MA5710
Mathematical Modelling in Industry

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ED15B013

Image Diffusion Filters Report

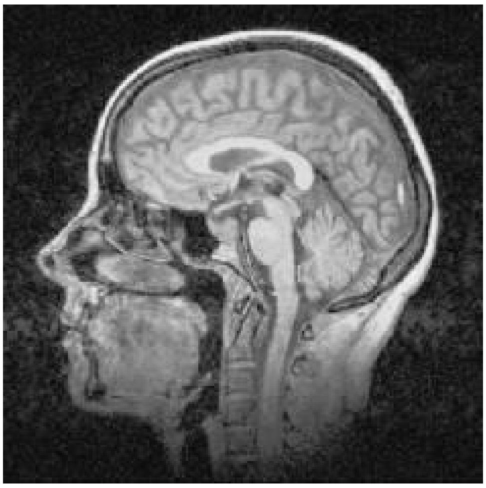
1 Edge Enhanced Diffusion

$$\text{Diffusivity } g(s^2) = \frac{1}{1 + \frac{s^2}{\lambda^2}}$$

$$\text{Flux Function } \Phi(s) = \frac{s}{1 + \frac{s^2}{\lambda^2}}$$

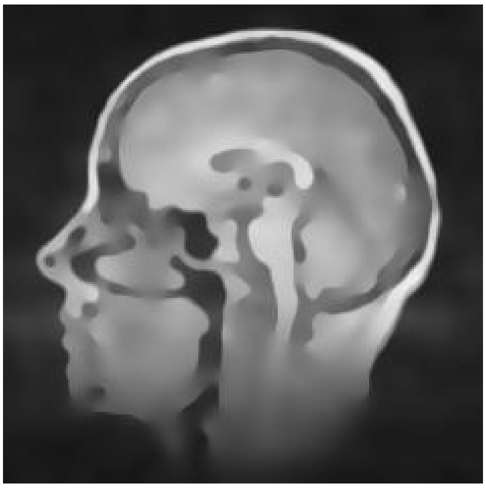
Code is available in the zip file attached with the mail.

1.1 Results



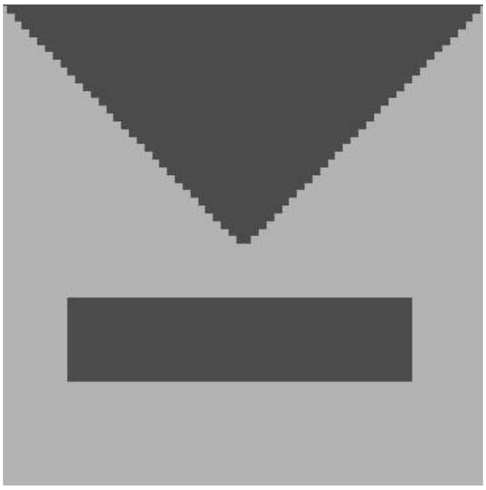
(a) Original Image

(b) Gaussian Noise, variance = 0.1

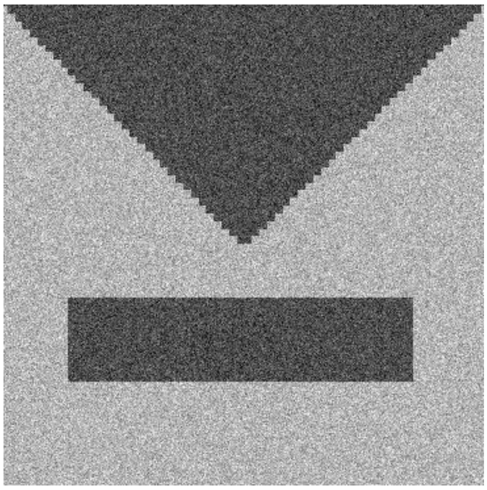


(c) Cleaned Image

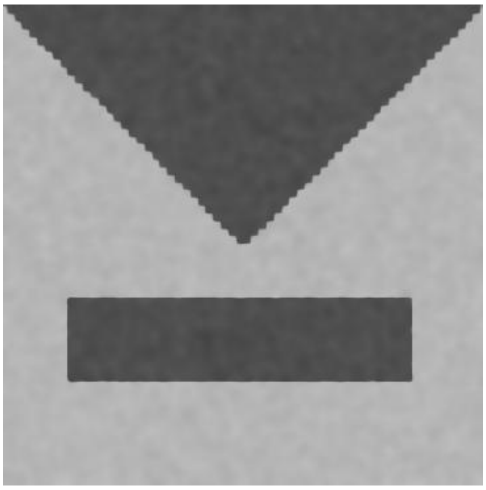
Figure 1: Tomo



(a) Original Image



(b) Gaussian Noise, variance = 0.1



(c) Cleaned Image

Figure 2: Triangle

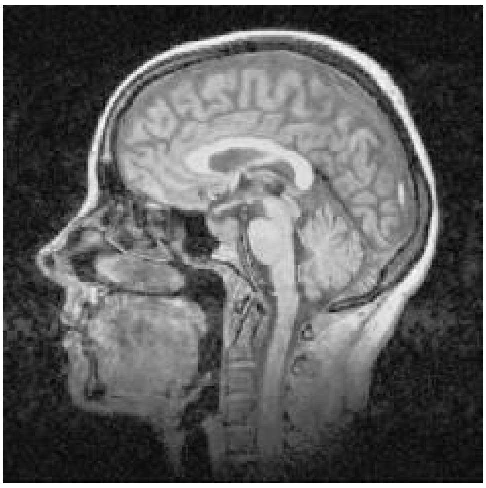
2 Coherence Enhancing Diffusion

$$\lambda_1 = \alpha$$

$$\lambda_2 = \begin{cases} \alpha & \text{if } \mu_1 = \mu_2 \\ \alpha + (1-\alpha)\frac{-C}{(\mu_1-\mu_2)^{2m}} & else \end{cases}$$

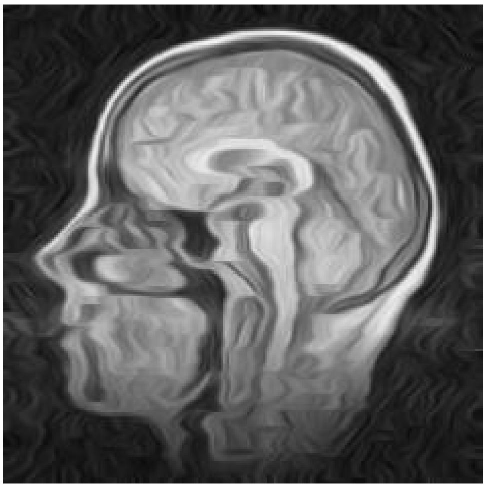
Code is available in the zip file attached with the mail.

