# Lecture 1
# Unit – I
# Probability

**Introduction to probability**

**Random experiments** Random Experiment: An experiment is said to be a random experiment, if it's out-come can't be predicted with certainty.

**Example** If a coin is tossed, we can't say, whether head or tail will appear. So it is a random experiment.

**Sample Space** The set of all possible out-comes of an experiment is called the sample space. It is denoted by 'S' and its number of elements are n(s).

**Example** In throwing a dice, the number that appears at top is any one of 1, 2, 3, 4, 5, 6. So here: S ={1,2,3,4,5,6} and n(s) = 6

Similarly in the case of a coin, S={Head,Tail} or {H,T} and n(s)=2.

The elements of the sample space are called sample point or event point.

**Event** Every subset of a sample space is an event. It is denoted by 'E'.

**Example** In throwing a dice S={1,2,3,4,5,6}, the appearance of an event number will be the event E={2,4,6}.

Clearly E is a sub set of S.

**Types of event** **Elementary event** If a random experiment is performed is performed, then each of its outcomes is known as elementary event.

**Sure event:** Let 'S' be a sample space. If E is a subset of or equal to S then E is called a sure event.

**Example:** In a throw of a dice, S={1,2,3,4,5,6}

Let $E_1$=Event of getting a number less than '7'.

So '$E_1$' is a sure event.

So, we can say, in a sure event n(E) = n(S)

**Impossible event:** an event is called impossible event if it can never occur whenever the experiment is performed. Eg event getting number 7

**Mutually exclusive or disjoint event:** If two or more events can't occur simultaneously, that is no two of them can occur together.

**Example** When a coin is tossed, the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events.

**Independent or mutually independent events** Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of the other event.

**Example** When a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.

**Difference between mutually exclusive an mutually independent events** Mutually exclusiveness id used when the events are taken from the same experiment, where as independence is used when the events are taken from different experiments.

**Equally likely event** Events are said to be equally likely, if we have no reason to believe that one is more likely to occur than the other.

**Example** When a dice is thrown, all the six faces {1, 2, 3, 4, 5, 6} are equally likely to come up.

**Exhaustive events** When every possible out come of an experiment is considered.

**Example** A dice is thrown, cases 1, 2, 3, 4, 5, 6 forms an exhaustive set of events

**Lecture 11**
**Unit – I**
**Probability**

**Mathematical or Classical definition of Probability**

It' 'S' be the sample-space, then the probability of occurrence of an event 'E' is defined as: P(E) = n (E) / n(S) = number of elements in 'E' / number of elements in sample space.

**Question** Find the probability of getting tails in tossing of a coin.

**Solution** Sample-space S = {H, T}, n(S) = 2

Event 'E' = {T}, n (E) = 1

P(E) = n (E) / n (S) =1/2

**Result** The probability of an event lies between 'O' and '1'.

**Result** The probability of an event lies between 'O' and '1'.

**Result** The probability of an event lies between 'O' and '1'.

**Proof** In sure event n(E) = n(S)

[Since Number of elements in Event 'E' will be equal to the number of element in sample-space.] By definition of Probability: P(S) = n (S)/ n (S) =1, P(S) = 1

**Result** If 'E' is any event and $E^1$ be the complement of event 'E', then P(E) + P($\bar{E}$) = 1.

**Proof** Let 'S' be the sample – space, then

n(E) + n($\bar{E}$) = n(S), n (E) / n (S) + n ($\bar{E}$) /n (S) =1, P(E) + P($\bar{E}$) = 1

**Question** A coin is tossed successively three times. Find the probability of getting exactly one head or two heads.

**Solution** Let 'S' be the sample – space. Then,

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

n (S) = 8

Let 'E' be the event of getting exactly one head or two heads.

Then:

E = { HHT, HTH, THH, TTH, THT, HTT }

n (E) = 6

Therefore: P(E) = n (E)/ n (S) = 6 / 8 = 3 / 4.

**Question** Three coins are tossed. What is the probability of getting (i) all heads, (ii) two heads, (iii) at least one head, (iv) at least two heads?

**Solution** Let 'S' be the sample – space. Then

S = { HHH, HHT, HTH, THH, HTT, THT, TTH, TTT }

(i) Let '$E_1$' = Event of getting all heads.

Then $E_1$ = {HHH}, n ($E_1$) = 1

P($E_1$) = n ($E_1$) / n(S) = 1 / 8

(ii) Let $E_2$ = Event of getting '2' heads.

