Reducing a DFA to a Minimal DFA

Input: DFA_{IN}

Assume DFA_{IN} never "gets stuck"

(add a dead state if necessary)

Output: DFA_{MIN}

An equivalent DFA with the minimum number of states.

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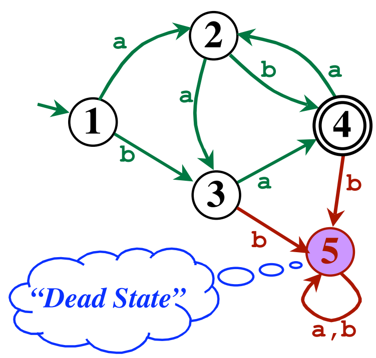

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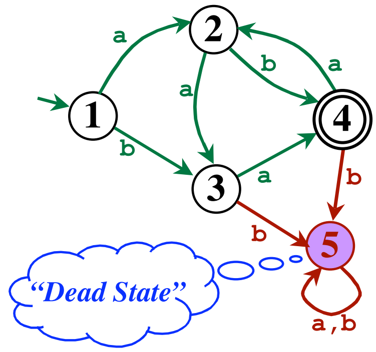

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An equivalent DFA with the minimum number of states.

Approach: Merge two states if the effectively do the same thing.

"Do the same thing?"

At EOF, is DFA_{IN} in an accepting state or not?

Lexical Analysis - Part 4

Sufficiently Different States

Merge states, if at all possible.

Are two states "*sufficiently different*"

... that they cannot be merged?

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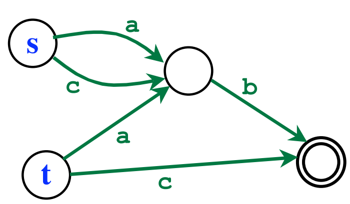
Example:



"`ab`" does not distinguish s and t.
But "`c`" distinguishes s and t.
Therefore, s and t cannot be merged.

Partitioning a Set

A partitioning of a set...

...breaks the set into non-overlapping subsets.

(The partition breaks the set into “groups”)

Example:

S = {A, B, C, D, E, F, G}

Π = {(A B) (C D E F) (G) }

Π_2 = {(A) (B C) (D E F G) }

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We can “refine” a partition...

Π_i = { (A B C) (D E) (F G) }

Π_{i+1} = { (A C) (B) (D) (E) (F G) }

Note:

{ (...) (...) (...) } means {{...}, {...}, {...} }

Hopcroft's Algorithm

Consider the set of states.

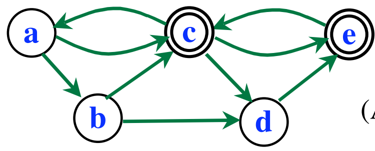
Partition it...

- Final States
- All Other States

Repeatedly "refine" the partioning.

Two states will be placed in different groups

... If they can be "distinguished"



(A B D) (C E)

Repeat until no group contains states that can be distinguished.

Each group in the partitioning becomes one state in a newly constructed DFA

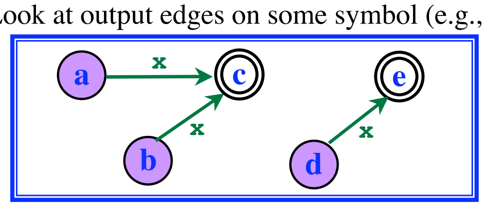
DFA_{MIN} = The minimal DFA

How to Refine a Partitioning?

$\Pi_i = \{ \underbrace{(A\ B\ D)}_{P_1}\ \underbrace{(C\ E)}_{P_2} \}$

Consider one group... (A B D)

Look at output edges on some symbol (e.g., "**x**")



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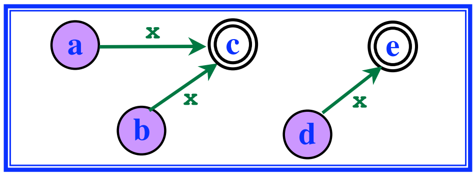

On "**x**", all states in P_1 go to states belonging to the same group.

