DFA Minimization

DFA Minimization using Myphill-Nerode Theorem

Algorithm

Input − DFA

Output − Minimized DFA

Step 1 − Draw a table for all pairs of states (Q_i, Q_j) not necessarily connected directly [All are unmarked initially]

Step 2 − Consider every state pair (Q_i, Q_j) in the DFA where $Q_i \in F$ and $Q_j \notin F$ or vice versa and mark them. [Here F is the set of final states]

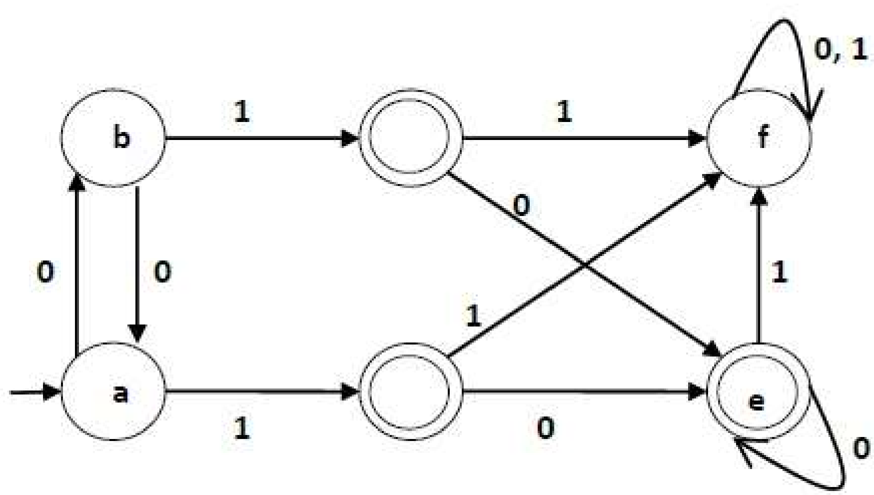
Step 3 − Repeat this step until we cannot mark anymore states −

If there is an unmarked pair (Q_i, Q_j), mark it if the pair {δ (Q_i, A), δ (Q_i, A)} is marked for some input alphabet.

Step 4 − Combine all the unmarked pair (Q_i, Q_j) and make them a single state in the reduced DFA.

Example

Let us use Algorithm 2 to minimize the DFA shown below.



Step 1 − We draw a table for all pair of states.

	a	b	c	d	e	f
a						
b						
c						
d						
e						
f						

Step 2 − We mark the state pairs.

	a	b	c	d	e	f
a						
b						
c	✓	✓				
d	✓	✓				
e	✓	✓				
f			✓	✓	✓	

Step 3 − We will try to mark the state pairs, with green colored check mark, transitively. If we input 1 to state ‘a’ and ‘f’, it will go to state ‘c’ and ‘f’ respectively. (c, f) is already marked, hence we will mark pair (a, f). Now, we input 1 to state ‘b’ and ‘f’; it will go to state ‘d’ and ‘f’ respectively. (d, f) is already marked, hence we will mark pair (b, f).

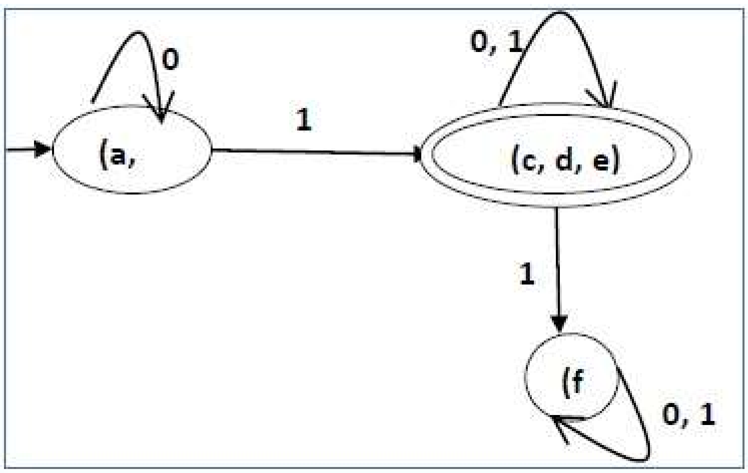
	a	b	c	d	e	f
a						
b						
c	✓	✓				
d	✓	✓				
e	✓	✓				
f	✓	✓	✓	✓	✓	

After step 3, we have got state combinations {a, b} {c, d} {c, e} {d, e} that are unmarked.

