Bounce Cosmology from a Compact Internal Degree of Freedom

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1 What is FRW?

The Friedmann–Robertson–Walker (FRW) metric is a simple mathematical description of the universe that assumes two things:

- The universe is **homogeneous**: looks the same everywhere.
- The universe is **isotropic**: looks the same in every direction.

The FRW metric is written as

$$ds^2 = -dt^2 + a(t)^2\,\gamma_{ij}\,dx^i dx^j, \tag{1}$$

where $a(t)$ is the **scale factor**, telling us how big the universe is at time t. The parameter $H = \dot{a}/a$ is the **Hubble parameter**, which measures how fast the universe is expanding or contracting.

2 What is a Compact Rotor?

In the X–θ framework, every particle carries an internal angle θ that is compact: it lives on a circle (S^1). Think of this like a clock hand that rotates—even if the particle returns to its original position in space, the hand may point differently.

Mathematically, this internal rotor has an effective energy:

$$\mathcal{L}_\theta = \tfrac{I}{2} g^{\mu\nu}\,\partial_\mu\theta\,\partial_\nu\theta, \tag{2}$$

where I is like the "moment of inertia" of this internal clock. The conserved momentum is $\Pi_\theta = Ia^3\dot{\theta}$. The corresponding energy density and pressure are

$$\rho_\theta = \frac{\Pi_\theta^2}{2Ia^6}, \qquad p_\theta = \rho_\theta. \tag{3}$$

This behaves like a **stiff fluid** with equation of state $w = 1$.

3 What is Curvature Backreaction?

In the extended space $Q = \mathbb{R}^3 \times S^1$, the geometry has a special term called the mixed curvature $F_{i\theta}$. This describes how motion in space (i) and motion in the internal circle (θ) interact.

- Ordinary curvature in FRW describes how space bends due to mass and energy.

- The new mixed curvature describes how the internal θ rotation can push back on space.

This feedback is called a **backreaction** because it is the response of the geometry to the presence of the internal degree of freedom.

After coarse-graining, the Friedmann equation becomes

$$H^2 = \frac{8\pi G}{3}\Big(\rho_{\text{std}} + \rho_\theta\Big) - \frac{\sigma^2}{a^6} - \frac{k}{a^2}. \tag{4}$$

Here:

- ρ_{std} = energy from normal matter and radiation.
- ρ_θ = stiff energy from the compact rotor.
- $-\sigma^2/a^6$ = new repulsive effect from curvature backreaction.
- k = curvature index (0 flat, +1 closed, −1 open).

The key term is $-\sigma^2/a^6$: it behaves like a centrifugal barrier, preventing collapse to a singularity.

4 Bounce Conditions

A **bounce** occurs when the universe shrinks to a minimum size and then expands again. Mathematically:

$$H = 0, \qquad \dot{H} > 0. \tag{5}$$

This means the expansion rate is zero but turning upward.

The condition simplifies to

$$0 = \frac{8\pi G}{3}\frac{\Pi_\theta^2}{2Ia_{\text{min}}^6} - \frac{\sigma^2}{a_{\text{min}}^6} - \frac{k}{a_{\text{min}}^2}. \tag{6}$$

4.1 Closed FRW with Dominant a^{-6}

If curvature is negligible, the balance is simply:

$$\frac{8\pi G}{3}\frac{\Pi_\theta^2}{2I} = \sigma^2. \tag{7}$$

This fixes a finite a_{min}.

4.2 Curvature-Assisted Bounce

For a closed universe ($k = +1$), the minimum scale factor is

$$a_{\text{min}} = \left[\frac{\frac{8\pi G}{3}\frac{\Pi_\theta^2}{2I} - \sigma^2}{1}\right]^{1/4}. \tag{8}$$

5 Verification of $\dot{H} > 0$

The Raychaudhuri (acceleration) equation is

$$\dot{H} = -4\pi G(\rho_{\text{tot}} + p_{\text{tot}}) + \frac{k}{a^2} + \frac{3\sigma^2}{a^6}. \tag{9}$$

At $a = a_{\text{min}}$, this evaluates to a positive value, confirming the bounce.

6 Summary in Plain Words

In standard GR, the Big Bang is a singularity: the universe shrinks to zero size and density becomes infinite.

Quantum mechanics hints that infinities should not exist, but does not provide a mechanism. The X–θ framework adds:

- An internal cyclic angle θ (compact rotor), behaving like stiff matter.
- A mixed curvature backreaction, producing a repulsive $-\sigma^2/a^6$ term.
- The competition of these effects halts collapse at a finite $a_{\text{min}} > 0$, replacing the singularity with a bounce.

For undergraduates: imagine a spinning top. Its spin prevents it from falling flat immediately. Similarly, the hidden θ rotation prevents the universe from collapsing to a point.