Lexical Analysis - Part 4

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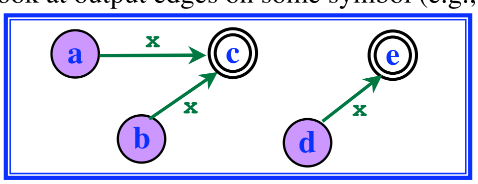

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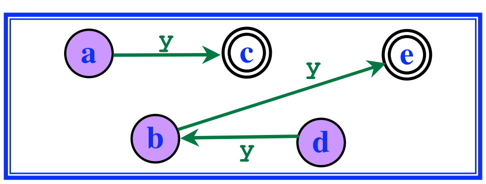

Now consider another symbol (e.g., "**y**")

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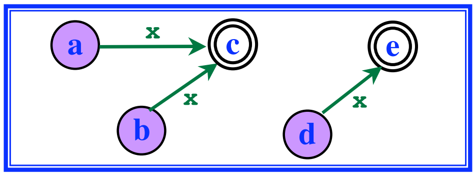

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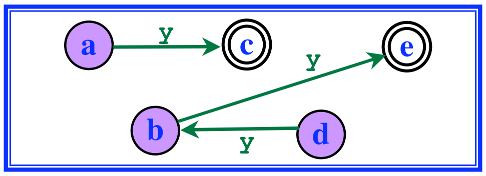

Now consider another symbol (e.g., "**y**")
D is distinguished from A and B!

How to Refine a Partitioning?

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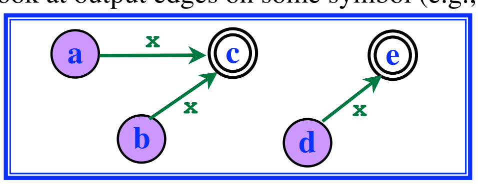

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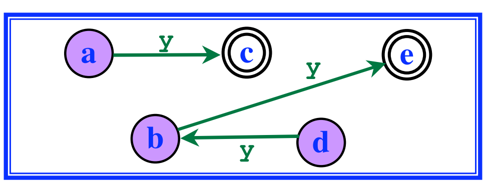
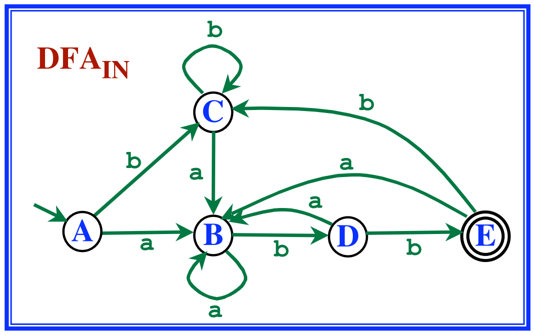


Now consider another symbol (e.g., "**y**")
D is distinguished from A and B!
So refine the partition!

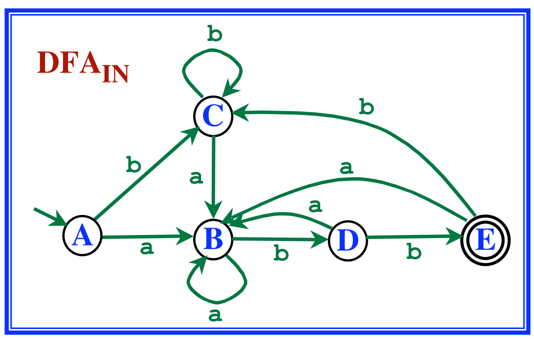
$\Pi_{i+1} = \{ \underbrace{(A\ B)}_{P_3}\ \underbrace{(D)}_{P_4}\ \underbrace{(C\ E)}_{P_2} \}$

Example



Initial Partitioning: Π_1 = (A B C D) (E)

Example



Initial Partitioning: Π_1 = (A B C D) (E)

Consider (A B C D)

Consider (E)

Example

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Consider "**a**"

Consider "**b**"

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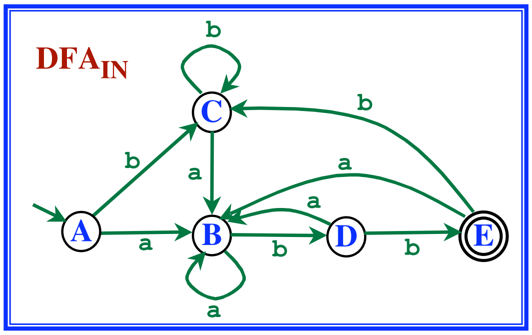
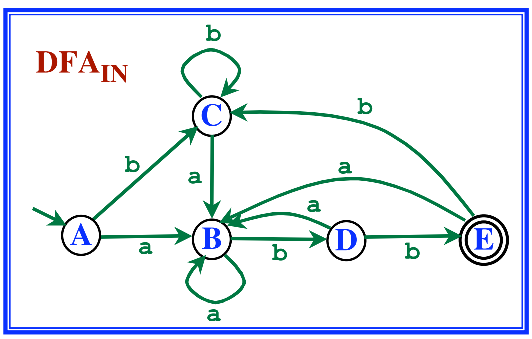


Lexical Analysis - Part 4

Example

Initial Partitioning: Π_1 = (A B C D) (E)
Consider (A B C D)
Consider "**a**"
Break apart?
Consider "**b**"