2.1 Results



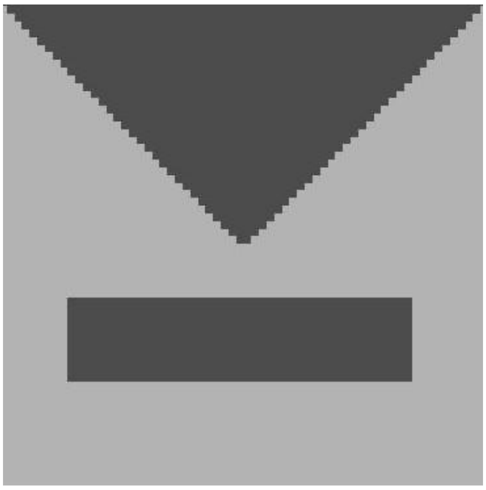
(a) Original Image

(b) Gaussian Noise, variance = 0.01

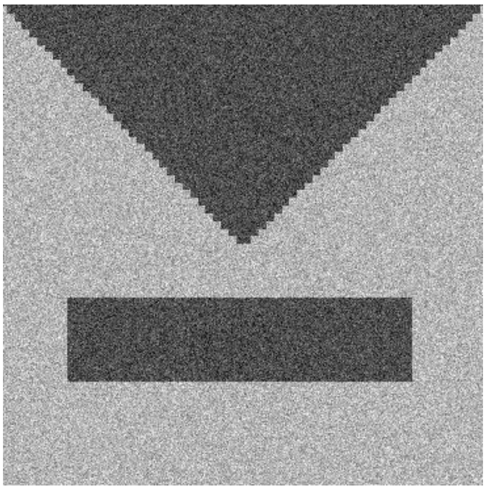


(c) Cleaned Image

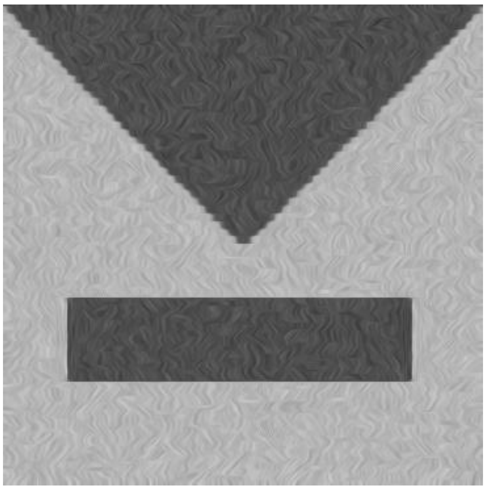
Figure 3: Tomo



(a) Original Image



(b) Gaussian Noise, variance = 0.01

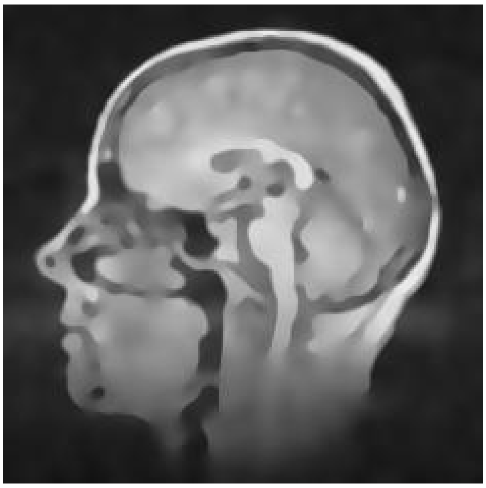


(c) Cleaned Image

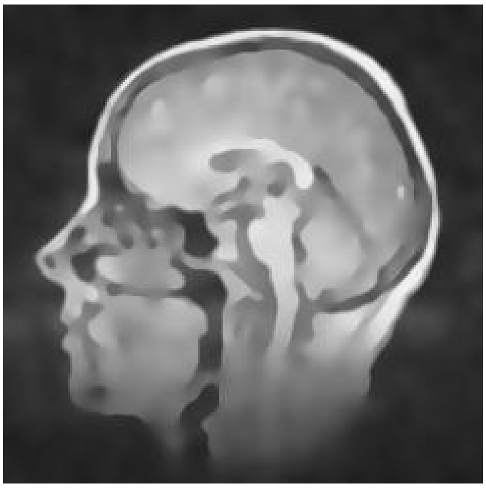
Figure 4: Triangle

3 Different Level of Noise

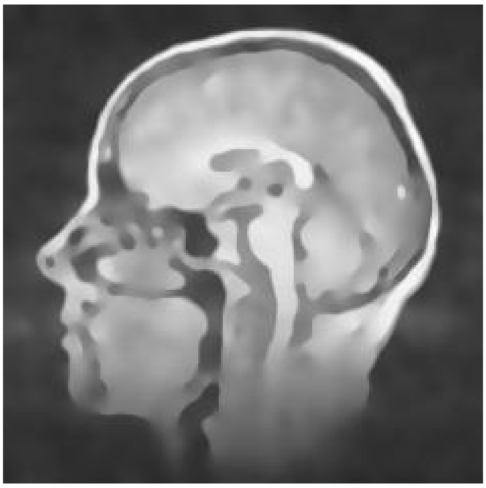
3.1 EED



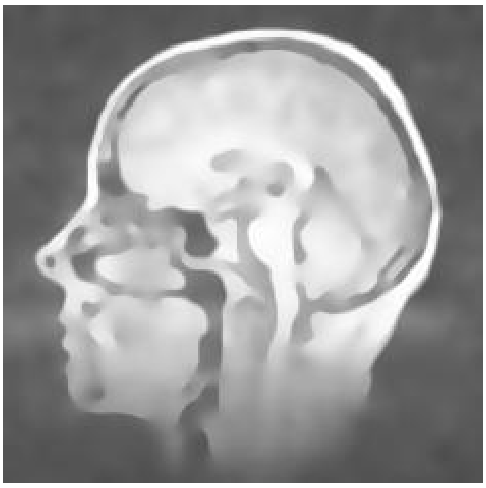
(a) variance = 0.01



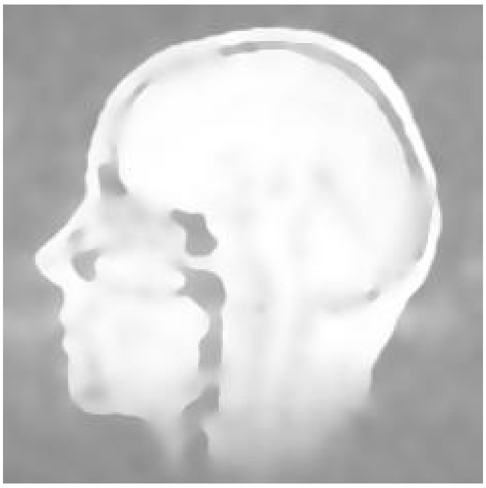
(b) variance = 0.05



(c) variance = 0.10

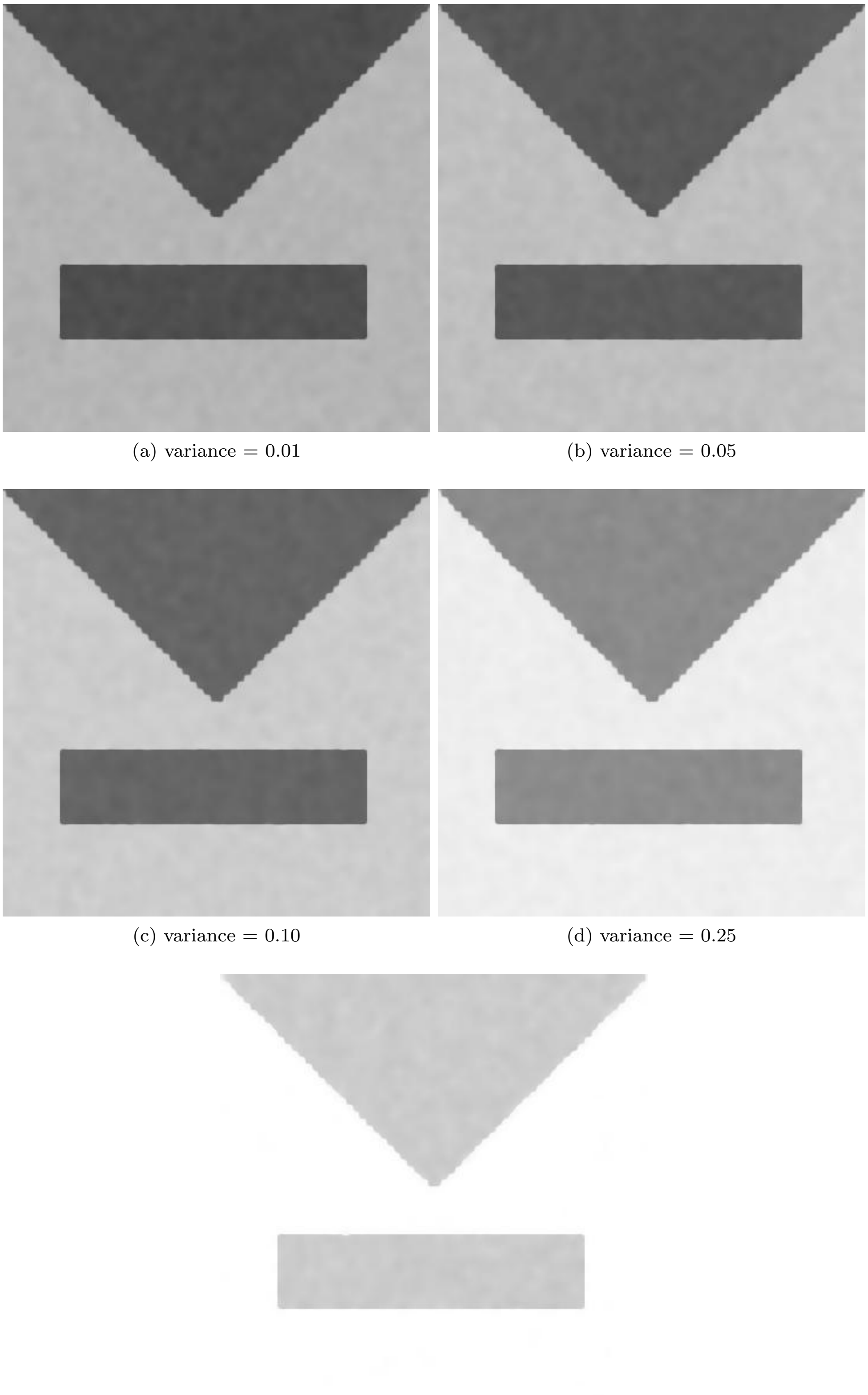


(d) variance = 0.25



(e) variance = 0.50

Figure 5: Tomo - EED for Different Levels of Gaussian Noise



