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Suspension Control Report

1 Quarter Car Model

Derivation of Equations of Motion

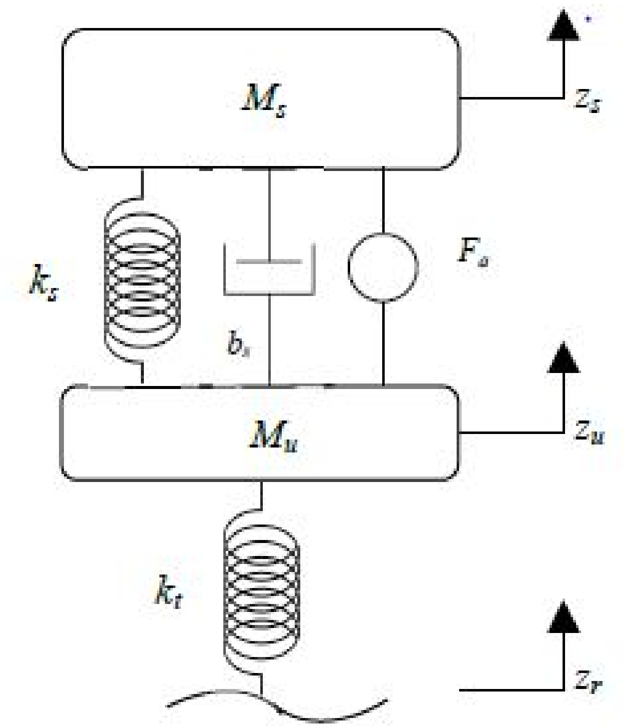


Figure 1: Quarter Car Model

- Euler Lagrange Formulation

Total Kinetic Energy $(T) = \frac{1}{2} m_s \dot{z}_s^2 + \frac{1}{2} m_u \dot{z}_u^2$

Total Potential Energy $(V) = \frac{1}{2} K_s (z_s - z_u)^2 + \frac{1}{2} K_t (z_u - z_r)^2$

Total Dissipation Energy $(D) = \frac{1}{2} b_s (\dot{z}_s - \dot{z}_u)^2$

Euler Lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = F_{ext_i}$$

For $q_i = z_s$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}_s}\right) - \frac{\partial T}{\partial z_s} + \frac{\partial V}{\partial z_s} + \frac{\partial D}{\partial \dot{z}_s} = F_a$$

$$m_s \ddot{z}_s + b_s (\dot{z}_s - \dot{z}_u) + K_s (z_s - z_u) = F_a$$

For $q_i = z_u$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}_u}\right) - \frac{\partial T}{\partial z_u} + \frac{\partial V}{\partial z_u} + \frac{\partial D}{\partial \dot{z}_u} = - F_a$$

$$m_u \ddot{z}_u - b_s (\dot{z}_s - \dot{z}_u) - K_s (z_s - z_u) + K_t (z_u - z_r) = - F_a$$

- Newton Euler Formulation

For m_s $\quad \Sigma F_z = m_s \ddot{z}_s$

$\Rightarrow m_s \ddot{z}_s = F_a - K_s (z_s - z_u) - b_s (\dot{z}_s - \dot{z}_u)$

For m_u $\quad \Sigma F_z = m_u \ddot{z}_u$

$$m_u \ddot{z}_u = - F_a - K_t (z_u - z_r) + b_s (\dot{z}_s - \dot{z}_u) + K_s (z_s - z_u)$$

Equations of motion for Quarter Car model are:

$$M_s \ddot{z}_s + b_s(\dot{z}_s - \dot{z}_u) + K_s(z_s - z_u) = F_a$$

$$M_u \ddot{z}_u + K_t(z_u - z_r) - b_s(\dot{z}_s - \dot{z}_u) - K_s(z_s - z_u) = -F_a$$

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \begin{bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{bmatrix} + \begin{bmatrix} b_s & -b_s \\ -b_s & b_s \end{bmatrix} \begin{bmatrix} \dot{z}_s \\ \dot{z}_u \end{bmatrix} + \begin{bmatrix} K_s & -K_s \\ -K_s & K_s + K_t \end{bmatrix} \begin{bmatrix} z_s \\ z_u \end{bmatrix} = \begin{bmatrix} 0 \\ K_t \end{bmatrix} z_r + \begin{bmatrix} 1 \\ -1 \end{bmatrix} F_a$$

$$x_1 = z_s - z_u$$

$$x_2 = \dot{z}_s$$

$$x_3 = z_u - z_r$$

$$x_4 = \dot{z}_u$$

$$\dot{x}_1 = x_2 - x_4$$

$$\dot{x}_2 = \frac{F_a}{M_s} - \frac{K_s}{M_s} x_1 - \frac{b_s}{M_s} x_2 + \frac{b_s}{M_s} x_4$$

$$\dot{x}_3 = x_4 - \dot{z}_r$$

$$\dot{x}_4 = -\frac{F_a}{M_u} + \frac{K_s}{M_u} x_1 + \frac{b_s}{M_u} x_2 - \frac{K_t}{M_u} x_3 - \frac{b_s}{M_u} x_4$$

Rewriting in state space form

$$\dot{\underset{\sim}{x}}(t) = \underset{\sim}{A}\, \underset{\sim}{x}(t) + \underset{\sim}{b}\, F_a(t) + \underset{\sim}{L}\, \dot{z}_r(t)$$

Where,

$$\underset{\sim}{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \qquad \underset{\sim}{A} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_s}{M_s} & -\frac{b_s}{M_s} & 0 & \frac{b_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_u} & \frac{b_s}{M_u} & -\frac{K_t}{M_u} & -\frac{b_s}{M_u} \end{bmatrix}$$

$$\underset{\sim}{b} = \begin{bmatrix} 0 \\ \frac{1}{M_s} \\ 0 \\ \frac{-1}{M_u} \end{bmatrix} \qquad \underset{\sim}{L} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\dot{\underset{\sim}{x}}(t) = \underset{\sim}{A}\,\underset{\sim}{x}(t) + \underset{\sim}{b}\,F_a(t) + \underset{\sim}{l}\,\dot{z}_r(t)$$

Taking Laplace Transform both sides
Taking zero initial conditions and zero input

$$s\underset{\sim}{X}(s) = \underset{\sim}{A}\,\underset{\sim}{X}(s) + \underset{\sim}{l}\,L(\dot{z}_r(t))$$

$$(s\underset{\sim}{I} - \underset{\sim}{A})\,\underset{\sim}{X}(s) = \underset{\sim}{l}\,L(\dot{z}_r(t))$$

$$\underset{\sim}{X}(s) = (s\underset{\sim}{I} - \underset{\sim}{A})^{-1}\,\underset{\sim}{l}\,L(\dot{z}_r(t)) \longrightarrow (1)$$

i) Acceleration transfer function : $T_a(s) = \dfrac{L(\ddot{z}_s(t))}{L(\dot{z}_r(t))}$

Taking dot product with $[0\ 1\ 0\ 0]^T$ on both sides in (1) and solving

$$T_a(s) = \frac{k_t s(b_s s + k_s)}{m_u m_s s^4 + (m_u + m_s) b_s s^3 + (m_u k_s + m_s k_s + m_s k_t)s^2 + b_s k_t s + k_s k_t}$$

ii) Rattle space transfer function : $T_r(s) = \dfrac{L(z_s(t) - z_u(t))}{L(\dot{z}_r(t))}$

Taking dot product with $[1\ 0\ 0\ 0]^T$ on both sides in (1) and solving

$$T_r(s) = \frac{-k_t m_s s}{m_u m_s s^4 + (m_u + m_s) b_s s^3 + (m_u k_s + m_s k_s + m_s k_t)s^2 + b_s k_t s + k_s k_t}$$

ii) Tyre deflection transfer function : $T_t(s) = \dfrac{L(z_u(t) - z_r(t))}{L(\dot{z}_r(t))}$

Taking dot product with $[0\ 0\ 1\ 0]^T$ on both sides in (1) and solving

$$T_t(s) = \frac{-[m_u m_s s^3 + (m_u + m_s) b_s s^2 + (m_u + m_s) k_s s]}{m_u m_s s^4 + (m_u + m_s) b_s s^3 + (m_u k_s + m_s k_s + m_s k_t)s^2 + b_s k_t s + k_s k_t}$$

