Examples

26. A bag A contains 5 white and 6 black balls. Another bag B contains 4 white and 3 black balls. A ball is transferred from bag A to the bag B and then ball is taken out of the second bag. Find the probability of this ball being black.

Solution:

Consider the following events

E_1 : White ball is transferred from first bag A to second bag B

E_2 : Black ball is transferred from first bag A to second bag B

E : Black ball is drawn from second bag B

Now, as first bag contains 5 white and 6 black balls

$\therefore P(E_1) = 5/11$ Similarly, $P(E_2) = 6/11$

If the event E_1 has already occurred (i.e. white ball is transferred from A to B), then in Bag B now we have 5 white and 3 black balls

$$P(E/E_1) = \frac{3}{8}$$

If the event E_2 has already occurred (i.e. black ball is transferred from A to B), then in Bag B now we have 4 white and 4 black balls

$$P(E/E_2) = \frac{4}{8}$$

Also from Law of Total Probability, we have

$$P(E) = P(E_1)P(E/E_1) + P(E_2)P(E/E_2) = \frac{5}{11}\times\frac{3}{8} + \frac{6}{11}\times\frac{4}{8} = \frac{15}{88} + \frac{24}{88} = \frac{39}{88}$$

27. Two third of the students in a class are boys and rest girls. It is known that the probability of a girl getting a first class is 0.32 and that of a boy getting a first class is 0.25. Find the probability that a student chosen at random will get a first class marks in the subject.

Solution:

Consider the following events

E_1 : boy is selected from the class

E_2 : girl is selected from the class

E : selected student gets first class marks in the subject

$$P(E_1) = \frac{2}{3}; P(E_2) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(E/E_1) = 0.25; P(E/E_2) = 0.32$$

By Theorem of Total Probability, we have

$$P(E) = P(E_1)P(E/E_1) + P(E_2)P(E/E_2) = \frac{2}{3}\times 0.25 + \frac{1}{3}\times 0.32 = \frac{25\times 2}{300} + \frac{32}{300} = \frac{82}{300} = \frac{41}{150}$$

28. A purse I contains 2 silver and 4 copper coins. A second purse II contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin?

Solution:

Consider the following events

E_1 : Purse I is chosen

E_2 : Purse II is chosen

E : Silver coin is drawn from the purse

As in total, there are two purses I and II $\therefore$ $P(E_1) = P(E_2) = \frac{1}{2}$

If event E_1 has already occurred (i.e. purse I is chosen), then

$$P(E/E_1) = \frac{2}{6} \text{ [Purse I contains 2 silver and 4 copper coins]}$$

If event E_2 has already occurred (i.e. purse II is chosen), then

$$P(E/E_2) = \frac{4}{7} \text{ [Purse II contains 4 silver and 3 copper coins]}$$

From Law of Total Probability, we have

$$P(E) = P(E_1)P(E/E_1)+P(E_2)P(E/E_2) = \frac{1}{2}\times\frac{2}{6}+\frac{1}{2}\times\frac{4}{7} = \frac{1}{6}+\frac{2}{7} = \frac{19}{42}$$

29. The contents of three bags I, II and III are as follows:
Bag I: 2 white, 4 blue and 4 red balls.
Bag II: 3 white, 1 blue and 2 red balls.
Bag III: 3 white, 2 blue and 3 red balls.
A bag is chosen at random and two balls are drawn. What is the probability that the balls drawn are of colour blue and red?

Solution:

Consider the following events

E_1 : Bag I is chosen $\quad$ E_2 : Bag II is chosen

E_3 : Bag III is chosen $\quad$ E : Two balls drawn are of colour blue and red

As there are total of 3 bags, therefore, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

If event E_1 has already occurred, then $P(E/E_1) = \frac{{}^4C_1 \times {}^4C_1}{{}^{10}C_2}$

[In bag I, there are total 10 balls out of which 4 are blue and 4 are re

$$= \frac{4\times4}{45} = \frac{16}{45}$$

If event E_2 has already occurred, then $P(E/E_2) = \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{1\times2}{15} = \frac{2}{15}$

[In bag II, there are total 9 balls out of which 1 is blue and 2 are r

If event E_3 has already occurred, then $P(E/E_3) = \frac{{}^2C_1 \times {}^3C_2}{{}^8C_2} = \frac{2\times3}{28} = \frac{3}{14}$