Then: $E_2$ = {HHT, HTH, THH }

n($E_2$) = 3, P ($E_2$) = 3 / 8

(iii) Let $E_3$ = Event of getting at least one head.

Then: $E_3$ = { HHH, HHT, HTH, THH, HTT, THT, TTH }

n($E_3$) = 7

$P(E_3) = \quad 7/8$

(iv) Let $E_4$ = Event of getting at least one head.

Then: $E_4$ = { HHH, HHT, HTH, THH, }

$n(E_4) = 4$, $P(E_4) = 4/8 = ½$

**Lecture 12**
**Unit – I**
**Probability**

**Algebra of Events** In a random experiment, let ‘S’ be the sample – space.
Let A $\subseteq$ S and B $\subseteq$ S, where ‘A’ and ‘B’ are events.
Thus we say that : (i) (AUB), is an event occurs only when at least of ‘A’ and ‘B’ occurs. $\Rightarrow$ (AUB) means (A or B).

**Example** If A = { 2,4,6,} and B = {1, 6}, than the event ‘A’ or ‘B’ occurs, if ‘A’ or ‘B’ or both occur i.e. at least one of ‘A’ and ‘B’ occurs. Clearly ‘A’ or ‘B’ occur, if the outcome is any one of the outcomes 1, 2, 4, 6. That is AUB.

**Question** What is the probability, that a number selected from 1, 2, 3, --- 2, 5, is a prime number, when each of the numbers is equally likely to be selected.

**Solution** S = { 1, 2, 3, ---- , 2, 5}, n(S) = 25
And E = { 2, 3, 5, 7, 11, 13, 17, 19, 23 }, n(E) = 9
Hence P(E) = n(E) / n(S) = 9 / 25.

**Question** Two dice are thrown simultaneously. Find the probability of getting :

(i) The same number on both dice,
(ii) An even number as the sum,
(iii) A prime number as the sum,
(iv) A multiple of ‘3’ as the sum,
(v) A total of at least 0,
(vi) A doublet of even numbers,
(vii) A multiple of ‘2’ on one dice and a multiple of ‘3’ on the other dice.

**Solution** Here:

S = { (1,1), (1,2) ------, (1,6), (2,1), (2,2), ---- (2,6), (3,1), (3,2), -----, (3,6), (4,1), (4,2), -- ------ (4,6), (5,1), (5,2), ----- (5,6), (6,1,), (6,2), --------------- (6,6) }
n(S) = 6 x 6 = 36

(i) Let $E_1$ = Event of getting same number on both side:
$\Rightarrow E_1$ = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}, $n(E_1)$ = 6,
$P(E_1) = n(E_1)/n(S) = 6/36 = 1/6$

(ii) Let $E_2$ = Event of getting an even number as the sum.
$E_2$ = { (1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,5), (6,2), (6,4), (6,6) }
$n(E_2) = 18$ hence $P(E_2) = n(E_2)/n(S) = 18/36 = 1/2$

(iii) Let $E_3$ = Event of getting a prime number as the sum..
$E_3$ = { (1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)}, $n(E_3) = 15$, $P(E_2) = n(E_3) / n(S) = 15/36 = 5/12$

(iv) Let $E_4$ = Event of getting a multiple of ‘3’ as the sum.
$E_4$ = { (1,2), (1,5), (2,1), (2,4), (3,3), (3,6), (4,2), (4,5), (5,1), (5,4), (6,3), (6,6),}$n(E_4) = 12$, $P(E_4) = n(E_4)/n(S) = 12/36 = 1/3$

(v) Let $E_5$ = Event of getting a total of at least 10.
$E_5$ = { (4,6), (5,5), (5,6), (6,4), (6,5), (6,6), }
$n(E_5) = 6$, $P(E_5) = n(E_5)/n(S) = 6/36 = 1/6$

(vi) Let $E_6$ = Event of getting a doublet of even numbers.
$E_6$ = { (2,2), (4,4), (6,6), }

$n(E_6) = 3$

$P(E_6) = n(E_6)/ n(S) = 3/36 = 1/12$

(vii) Let $E_7$ = Even of getting a multiple of '2" on one dice and a multiple of '3' on the other dice.

