ED5330
Control of Automotive Systems

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ED15013

Heading Angle Control Report

Problem Statement

To design a heading angle controller for an autonomous ground vehicle taking into account the vehicle dynamics.

1 System Modelling and Open Loop Response Analysis

1.1 Derivation of Governing Equations

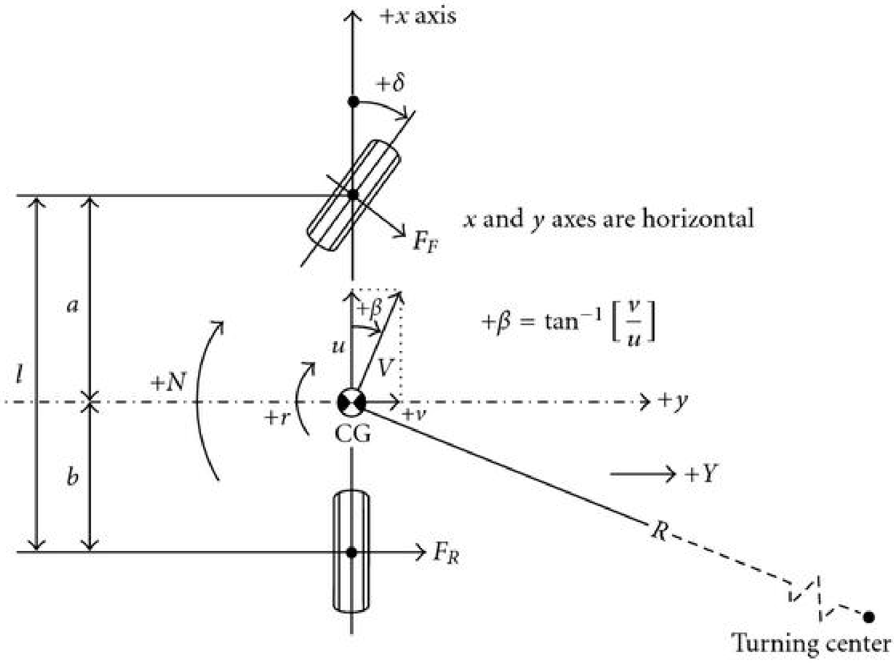


Figure 1: Bicycle Model
Reference: https://www.hindawi.com/journals/ijcgt/2009/952524/

From Geometry:

$$\alpha_f = \frac{v + L_f r}{u} - \delta$$

$$\alpha_r = \beta - \frac{L_r r}{u}$$

Balancing forces along lateral direction, and taking moment about center of mass.

$$m(\dot{v} + ur) = F_{yf} + F_{yr}$$

$$I_z \dot{r} = L_f F_{yf} - L_r F_{yr}$$

$$
\begin{aligned}
F_y = m\dot{v} &= -C_f\alpha_f - C_r\alpha_r \\
&= -C_f(\frac{v+L_f r}{u} - \delta) - C_r(\beta - \frac{L_r r}{u}) \\
&= (-\frac{L_f}{u}C_f + \frac{L_r}{u}C_r)r - (C_f + C_r)\beta + C_f\delta
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
M_z = I_z\dot{r} &= L_f F_{yf} - L_r F_{yr} \\
&= -L_f C_f \alpha_f + L_r C_r \alpha_r \\
&= (-\frac{{L_f}^2}{u}C_f - \frac{{L_r}^2}{u}Cr)r - (L_f C_f - L_r C_r)\beta + L_f C_f \delta
\end{aligned}
\tag{2}
$$

Writing this in state space form:

$$
\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\frac{C_f+C_r}{mu} & \frac{-L_f C_f + L_r C_r}{mu} - u \\ \frac{-L_f C_f + L_r C_r}{I_z u} & -(\frac{{L_f}^2 C_f + {L_r}^2 C_r}{I_z u}) \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_f}{m} \\ \frac{L_f C_f}{I_z} \end{bmatrix} \delta
$$

$$
\boldsymbol{A} = \begin{bmatrix} -\frac{C_f+C_r}{mu} & \frac{-L_f C_f + L_r C_r}{mu} - u \\ \frac{-L_f C_f + L_r C_r}{I_z u} & -(\frac{{L_f}^2 C_f + {L_r}^2 C_r}{I_z u}) \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} \frac{C_f}{m} \\ \frac{L_f C_f}{I_z} \end{bmatrix}
$$