[In bag III, there are total 8 balls out of which 2 are blue and 3 are

By Law of Total Probability, we have

$$P(E) = P(E_1)P(E/E_1)+P(E_2)P(E/E_2)+P(E_3)P(E/E_3)$$

$$= \frac{1}{3}\times\frac{16}{45}+\frac{1}{3}\times\frac{2}{15}+\frac{1}{3}\times\frac{3}{14} = \frac{1}{3}\left[\frac{16}{45}+\frac{2}{15}+\frac{3}{14}\right] = \frac{443}{1890}$$

2.5 [Baye's Theorem]

Baye's Theorem:

Let S be the sample space. Let $E_1, E_2 ....E_n$ be mutually exclusive and exhaustive events associat random experiment (Clearly, E_i's are 'n' in number).

If E is any event that occurs within E_1 or E_2..........or E_n then $P\left(\frac{E_i}{E}\right) = \frac{P(E_i)P\left(\frac{E}{E_i}\right)}{\sum_{i=1}^{n} P(E_i)P\left(\frac{E}{E_i}\right)}$ for i = 1, 2, ...,

Proof:

As E_i's (i = 1, 2, ..., n) are mutually exclusive and exhaustive events, therefore, by definition

$$S = E_1 \cup E_2 \cup E_3 ....\cup E_n \text{ and } E_i \cap E_j = \varnothing \text{ for } i \neq j$$

Also $E = E \cap S$ [S is sample space and E is any event]

$$\Rightarrow E = E \cap [E_1 \cup E_2 \cup ....\cup E_n]$$

$$E = (E \cap E_1) \cup (E \cap E_2) \cup ....\cup (E \cap E_n)$$

$$\Rightarrow P(E) = P[(E \cap E_1) \cup (E \cap E_2) \cup ....\cup (E \cap E_n)]$$

$$\Rightarrow P(E) = P(E \cap E_1) + P(E \cap E_2) + ...P(E \cap E_n)$$

$$\Rightarrow P(E) = \sum_{i=1}^{n} P(E_i)P\left(\frac{E}{E_i}\right) \quad ....(i) \qquad \left[\because P(E \cap E_i) = P(E_i)P\left(\frac{E}{E_i}\right)\right]$$

[Multiplication theorem on probability]

By Multiplication theorem on probability, we have

$$P(E \cap E_i) = P(E)P\left(\frac{E_i}{E}\right) \quad i = 1 \text{ to } (n)$$

$$\Rightarrow P\left(\frac{E_i}{E}\right) = \frac{P(E \cap E_i)}{P(E)} \quad i = 1 \text{ to } n \Rightarrow P\left(\frac{E_i}{E}\right) = \frac{P(E_i)P\left(\frac{E}{E_i}\right)}{P(E)} \quad \text{R[Multiplication theorem on probability]}$$

$$P\left(\frac{E_i}{E}\right) = \frac{P(E_i)P\left(\frac{E}{E_i}\right)}{\sum P(E_i)P\left(\frac{E}{E_i}\right)} \quad \text{[From (i)] } i = 1 \text{ to } n \qquad P\left(\frac{E_i}{E}\right) = \frac{P(E_i)P\left(\frac{E}{E_i}\right)}{\sum_{i=1}^{n} P(E_i)P\left(\frac{E}{E_i}\right)} \quad i = 1 \text{ to } n$$

Examples

30. A company has two plants to manufacture scooters. Plant I manufactures 70% of the scooters and Plant II manufactures 30%. At Plant I, 80% of the scooters are rated as of standard quality and at Plant II, 90% of the scooters are rated as of standard quality. A scooter is chosen at random and is found to be of standard quality. What is the probability that it has come from Plant I?