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix}\begin{bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{bmatrix} + \begin{bmatrix} b_s & -b_s \\ -b_s & b_s \end{bmatrix}\begin{bmatrix} \dot{z}_s \\ \dot{z}_u \end{bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s+k_t \end{bmatrix}\begin{bmatrix} z_s \\ z_u \end{bmatrix} = \begin{bmatrix} 0 \\ k_t \end{bmatrix} z_r + \begin{bmatrix} 1 \\ -1 \end{bmatrix} F_a$$

$$\underset{\sim}{M} = \begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \qquad \underset{\sim}{K} = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s+k_t \end{bmatrix}$$

Let ω be the natural frequency of the system

ω satisfies

$$\det\left(\underset{\sim}{M}\omega^2 - \underset{\sim}{K}\right) = 0$$

$$\det\left(\begin{bmatrix} m_s\omega^2 - k_s & k_s \\ k_s & m_u\omega^2 - k_s - k_t \end{bmatrix}\right) = 0$$

$$m_u m_s \omega^4 + \left(-m_s k_s - m_s k_t - m_u k_s\right)\omega^2 + k_s k_t = 0$$

$m_s = 300$ Kg

$m_u = 50$ Kg

$k_s = 15000$ N/m

$k_t = 150000$ N/m

$b_s = 900$ Ns/m

$$1.5 \times 10^4 \omega^4 - 5.025 \times 10^7 \omega^2 + 2.25 \times 10^9 = 0$$

$\omega_1 = 57.4857$ rad s^{-1}

$\omega_2 = 6.7373$ rad s^{-1}

Bode Plots of Transfer Functions

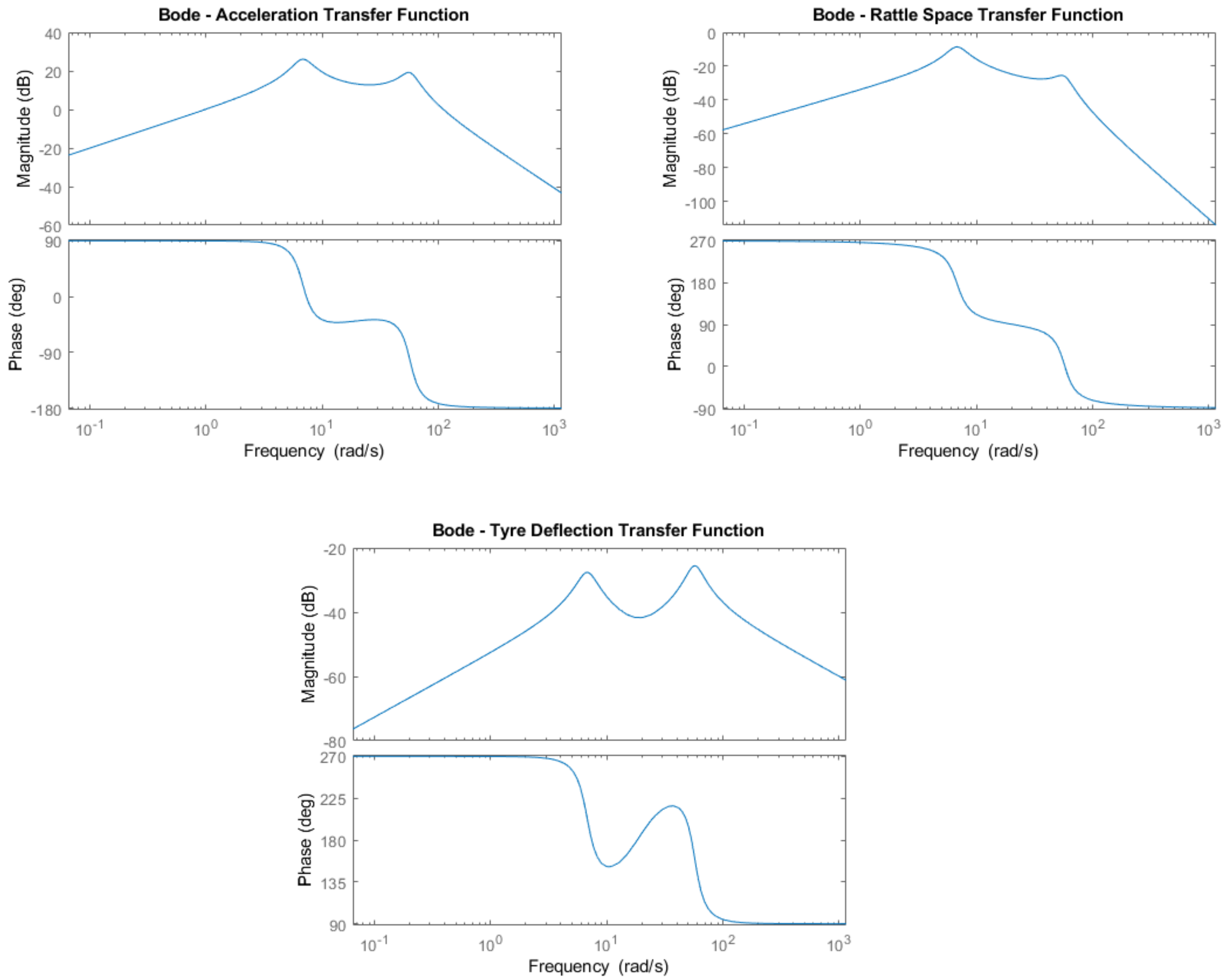


Figure 2: Transfer Functions

Transfer Function	Gm (dB)	Pm (degrees)
$T_a(s)$	Inf	5.6149
$T_r(s)$	2.6823	Inf
$T_t(s)$	20.3474	Inf

Observations

- The resonant frequencies of all the three transfer function are close to 7 rad/s and 60 rad/s
- Acceleration is amplified of inputs of frequency range 10 rad/s to 100 rad/s