Since we need r as output

$$
\boldsymbol{c} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
$$

Plant Transfer function:

$$
P(s) = \frac{\Theta(s)}{\delta(s)} = \boldsymbol{c}.(sI - \boldsymbol{A})^{-1}\boldsymbol{b} \times \frac{1}{s}
$$

We need to divide by s because $\theta(t) = \int_0^t r(\tau)$, which is the heading angle.
Using integration rule for Laplace transform $\Theta(s) = \frac{R(s)}{s}$

For $u = 2.5m/s$, and using the given vehicle parameters, we get the following as the transfer function:

$$
P(s) = \frac{44.97s + 1618}{s^3 + 71.95s^2 + 1294s}
$$

1.2 Plots

Figure 2 shows the Bode plot for the above mentioned plant transfer function. Figure 3 shows the outputs for various inputs.

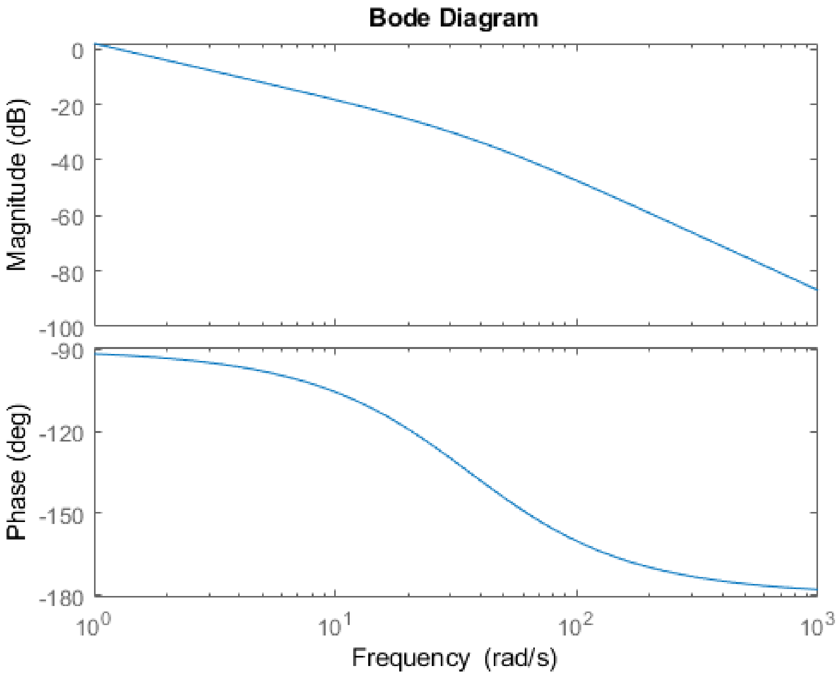


Figure 2: Bode Diagram

1.3 Observations

The frequency response curves show that if the input is a sinusoid, the output is a sinusoid of the same frequency, but at a different phase and magnitude from that of the input. This shows that the system is Linear Time Invariant.

It can be seen from the pulse input that the heading angle increases because of the pulse and then becomes constant after the pulse is over.

In the case of step input, since the steering angle is kept constant, therefore the heading angle will keep on increasing.

The output of pulse is step and the output of step is a ramp signal.

The poles of P(s) are {0, -35.9932, -35.9524}. Since, one pole is at the origin, the system is not BIBO stable, as a step input can make the system unstable, which can be seen in the figure.

The Bode Diagram shows that as the frequency of the input increases, the magnitude of the output decreases, as well as the phase delay between the steering input and the heading angle output increases. This can be seen from the frequency response graphs.