Solution:

Consider the following events

E_1 : Plant I is chosen

E_2 : Plant II is chosen

E: Standard quality of scooter is chosen

$$P(E_1) = \frac{70}{100} = \frac{7}{10} \quad ; P(E_2) = \frac{30}{100} = \frac{3}{10} \ ;$$

also $$P\left(\frac{E}{E_1}\right) = \frac{80}{100} = \frac{8}{10} \quad ; \quad P\left(\frac{E}{E_2}\right) = \frac{90}{100} = \frac{9}{10}$$

Now we have to find the probability that if the scooter chosen is of standard quality, then it has come from plant I.
Required probability:

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)} = \frac{\frac{7}{10} \times \frac{8}{10}}{\frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{9}{10}} = \frac{\frac{56}{100}}{\frac{56}{100} + \frac{27}{100}} = \frac{56}{83}$$

1. In 1995, there will be three candidates for the position of principal - P_1, P_2 and P_3. The chances of their selection are in the proportion 4:2:3 respectively. The probability that if principal P_1 is selected, will introduce co-education in the college is 0.3. The probability of P_2 and P_3 doing the same thing are 0.5 and 0.8. What is the probability that principal P_3 introduces co-education in the college?

olution:

Consider the following events

E_1 : Principal P_1 is selected.

E_2 : Principal P_2 is selected.

E_3 : Principal P_3 is selected.

E : Co-education in the college is introduced

$$P(E_1) = \frac{4}{9} ; P(E_2) = \frac{2}{9} ; P(E_3) = \frac{3}{9}$$

also $$P\left(\frac{E}{E_1}\right) = 0.3 = \frac{3}{10} \ ; \ P\left(\frac{E}{E_2}\right) = 0.5 = \frac{5}{10} \ ; \ P\left(\frac{E}{E_3}\right) = 0.8 = \frac{8}{10}$$

We have to find the probability that principal P_3 introduces co-education in the college?

$$\therefore \text{Required probability} = P\left(\frac{E_3}{E}\right) = \frac{P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{3}{9}\times\frac{8}{10}}{\frac{3}{9}\times\frac{8}{10}+\frac{4}{9}\times\frac{3}{10}+\frac{2}{9}\times\frac{5}{10}} = \frac{\frac{24}{90}}{\frac{24}{90}+\frac{12}{90}+\frac{10}{90}} = \frac{24}{46} = \frac{12}{23}$$

32. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective. From which machine, the defective bolt is expected to have manufactured.

Solution:

Consider the following events

E_1 : Selected bolt is manufactured by machine A

E_2 : Selected bolt is manufactured by machine B

E_3 : Selected bolt is manufactured by machine C

E : Defective bolt is drawn

$$P(E_1) = \frac{25}{100};\ P(E_2) = \frac{35}{100};\ P(E_3) = \frac{40}{100}$$

$$P\left(\frac{E}{E_1}\right) = \frac{5}{100};\ P\left(\frac{E}{E_2}\right) = \frac{4}{100};\ P\left(\frac{E}{E_3}\right) = \frac{2}{100}$$

Now P (given that bolt is defective, it is manufactured by machine A)

$$\text{Similarly } P\left(\frac{E_1}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{25}{100}\times\frac{5}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{40}{100}\times\frac{2}{100}} = \frac{125}{345} = \frac{25}{69}$$

$$\text{Similarly } P\left(\frac{E_2}{E}\right) = \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{35}{100}\times\frac{4}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{40}{100}\times\frac{2}{100}} = \frac{140}{345} = \frac{28}{69}$$

$$\text{Similarly } P\left(\frac{E_3}{E}\right) = \frac{P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{40}{100}\times\frac{2}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{40}{100}\times\frac{2}{100}} = \frac{80}{345} = \frac{16}{69}$$

Since $P\left(\frac{E_2}{E}\right)$ is maximum $\therefore$ defected bolt is expected to have been manufactured by machine B.

33. If a machine is correctly set up, it will produce 90% of acceptable items. If it is incorrectly set up, it will produce 30% of acceptable items. Past experience shows that 80% of the set-ups are correctly done. If after a certain set-up, first item produced is acceptable, what is the probability that the machine is correctly set up?

Solution:

Consider the following events

E_1 : Set up was correct

E_2 : Set up was wrong

E_3 : Item is acceptable

Now, $P(E_1) = \frac{80}{100}$; $P(E_2) = 1 - \frac{80}{100} = \frac{20}{100}$

$P\left(\frac{E}{E_1}\right) = P$ (If set up is correct, items are accepted) $= \frac{90}{100}$

$P\left(\frac{E}{E_2}\right) = P$ (If set up is wrong, items are rejected) $= \frac{30}{100}$

We have to find the probability if items produced is accepted, then set up of machine was correct.

$$\therefore \text{Required probability} = P\left(\frac{E_1}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$= \frac{\frac{80}{100}\times\frac{90}{100}}{\frac{80}{100}\times\frac{90}{100}+\frac{20}{100}\times\frac{30}{100}} = \frac{7200}{7800} = \frac{72}{78} = \frac{36}{39} = \frac{12}{13}$$

34. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it is 1/8. Find the probability that he knew the answer to the question, given that he correctly answered it.

