ECE250: Signals and Systems
Assignment 2

Issued on: September 28, 2022

Max-Marks : 50

Due by: October 4, 2022 (5:30 pm)

Guidelines for submission

Theory Problems:

- Submit a hard copy of your solutions in the wooden box kept on the 3rd Floor of Old Academic Block (right side of the lift).
- Write your Name, Roll No. and Group No. (as assigned for your tutorials) on the hard copy of your solutions.
- Do all questions in sequence.
- Use A4 sheets (Plain)
- Staple your sheets properly

Programming Problems:

- Use Matlab or python to solve the programming problems.
- For your solutions, you need to submit a zipped file on Google classroom with the following:
  - program files (.m) or (.ipynb) with all dependencies.
  - a report (.pdf) with your coding outputs and generated plots. The report should be self-complete with all your assumptions and inferences clearly specified.
- Before submission, please name your zipped file as: "A1_GroupNo_RollNo_Name.zip".
- Codes/reports submitted without a zipped file or without following the naming convention will NOT be checked.

Theory Problems:

1) [CO2] (8 points) Consider an LTI system with input and output related through the equation (1)

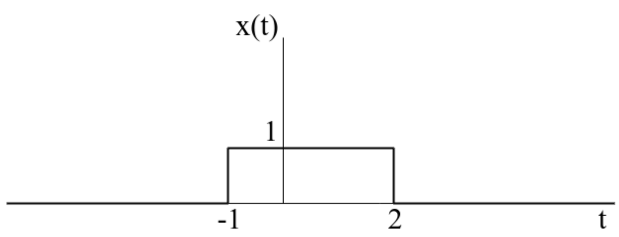

Figure 1: Signal $x(t)$

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau \tag{1}$$

i) (4 pts) What is the impulse response $h(t)$ for this system.

ii) (4 pts) Determine the response of the system when the input $x(t)$ is as shown in Figure 1.

2) [CO2] (8 points) Consider a system whose input $x(t)$ and output $y(t)$ satisfy the first-order differential equation (2).

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \tag{2}$$

The system satisfies the condition of initial rest.

i) (1 pt) Determine the system output $y_1(t)$ when the input is $x_1(t) = e^{3t}u(t)$.

ii) (3 pts) Determine the system output $y_2(t)$ when the input is $x_2(t) = \alpha e^{3t}u(t) + \beta e^{2t}u(t)$, where α and β are real numbers.

iii) (1 pt) Determine the system output $y_3(t)$ when the input is $x_3(t) = Ke^{2t}u(t)$.

iv) (3 pts) Determine the system output $y_4(t)$ when the input is $x_4(t) = Ke^{2(t-T)}u(t-T)$. Show that $y_4(t) = y_3(t-T)$.

3) [CO2] (10 points) We are given a certain linear time-invariant system with impulse response $h_0(t)$. We are told that when the input is $x_0(t)$ the output is $y_0(t)$, which is sketched in Figure 2. We are given the following set of inputs to linear time invariant systems with the indicated impulse responses:

i) (2 pts) $x(t) = 2x_0(t)$; $h(t) = h_0(t)$

ii) (2 pts) $x(t) = x_0(t) - x_0(t-2)$; $h(t) = h_0(t)$

iii) (2 pts) $x(t) = x_0(t-2)$; $h(t) = h_0(t+1)$

iv) (2 pts) $x(t) = x_0(-t) - x_0(t-2)$; $h(t) = h_0(t)$

v) (2 pts) $x(t) = x_0(-t) - x_0(t-2)$; $h(t) = h_0(-t)$

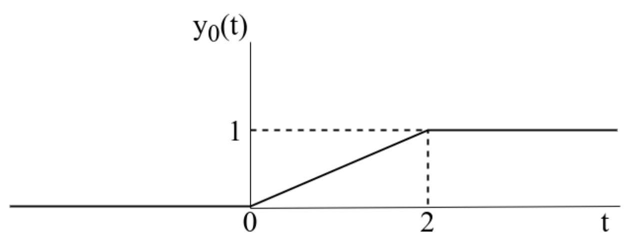

Figure 2: Signal $y_0(t)$

In each of these cases, determine whether or not we have enough information to determine the output $y(t)$ when the input $x(t)$ and the system has impulse response $h(t)$. If it is possible to determine y(t), provide an accurate sketch of it with numerical values clearly indicated on the graph.

4) [CO2] (9 points) Evaluate $y[n] = x[n] \circledast h[n]$, where $x[n]$ and $h[n]$ are shown in Figure 3.

i) (5 pts) by an analytical technique.

ii) (4 pts) by a graphical method.

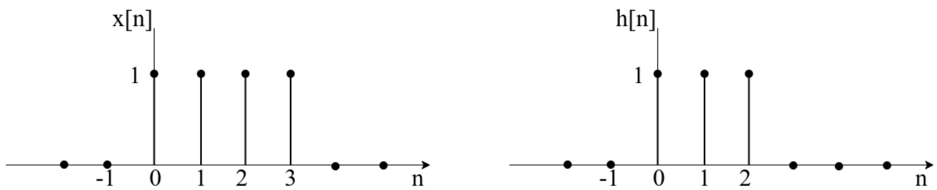

Figure 3: Signals $x[n]$ and $y[n]$

5) [CO2] (5 points) Two signals $s_1(t)$ and $s_2(t)$ are defined as below:

$$s_1(t) = \begin{cases} e^t, & \text{if } 0 \le t < 1 \\ e^{2-t}, & \text{if } 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

$$s_2(t) = \begin{cases} e^{-t}, & \text{if } 0 \le t \le 4 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Evaluate $g(t) = s_1(t) \circledast s_2(t)$, where $\circledast$ denotes the convolution operator.

Programming Problems:

1. [CO2] (10 points) A system S is represented by its impulse response:

$$h(t) = \frac{1}{4}\left(e^{-2t} - e^{-t}\right)u(t) \tag{5}$$

a) (5 pts) Find the response of the system, if the input is $x(t) = cos(t)u(t)$. Plot the signals $x(t)$, $h(t)$ and $y(t)$ for $t = [0, 20]$.
b) (5 pts) Find the response of the system if $h(t) = \frac{1}{2}(e^{-t} - e^{-4t})u(t)$ and $x(t) = e^{-t}sin(t)u(t)$. Plot the signals $x(t)$, $h(t)$ and $y(t)$ for $t = [0, 20]$.