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Date: November 14, 2023

QUESTION 1

- This is solved by taking the cluster which has its mean closest to the test point x_n

$$\arg\min_k ||x_n - \mu_k||^2$$

- If we assume the test point to be closest to a cluster mean, denoted by μ_k then we can get the update equation for μ_k by taking derivative of $\mathcal{L}$ w.r.t. μ_k

$$\frac{\partial \mathcal{L}}{\partial \mu_k} = -2||x_n - \mu_k||$$

  Which can be put into the update equation as

$$\mu_k = \mu_k + \eta||x_n - \mu_k|| \tag{1}$$

- In 1 we have taken all constants to be part of the step-size η. A good choice of η would be a small value that decreases monotonically as the steps progress. By taking a small step size the cluster means will slowly progress towards the expected means and remain unaffected by noisy input datapoints.

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QUESTION 2

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My solution to problem 2

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QUESTION 3

Let us take $\mathbf{S}' = \frac{1}{N}\mathbf{X}\mathbf{X}^T$. 2 represents the equation to calculate eigenvalue λ' and eigenvector $\mathbf{v}$ of $\mathbf{S}'$

$$\mathbf{S}'\mathbf{v} = \lambda'\mathbf{v} \tag{2}$$

Now if we take the value of $\mathbf{S}'$ and pre-multiply with $\mathbf{X}^T$ and readjust the values we get:

$$\frac{1}{N}\mathbf{X}\mathbf{X}^T\mathbf{v} = \lambda'\mathbf{v} \tag{3}$$

$$\frac{1}{N}\mathbf{X}^T\mathbf{X}\mathbf{X}^T\mathbf{v} = \lambda'\mathbf{X}^T\mathbf{v} \tag{4}$$

$$\mathbf{S}\mathbf{u} = \lambda'\mathbf{u} \tag{5}$$

Thus the eigenvalue remains the same in both forms, only the eigenvectors change, which can be see in blue from Equation 4 and 5. By computing eigenvectors this way we can reduce the complexity of calculating eigenvalues for a $D \times D$ matrix to $N \times N$ matrix, which is feasible in this case as $D < N$.

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QUESTION 4

My solution to problem 4

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QUESTION 5

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My solution to problem 5

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QUESTION 6

Solve this and if you can then give me 100