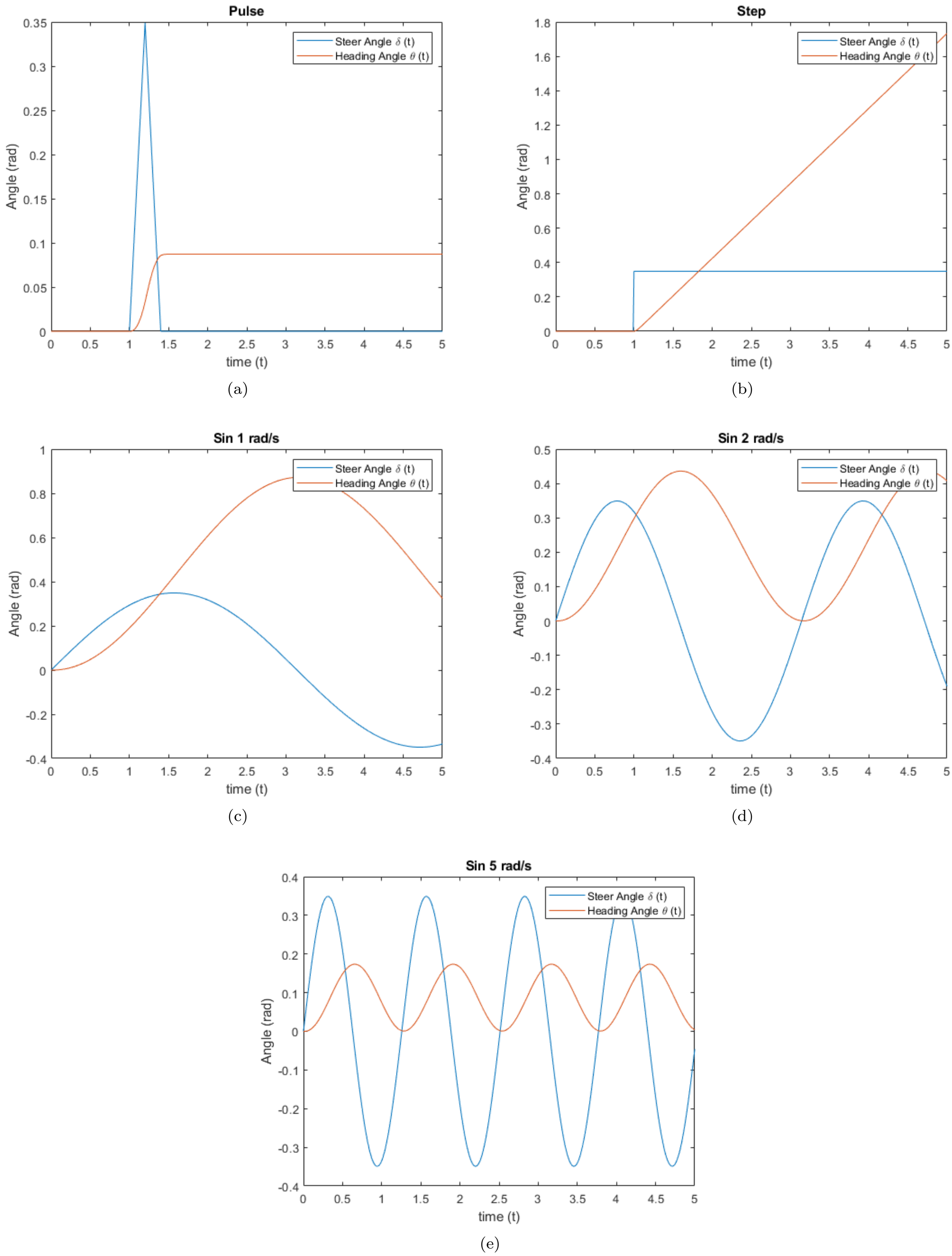


Figure 3: Output for various inputs

2 Controller Design

For a closed loop system:

$$G(s) = C(s)P(s)$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

For unity negative feedback $H(s) = 1$

2.1 Performance Criteria

2.1.1 Maximum Peak Overshoot $\leq 10\%$

$$e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \leq 0.1 \quad \Rightarrow \quad \zeta \geq 0.5910 \quad \Rightarrow \quad \beta \leq 53.77^o$$

Since,

$$\zeta = \cos\beta$$

2.1.2 Steady State Error

Error for step input:

$$E(s) = \frac{1}{s}\left(\frac{1}{1 + G(s)H(s)}\right)$$

$$C(s) = K_p + \frac{K_i}{s}$$

For any K_p and K_i

Hence,

$$E(s) = \frac{s^3 + 17.95s^2 + 1294s}{s^4 + 17.95s^3 + (44.95K_p + 1295)s^2 + (1618K_p + 44.97K_i) + 1618K_i}$$

Using final value theorem:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

Therefore, steady state error will be 0 for all K_p and K_i

2.2 P Controller

$$C(s) = K_p$$

$$Open\ Loop\ Transfer\ Function \;=\; G(s)H(s) \;=\; K_p \frac{44.97s + 1618}{s^3 + 71.95s^2 + 1294s}$$

