Problem 1(a):

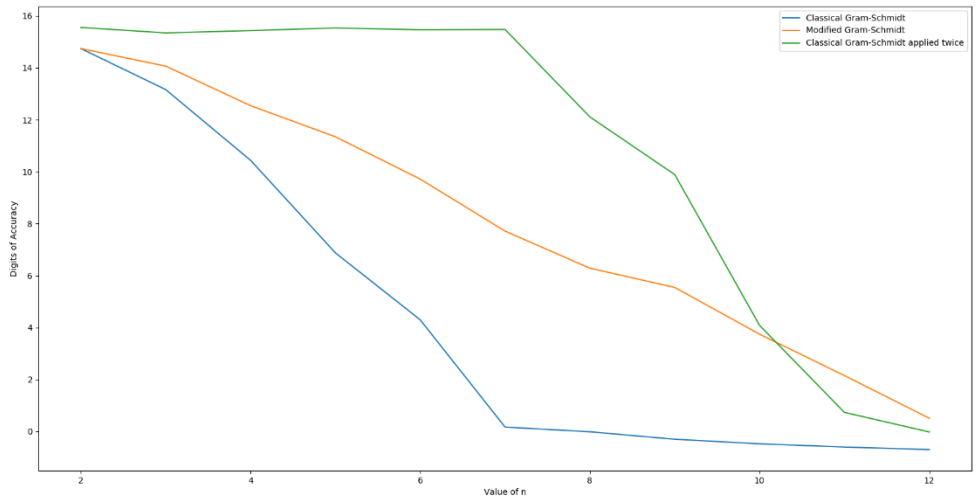

fig: Digits of accuracy using factorization methods

Problem 1(b):

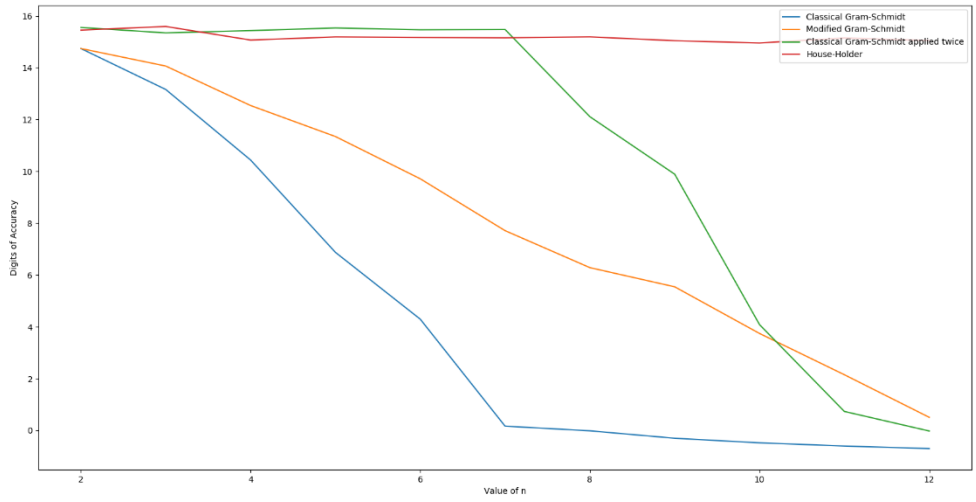

Fig: Digits of accuracy using factorization methods

Problem 1(c):

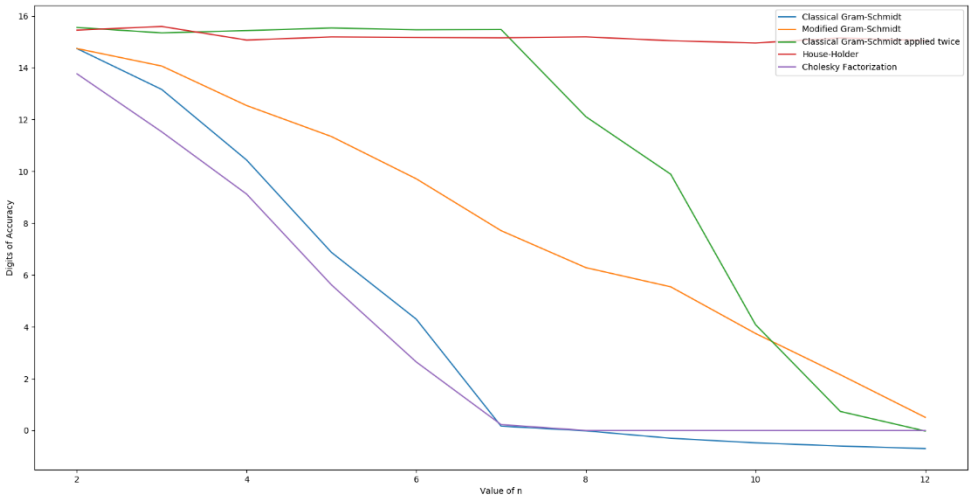

Fig: Digits of accuracy using factorization methods

Problem 1(d):

1. Modified Gram-Schmidt is numerically more stable as compared to the standard.
2. Gram-Schmidt applied twice appears to refine the solution. Values of Q^TQ are closer to identity matrix.
3. Cholesky factorization uses square roots in numerical calculations is not stable for larger values of n. It was observed that the iterative implementation and the numpy package for Cholesky decomposition are both unstable for values of n greater than 7. Hence the plot must be interpreted with this caveat.
4. House-Holder performs consistently performs better because it computes Q as a product of accurate House-Holder reflections, while Gram-Schmidt directly orthogonalizes the columns of A.

Sources:

https://math.stackexchange.com/questions/770882/why-is-householder-computationally-more-stable-than-modified-gram-schmidt