Consider (E)



Example

Initial Partitioning: Π_1 = (A B C D) (E)

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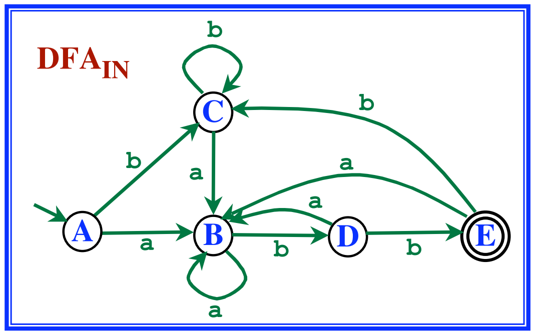
Break apart? No

Consider "**b**"

Break apart?

Consider (E)



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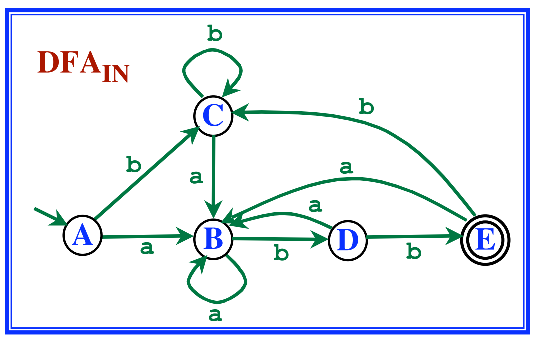
Break apart? (A B C) (D)

Consider (E)



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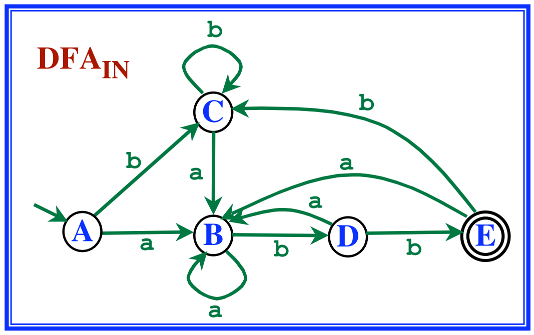

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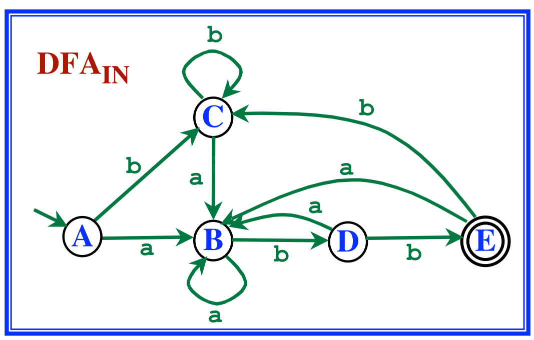

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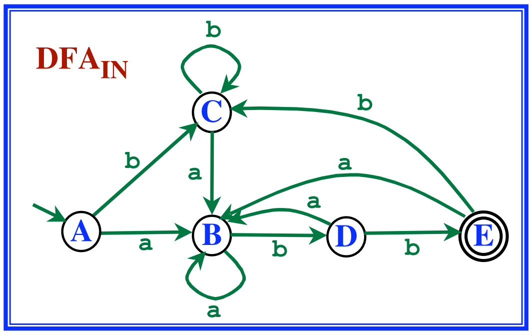

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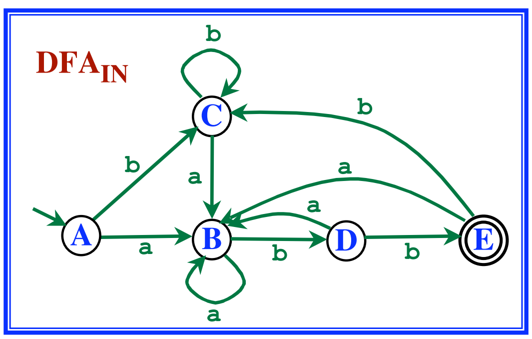

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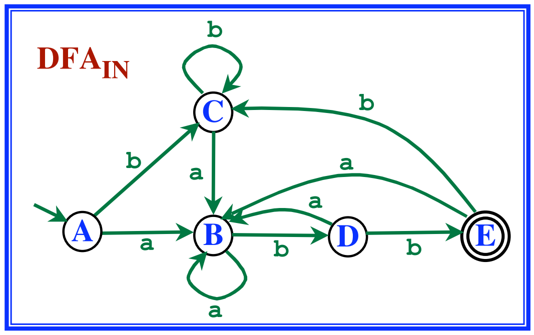