We can recombine {c, d} {c, e} {d, e} into {c, d, e}

Hence we got two combined states as − {a, b} and {c, d, e}

So the final minimized DFA will contain three states {f}, {a, b} and {c, d, e}



DFA Minimization using Equivalence Theorem

If X and Y are two states in a DFA, we can combine these two states into {X, Y} if they are not distinguishable. Two states are distinguishable, if there is at least one string S, such that one of δ (X, S) and δ (Y, S) is accepting and another is not accepting. Hence, a DFA is minimal if and only if all the states are distinguishable.

Algorithm 3

Step 1 − All the states **Q** are divided in two partitions − **final states** and **non-final states** and are denoted by $\mathbf{P_0}$. All the states in a partition are 0^{th} equivalent. Take a counter **k** and initialize it with 0.

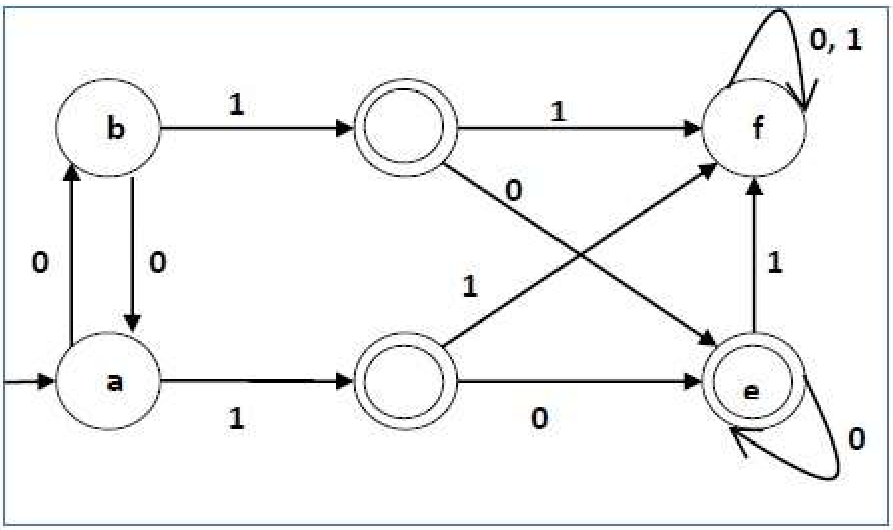
Step 2 − Increment k by 1. For each partition in P_k, divide the states in P_k into two partitions if they are k-distinguishable. Two states within this partition X and Y are k-distinguishable if there is an input **S** such that **δ(X, S)** and **δ(Y, S)** are (k-1)-distinguishable.

Step 3 − If $P_k \neq P_{k-1}$, repeat Step 2, otherwise go to Step 4.

Step 4 − Combine k^{th} equivalent sets and make them the new states of the reduced DFA.

Example

Let us consider the following DFA −



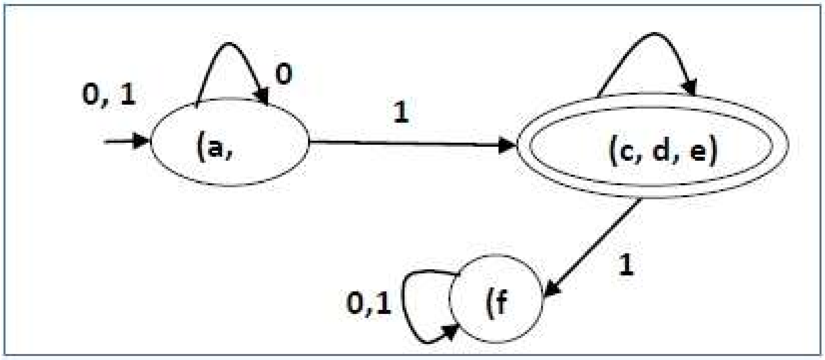
q	δ(q,0)	δ(q,1)
a	b	c
b	a	d
c	e	f
d	e	f
e	e	f
f	f	f

Let us apply the above algorithm to the above DFA −

- P_0 = {(c,d,e), (a,b,f)}
- P_1 = {(c,d,e), (a,b),(f)}
- P_2 = {(c,d,e), (a,b),(f)}

Hence, $P_1 = P_2$.

There are three states in the reduced DFA. The reduced DFA is as follows −



Q	δ(q,0)	δ(q,1)
(a, b)	(a, b)	(c,d,e)
(c,d,e)	(c,d,e)	(f)
(f)	(f)	(f)