Effect of Suspension Stiffness

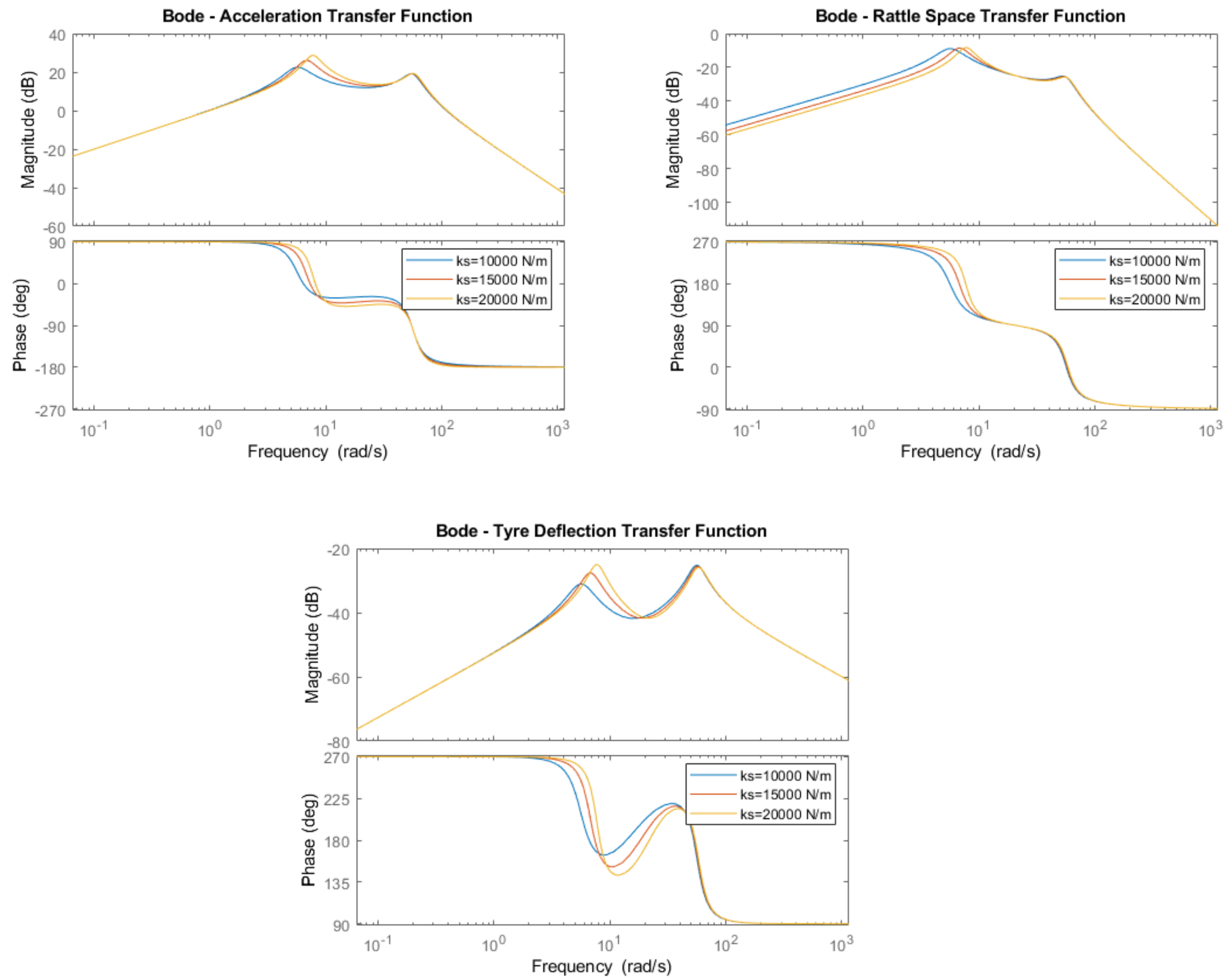


Figure 3: Effect of Suspension Stiffness

	Kt = 10000 N/m		Kt = 15000 N/m		Kt = 20000 N/m	
Transfer Functions	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)
Ta(s)	Inf	8.3985	Inf	5.6149	5.7989	2.8980
Tr(s)	2.7814	Inf	2.6823	Inf	2.5892	Inf
Tt(s)	19.4032	Inf	20.3474	Inf	19.0012	Inf

Observations

- The lower resonant frequency increases for all the transfer functions with an increase in suspension stiffness
- Higher suspension stiffness decreases the range for which rattle space transfer function amplifies the input
- Low frequency amplification for tyre deflection increases

Effect of Suspension Damping

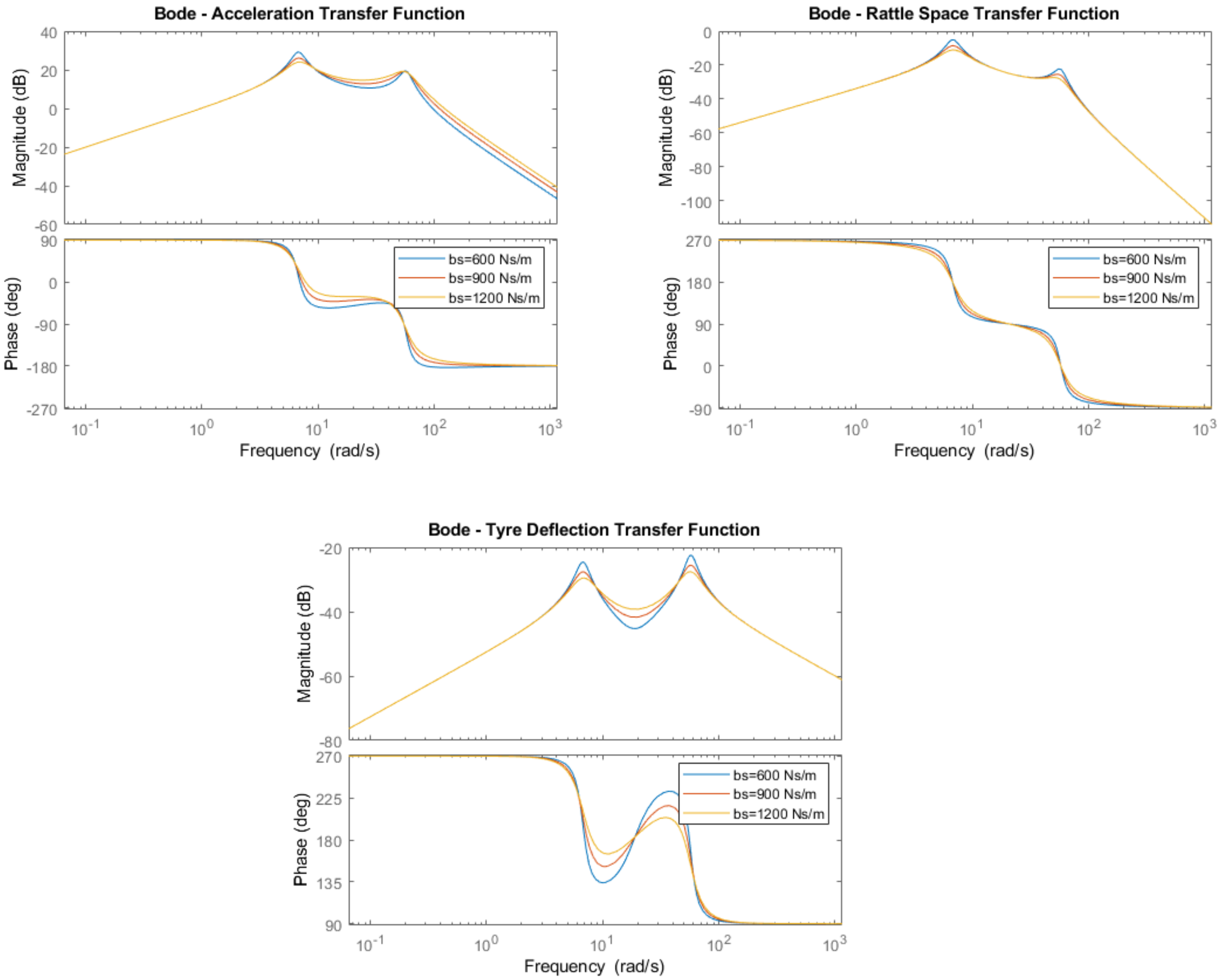


Figure 4: Effect of Suspension Damping

	bs = 600 Ns/m		bs = 900 Ns/m		bs = 1200 Ns/m	
Transfer Functions	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)
Ta(s)	0.6155	-2.4007	Inf	5.6149	Inf	10.1508
Tr(s)	1.7881	Inf	2.6823	Inf	3.5764	Inf
Tt(s)	13.6045	Inf	20.3474	Inf	26.9942	Inf

Observations

- The amplification for resonant frequencies decreases for all three transfer function for increase in suspension damping
- Range of frequencies for which acceleration is amplified increases
- System is unstable for bs = 600 Ns/m