(a) variance = 0.01

(b) variance = 0.05

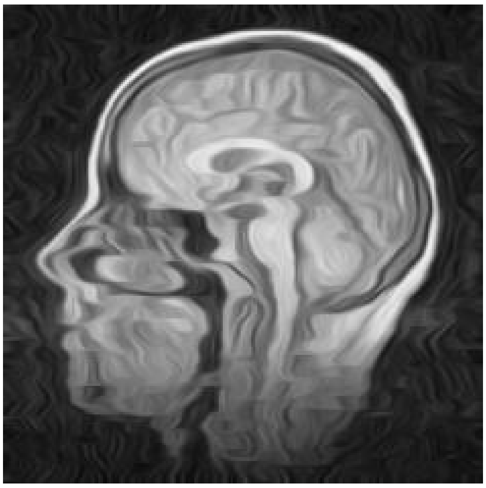
(c) variance = 0.10

(d) variance = 0.25

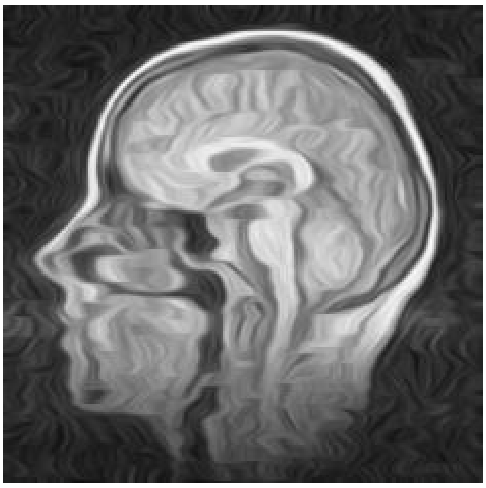
(e) variance = 0.50

Figure 6: Triangle - EED for Different Levels of Gaussian Noise

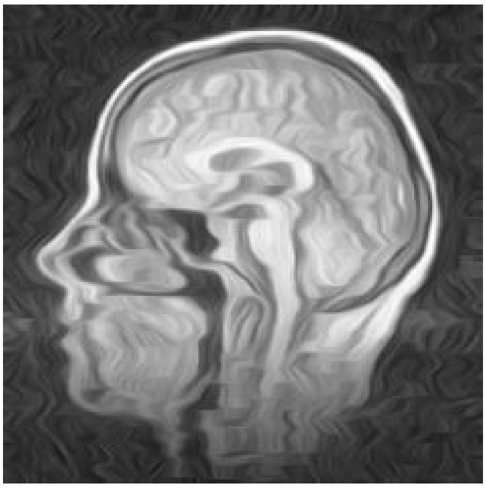
3.2 CED



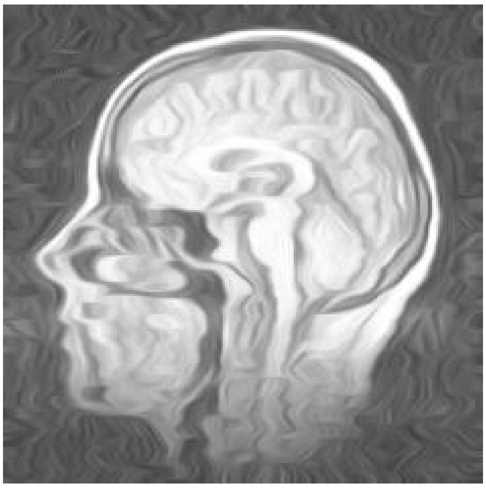
(a) variance = 0.01



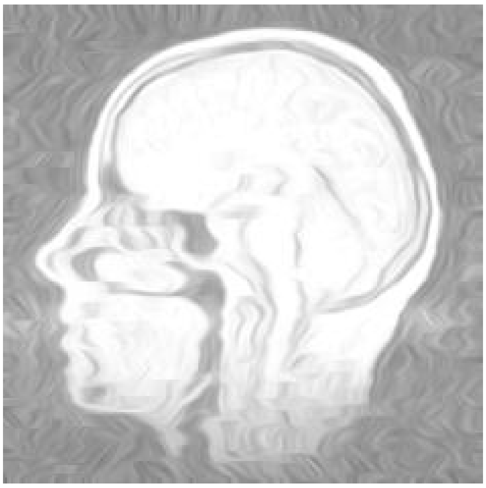
(b) variance = 0.05



(c) variance = 0.10

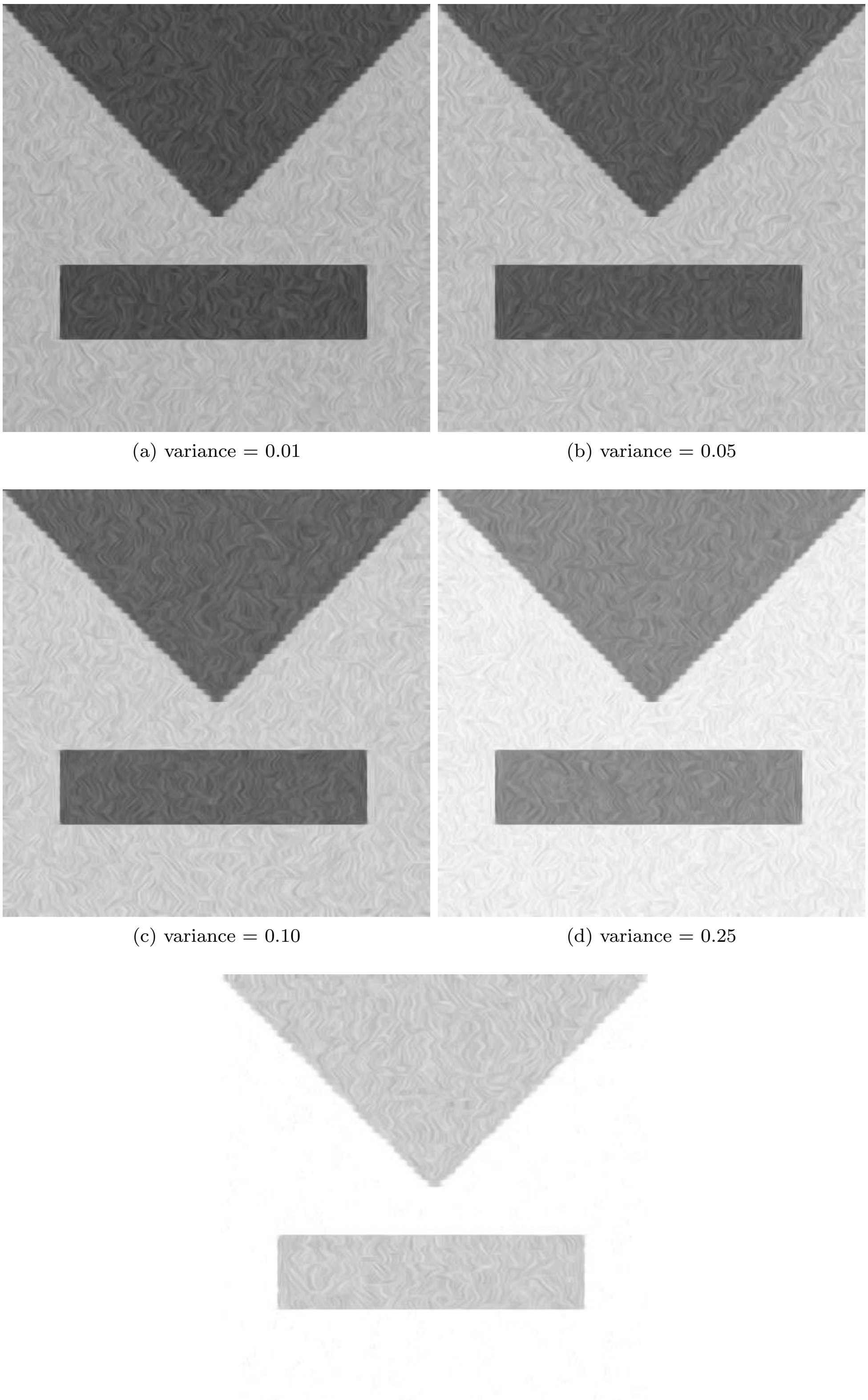


(d) variance = 0.25



(e) variance = 0.50

Figure 7: Tomo - CED for Different Levels of Gaussian Noise



(a) variance = 0.01

(b) variance = 0.05

(c) variance = 0.10

(d) variance = 0.25

(e) variance = 0.50

Figure 8: Triangle - CED for Different Levels of Gaussian Noise

