# Math Review Part II
# Problem Set 1: Real Analysis

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1. The **inverse** image of a set $T \subset Y$ under a function $f : X \to Y$ is defined as

$$f^{-}(T) = \{x \in X : f(x) \in T\}$$

   Let $T_1, T_2 \subset Y$, prove that $f^{-}(T_1 \cup T_2) = f^{-}(T_1) \cup f^{-}(T_2)$.

2. Prove that in a metric space $(X, d)$, any open ball is an open set.

3. Let $(X, d)$ be a metric space. Suppose $x_n \to x$ and $x_n \to x'$, then prove that $x = x'$.

4. Prove the following statement:
   Let $(x_n)$ be a sequence in $(\mathbb{R}^k, d_2)$. The sequence $(x_n)$ converges to $x \in \mathbb{R}^k$ *iff* the sequence $(x_n^i)$ converges to $x^i$ in $(\mathbb{R}, d_2)$ for any $i \in \{1, 2, ..., k\}$.

5. In $(\mathbb{R}, d_2)$, let there be two convergent sequences $x_n \to x$ and $y_n \to y$. Then prove that $x_n y_n \to xy$.

6. Let $(X, d)$ be a metric space, and $S$ a subset of $X$. If $S$ is compact in $(X, d)$, then $S$ is closed in $(X, d)$.

7. Let $(X, d)$ be a metric space, and $S \subset Y \subset X$. If $S$ is closed in $(X, d)$ and $Y$ is compact in $(X, d)$, then $S$ is compact in $(X, d)$.

8. Prove that any closed interval $[a, b]$ is compact in $(\mathbb{R}, d_2)$.

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