2.2.1 Stability Criteria

For stability, the roots of closed loop characteristic equation should lie on the Right Half Plane

$$1 + G(s)H(s) = 0$$

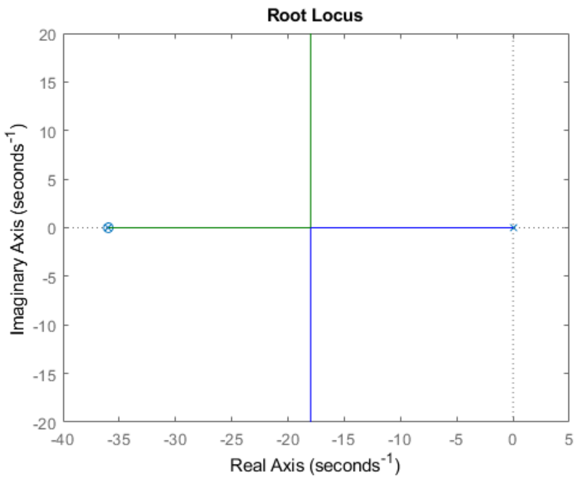
$$s^3 + 17.95s^2 + (44.95K_p + 1295)s + 1618K_p = 0$$

According to Routh's criteria, coefficients of s^2 and s, and constant should be greater than zero. Therefore we get:

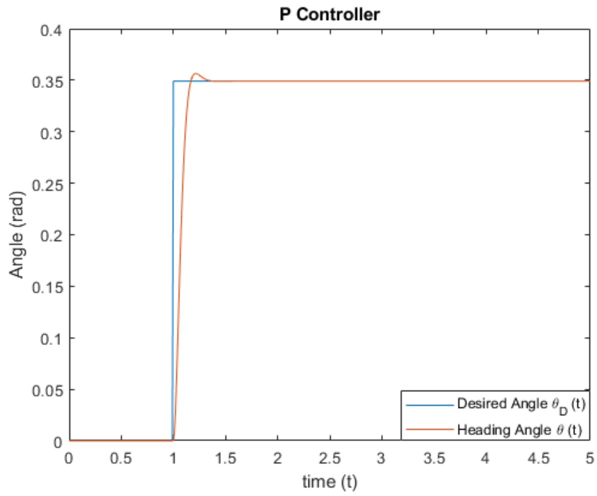
$$K_p > 0$$

2.2.2 Root Locus and Output

The root locus is plotted on MATLAB and a value of $K_p = 12$ is chosen, as it satisfies the performance criteria, as well as gives a good settling time.
$MP = 2.1323\%$



(a) Root Locus



(b) Output, MP = 2.1323 %, K_p = 12

Figure 4: P Controller, Unactuated Steering, u = 2.5 m/s

2.3 PI Controller

$$C(s) = K_p + \frac{K_i}{s}$$

$$Open\ Loop\ Transfer\ Function \ = \ G(s)H(s) \ = \ \left(K_p + \frac{K_i}{s}\right)\frac{44.97s + 1618}{s^3 + 71.95s^2 + 1294s}$$

2.3.1 Stability Criteria

For stability, the roots of closed loop characteristic equation should lie on the Right Half Plane

$$1 + G(s)H(s) = 0$$

$$s^4 + 17.95s^3 + (44.95K_p + 1295)s^2 + (1618K_p + 44.97K_i) + 1618K_i = 0$$

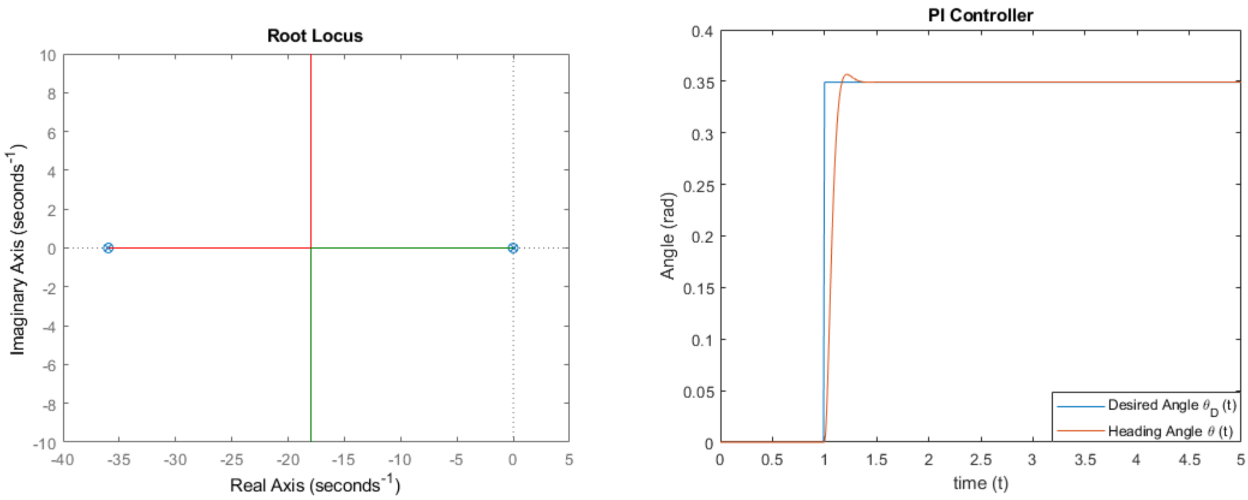
According to Routh's criteria, coefficients of s^3, s^2 and s, and constant should be greater than zero. Therefore we get:

$$K_i, K_p > 0$$

2.3.2 Root Locus and Output

$\frac{K_i}{K_p} = 0.01s^{-1}$ is chosen. The root locus is plotted on MATLAB and values of $K_p = 12$ is chosen. Hence, $K_i = 0.12s^{-1}$, these values satisfy the performance criteria, as well as gives a good settling time.

$MP = 2.1333\%$



(a) Root Locus

(b) Output, MP = 2.1333 %, K_p = 12, K_i = 0.12 s^{-1}

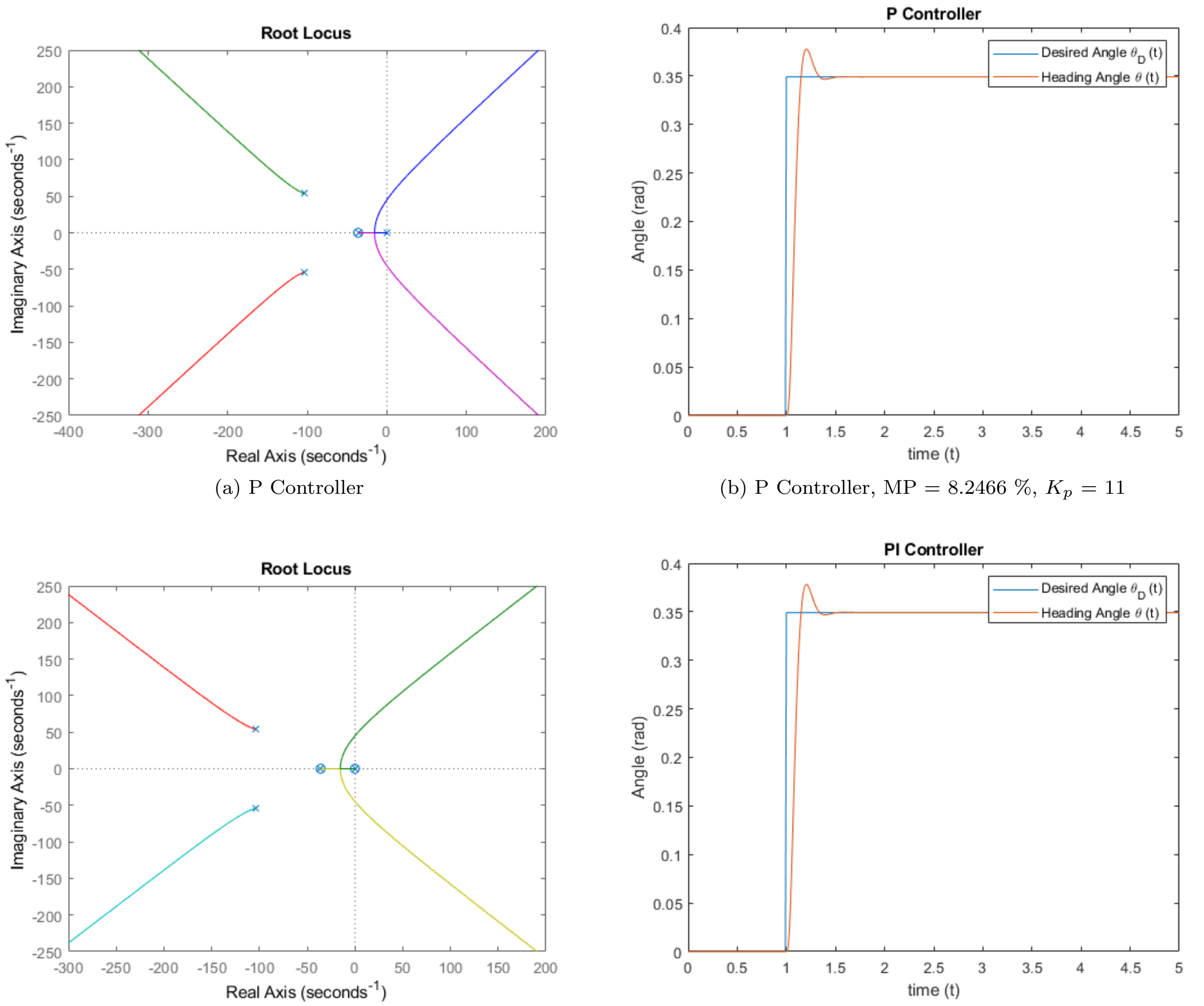
Figure 5: PI Controller, Unactuated Steering, u = 2.5 m/s