3.3 PSNR and MSE for different levels of noise

		EED		CED	
	Variance	PSNR	MSE	PSNR	MSE
Tomo	0.01	25.8355	169.6412	26.9776	130.4116
	0.05	23.0151	324.7666	23.5480	287.2654
	0.10	19.0493	809.3756	19.3409	756.8116
	0.25	12.0768	4030.9103	12.1418	3970.9902
	0.50	6.7827	13639.9816	6.8137	13542.8743
Triangle	0.01	34.2346	24.5256	30.3502	59.9872
	0.05	25.6495	177.0632	24.8210	214.2786
	0.10	19.9438	658.7266	19.7339	691.3343
	0.25	12.4575	3692.5849	12.4292	3716.7462
	0.50	8.2352	9762.5582	8.2385	9755.0832

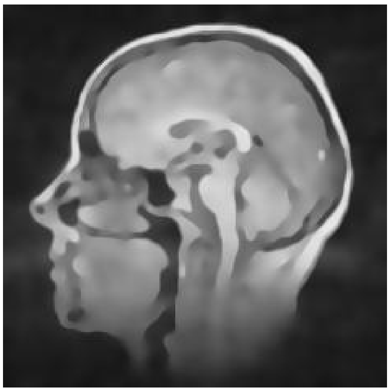
3.3.1 Noise Level and Performance

It can bee seen from the data that as the noise level increases, the PSNR value decreases, as well as the MSE value increases. This shows that as the noise level increases, the cleaning performance decreases.