Solution:

Consider the following events

E_1 : Examinee guesses the answer

E_2 : Examinee copies the answer

E_3 : Examinee knows the answer

E : Answer is right

Now,

$P(E_1) = \frac{1}{3}$; $P(E_2) = \frac{1}{6}$ $\quad \therefore P(E_3) = 1-(P(E_1)+P(E_2)) = 1-\left(\frac{1}{3}+\frac{1}{6}\right) = \frac{1}{2}$

Also $P\left(\frac{E}{E_1}\right) = \frac{1}{4}$ [To guess he had four choices] $P\left(\frac{E}{E_2}\right) = \frac{1}{8}$; $P\left(\frac{E}{E_3}\right) = 1$

[If he knows the answer, probability of answering the question correctly is 1]

Now Required probability $= P\left(\frac{E_3}{E}\right)$

$$= \frac{P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{1}{2}\times 1}{\frac{1}{3}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{8}+\frac{1}{2}\times 1} = \frac{\frac{1}{2}}{\frac{1}{12}+\frac{1}{48}+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{29}{48}} = \frac{48}{58} = \frac{24}{29}$$

35. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution:

Consider the following events

E_1 : Six occurs

E_2 : Six does not occur

E : Man reports the number to be six

$$P(E_1)=\frac{1}{6};\ P(E_2)=1-\frac{1}{6}=\frac{5}{6}$$

$P\left(\frac{E}{E_1}\right)$ = P (Given that six occurs, then man reports it to be 6)

= P (man speaks truth)=3/4

$P\left(\frac{E}{E_2}\right)$ = P (Given that six does not occurs, man reports it to be 6)

= P (man does not speak the truth)= 1–(3/4) = 1/4

Now = P (there is actually a six)

= P (Given that man reports the number to be 6, occurs or a die)

$$P\left(\frac{E_1}{E}\right)=\frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}=\frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}=\frac{\frac{3}{24}}{\frac{3}{24}+\frac{5}{24}}=\frac{3}{8}$$

36. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are four to be of hearts. Find the probability of the missing card to be a of heart.

Solution:

Consider the following events

E_1 : missing card is of spade

E_2 : missing card is of club

E_3 : missing card is of heart

E_4 : missing card is of diamonds

E: drawing two heart cards from the remaining cards

Now $P(E_1)=P(E_2)=P(E_3)=P(E_4)=\frac{13}{52}=\frac{1}{4}$

Also

$P\left(\frac{E}{E_1}\right)$ = P (If missing card is of spade, drawing 2 heart cards from remaining)

$$=\frac{^{13}C_2}{^{51}C_2}$$

$P\left(\frac{E}{E_2}\right)$ = P (If missing card is of club, drawing 2 heart cards from remaining)

$$=\frac{^{13}C_2}{^{51}C_2}$$

$P\left(\frac{E}{E_3}\right)$ = P (If missing card is of heart, drawing 2 heart cards from remaining)

$$=\frac{^{12}C_2}{^{51}C_2}$$

$P\left(\frac{E}{E_4}\right)$ = P (If missing card is of diamond, drawing 2 heart cards from remaining)

$$=\frac{^{13}C_2}{^{51}C_2}$$

Required probability = P (If 2 heart cards are drawn from remaining cards, then missing card is of heart)

$$P\left(\frac{E_3}{E}\right)=\frac{P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$=\frac{\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}}{\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}+\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}+\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}+\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}}$$

$$=\frac{^{12}C_2}{^{13}C_2+^{13}C_2+^{12}C_2+^{13}C_2}=\frac{66}{78+78+66+78}=\frac{66}{300}=\frac{11}{50}$$

37. There are three coins. One is two headed coin, another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads. What is the probability that it was the two headed coin?