3 Effect of Steering Actuator Dynamics

$$G_m(s) = \frac{604}{0.044s^2 + 9.164s + 604}$$

$$P_{act}(s) = P(s) * G_m(s) = \frac{44.97s + 1618}{s^3 + 71.95s^2 + 1294s} \times \frac{604}{0.044s^2 + 9.164s + 604}$$

3.1 Plots



(a) P Controller

(b) P Controller, MP = 8.2466 %, K_p = 11

(c) PI Controller

(d) PI Controller, MP = 8.2483 %, K_p = 11, K_i = 0.11 s^{-1}

Figure 6: Actuated Steering, u = 2.5 m/s

3.2 Observation

The Maximum peak overshoot for a similar range of settling time increases considerably after steering actuator has been added. The root locus of the system also changes, since the order of the system was increased.

Poles affect the stability of a system and zeros decide the characteristics and shape of the output curve. The steering actuator changes the zeros of the old system as well as adds some new ones. Hence, the output changes.

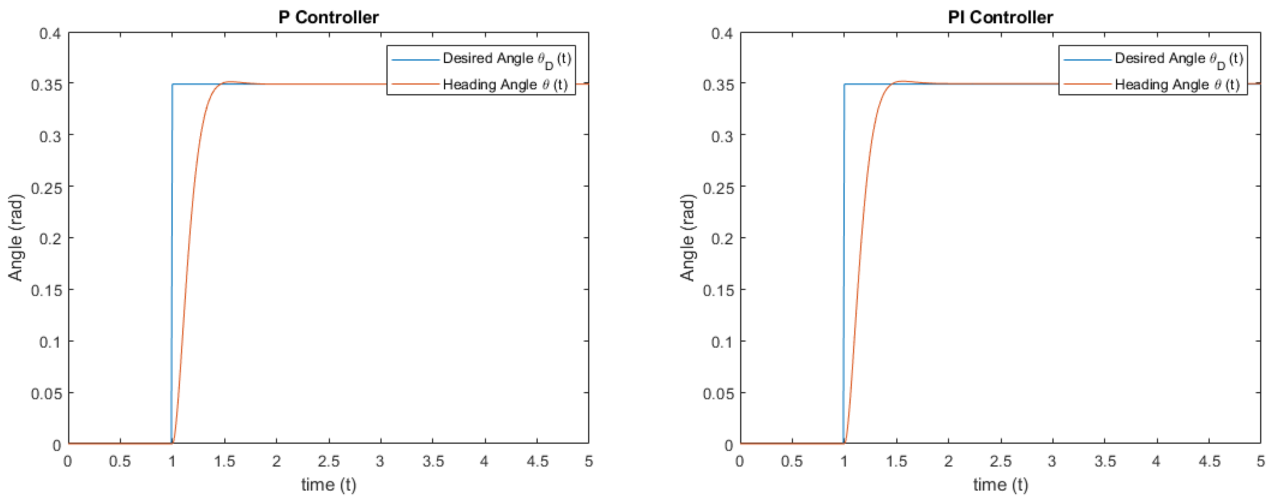
4 Effect of Variation in Vehicle Longitudinal Speed