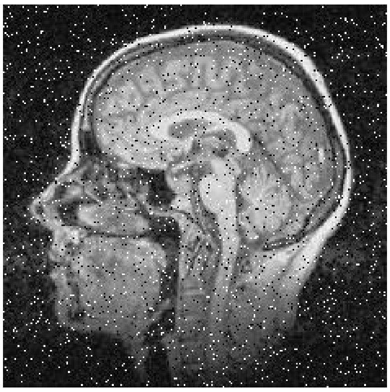
4 Different Kind of Noises

4.1 EED

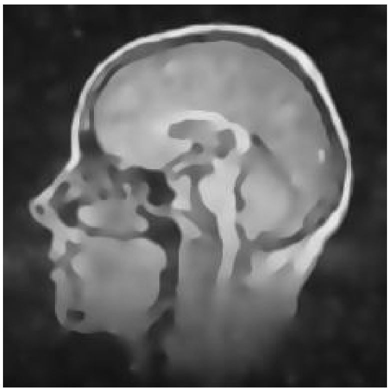
(a) Gaussian



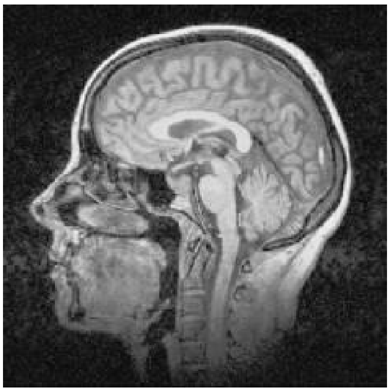
(b) Gaussian - Cleaned



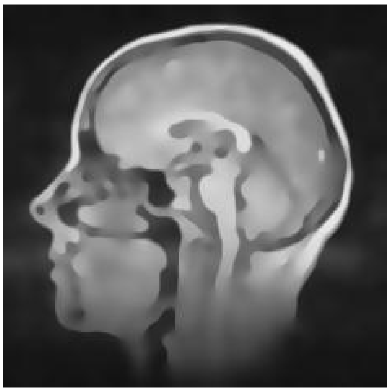
(c) Salt and Pepper



(d) Salt and Pepper - Cleaned

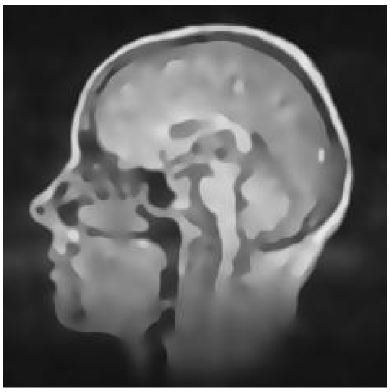


(e) Poisson



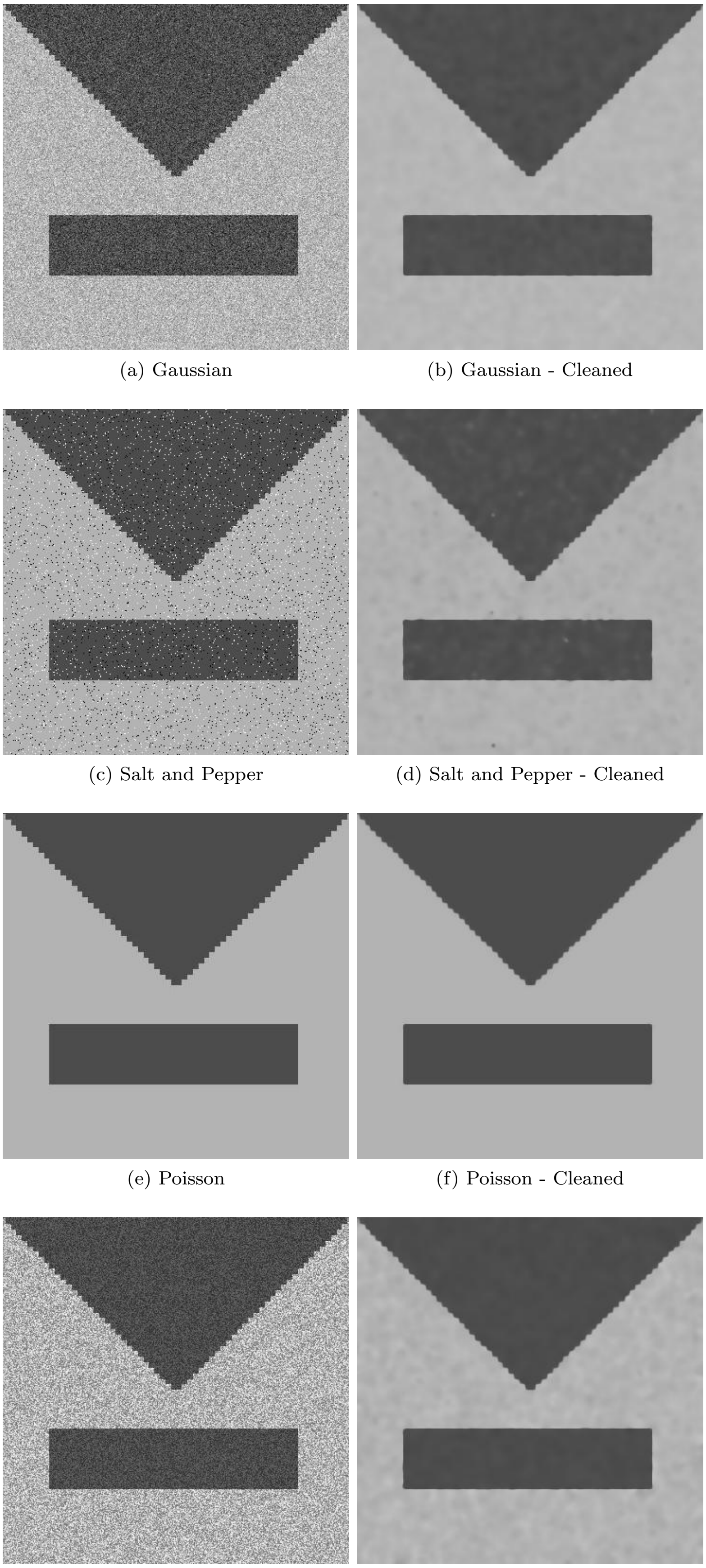
(f) Poisson - Cleaned

(g) Speckle



(h) Speckle - Cleaned

Figure 9: Tomo - EED for Different Kind of Noises



(a) Gaussian (b) Gaussian - Cleaned

(c) Salt and Pepper (d) Salt and Pepper - Cleaned

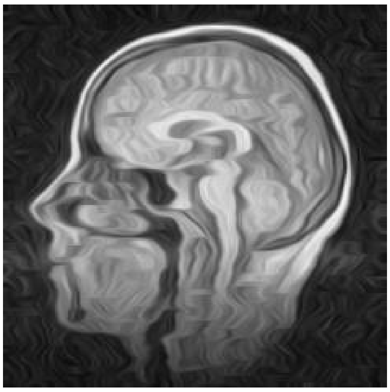
(e) Poisson (f) Poisson - Cleaned

(g) Speckle (h) Speckle - Cleaned

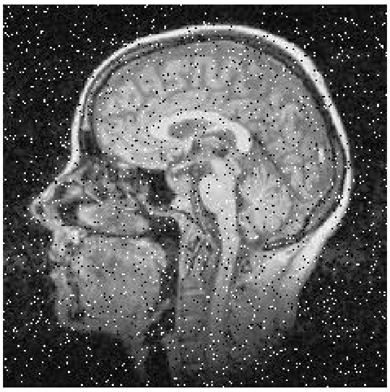
Figure 10: Triangle - EED for Different Kind of Noises

