

Problem 1(a):

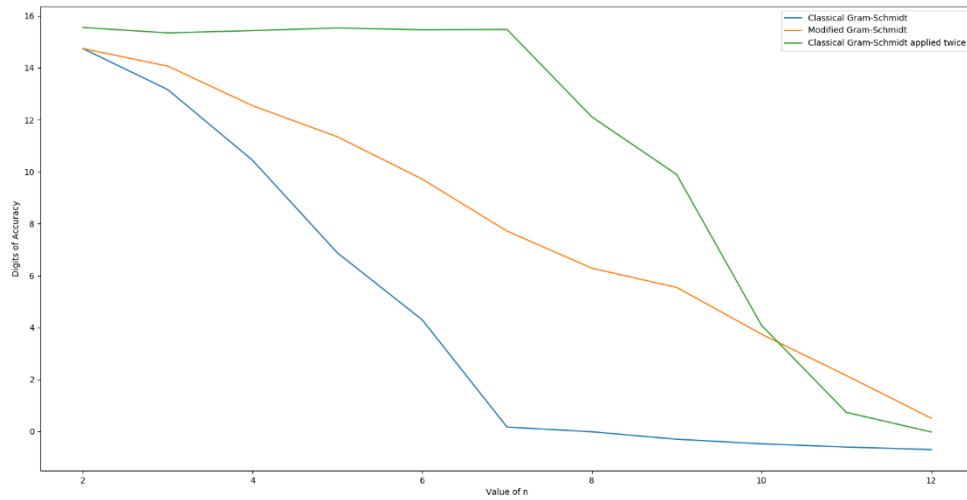


fig: Digits of accuracy using factorization methods

Problem 1(b):

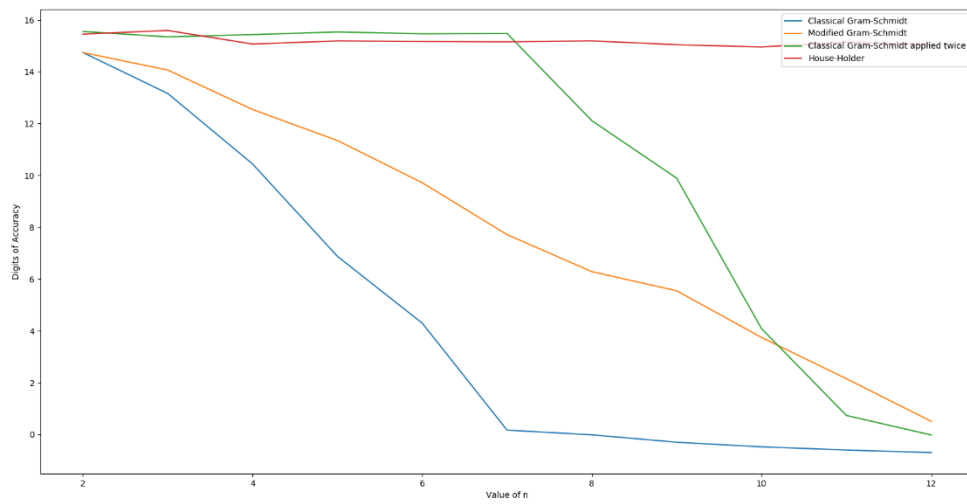


Fig: Digits of accuracy using factorization methods

Problem 1(c):

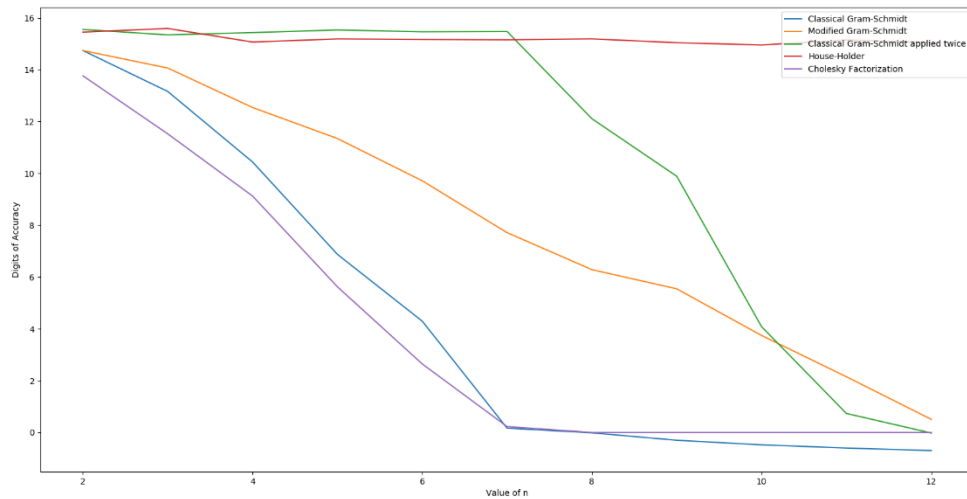


Fig: Digits of accuracy using factorization methods

Problem 1(d):

1. Modified Gram-Schmidt is numerically more stable as compared to the standard.
2. Gram-Schmidt applied twice appears to refine the solution. Values of $Q^T Q$ are closer to identity matrix.
3. Cholesky factorization uses square roots in numerical calculations is not stable for larger values of n.
4. House-Holder performs consistently better because it computes Q as a product of accurate House-Holder reflections, while Gram-Schmidt directly orthogonalizes the columns of A.

Sources:

<https://math.stackexchange.com/questions/770882/why-is-householder-computationally-more-stable-than-modified-gram-schmidt>