Effect of Tyre Stiffness

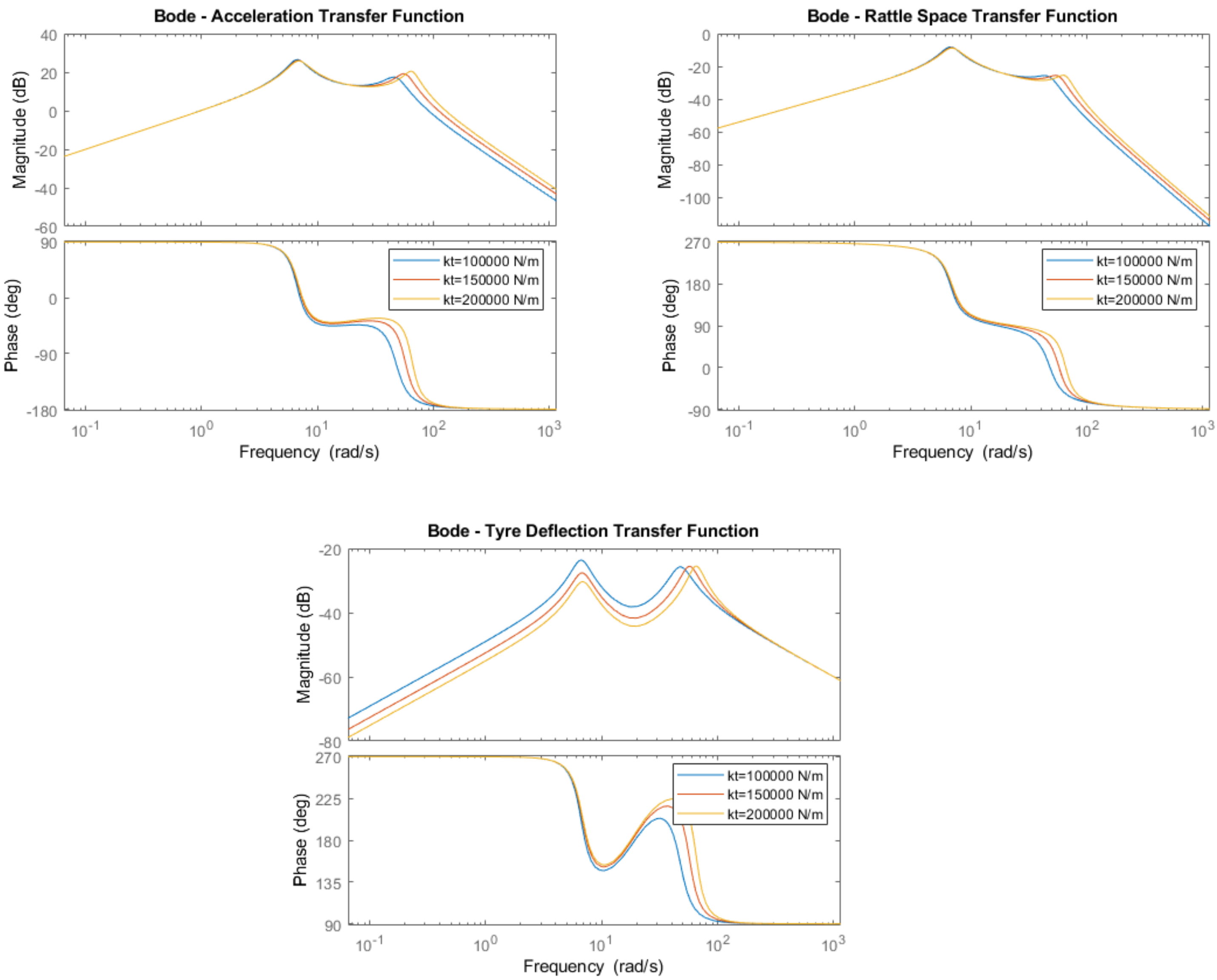


Figure 5: Effect of Tyre Stiffness

	Kt = 100000 N/m		Kt = 150000 N/m		Kt = 200000 N/m	
Transfer Functions	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)
Ta(s)	Inf	7.0021	Inf	5.6149	Inf	4.8178
Tr(s)	2.5448	Inf	2.6823	Inf	2.7560	Inf
Tt(s)	16.4825	Inf	20.3474	Inf	19.6908	Inf

Observations

- Increase in tyre stiffness decreases the tyre deflection at frequencies less than 110 rad/s
- Frequency range of amplification for both acceleration and rattle space increases
- There is increase in acceleration for high frequencies

$$J = \int_0^\infty \left[\ddot{z}_s^2 + f_1 (z_s - z_u)^2 + f_2 \dot{z}_s^2 + f_3 (z_u - z_r)^2 + f_4 \dot{z}_u^2 \right] dt$$

$$= \int_0^\infty \left[\dot{x}_2^2 + f_1 x_1^2 + f_2 x_2^2 + f_3 x_3^2 + f_4 x_4^2 \right] dt$$

$$\dot{x}_2 = \frac{F_a}{M_s} - \frac{K_s}{M_s} x_1 - \frac{b_s}{M_s} x_2 + \frac{b_s}{M_s} x_4$$

$$\dot{x}_2^2 = \frac{F_a^2}{M_s^2} + \frac{K_s^2}{M_s^2} x_1^2 + \frac{b_s^2}{M_s^2} x_2^2 + \frac{b_s^2}{M_s^2} x_4^2$$
$$- \frac{2 F_a K_s}{M_s^2} x_1 - \frac{2 F_a b_s}{M_s^2} x_2 + \frac{2 F_a b_s}{M_s^2} x_4$$
$$+ \frac{2 K_s b_s}{M_s^2} x_1 x_2 - \frac{2 b_s^2}{M_s^2} x_2 x_4 - \frac{2 K_s b_s}{M_s^2} x_1 x_4$$

$$\dot{x}_2^2 + f_1 x_1^2 + f_2 x_2^2 + f_3 x_3^2 + f_4 x_4^2$$
$$= \left(f_1 + \frac{K_s^2}{M_s^2}\right) x_1^2 + \left(f_2 + \frac{b_s^2}{M_s^2}\right) x_2^2 + f_3 x_3^2 + \left(f_4 + \frac{b_s^2}{M_s^2}\right) x_4^2$$
$$+ \frac{2 K_s b_s}{M_s^2} x_1 x_2 - \frac{2 b_s^2}{M_s^2} x_2 x_4 - \frac{2 K_s b_s}{M_s^2} x_1 x_4$$
$$+ \left(-\frac{2 K_s}{M_s^2} x_1 - \frac{2 b_s}{M_s^2} x_2 + \frac{2 b_s x_4}{M_s^2}\right) F_a + \frac{F_a^2}{M_s^2}$$

$$= \underset{\sim}{x} \cdot \underset{\sim}{Q}\, \underset{\sim}{x} + 2 \underset{\sim}{x} \cdot \underset{\sim}{N} F_a + F_a R F_a$$

Where
$$\underset{\sim}{Q} = \begin{bmatrix} f_1 + \frac{K_s^2}{M_s^2} & \frac{K_s b_s}{M_s^2} & 0 & -\frac{K_s b_s}{M_s^2} \\ \frac{K_s b_s}{M_s^2} & f_2 + \frac{b_s^2}{M_s^2} & 0 & -\frac{b_s^2}{M_s^2} \\ 0 & 0 & f_3 & 0 \\ -\frac{K_s b_s}{M_s^2} & -\frac{b_s^2}{M_s^2} & 0 & f_4 + \frac{b_s^2}{M_s^2} \end{bmatrix}$$

$$\underset{\sim}{N} = \begin{bmatrix} -\frac{K_s}{M_s^2} & -\frac{b_s}{M_s^2} & 0 & \frac{b_s}{M_s^2} \end{bmatrix}^T \qquad R = \frac{1}{M_s^2}$$