4.2 CED

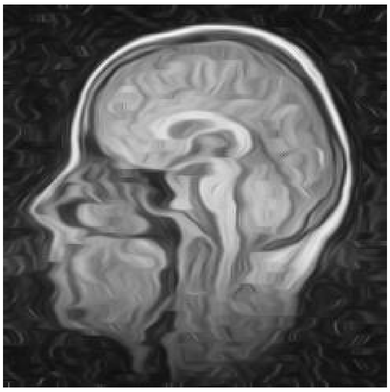
(a) Gaussian



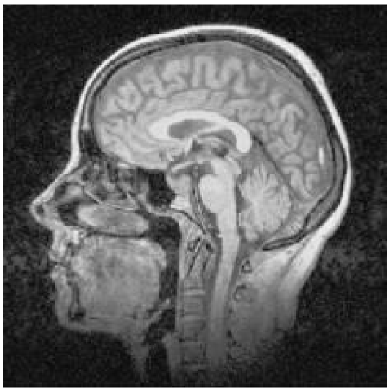
(b) Gaussian - Cleaned



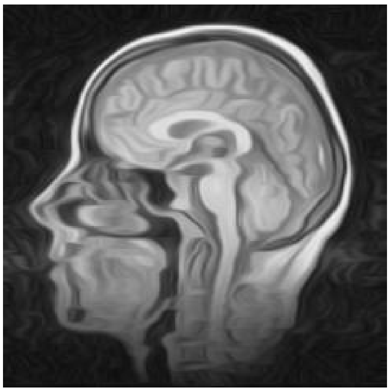
(c) Salt and Pepper



(d) Salt and Pepper - Cleaned

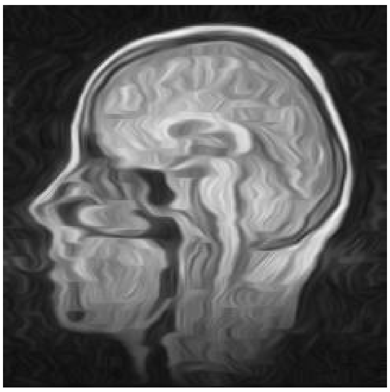


(e) Poisson



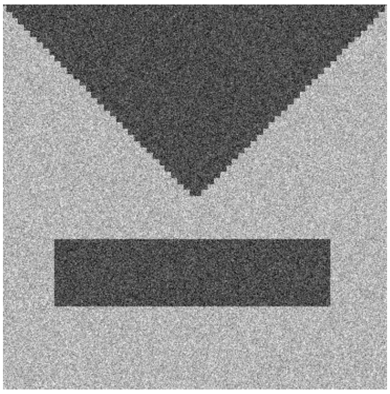
(f) Poisson - Cleaned

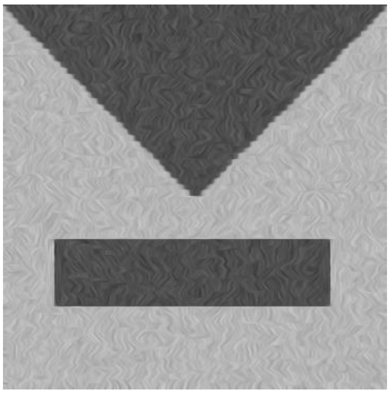
(g) Speckle

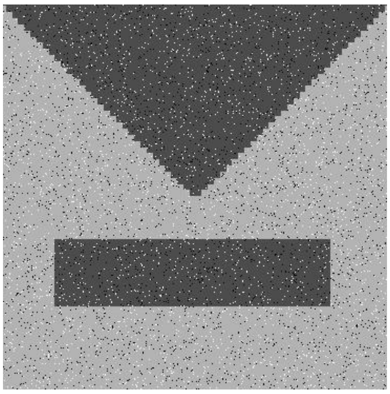


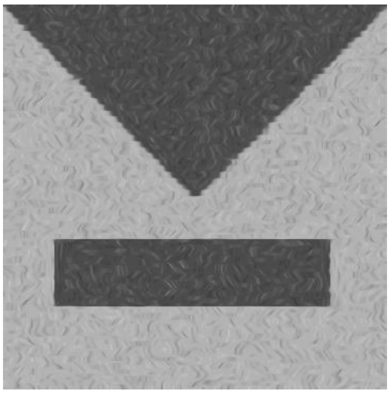
(h) Speckle - Cleaned

Figure 11: Tomo - CED for Different Kind of Noises


(a) Gaussian

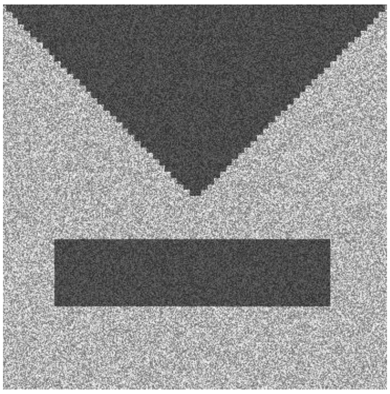

(b) Gaussian - Cleaned


(c) Salt and Pepper


(d) Salt and Pepper - Cleaned


(e) Poisson


(f) Poisson - Cleaned


(g) Speckle

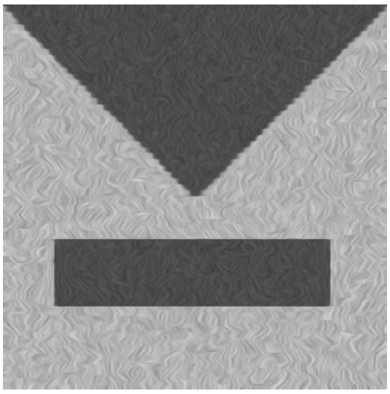

(h) Speckle - Cleaned

Figure 12: Triangle - CED for Different Kind of Noises

4.3 PSNR and MSE for different kind of noises

		EED		CED	
	Variance	PSNR	MSE	PSNR	MSE
Tomo	Gaussian	25.8103	170.6273	26.9618	130.8877
	Salt and Pepper	25.4032	187.3964	26.3845	149.4973
	Poisson	26.2790	153.1741	29.0430	81.0546
	Speckle	25.9982	163.4048	27.3422	119.9106
Triangle	Gaussian	34.3071	24.1199	30.3531	59.9468
	Salt and Pepper	33.3016	30.4031	29.7614	68.6973
	Poisson	37.0701	12.7664	35.9050	16.6948
	Speckle	34.3475	23.8965	29.2987	76.4201

5 Perception of PSNR and MSE

$$MSE = \frac{1}{MN}\sum_{n=1}^{N}\sum_{m=1}^{M}[imgRef_{n,m} - img_{n,m}]^2$$

The with mean-squared error (MSE) is that it depends strongly on the image intensity scaling. A mean-squared error of 100 for an 8-bit image (with pixel values in the range 0-255) is a very high error; but the same error is barely noticeable for 10-bit image (pixel values in range 0-1023).

Peak Signal-to-Noise Ratio (PSNR) avoids this problem by scaling the MSE according to the image range:

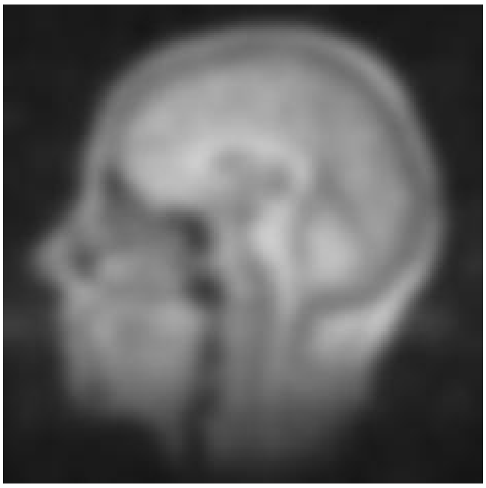
$$PSNR = 10\ log_{10}\frac{S^2}{MSE}$$

where S is the maximum pixel value.

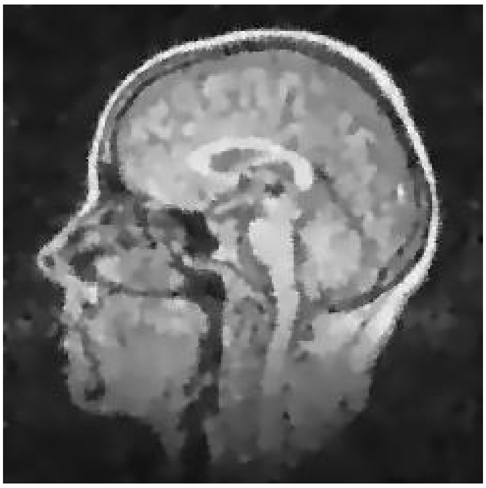
PSNR is measured in decibels (dB). The PSNR measure is also not ideal, but is in common use. Its main failing is that the signal strength is estimated as S^2, rather than the actual signal strength for the image. PSNR is a good measure for comparing restoration results for the same image, but between-image comparisons of PSNR are meaningless. One image with 30 dB PSNR may look much better than another image with 40 dB PSNR.