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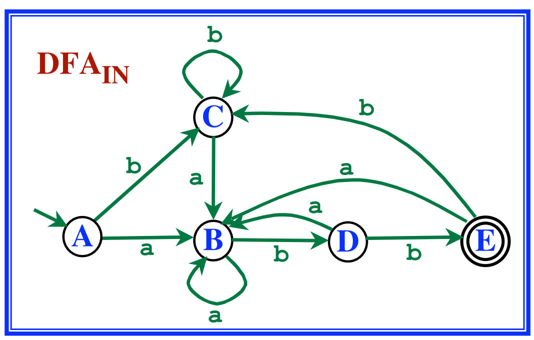

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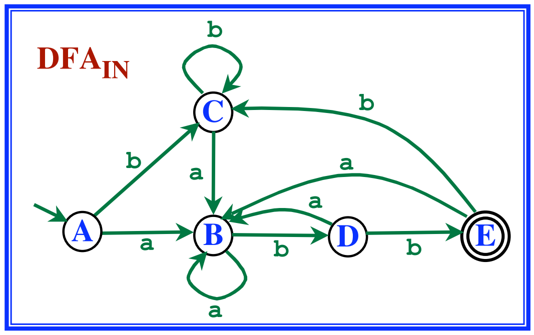
Example



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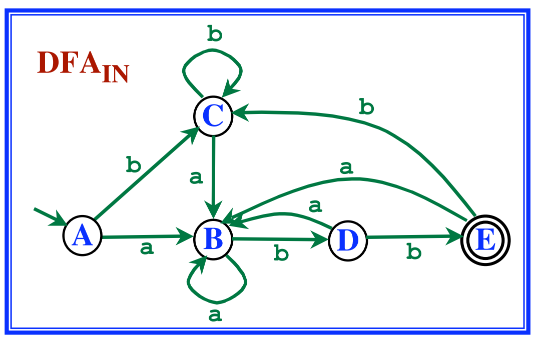

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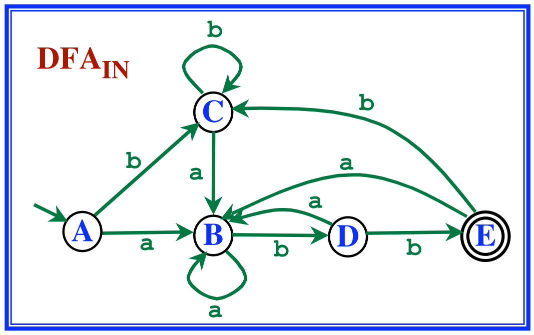