Active and Passive Suspension

$\rho = [0.2,\ 0.1,\ 0.2,\ 0.1]$

$$K = [-14865.83,\ -600.87,\ 39.44,\ 805.08]$$

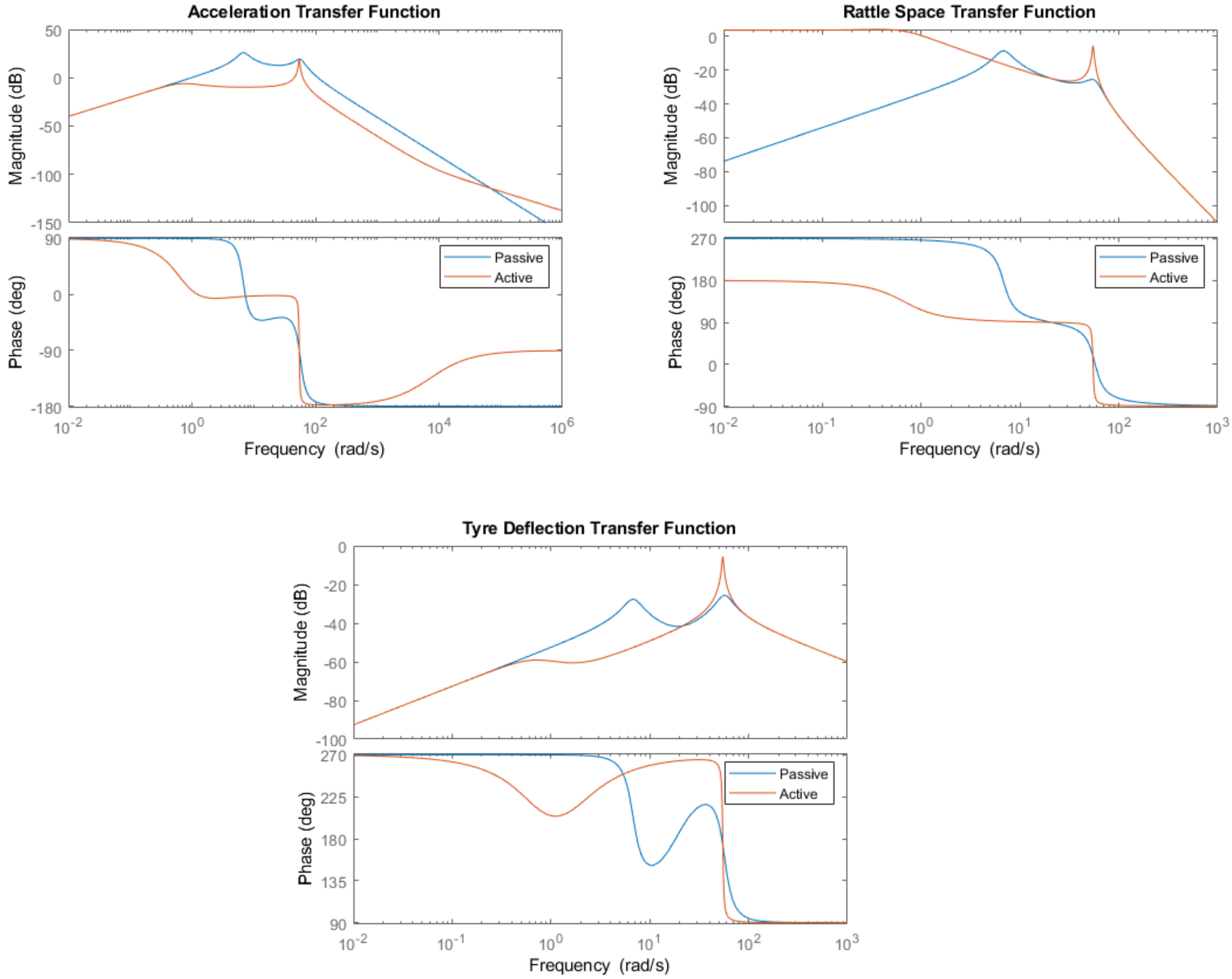


Figure 6: Transfer functions for active and passive suspensions

	Passive Suspension		Active Suspension	
Transfer Functions	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)
Ta(s)	Inf	5.6149	Inf	7.3320
Tr(s)	2.6823	Inf	0.6570	-65.5289
Tt(s)	20.3474	Inf	1.8991	Inf

Observations

- Rattle space transfer function is unstable for active suspension in this case
- Active suspension reduced acceleration and tyre deflection for lower frequencies
- Active suspension reduces the bandwidth of T_r and T_t

$\rho = [20000,\ 100,\ 20000,\ 100]$

$K = [27426.41,\ 5239.22,\ -19057.84,\ -2089.97]$

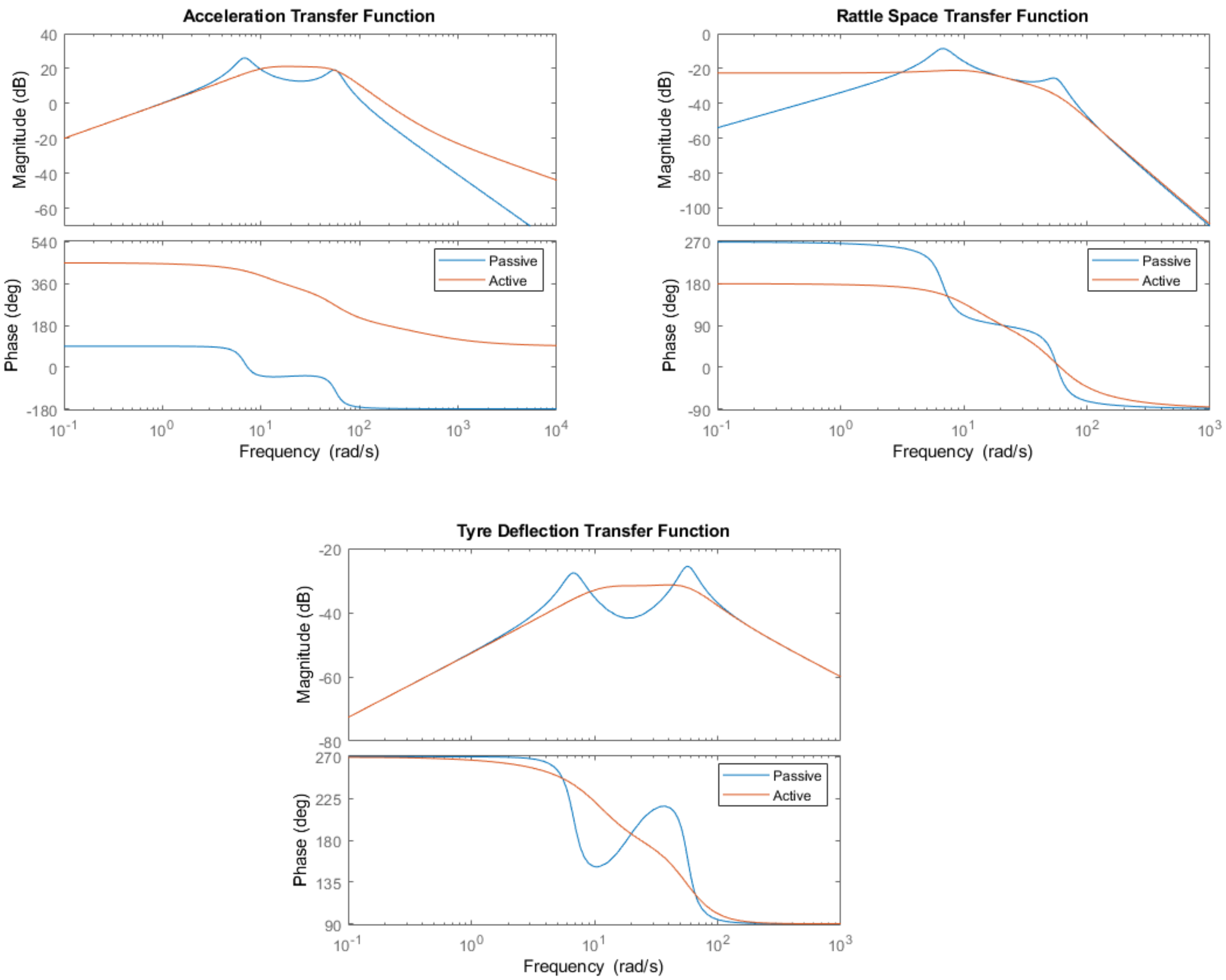


Figure 7: Transfer functions for active and passive suspensions

	Passive Suspension		Active Suspension	
Transfer Functions	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)
Ta(s)	Inf	5.6149	1.0403	0.8762
Tr(s)	2.6823	Inf	13.4719	Inf
Tt(s)	20.3474	Inf	38.0520	Inf

Observations

- Active suspension reduced acceleration and tyre deflection at resonant frequencies
- Active suspension reduces the bandwidth of T_r