6 Performance of Various Methods

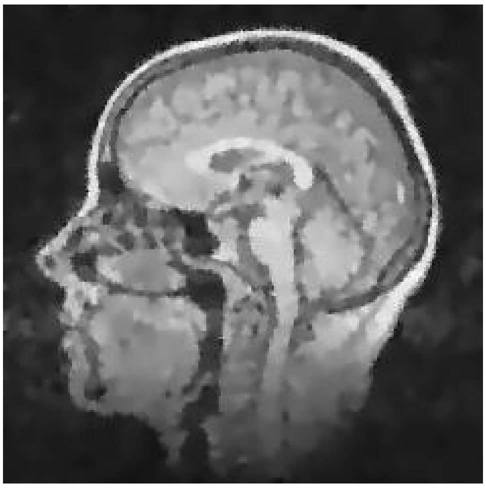
6.1 Images



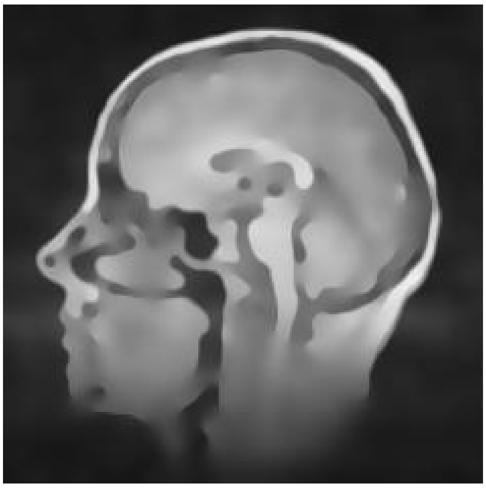
(a) LD



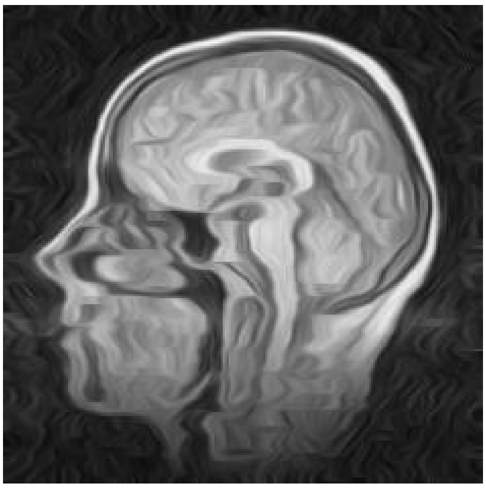
(b) PM



(c) PMC

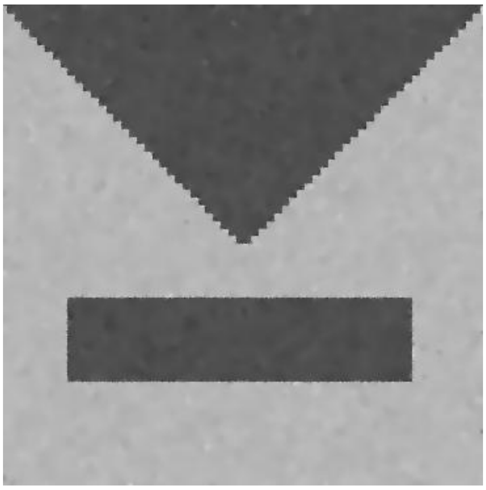


(d) EED

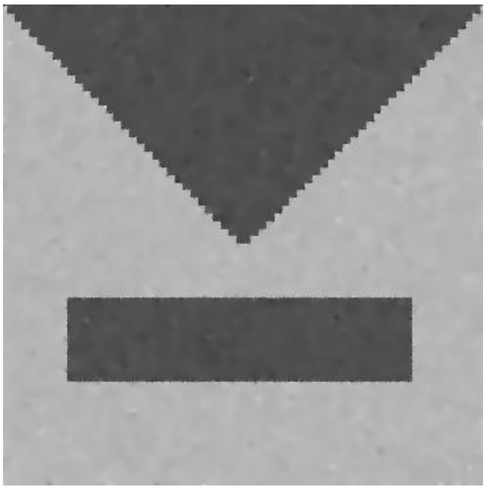


(e) CED

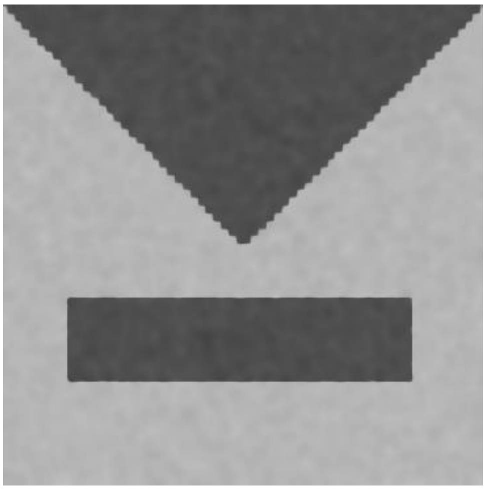
(f) LD



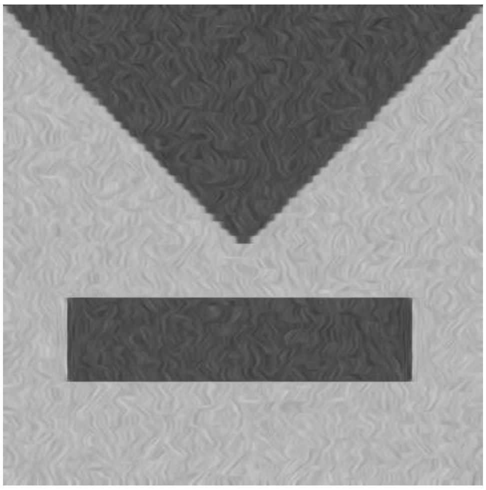
(g) PM



(h) PMC



(i) EED



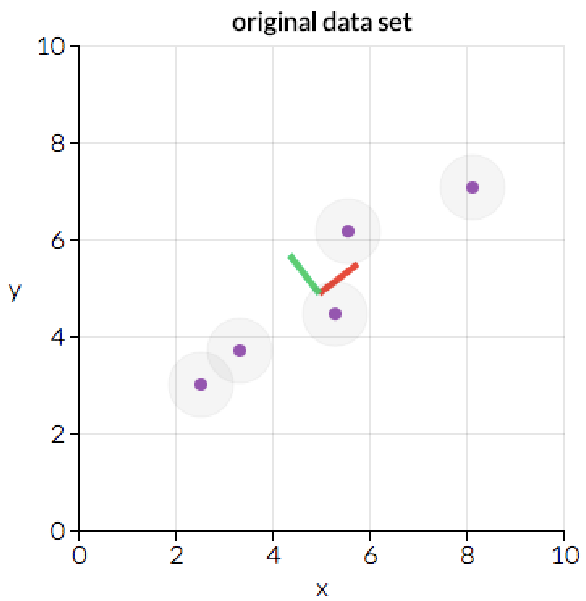
(j) CED

6.2 PSNR and MSE for different methods

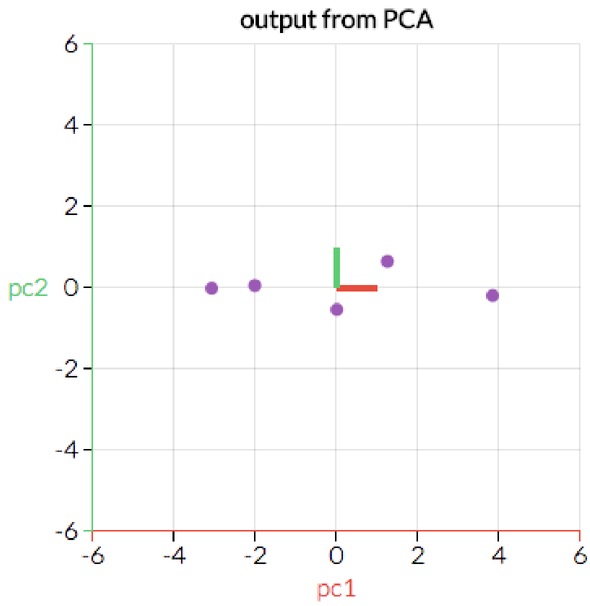
	Method	PSNR	MSE
Tomo	LD	19.8133	678.8125
	PM	27.0216	129.0993
	PMC	27.0295	128.8645
	EED	25.1047	200.7269
	CED	26.9325	131.7735
Triangle	LD	27.3086	120.8433
	PM	34.5369	22.8767
	PMC	34.6929	22.0696
	EED	34.4357	23.4159
	CED	30.3191	60.4184

7 Black Box Model

Principal Component Analysis is used to model the black box model. In this method, the data is resolved in the directions of the principal components. The directions in which the variance is the maximum is taken as the useful directions. The directions in which the variance is the least are ignored, since it is assumed that the noise is present in these directions. The data is then resolved back to the original basis.