$E_7 = \{ (2,3), (2,6), (4,3), (4,6), (6,3), (3,2), (3,4), (3,6), (6,2), (6,4) \}$

$n(E_7) = 11$, $P(E_7) = n(E_7) / n(S) = 11/36$.

**Question** What is the probability, that a leap year selected at random will contain 53 Sundays?

**Solution** A leap year has 366 days, therefore 52 weeks i.e. 52 Sunday and 2 days.

The remaining 2 days may be any of the following :

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

For having 53 Sundays in a year, one of the remaining 2 days must be a Sunday.

$n(S) = 7$

$n(E) = 2$

$P(E) = n(E) / n(S) = 2 / 7$.

**Question** A bag contains '6' red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability, that

(i) '1' is red and '2' are white, (ii) '2' are blue and 1 is red, (iii) none is red.

**Solution** We have to select '3' balls, from 18 balls (6+4+8)

$n(S) = {}^{18}C_3 = 18! / (3! \times 15!) = (18 \times 17 \times 16) / (3 \times 2 \times 1) = 816$

(i) Let $E_1$ = Event of getting '1' ball is red and '2' are white

Total number of ways = $n(E_1) = {}^6C_1 \times {}^4C_2 = 6! / (1! \times 5!) \times 4! / (2! \times 2!) = 6 \times 4 / 2 = 36$

$P(E_1) = n(E_1) / n(S) = 36/816 = 3/68$

(ii) Let $E_2$ = Event of getting '2' balls are blue and '1' is red.

= Total no. of ways, $n(E_2) = {}^8C_2 \times {}^6C_1$

$= 8! / (2! \times 6!) \times 6! / (1! \times 5!) = (8 \times 7) / 2 \times 6 / 1 = 168$

$P(E_2) = 168 / 816 = 7/34$

(iii) Let $E_3$ = Event of getting '3' non – red balls. So now we have to choose all the three balls from 4 white and 8 blue balls.

Total number of ways :

$n(E_3) = {}^{12}C_3 = 12! / (3! \times 9!) = (12 \times 11 \times 10) / (3 \times 2 \times 1) = 220$

$P(E_3) = n(E_3) / n(S) = 220 / 816 = 55/204$.

**Question** A box contains 12 bulbs of which '4' are defective. All bulbs took alike. Three bulbs are drawn randomly.

What is the probability that :

(i) all the '3' bulbs are defective?

(ii) At least '2' of the bulbs chosen are defective?

(iii) At most '2' of the bulbs chosen are defective?

**Solution** We have to select '3' bulbs from 12 bulbs.

$n(S) = {}^{12}C_3 = 12! / (3! \times 9!) = (12 \times 11 \times 10) / (3 \times 2 \times 1) = 220$

(i) Let $E_1$ = All the '3' bulbs are defective.

All bulbs have been chosen, from ‘4’ defective bulbs.
$n(E_1) = {}^{4}C_3 = 4! / (3! \times 1!) = 4$
$P(E_1) = n(E_1) / n(S) = 4 / 220 = 1/55$
(ii) Let $E_2$ = Event drawing at least 2 defective bulbs. So here, we can get ‘2’ defective and 1 non-defective bulbs or 3 defective bulbs.
$n(E_2) = {}^{4}C_2 \times {}^{8}C_1 + {}^{4}C_3$ [Non-defective bulbs = 8]
$= 4! / (2! \times 2!) \times 8! / (1! \times 7!) + 4! / (3! \times 1!) = 4\times3 / 2 \times 8/1 + 4/1 = 48+4$
$n(E_2) = 52$
$P(E_2) = n(E_2) / n(S) = 52/220 = 13/55$
(iii) Let $E_3$ = Event of drawing at most ‘2’ defective bulbs. So here, we can get no defective bulbs or 1 is defective and ‘2’ is non-defective or ‘2’ defective bulbs.
$n(E_3) = {}^{8}C_3 + {}^{4}C_1 \times {}^{8}C_2 + {}^{4}C_2 \times {}^{8}C_1$
= 8? / (3? x 5?) + 4? / (1? x 3?) x 8? / (2? x 6?) + 4? / (2? x 2?) x 8? / (1? x 7?)
$= (8\times7\times6) / (3\times2\times1) + 4 \times (8\times7)/2 + (4 \times 3) / 2 + 8/1 = 216$
$P(E_3) = n(E_3) / n(S) = 216 / 220 = 54 / 55.$