$\rho = [200,\ 10,\ 200,\ 10]$

$K = [-10757.36,\ 963.82,\ -1664.04,\ -48.88]$

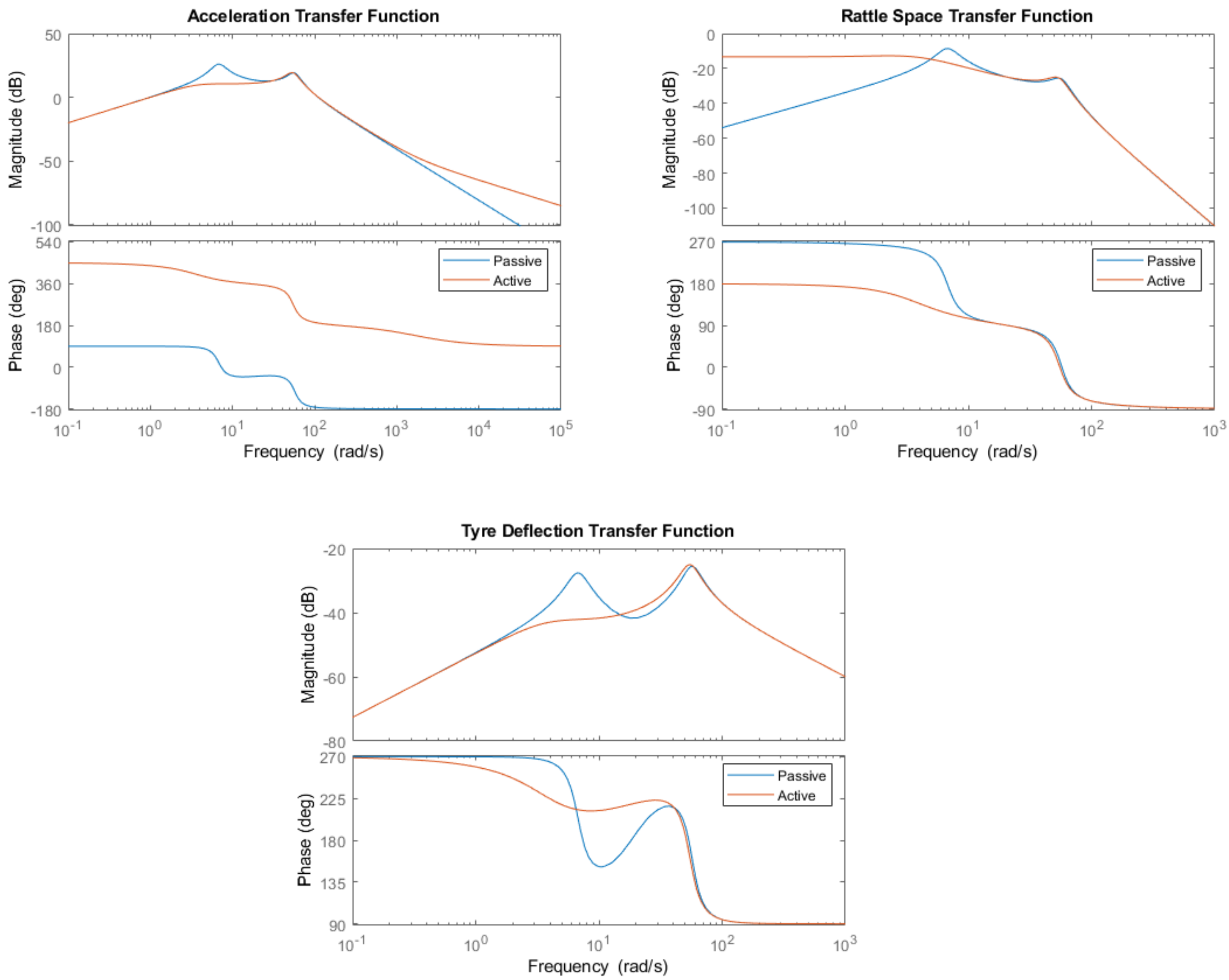


Figure 8: Transfer functions for active and passive suspensions

	Passive Suspension		Active Suspension	
Transfer Functions	Gm (dB)	Pm (degrees)	Gm (dB)	Pm (degrees)
Ta(s)	Inf	5.6149	3.7153	9.9724
Tr(s)	2.6823	Inf	4.6371	Inf
Tt(s)	20.3474	Inf	18.8429	Inf

Observations

- Low frequency amplification for T_a and T_t decreases on using this active suspension
- Active suspension reduces the bandwidth of T_r and T_t

2 Half Car Model

Derivation of Equations of Motion

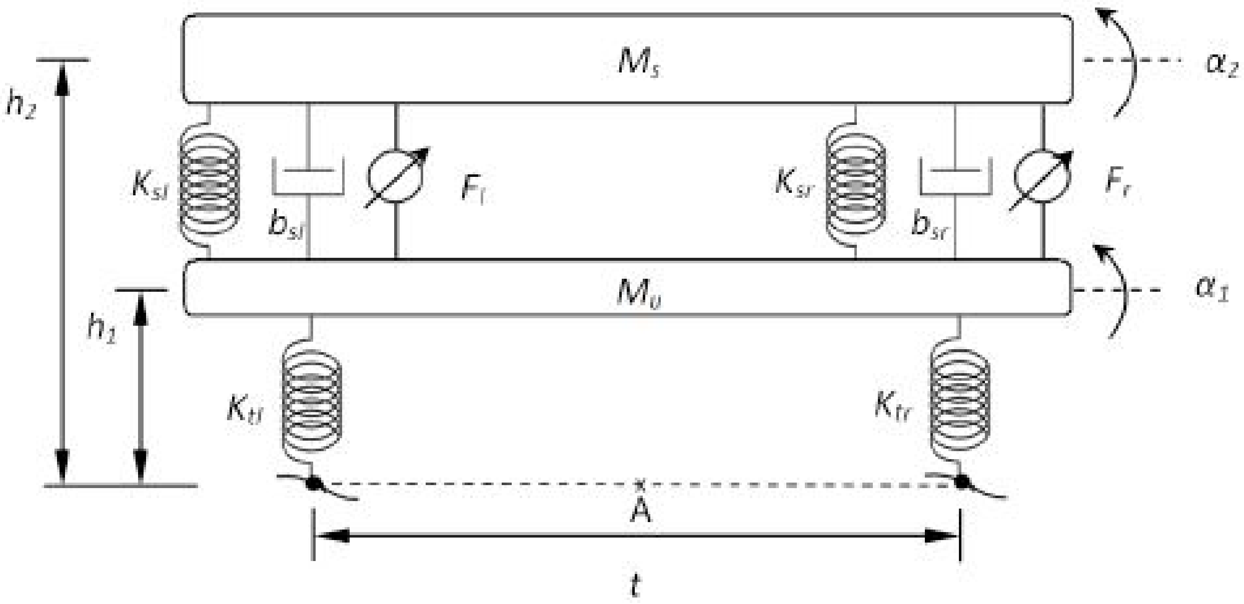
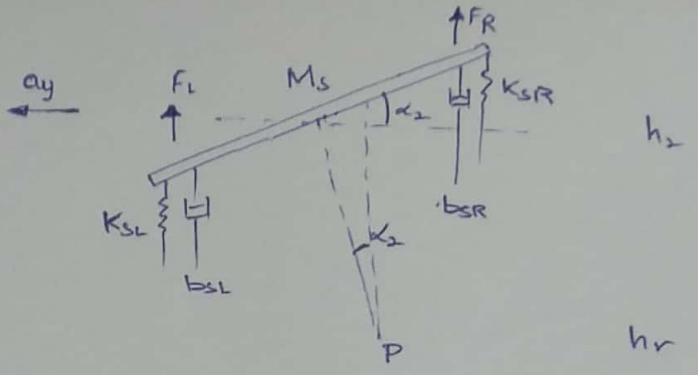


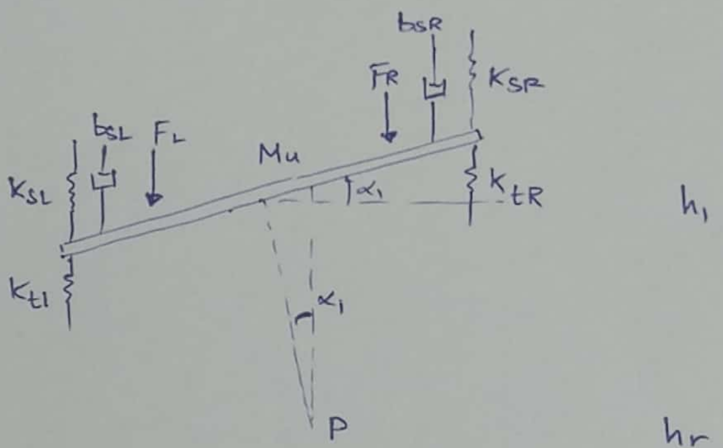
Figure 9: Half Car Model



Let the roll center height be h_r
P is the roll center

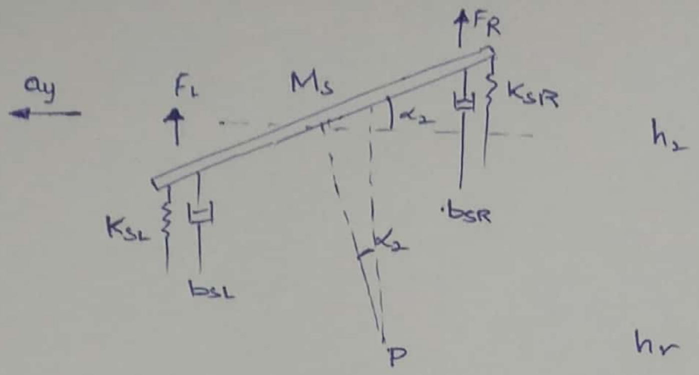
Taking moment about P

$$\left[I_s + M_s(h_2-h_r)^2\right]\ddot{\alpha}_2 = -(K_{SL}+K_{SR})(\alpha_2-\alpha_1)\frac{t}{2}\times\frac{t}{2} - (b_{SL}+b_{SR})(\dot{\alpha}_2-\dot{\alpha}_1)\frac{t}{2}\times\frac{t}{2} + (F_R-F_L)\frac{t}{2} + M_s g(h_2-h_r)\alpha_2\cos(\alpha_2) + M_s a_y(h_2-h_r) \longrightarrow (1)$$