(k) Figure 1



(l) Figure 2

It can be seen in Figure 2, the the noise is assumed to be in the vertical direction. Therefore, that component of the data-points is set to zero.

7.1 Algorithm

- Dataset (Image) is decomposed as follows: $X = U\Sigma W^T$

  Here, Σ is a rectangular diagonal matrix, made of singular values of X, U is made of left singular vectors of X, W is made of right singular vectors of X

  This decomposition follows the relation:
  $X^T X = W\Sigma^T \Sigma W^T$
  Hence, W is made of eigenvectors of $X^T X$

- columns of W are arranged in descending order of the magnitude of corresponding eigenvalues

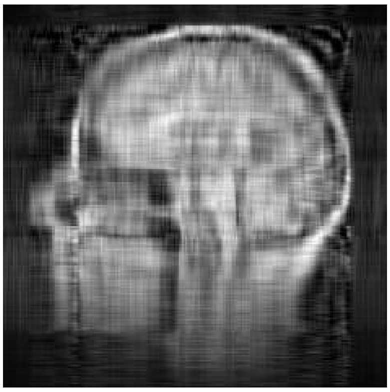
- According to some heuristic, L eignevectors are selected

- Score matrix T_L is calculated
  $T_L = XW_L$

- The denoised dataset (image) is calculated as follows:
  $X_{denoised} = T_L W_L^T$

7.2 Results

7.2.1 Images



(m) Components Selected = 10

(n) Components Selected = 20

(o) Components Selected = 30

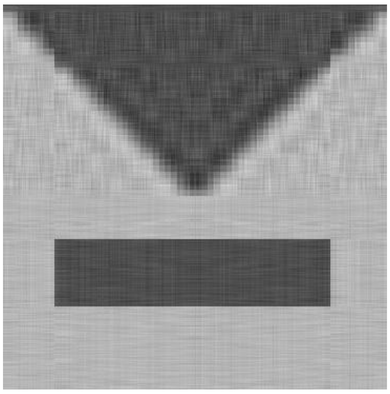
(p) Components Selected = 40

(q) Components Selected = 50

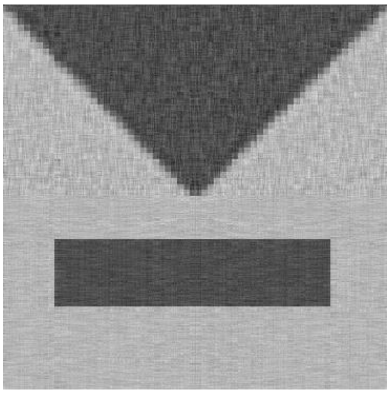
(r) Components Selected = 60

(s) Image with Noise

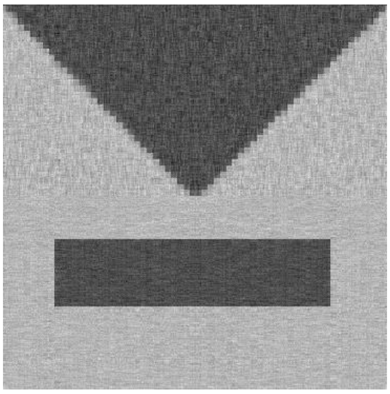
Figure 13: Tomo - Different number of Principal Components Selected



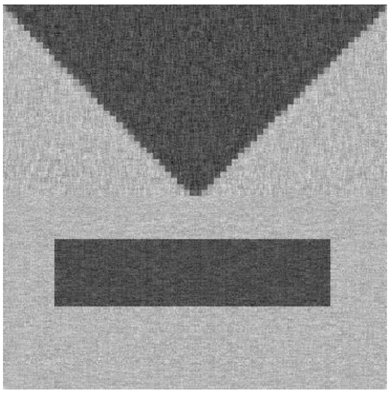
(a) Components Selected = 10



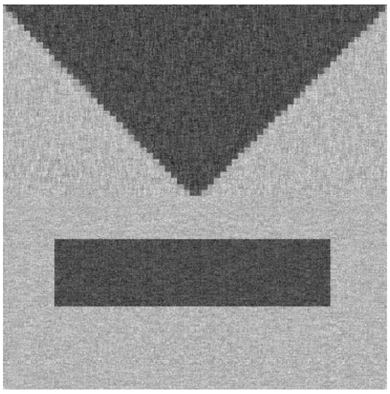
(b) Components Selected = 20



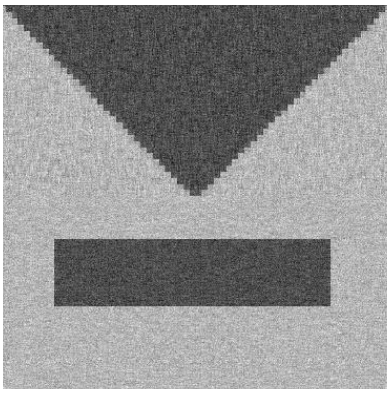
(c) Components Selected = 30



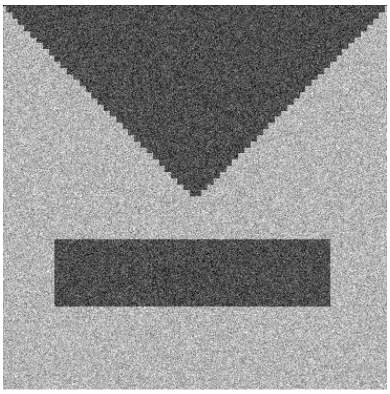
(d) Components Selected = 40



(e) Components Selected = 50



(f) Components Selected = 60



(g) Image with Noise

Figure 14: Triangle - Different number of Principal Components Selected

7.2.2 PSNR and MSE for different number of principal components selected

	Components Selected	PNSR	MSE
Tomo	10	21.4494	465.7379
	20	23.5372	287.9757
	30	23.7602	273.5660
	40	23.2718	306.1257
	50	22.6665	351.9125
	60	22.0942	401.4793
Triangle	10	27.9348	104.6158
	20	27.2315	123.0058
	30	25.9287	166.0407
	40	24.6550	222.6273
	50	23.6895	278.0575
	60	22.9475	329.8640

7.3 Problems with PCA

There is a trade off while selecting components. As the number of selected components increases, the chances of capturing components with errors also increases. If too less components are selected, then important image information is lost.

References

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- Two-stage image denoising by principal component analysis with local pixel grouping - L.Zhang, W.Dong, D.Zhang, G.Shi
- PCA based image denoising - Y.M.M. Babu, M.V.Subramanyam, M.N.G. Prasad
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