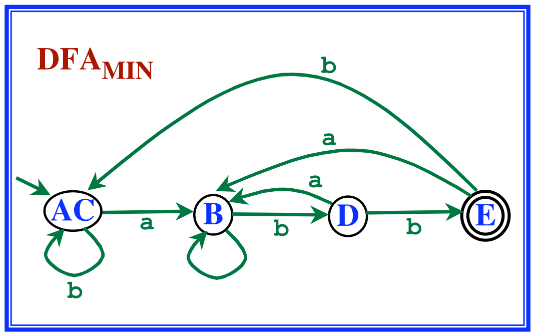

Hopcroft's Algorithm

```
Add dead state and transitions to it if necessary.
   (Now, every state has an outgoing edge on every symbol.)

Π = initial partitioning
loop
  Π_NEW = Refine(Π)
  if (Π_NEW = Π) then break
  Π = Π_NEW
endLoop
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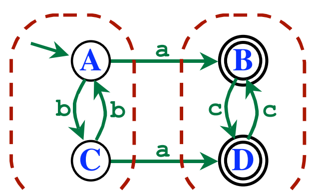

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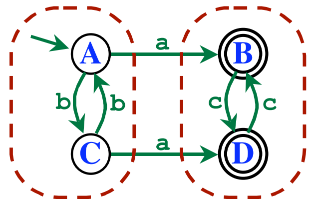

Construct DFA_{MIN}
- Each group in Π becomes a state
- Choose one state in each group (throw all other states away)
- Preserve the edges out of the chosen state

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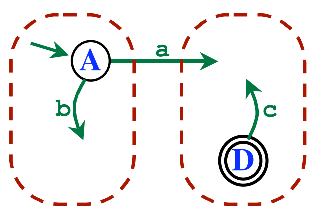

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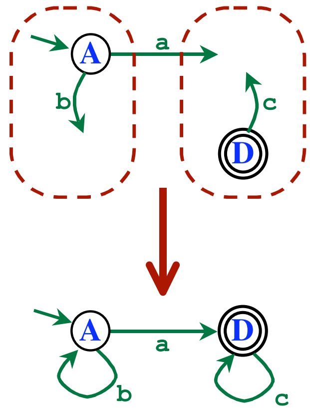

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Construct DFA_{MIN}
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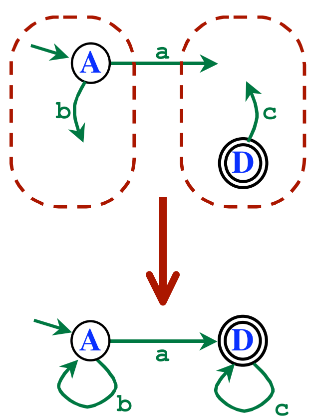
- Preserve the edges out of the chosen state
- Deal with start state and final states



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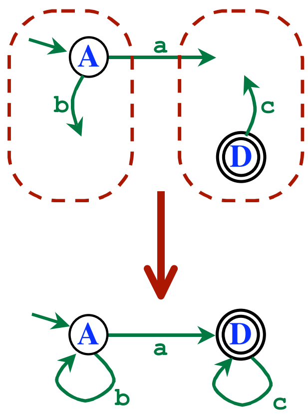
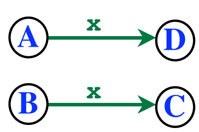
Construct DFA_{MIN}
- Each group in Π becomes a state
- Choose one state in each group (throw all other states away)
- Preserve the edges out of the chosen state
- Deal with start state and final states
- If desired...
  Remove dead state
  Remove any state unreachable from the start state



Π_{NEW} = Refine(Π)

Π_{NEW} = {}
<u>for</u> each group G in Π <u>do</u>
Example: Π = (A B C E) (D F)
  Break G into sub-groups
  (A B C E) → (A C) (B E)
  as follows:
  Put S and T into different subgroups if...
  For any symbol a∈Σ, S and T go to states in two different groups in Π



Must split A and B into different groups

  Add the sub-groups to Π_{NEW}
<u>endFor</u>
<u>return</u> Π_{NEW}

Π_{NEW} = { }
Π_{NEW} = { (A C) (B E) }
Π_{NEW} = { (A C) (B E) (D F) }

Summarizing...