**Question** In a lottery of 50 tickets numbered from ‘1’ to ‘50’ two tickets are drawn simultaneously. Find the probability that:
(i) Both the tickets drawn have prime number on them,
(ii) None of the tickets drawn have a prime number on it.
**Solution** We want to select ‘2’ tickets from 50 tickets.
$n(S) = {}^{50}C_2 = 50! / (2! \times 48!) = (50\times49) / 2 = 1225$
(i) Let $E_1$ = Event that both the tickets have prime numbers Prime numbers between ‘1’ to ‘50’ are : 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47.
Total Numbers = 15. We have to select ‘2’ numbers from these 15 numbers.
$n(E_1) = {}^{15}C_2$ = 15? / (2! x 13!) = (15x47) / 2 = 105, $\Rightarrow P(E_1) = n(E_1) / n(S) = 105/1225 = 21/245.$
(ii) Non prime numbers between ‘1’ to ‘50’ = 50-15 = 35
Let $E_2$ = Event that both the tickets have non-prime numbers.
Now we have to select ‘2’ numbers, from ‘35’ numbers.
$n(E_2) = {}^{35}C_2 = 35! / (2! \times 33!) = (35\times34) / 2 = 595$, $\Rightarrow P(E_2) = n(E_2) / n(S) = 595 / 1225 = 17/35$

**Lecture 13**
**Unit – I**
**Probability**

**Question** If from a pack of '52' playing cards one card is drawn at random, what is the probability that it is either a kind or a queen?

**Solution** n(S) = Total number of ways of selecting 1 card out of 52 cards.
$= {}^{52}C_1 = 52$
n(E) = Total number of selections of a card, which is either a kind or a queen.
$= {}^4C_1 + {}^4C_1 = 4 + 4 = 8$
P(E) = n(E) / n(S) = 8 / 52 = 2 / 3

**Question** From a pack of 52 playing cards, three cards are drawn at random. Find the probability of drawing a king, a queen and a jack.

**Solution** Here $n(S) = {}^{52}C_3$ = 52? / (3? x 49?) = (52x51x50) / (3x2x1)
= 52x17x25
$n(E) = {}^4C_1.\ {}^4C_1.\ {}^4C_1$
= 4! / (1! x 3!) x 4! / (1! x 3!) = 4! / (1! x 3!)
n(E) = 4 x 4 x 4, P(E) = n(E) / n(S) = (4x4x4) / (52x17x25) = 16 / 5525

**Question** In solving any problem, odds against A are 4 to 3 and odds in favour of B in solving the same problem are 7 to 5. The probability that the problem will be solved is

**Solution** Let $E$ be the event that problem solved by A and
$F$ be the event that problem is solved by the B.
It is given that $P(\overline{E}) = 4/(4 + 3) = 4/7$ and $P(F) = 7/(7 + 5) = 7/12$.
$P(E) = 1 - P(\overline{E}) = 1 - 4/7 = 3/7$.
E and F are independent events then $P(E \cap F) = P(E) \times P(F) = (3/7) \times (7/12) = 1/4$.
The probability that the problem will be solved is
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= 3/7 + 7/12 - 1/4$
$= 16/21$.

**Question** If '12' persons are seated at a round table, what is the probability that two particulars persons sit together?

**Solution** We have to arrange 12 persons along a round table.
So if 'S" be the sample – space, then n(S) = (12-1)? = 11?
n(S) = 11!
Now we have to arrange the persons in away, such that '2' particulars person sit together.
Regarding that 2 persons as one person, we have to arrange 11 persons.
Total no. of ways = (11-1)! = 10! ways.
That '2' persons can be arranged among themselves in 2! ways.
So, total no. of ways, of arranging 12 persons, along a round table, so that two particular person sit together : = 10! x 2!, n(E) = 10! x 2!, P(E) = n(E) / n(S) = (10! x 2!) / 11! = 2 / 11

**Question** 6 boys and 6 girls sit in a row randomly, find the probability that all the '6' girls sit together.

**Solution** We have to arrange '6' boys and '6' girls in a row.
n(S) = 12!
Now, we have to arrange '6' girls in a way, such that all of them should sit together.