Taking moment about P

$$\left[I_u + M_u(h_1-h_r)^2\right]\ddot{\alpha}_1 = (K_{SL}+K_{SR})(\alpha_2-\alpha_1)\frac{t}{2}\times\frac{t}{2} + (b_{SL}+b_{SR})(\dot{\alpha}_2-\dot{\alpha}_1)\frac{t}{2}\times\frac{t}{2} - (F_R-F_L)\frac{t}{2} + M_u g(h_1-h_r)\alpha_1\cos(\alpha_1) + M_s a_y(h_1-h_r) - (K_{tL}+K_{tR})\alpha_1\frac{t}{2}\times\frac{t}{2} \longrightarrow (2)$$

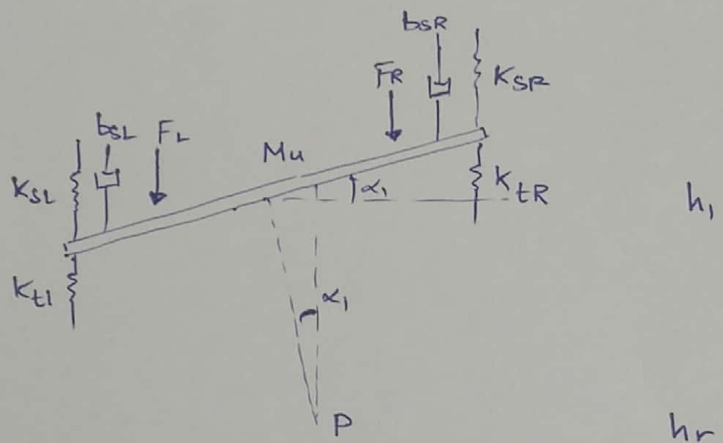


Let the roll center height be h_r

P is the roll center

Taking moment about P

$$\left[I_s + M_s(h_2-h_r)^2\right]\ddot{\alpha}_2 = -(K_{SL}+K_{SR})(\alpha_2-\alpha_1)\frac{t}{2}\times\frac{t}{2} - (b_{SL}+b_{SR})(\dot{\alpha}_2-\dot{\alpha}_1)\frac{t}{2}\times\frac{t}{2}$$
$$+ (F_R-F_L)\frac{t}{2} + M_s g(h_2-h_r)\alpha_2\cos(\alpha_2)$$
$$+ M_s a_y(h_2-h_r) \longrightarrow (1)$$



Taking moment about P

$$\left[I_u + M_u(h_1-h_r)^2\right]\ddot{\alpha}_1 = (K_{SL}+K_{SR})(\alpha_2-\alpha_1)\frac{t}{2}\times\frac{t}{2} + (b_{SL}+b_{SR})(\dot{\alpha}_2-\dot{\alpha}_1)\frac{t}{2}\times\frac{t}{2}$$
$$-(F_R-F_L)\frac{t}{2} + M_u g(h_1-h_r)\alpha_1\cos(\alpha_1)$$
$$+ M_u a_y(h_1-h_r) - (K_{tL}+K_{tR})\alpha_1\frac{t}{2}\times\frac{t}{2} \longrightarrow (2)$$

Taking $h_r = 0$ and $\alpha_1, \alpha_2 \approx 0$

$$(I_s + M_s h_2^2)\ddot{\alpha}_2 = -(K_{SL}+K_{SR})(\alpha_2-\alpha_1)\frac{t^2}{4} - (b_{SL}+b_{SR})(\dot{\alpha}_2-\dot{\alpha}_1)\frac{t^2}{4} + (F_R-F_L)\frac{t}{2}$$
$$+ M_s g h_2\alpha_2 + M_s a_y h_2 \longrightarrow (3)$$

$$(I_u + M_u h_1^2)\ddot{\alpha}_1 = (K_{SL}+K_{SR})(\alpha_2-\alpha_1)\frac{t^2}{4} + (b_{SL}+b_{SR})(\dot{\alpha}_2-\dot{\alpha}_1)\frac{t^2}{4} - (K_{tL}+K_{tR})\alpha_1\frac{t^2}{4}$$
$$-(F_R-F_L)\frac{t}{2} + M_u g h_1\alpha_1 + M_u a_y h_1$$

$$\dot{\underset{\sim}{x}} = \underset{\sim}{A}\,\underset{\sim}{x} + \underset{\sim}{B}\,\underset{\sim}{f} + \underset{\sim}{d}\,a_y$$

$$\underset{\sim}{x} = [\alpha_1 \;\; \dot{\alpha}_1 \;\; \alpha_2 \;\; \dot{\alpha}_2]^T$$

$$\underset{\sim}{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{[-(k_{tL}+k_{tR}+k_{sL}+k_{sR})\frac{t^2}{4} + M_u g h_1]}{I_u + M_u h_1} & \dfrac{-[b_{sL}+b_{sR}]\frac{t^2}{4}}{I_u + M_u h_1^2} & \dfrac{[k_{sL}+k_{sR}]\frac{t^2}{4}}{I_u + M_u h_1^2} & \dfrac{[b_{sL}+b_{sR}]\frac{t^2}{4}}{I_u + M_u h_1^2} \\ 0 & 0 & 0 & 1 \\ \dfrac{[k_{sL}+k_{sR}]\frac{t^2}{4}}{I_s + M_s h_2^2} & \dfrac{[b_{sL}+b_{sR}]\frac{t^2}{4}}{I_s + M_s h_2^2} & \dfrac{[-(k_{sL}+k_{sR})\frac{t^2}{4} + M_s g h_2]}{I_s + M_s h_2^2} & \dfrac{-[b_{sL}+b_{sR}]\frac{t^2}{4}}{I_s + M_s h_2^2} \end{bmatrix}$$

$$\underset{\sim}{f} = [F_R \quad F_L]^T$$

$$\underset{\sim}{B} = \begin{bmatrix} 0 & 0 \\ \dfrac{-t}{2(I_u + M_u h_1^2)} & \dfrac{t}{2(I_u + M_u h_1^2)} \\ 0 & 0 \\ \dfrac{t}{2(I_s + M_s h_2^2)} & \dfrac{-t}{2(I_s + M_s h_2^2)} \end{bmatrix} \qquad \underset{\sim}{d} = \begin{bmatrix} 0 \\ \dfrac{m_u h_1}{\cancel{2}(I_u + M_u h_1^2)} \\ 0 \\ \dfrac{m_s h_2}{\cancel{2}(I_s + M_s h_2^2)} \end{bmatrix}$$

$$J = \int_0^\infty \left(f_1 \alpha_1^2 + f_2 \dot{\alpha}_1^2 + f_3 \alpha_2^2 + f_4 \dot{\alpha}_2^2 + f_5 F_R^2 + f_6 F_L^2 \right) dt$$

$$= \int_0^\infty \left(\underset{\sim}{x} \cdot \underset{\sim}{Q} \underset{\sim}{x} + 2 \underset{\sim}{x} \cdot \underset{\sim}{N} \underset{\sim}{F} + \underset{\sim}{F} \cdot \underset{\sim}{R} \underset{\sim}{F} \right) dt$$

Where

$$\underset{\sim}{Q} = \begin{bmatrix} f_1 & 0 & 0 & 0 \\ 0 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & 0 \\ 0 & 0 & 0 & f_4 \end{bmatrix} \qquad \underset{\sim}{R} = \begin{bmatrix} f_5 & 0 \\ 0 & f_6 \end{bmatrix}$$

$$\underset{\sim}{N} = \underset{\sim}{0}$$

Controller Gains for the given parameters

$$K = \begin{bmatrix} -2.7977 & -0.0697 & 4.4914 & 0.4726 \\ 2.7977 & 0.0697 & -4.4914 & -0.4726 \end{bmatrix} \times 10^6$$

