarter and calendar year. See Section A.2 for details on data and Section A.4.2 for calibration. The list of acronyms is detailed in Table 1.

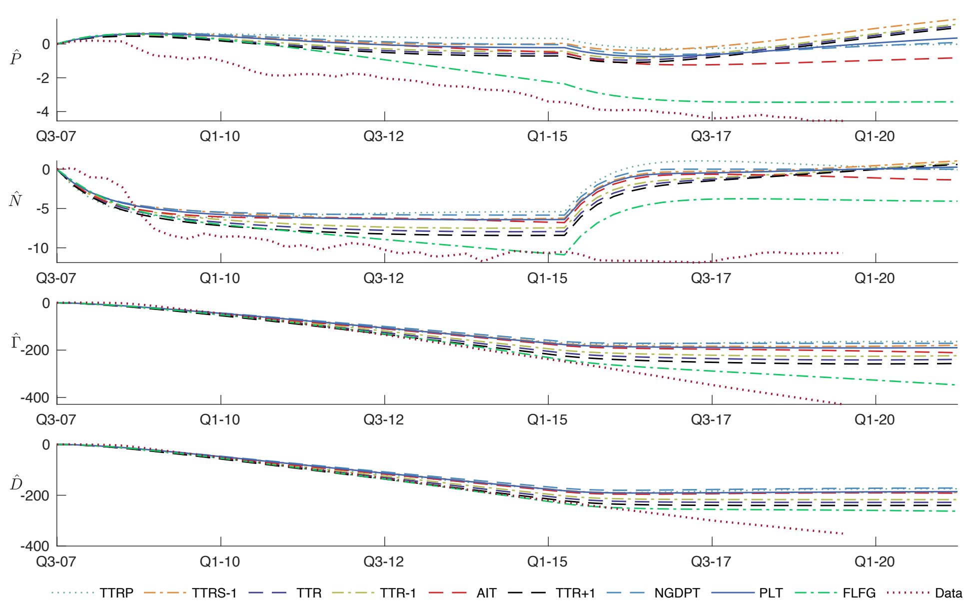
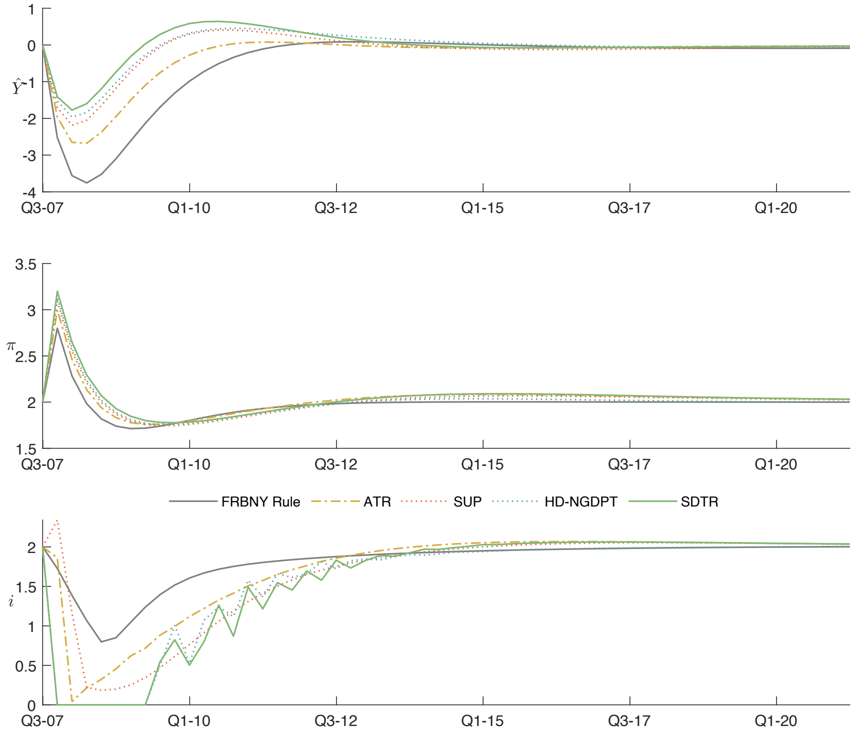
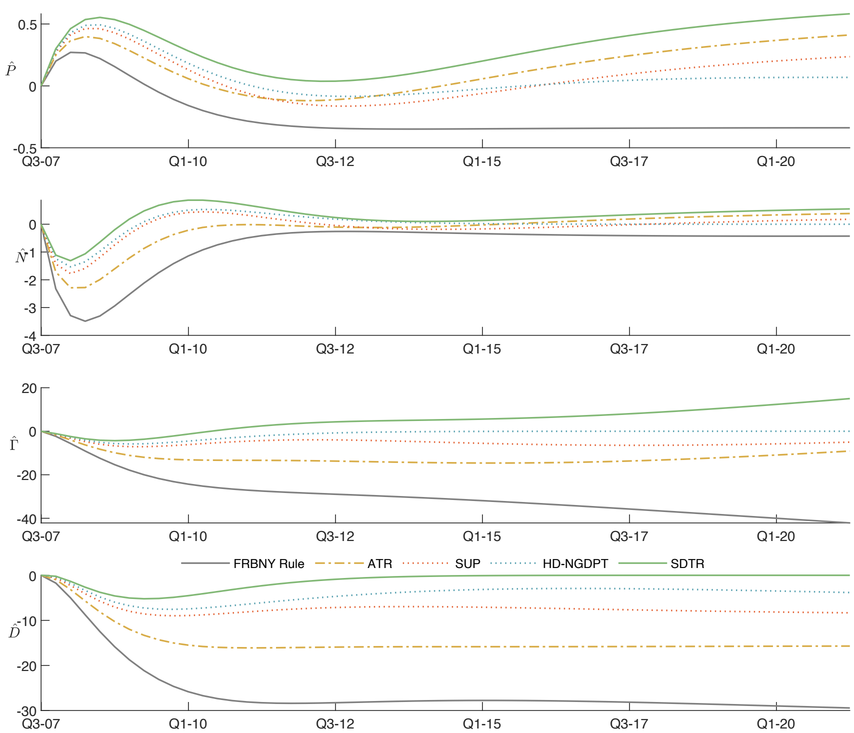