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Fast:
- Get Next Char
- Evaluate Move Function
  e.g., Array Lookup
- Change State Variable
- Test for Accepting State
- Test for EOF
- Repeat

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- Algorithm for Simulating NFA
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  • Change State Variable
  • Test for Accepting State
  • Test for EOF
  • Repeat

- Scanner Generators

  Create an efficient Lexer from regular expressions!

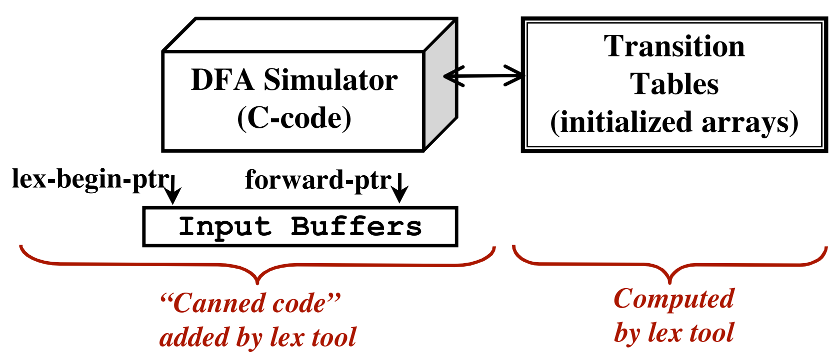
Scanner Generator: LEX

Input:

r_1 { $action_1$ }
r_2 { $action_2$ }
...
r_N { $action_n$ }

Requirements:

- Choose the largest lexeme that matches.
- If more than one r_i matches, choose the first one.



Input:

`a`	{ Action-1 }
`abb`	{ Action-2 }
`a*b+`	{ Action-3 }

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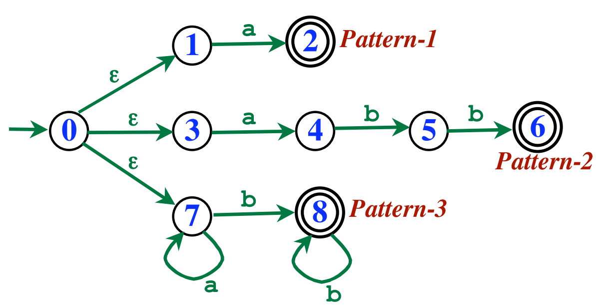
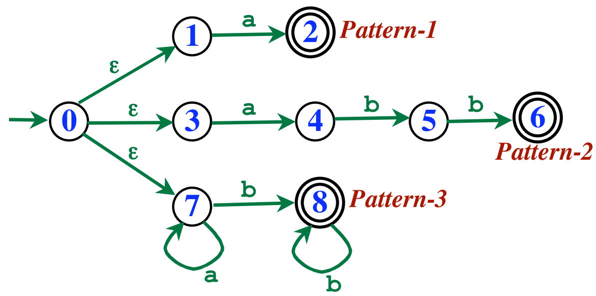
Create NFA:

`a | abb | a*b+`



Input:

`a` { Action-1 }
`abb` { Action-2 }
`a*b+` { Action-3 }



Create NFA:

`a | abb | a*b+`

Example Input: `"aabc..."`

Start: { 0, 1, 3, 7 }
Input: `"a"`
{ **2**, 4, 7 }

Match!
Pattern: 1
Length: 1

Input: `"a"`
{ 7 }
Input: `"b"`
{ **8** }

Match!
Pattern: 3
Length: 3

Input: `"c"`
{ }

Done!
Identify the last match.
Execute the corresponding action & adjust pointers

Approach

- Find the NFA for
  r_1 | r_2 | ... | r_N
- Convert to a DFA.
- Each state of the DFA corresponds to a set of NFA states.
- A state is final if any NFA state in it was a final state.
- If several, choose the lowest numbered pattern to be the one accepted.
- During simulation, keep following edges until you get stuck.
- As the scanning proceeds...
  Every time you enter a final state...
    Remember:
      The current value of buffer pointers
      Which pattern was recognized
- Upon termination...
  Use that information to...
    Adjust the buffer pointers
    Execute the desired action

Example

Input:

```
a     { Action-1 }
abb   { Action-2 }
a*b+  { Action-3 }
```


Example

Input:

```
a     { Action-1 }
abb   { Action-2 }
a*b+  { Action-3 }
```

Create NFA:

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a | abb | a*b+
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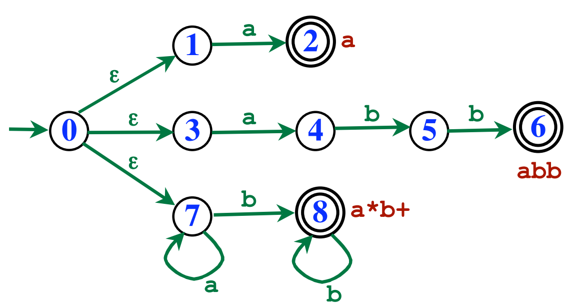


Example

Input:

`a` { Action-1 }

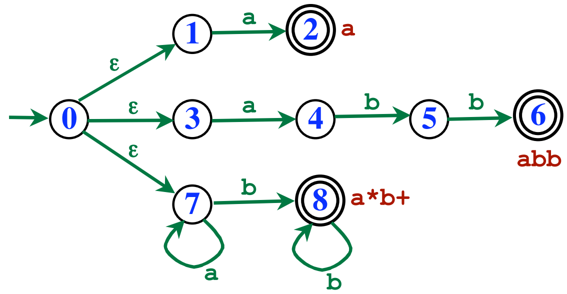
`abb` { Action-2 }

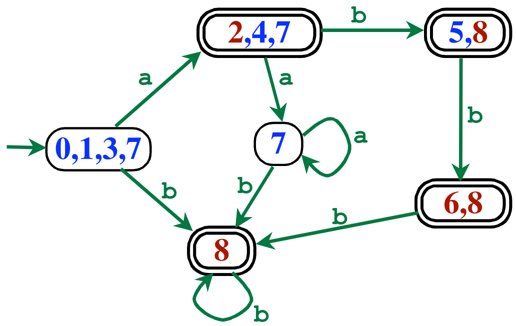
`a*b+` { Action-3 }

Create NFA:

`a | abb | a*b+`

Construct Minimal DFA





Lexical Analysis - Part 4

Example

Input:

`a` { Action-1 }

`abb` { Action-2 }

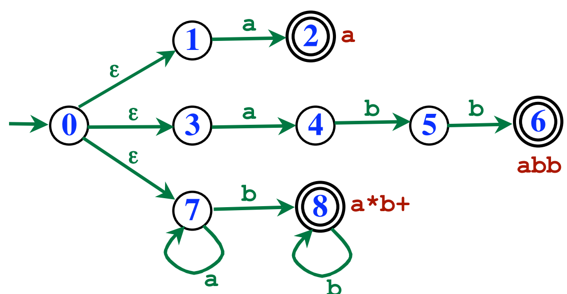
`a*b+` { Action-3 }

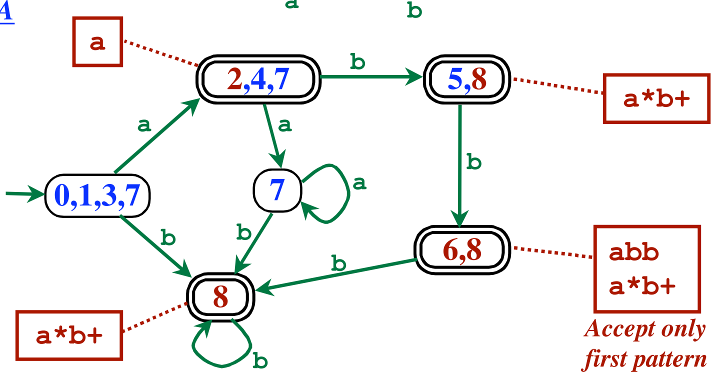
Create NFA:

`a | abb | a*b+`

Construct Minimal DFA

Attach Actions





Example

Input:

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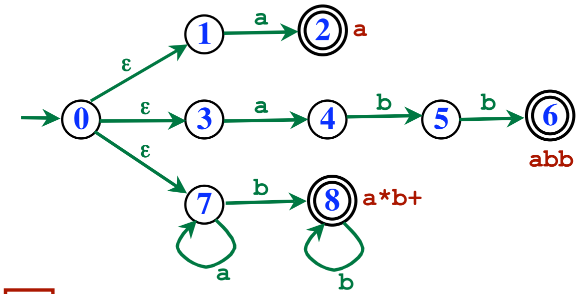
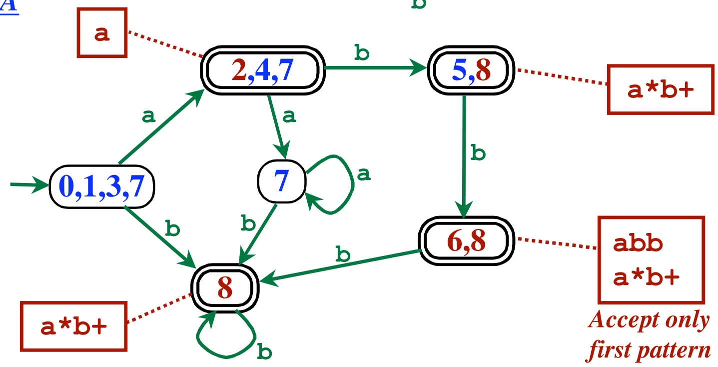


Construct Minimal DFA

Attach Actions

Example Strings:

`a`
`ab`
`abbbbb`
`abb`

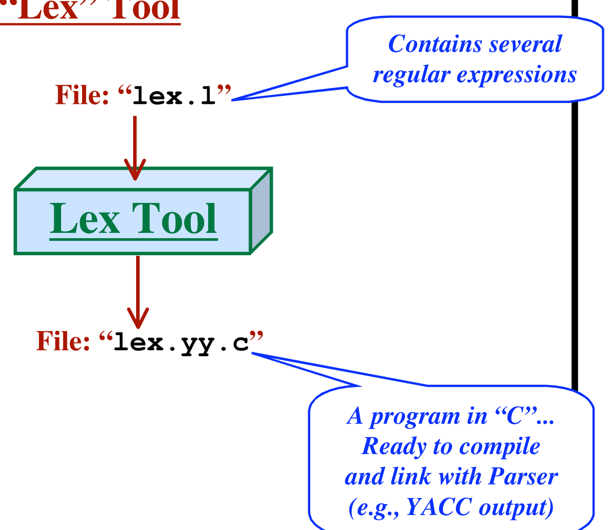


The "Lex" Tool

Oldest, most well-known
For Unix/C Environment

In UNIX:

```
%lex lex.l
%cc lex.yy.c
```



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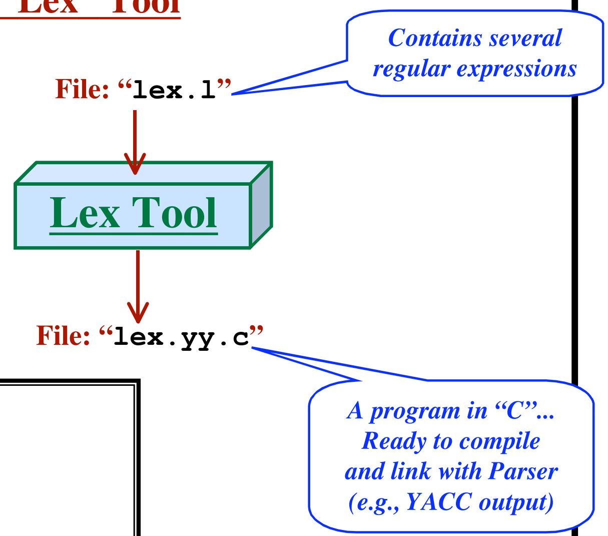

In UNIX:

```
%lex lex.l
%cc lex.yy.c
```

Input File Format:

```
%{
        ...Any "C" Code...
}%
        ...Regular Definitions...
%%
        ...Regular Expressions with Actions...
%%
        ...Any "C" Code...
```


Regular Expressions in Lex

`abc` Concatenation; Most characters stand for themselves

Meta Charaters:

`|`	Usual meanings
`*`	Example: `(a|b)*c*`
`()`	
`+`	One or more, e.g., `ab+c`
`?`	Optional, e.g., `ab?c`
`[x-y]`	Character classes, e.g., `[a-z][a-zA-Z0-9]*`
`[^x-y]`	Anything but `[x-y]`
`\x`	The usual escape sequences, e.g., `\n`
`.`	Any character except '\n'
`^`	Beginning of line
`$`	End of line
`"..."`	To use the meta characters literally, Example: PCAT comments: `"(*".*"*)"`
`{...}`	Defined names, e.g., `{letter}`
`/`	Look-ahead Example: `ab/cd` (Matches `ab`, but only when followed by `cd`)

Look-Ahead Operator, /

abb/cd

"Matches **abb**, but only if followed by **cd**."

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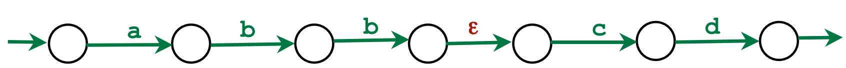
abb/cd

"Matches **abb**, but only if followed by **cd**."

Add a special **ε** edge for **/**



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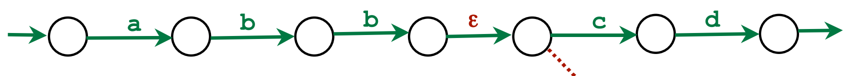


“/” Encountered
Save buffer pointers

Mark the following state to make a note of...
- The pattern in question
- The current value of the buffer pointers

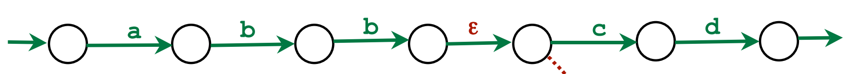
...whenever this state is encountered during scanning.

Look-Ahead Operator, /

abb/cd

“Matches **abb**, but only if followed by **cd**.”

Add a special **ε** edge for **/**



“/” Encountered
Save buffer pointers

Mark the following state to make a note of...
- The pattern in question
- The current value of the buffer pointers

...whenever this state is encountered during scanning.

When a pattern is finally matched, check these notes.
- If we passed through a “/” state for the pattern accepted,
  Use the stored buffer positions,
  instead of the final positions
  to describe the lexeme matched.

Lex: Input File Format

```
%{
    ...Any "C" Code...
}%
    ...Regular Definitions...
%%
    ...Regular Expressions with Actions...
%%
    ...Any "C" Code...
```


Lex: Input File Format

```
%{
    ...Any "C" Code...
    #define ID    13
    #define NUM   14
    #define PLUS  15
    #define MINUS 16
    ...
    #define WHILE 37
    #define IF    38
    ...
}%
    ...Regular Definitions...
%%
    ...Regular Expressions with Actions...
%%
    ...Any "C" Code...
    ...
    int lookup (char * p) {...}
    int enter (char * p, int i) {...}
    ...
```

Any "C" code;
Copied without changes
to beginning of the output file

Any "C" code; added to end of file
(typically, auxillary support routines)

Lex: Input File Format

```
%{
    ...Any "C" Code...
}%
    ...Regular Definitions...
    delim     [ \t\n]
    white     {delim}+
    letter    [a-zA-Z]
    digit     [0-9]
    id        {letter}({letter}|{digit})*
    num       {digit}+(\.{digit}+)?
%%
    ...Regular Expressions with Actions...
%%
    ...Any "C" Code...
```

Defined Names

Blank: Every character is Itself literally

Defined names can be used in regular expressions

Lex: Input File Format

```
%{
    ...Any "C" Code...
}%
    ...Regular Definitions...
%%
    ...Regular Expressions with Actions...
    "+"       {return PLUS;}
    "-"       {return MINUS;}
    ...
    while     {return WHILE;}
    if        {return IF;}
    ...
    {white}   {}
    ...
    {num}     {yylval = ...; return NUM;}
    {id}      {yylval = ...lookup(...)...; return ID;}
%%
    ...Any "C" Code...
```

Regular expressions

Any "C" code. Include "return" to give the token to parser.

No return means "do nothing". (This "token" is recognized but not returned to parser)

`yylval` *is where token attribute info is stored.*

You may use these variables to access the lexeme:

```
char * yytext;
int yyleng;
```