Regarding all the 6 girls as one person, we have to arrange 7 person in a row.
Total no. of ways = 7!
But 6 girls can be arranged among themselves in 6! ways.
n(E) = 7! x 6!
P(E) = n(E) / n(S) = (7! x 6!) / 12! = (6x5x4x3x2x1) / (12x11x10x9x8)
P(E) = 1 / 132

**<u>Question</u>** A bag contains 30 tickets, numbered from '1' to '30'. Five tickets are drawn at random and arranged in ascending order. Find the probability that the third number is 20.

**<u>Solution</u>** Total number of ways of selecting '5' tickets from 30 tickets = $^{30}C_5$
n(S) = $^{30}C_5$ = 30! / (5! x 25!) = (30 x 29 x 28 x 27 x 26) / (5 x 4 x 3 x 2 x 1)
n(S) = 29 x 27 x 26 x 7
Suppose the '5' tickets are a1, a2,20, a4, a5
They are arranged in ascending order.
a1, a2 $\subseteq$ {1, 2, 3, ------- , 19} and a4, a5 $\subseteq$ { 21, 22, 23, -----, 30}
We have to select '2' tickets from first '19' tickets and '2' tickets from last 10 tickets.
n(E) = $^{19}C_2$ x $^{10}C_2$
= 19! / (2! x 17!) = 10! / (2! x 8!) = (19 x 18) / 2 = (10 x 9) / 2
= 19 x 9 x 5 x 9
P(E) = n(E) / n(S) = (19x9x5x9) / (29x27x26x7) = 285 / 5278

**<u>Odds are Favour and Odds against an Event:</u>**

Let 'S" be the sample space and 'E' be an event. Let 'E' devotes the complement of event 'E', then.

(i) Odds in favour of event 'E' = n(E) / n($E^1$)
(ii) Odds in against of an event 'E' = n($E^1$) / n(E)

**<u>Note</u>** Odds in favour of 'E' = n(E) / n($E^1$)
= [n(E) / n(S)] / [n($E^1$) / n(S)] = P(E) / P($E^1$)
Similarly odds in against of 'E' = P($E^1$) / P(E)

# Lecture 14
# Unit – I
# Probability

**Definition of conditional probability** Let A and B be the two events associated with random experiments. Then the probability of occurrence A under the condition that event B has already occurred is called conditional probability. It is denoted by P(A/B).

**Theorem of compound probability OR Multiplication Rule** When two events A and B are independent, the probability of occurring of both the events is

$$P(A \text{ and } B) = P(A)P(B/A).$$

**Question** A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

**Solution** The probability of selecting a black marble and then a white marble is 0.34. The probability of selecting a black marble on the first draw is 0.47.

$$P(\text{White}/\text{ Black}) = \frac{P(\text{Black and White})}{P(\text{Black})} = 0.72.$$

**Question** The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

**Solution** The probability that it is Friday and that a student is absent is 0.03. The probability that it is Friday is 0.2. $P(\text{Absent}|\text{Friday}) = \frac{P(\text{Friday and Absent}}{P(\text{Friday})} = 0.15.$

**Question** A card is drawn from an ordinary deck and we are told that it is red, what is the probability that the card is greater than 2 but less than 9.

**Solution** Let A be the event of getting a card greater than 2 but less than 9.

B be the event of getting a red card. We have to find the probability of A given that B has occurred. That is, we have to find P (A/B). Among the red cards, the number of outcomes which are favourable to A are 12. That is $n(A \cap B) = 12$. Therefore P(A/B) = 6/13.

**Question** A pair of dice is thrown. If it is known that one die shows a 4, what is the probability that

a) the other die shows a 5 b) the total of both the die is greater than 7

**Solution** Let A be the event that one die shows up 4. Then the outcomes which are favourable to A are {(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) (1, 4), (2, 4), (3, 4), (5, 4), (6, 4)}, which implies that n(A) = 11.

a) Let B be the event of getting a 5 in one of the dies. Then the outcomes which are favourable to both A and B are {(4, 5), (5, 4)}, which implies that $n(A \cap B) = 2$. Therefore P(B/A) = 2/11.

b) Let C be the event of getting a total of both the die greater than 7. The out-comes which are favourable to both C and A {(4, 4), (4, 5), (4, 6), (5, 4), (6, 4)}. Therefore P(C/A) = 5/11

Note that in the above example P (B) and P (B/A) are different. P(B/A) =2/11, P(B) = 11/36. Similarly P (C) and P (C/A) are different.