Figure A.18: Dynamic response to a preference shock and a correlated cost push shock in FRBNY model, under additional policy rules. Lines show paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$). The two-state Markov shocks switch to low state in Q4-07 and revert to the absorbing state after 32 quarters (32nd contingency). The vertical axis for P reports percent deviations from its trend. The vertical axis for $\hat{N}$ reports deviations from detrended steady state, in percentage points (annualised figures). The vertical axes for $\hat{\Gamma}$ and $\hat{D}$ report deviations from initial levels (Q3-2007 = 0). The horizontal axis shows quarter and calendar year. The horizontal axis shows quarter and calendar year. See Section A.4.2 for calibration. The list of acronyms is detailed in Table 1.

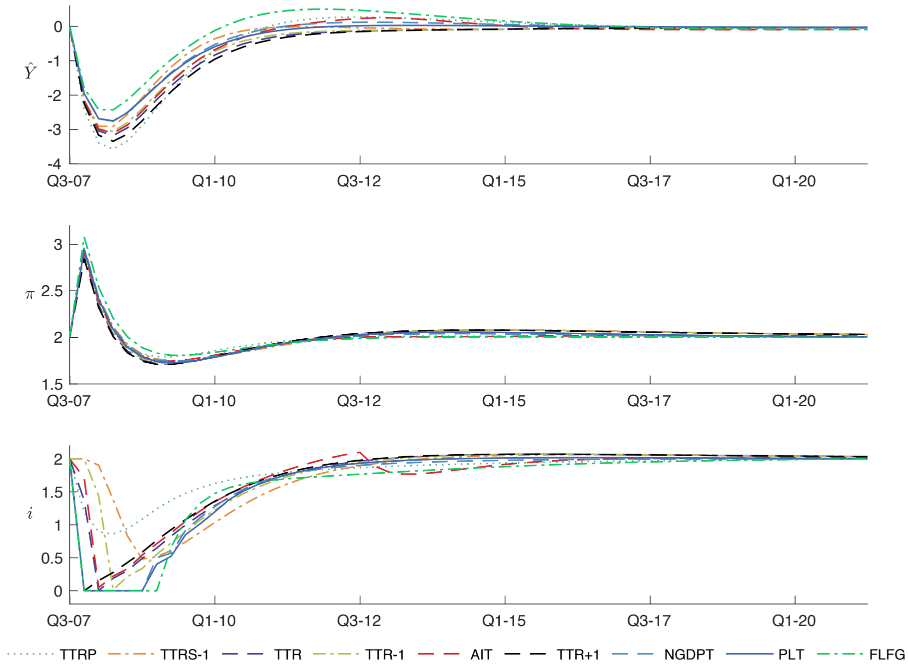


(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)