Impulse Input

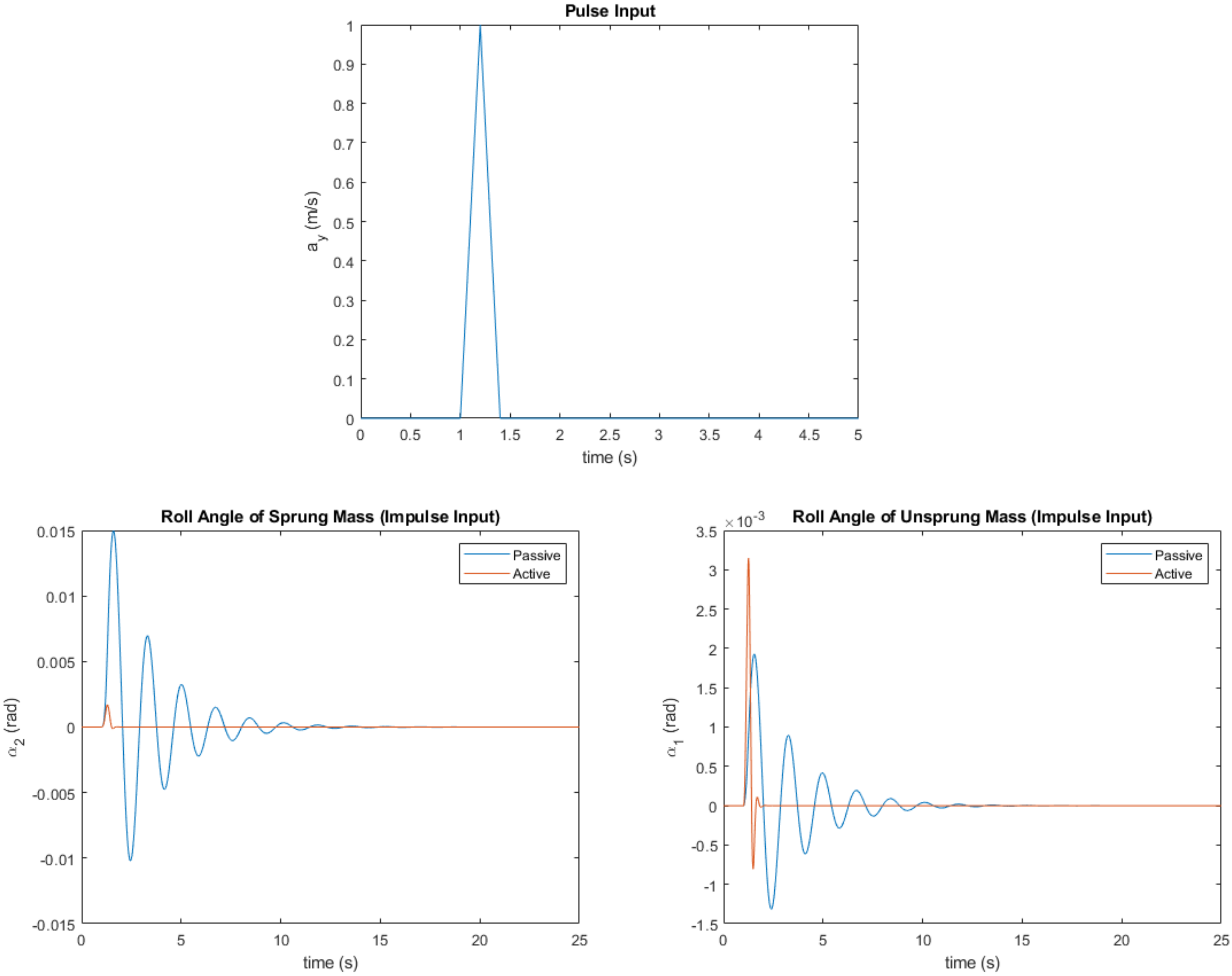


Figure 10: Outputs for Impulse Input

Rectangular Input

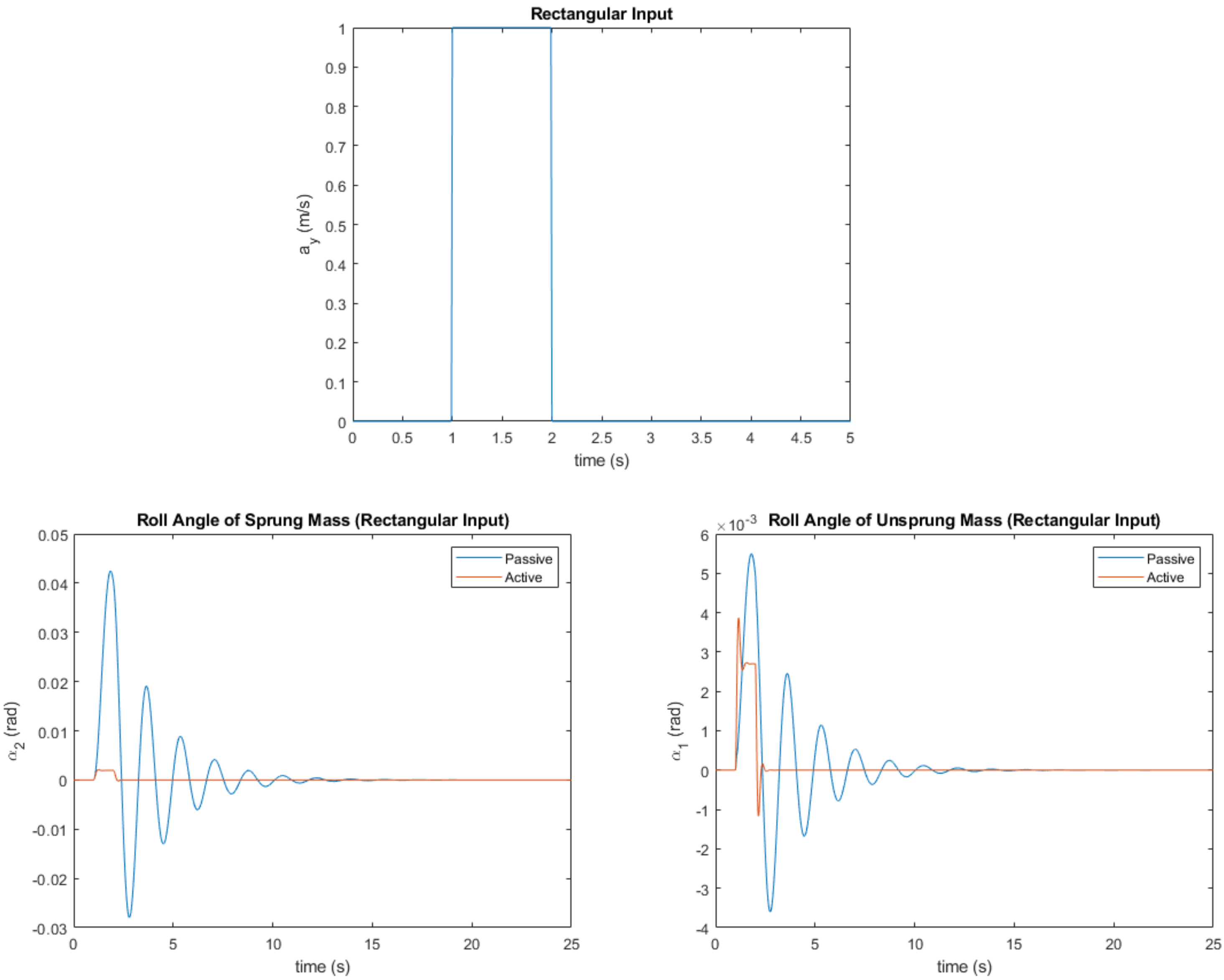


Figure 11: Outputs for Rectangular Input

Observations

- Active suspension is highly efficient in damping the roll angle of the sprung mass
- The unsprung mass experiences sudden roll at the initial stage of the input, especially for impulse input, but it is soon settles down