# Lecture 15
## Unit – I
## Probability

**Independent or mutually independent events** Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of the other event.

**Example** When a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.
Some other examples of independent events are:

Landing on heads after tossing a coin **AND** rolling a 5 on a single 6-sided die.

- Choosing a marble from a jar **AND** landing on heads after tossing a coin.
- Choosing a 3 from a deck of cards, replacing it, **AND** then choosing an ace as the second card.
- Rolling a 4 on a single 6-sided die, **AND** then rolling a 1 on a second roll of the die.

**Result** When two events, A and B, are independent, the probability of both occurring is: P(A and B) = P(A)P(B)

**Result** If A and B are independent events associated with random experiment, then (a) $A^C$ and B are independent events

(b) A and $B^C$ are independent events

(c) $A^C$ and $B^C$ are independent events

**Question** Two fair dice are rolled. Use a probability tree diagram to determine the probability of obtaining:
(a) two sixes,
(b) no sixes,
(c) exactly one six.

**Solution** We are only interested if the dice shows a six or not, so we have two outcomes, "six" and "not six". The probability of "six" is 1/6 and the probability of "not six" is 5/6.

We start by drawing a tree diagram to show the outcomes:
Then we write on the probability for each branch. Each set of branches should add up to 1

**Question** If the probability of $n$ independent events are $p_1$, $p_2$…..$p_n$, then the probability that at least one of the event will happen is……………..

**Solution** Let $E_i$ be the happening of ith event.

$P(E_1) = p_1$, $P(E_1) = p_1$, ………………. $P(E_n) = p_n$.

$P(\overline{E_1}) = 1 - P(E_1) = 1 - p_1$,

$P(\overline{E_2}) = 1 - P(E_2) = 1 - p_2$,

…………………..

$P(\overline{E_n}) = 1 - P(E_n) = 1 - p_n$,

The probability that at least one of the event will happen = $P(E_1 \cup E_2 \cup \ldots \cup E_n)$

$= 1 - P(\overline{E_1 \cup E_2 \cup \ldots\ldots\ldots\ldots \cup E_n})$

$= 1 - P(\overline{E_1} \cap \overline{E_2} \ldots\ldots\ldots\ldots \cap \overline{E_n}) = 1 - P(\overline{E_1}) \times P(\overline{E_2}) \times \ldots\ldots P(\overline{E_n})$ [ Since events are independent]

$= 1 - (1 - p_1)(1 - p_2)\ldots\ldots(1 - p_n)$.

**Question** Let A and B be two events such that P(A) = 3/4 and P(B) = 5/8 . Show that $3/8 \leq P(A \cap B) \leq 5/8$?

**Solution** It is given that P(A) = ¾, P(B) = 5/8 and we know that $A \cap B \subseteq B$, which implies that $P(A \cap B) \leq P(B)$. Thus, $P(A \cap B) \leq 5/8$ (1)

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = ¾ + 5/8 - P(A \cap B)$

$= 11/8 - P(A \cap B)$

We know that $P(A \cup B) \leq 1$, which implies that $11/8 - P(A \cap B) \leq 1$,

which gives that $-P(A \cap B) \leq -3/8$, $3/8 \leq P(A \cap B)$. (2)

From (1) and (2)

$3/8 \leq P(A \cap B) \leq 5/8$.

**Question** A problem in Discrete Mathematics is given to three students whose chances of solving it are 1/2, 1/3, ¼ respectively. What is the probability that only one of them solves it correctly?

**Solution** Let P($A$), P($B$), P($C$) be the probability that problem is solved by the students $A$, $B$, $C$.

Therefore, P($A$) =1/2, P($B$) = 1/3, P($C$) = 1/4.

$P(\overline{A}) = 1 - 1/2 = 1/2$, $P(\overline{B}) = 1 - 1/3 = 2/3$ $P(\overline{C}) = 1 - 1/4 = 3/4$

Case I Problem is solved by the student A

$P(A \cap \overline{B} \cap \overline{C}) = P(A) \times P(\overline{B}) \times P(\overline{C}) = 1/2 \times 2/3 \times 3/4 = 1/4$.

Case II Problem is solved by the student A

$P(\overline{A} \cap B \cap \overline{C}) = P(\overline{A}) \times P(B) \times P(\overline{C}) = 1/2 \times 1/3 \times 3/4 = 1/8$.

Case III Problem is solved by the student A

$P(\overline{A} \cap \overline{B} \cap C) = P(\overline{A}) \times P(\overline{B}) \times P(C) = 1/2 \times 2/3 \times 1/4 = 1/12$.