4.1 u = 5 m/s

4.1.1 Unactuated Steering

The plant transfer function:

$$P(s) = \frac{44.97s + 808.8}{s^3 + 35.97s^2 + 323.5s}$$



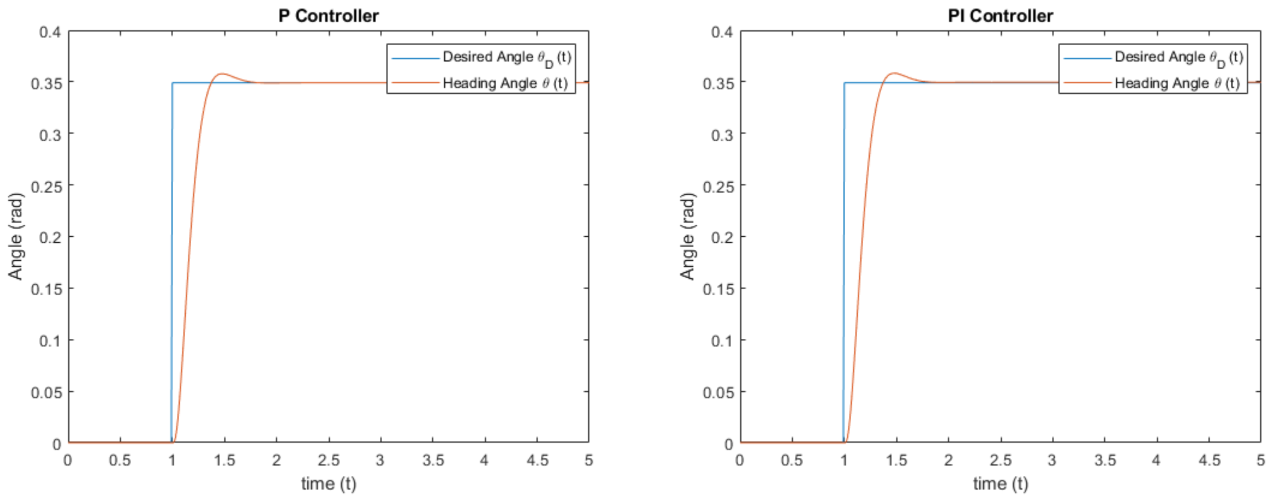
(a) P Controller, MP = 0.6532 %, $K_p = 2.5$ (b) PI Controller, MP = 0.6557 %, $K_p = 2.5$, $K_i = 0.025\ s^{-1}$

Figure 7: Unactuated Steering, u = 5 m/s

4.1.2 Actuated Steering

The plant transfer function:

$$P_{act}(s) = \frac{44.97s + 808.8}{s^3 + 35.97s^2 + 323.5s} \times \frac{604}{0.044s^2 + 9.164s + 604}$$



(a) P Controller, MP = 2.4970 %, $K_p = 2.5$ (b) PI Controller, MP = 2.4988 %, $K_p = 2.5$, $K_i = 0.025\ s^{-1}$