Solution:

Consider the following events

E_1 : two headed coin is selected

E_2 : biased coin is selected

E_3 : unbiased coin is selected

E : getting head on the coin

Clearly, $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$

Also $P\left(\frac{E}{E_2}\right)=\frac{75}{100}=\frac{3}{4}$; $P\left(\frac{E}{E_1}\right)=1$

[given it is two headed coin, then probability of getting heads is 1]

$P\left(\frac{E}{E_3}\right)=\frac{1}{2}$ [given it is unbiased coin, then probability of getting heads is 1/2]

Required probability = $P\left(\frac{E_1}{E}\right)$

$$=\frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}}=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{6}}=\frac{\frac{1}{3}}{\frac{4+3+2}{12}}=\frac{\frac{1}{3}}{\frac{9}{12}}=\frac{12}{27}=\frac{4}{9}$$

38. A letter is known to have come either from LONDON or from CLIFTON. On the envelope last two consecutive letters 'ON' are visible. What is the probability that the letter has come from CLIFTON?

Solution:

Consider the following events

E_1 : letter comes from LONDON

E_2 : letter comes from CLIFTON

E : two consecutive letters 'ON' are visible

Now $P(E_1)=P(E_2)=\frac{1}{2}$ [As the letter come either from LONDON or from CLIFTON]

Now, if event E_1 has occurred, i.e. letter has come from LONDON. In LONDON, total there are 6 letters and 'ON' occurs twice. Considering 'ON' as one letter, letter 'ON' can be taken in 5 ways.

$\therefore P\left(\frac{E}{E_1}\right)=\frac{2}{5}$

Also, if event E_2 has occurred, i.e. letter has come from CLIFTON. In CLIFTON, total there are 7 letters. Considering 'ON' as one letter, letter 'ON' can be taken in 6 ways.

$$\therefore P\left(\frac{E}{E_2}\right)=\frac{1}{6}.$$

Now, Required probability = $P\left(\frac{E_2}{E}\right)$

$$=\frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}=\frac{\frac{1}{6}\times\frac{1}{2}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}=\frac{\frac{1}{12}}{\frac{1}{5}+\frac{1}{12}}=\frac{5}{17}$$

39. There are two bags A and B. A contains 'p' white and 2 black balls and B contains 2 white and 'p' black balls. One of the two bags is selected at random and two balls are drawn are white and the probability that the bag A was used to draw the balls is 6/7. Find the value of 'p'.

Solution:

Consider the following events

E_1 : Bag A is selected

E_2 : Bag B is drawn

E_3 : Ball selected is white in colour

Now $P(E_1)=P(E_2)=\frac{1}{2}$; $P\left(\frac{E}{E_1}\right)=\frac{{}^{p}C_2}{{}^{p+2}C_2}$ [In bag A, there are 'p' white and 2 black balls]

Similarly $P\left(\frac{E}{E_2}\right)=\frac{{}^{2}C_2}{{}^{p+2}C_2}$ [In bag B, there are 2 white and 'p' black balls]

Also $P\left(\frac{E_1}{E}\right)=\frac{6}{7}$ (given)

$$P\left(\frac{E_1}{E}\right)=\frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}$$ [By Baye's Theorem]

$$\frac{6}{7}=\frac{\frac{1}{2}\times\frac{{}^{p}C_2}{{}^{p+2}C_2}}{\frac{1}{2}\times\frac{{}^{p}C_2}{{}^{p+2}C_2}+\frac{1}{2}\times\frac{{}^{2}C_2}{{}^{p+2}C_2}}\ ;\ \frac{6}{7}=\frac{{}^{p}C_2}{{}^{p}C_2+1}\ ;\ \frac{6}{7}=\frac{\frac{p(p-1)}{2}}{\frac{p(p-1)}{2}+1}$$

$$\frac{6}{7}=\frac{p(p-1)}{p(p-1)+2}\Rightarrow\frac{6}{7}=\frac{p^2-p}{p^2-p+2}\ ;\ 6(p^2-p+2)=7p^2-7p$$

$$6p^2-6p+12=7p^2-7p\ ;\ p^2-p-12=0$$

$$\Rightarrow p^2-4p+3p-12=0=p(p-4)+3(p-4)=0\Rightarrow p=-3,p=4$$

Neglect $p=-3$ $\therefore$ Required value = 4