# Working of pi-calculation-simulator

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## Preface

This document was written after creation of the repository pi-calculation-simulator, only to provide a better experience to readers wishing to go through the proof of why the $\pi$-calculation algorithm is correct.

## Set-Up

A square $S$ of side $s$ appears on the screen. A circle $C$ of diameter $d$ is inscribed in $S$. This ensures that side $s$ of square $S$ is exactly equal to diameter $d$ of circle $C$, i.e.

$$s = d \tag{1}$$

Pairs $(x, y)$ are drawn randomly from a uniform distribution to place darts at.

## Explanation

The following section contains the explanation of why the algorithm works.

Let $N_t$ be the total number of darts to be thrown at $S$.
Let $N_0$ be the number of darts that land inside $C$.
We find that the ratio

$$\frac{N_0}{N_t} \propto \frac{\text{Area}(C)}{\text{Area}(S)} \tag{2}$$

Note that (using (1)),

$$\begin{aligned} \frac{\text{Area}(C)}{\text{Area}(S)} &= \frac{\pi d^2}{4} \cdot \frac{1}{s^2} \\ &= \frac{\pi d^2}{4} \cdot \frac{1}{d^2} \quad = \frac{\pi}{4} \end{aligned} \tag{3}$$

Note how for larger $N_t$, the following holds true, on account of covering larger area (using (3)):

$$\lim_{N_t \to \infty} \left( \frac{N_0}{N_t} \right) = \frac{\text{Area}(C)}{\text{Area}(S)} = \frac{\pi}{4} \tag{4}$$

This is the reason that $N_t > 1{,}000$ is suggested by the application. On rearranging (4),

$$\lim_{N_t \to \infty} 4 \cdot \left( \frac{N_0}{N_t} \right) = \pi \tag{5}$$

Hence the aforementioned (result (5)) is a close (convergent) approximation of $\pi$ for large $N_t$.

## Footnotes

This algorithm cannot be guaranteed to work. This can be attributed to the fact that each location $(x, y)$ is chosen randomly, where $x$ and $y$ are chosen randomly from uniform distributions. In the event (of minuscule probability) that most/all points lie outside/inside $C$, the algorithm is bound to fail.
Hence, it must only be looked at as an approximation.

## References


1. Calculating Pi with Darts (YouTube)
2. Computing PI by throwing darts