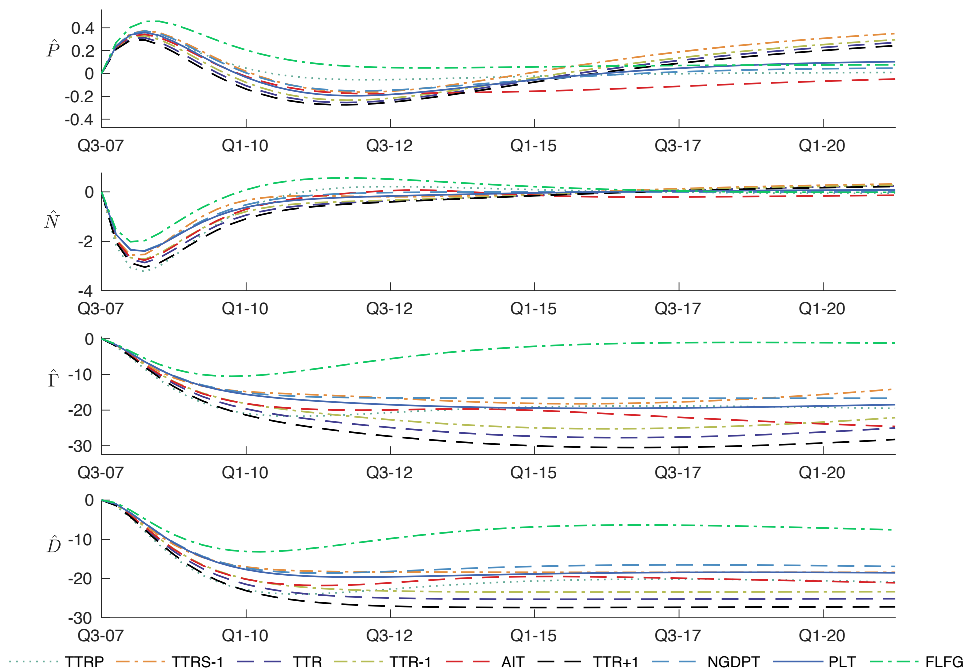


(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.19: Average impulse response to a preference shock and a correlated cost push shock in FRBNY model, under baseline policy rules. The vertical axis for $\hat{Y}$ reports deviations from detrended steady state, in percentage points (annualised figures). The vertical axes for π and i report annualised percentage points. The vertical axis for P reports percent deviations from its trend. The vertical axis for $\hat{N}$ reports deviations from detrended steady state, in percentage points (annualised figures). The vertical axes for $\hat{\Gamma}$ and $\hat{D}$ report deviations from initial levels (Q3-2007 = 0). The horizontal axis shows quarter and calendar year. See Section A.4.2 for calibration. The list of acronyms is detailed in Table 1. FRBNY rule refers to Equation (A.6).



(a) Paths for output ($\hat{Y}$), inflation (π) and nominal interest rate (i)



(b) Paths for price level ($\hat{P}$), nominal output ($\hat{N}$), cumulated nominal output ($\hat{\Gamma}$) and the Dual Mandate index ($\hat{D}$)

Figure A.20: Average impulse response to a preference shock and a correlated cost push shock in FRBNY model, under additional policy rules. The vertical axis for $\hat{Y}$ reports deviations from detrended steady state, in percentage points (annualised figures). The vertical axes for π and i report annualised percentage points. The vertical axis for P reports percent deviations from its trend. The vertical axis for $\hat{N}$ reports deviations from detrended steady state, in percentage points (annualised figures). The vertical axes for $\hat{\Gamma}$ and $\hat{D}$ report deviations from initial levels (Q3-2007 = 0). The horizontal axis shows quarter and calendar year. See Section A.4.2 for calibration. The list of acronyms is detailed in Table 1.