**Lecture 16**
**Unit – I**
**Probability**

## Law Of Total Probability

According to the theory of probability the rule of total probability is defined as that "the previous probability of an event (say A) is similar to the previous expected value of the posterior probability of that event A". Let us consider a random variable N, i.e. for any N,

$$P_r(A) = E[P_r(A \mid N)]$$

Where $P_r(A \mid N)$ is called as the conditional probability of the event A given N.

## More on the Rule of Total Probability

The rule of total probability of an event is also given by the formula

$$P(A) = P(A \cap B) + P(A \cap B')$$

According to the rule of multiplication the above rule of total probability can be expressed as

$$P(A) = P(A \mid B)\, P(B) + P(A \mid B').P(B')$$

Here P (A) denotes the probability that the event A occurs

$P(A \cap B)$ denotes the probability that both the event A and B occurs.

$P(A \cap B')$ denotes the probability that the event A and the event $B'$ occurs.

According to the rule of total probability

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)\, P(A) + P(B \mid A')\, P(A')}$$

**Question** A reasonable coin is tossed. If the coin lands on heads a bag is filled with one black ball and three white balls. If the coin landed on tails the bag is filled with one black ball and nine white balls. A ball is then selected from the bag. What is the probability that the ball selected is black?

**Solution**

Let *H = Heads, T = Tails* and *B = Black ball selected*. Then by the ***law of total probability***

$P(B) = P(B|H)P(H) + P(B|T)P(T)$

$= (0.25)(0.5) + (0.1)(0.5)$

$P(B) = 0.175$

**Question** Three packs contain red and green orb. pack 1 has 5 red orb and 5 green orb, Pack 2 has 7 red orb and 3 green orb and Pack 3 contains 6 red orb and 4 green orb. The probabilities of choosing a pack are `1/4` , `1/6` , `1/8` . What is the probability that the orb chosen is green?

**Solution** we begin by defining the following sets. Let,

G = the orb chosen is green.

$P_1$ = Pack 1 is selected

$P_2$ = Pack 2 is selected

$P_3$ = Pack 3 is selected

Using Law of total probability

Then $P(G|B_1) = 5/10$, $P(G|B_2) = 3/10$ and $P(G|B_3) = 4/10$.

P(G) = `((5/10)xx(1/4)) + ((3/10)xx(1/6))+((4/10)xx(1/8)) = 9/(40)`

**Bayes' theorem** Let S be a sample space associated with random experiment and $E_1, E_2, \ldots\ldots E_n$ be mutually exclusive and exhaustive events. If E is any event occurs within $E_i$, I = 1, 2,….n, then $P(E_i/E) = P(E/E_i)P(E_i)/\sum_{i=1}^{n} P(E/Ei)P(E_i)$; i = 1, 2,….n.

**Example** Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?

Solution*:* The sample space is defined by two mutually-exclusive events - it rains or it does not rain. Additionally, a third event occurs when the weatherman predicts rain. Notation for these events appears below.

Event $A_1$. It rains on Marie's wedding.

Event $A_2$. It does not rain on Marie's wedding

Event B. The weatherman predicts rain.

In terms of probabilities, we know the following:

$P( A_1 ) = 5/365 = 0.0136985$ [It rains 5 days out of the year.]

$P( A_2 ) = 360/365 = 0.9863014$ [It does not rain 360 days out of the year.]

$P( B \mid A_1 ) = 0.9$ [When it rains, the weatherman predicts rain 90% of the time.]

$P( B \mid A_2 ) = 0.1$ [When it does not rain, the weatherman predicts rain 10% of the time.]

We want to know $P( A_1 \mid B )$, the probability it will rain on the day of Marie's wedding, given a forecast for rain by the weatherman. The answer can be determined from Bayes' theorem, as shown below.

$P(A_1/B) = 0.111$

**Question** A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six?

**Solution** Consider the following events: $E_1$: Six occurs, $E_2$ : Six does not occur

E: Man reports the number to be six. $P(E_1) = 1/6$, $P(E_2) = 5/6$. $P(E/E_1) = ¾$, $P(E/ E_2) = ¼$.

$P(E_i/E) = P(E_1)\, P(E_1/E)/ \sum_{i=1}^{2} P(E/Ei)P(E_i) = 3/8$.