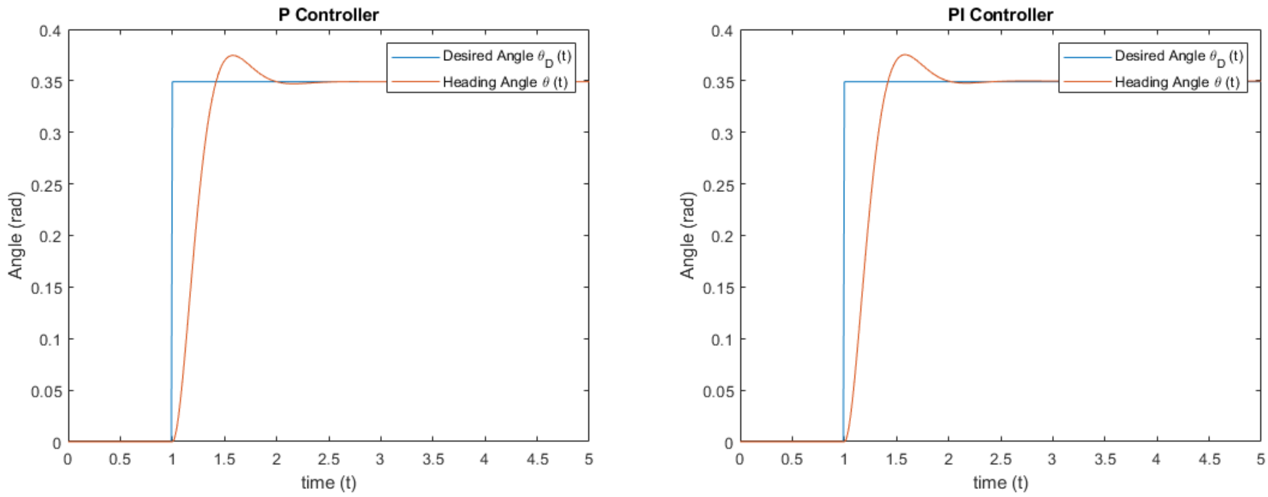
Figure 8: Actuated Steering, u = 5 m/s

4.2 u = 10 m/s

4.2.1 Unactuated Steering

The plant transfer function:

$$P(s) = \frac{44.97s + 404.4}{s^3 + 17.99s^2 + 80.88s}$$



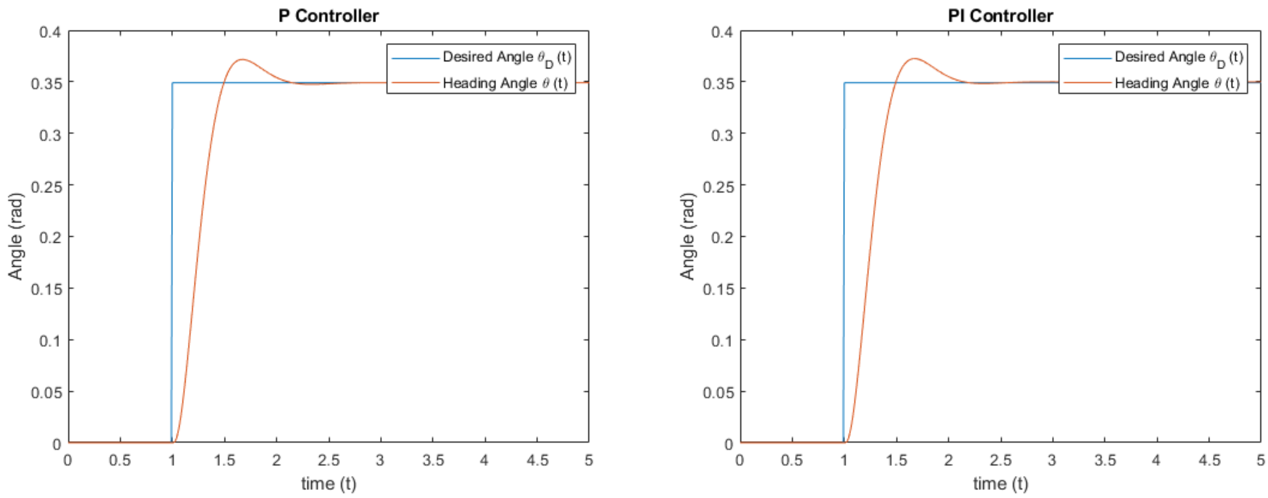
(a) P Controller, MP = 7.3366 %, K_p = 1.1 (b) PI Controller, MP = 7.3494 %, K_p = 1.1, K_i = 0.011 s^{-1}

Figure 9: Unactuated Steering, u = 10 m/s

4.2.2 Actuated Steering

The plant transfer function:

$$P_{act}(s) = \frac{44.97s + 404.4}{s^3 + 17.99s^2 + 80.88s} \times \frac{604}{0.044s^2 + 9.164s + 604}$$



(a) P Controller, MP = 6.4821 %, K_p = 0.9 (b) PI Controller, MP = 6.4918 %, K_p = 0.9, K_i = 0.009 s^{-1}

Figure 10: Actuated Steering, u = 10 m/s

4.3 Observations

As the speed increases, it gets difficult to get controller parameters that give low maximum peak overshoot and settling time simultaneously. This implies that if we want to reduce max peak overshoot, we have to compromise on settling time.

The performance of the system takes reduces at high speeds. Therefore, it is easier and more efficient, in terms of overshoot and settling time, to take a turn at lower speeds.

5 Summary

		P Controller	PI Controller	
Longitudinal Velocity (u) (m/s)	Steering Type	K_p	K_p	K_i (s^{-1})
2.5	Unactuated Actuated	12 11	12 11	0.12 0.11
5	Unactuated Actuated	2.5 2.5	2.5 2.5	0.025 0.025
10	Unactuated Actuated	1.1 0.9	1.1 0.9	0.011 0.009

Table 1: Controller Parameters